Towards holography for the IKKT matrix model

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work with Franz Ciceri

[2503.08771, and to appear]

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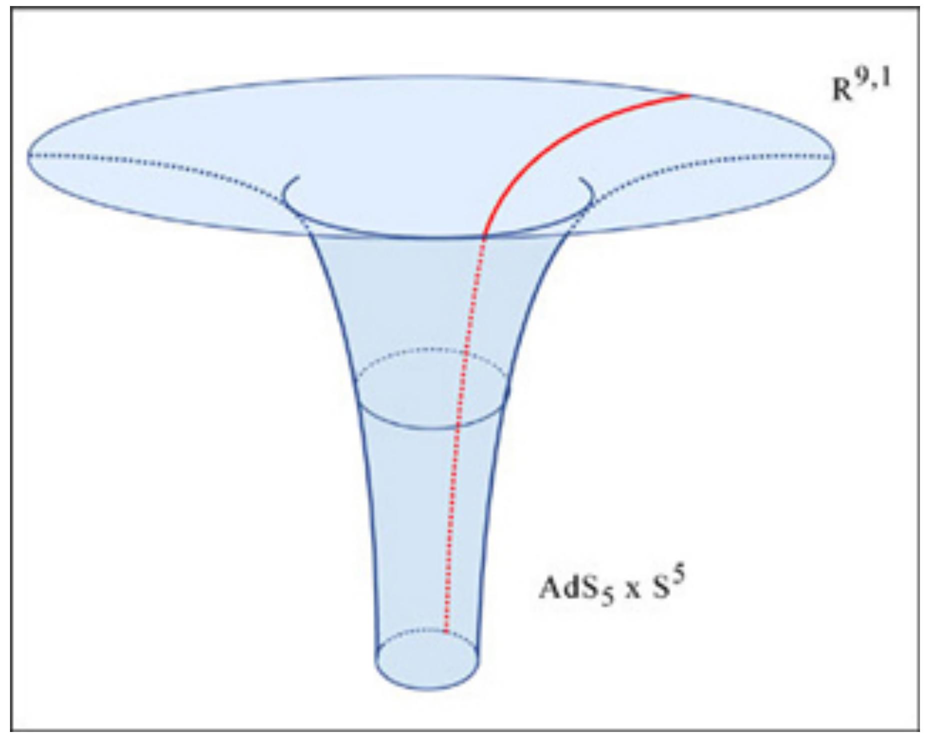
motivation: non-conformal gauge-gravity duality

standard AdS / CFT

[Maldacena, 1998]

$$\mathcal{N}=4 \quad \text{SYM} \longleftrightarrow \text{AdS}_5\times S^5$$



BPS multiplets of gauge invariant operators ⟷ supergravity KK states

lowest BPS multiplet ⟷ maximal supergravity in D=5
[Günaydin, Romans, Warner, 1984]

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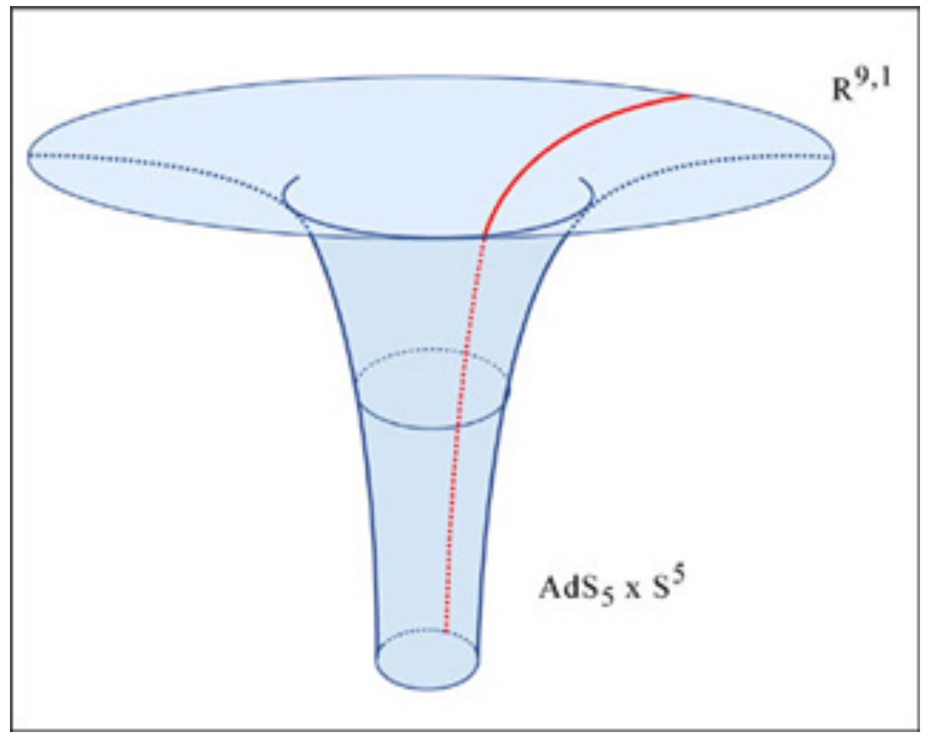

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generalizes to non-conformal Dp branes
domain wall / QFT correspondence

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz, 1998]
[Boonstra, Skenderis, Townsend, 1999]
[Kanitscheider, Skenderis, Taylor, 2008]

$\text{maximal} \quad \text{SYM} \longleftrightarrow \text{dW}_{p+1} \times S^{9-p}$

in $p+1$ dimensions

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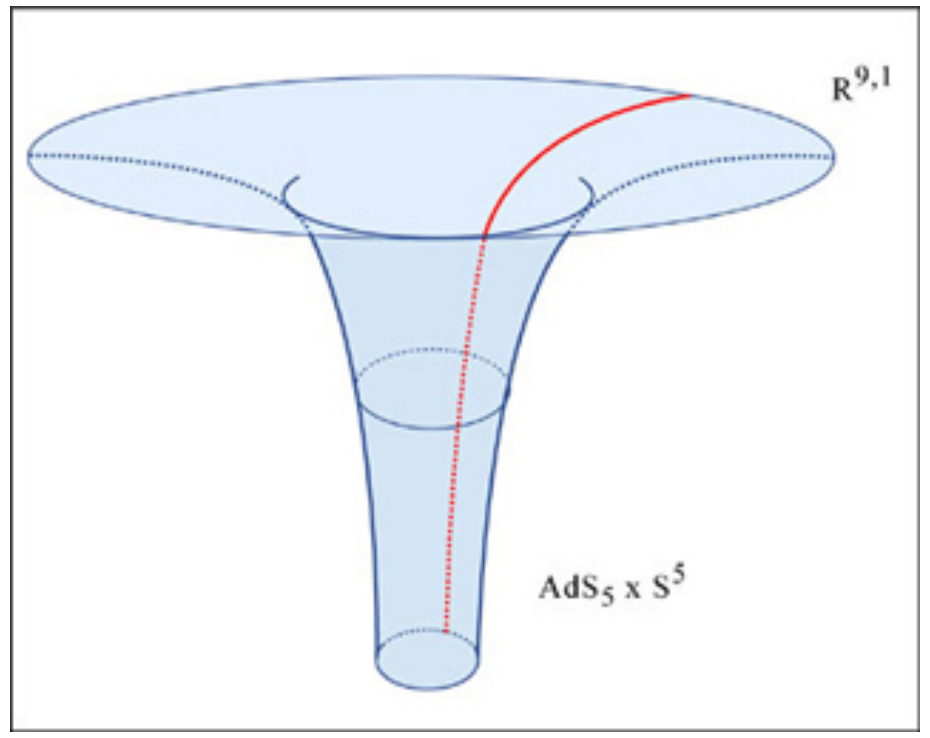

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in $p+1$ dimensions

→ p=−1 : IKKT matrix model & D(−1) instanton

plan: IKKT and a holographic dual

IKKT matrix model

- D=0 SYM
- multiplets of gauge invariant operators

D(-1) instanton background

- consistent truncation of Euclidean IIB on S^9
- "D=1 maximal supergravity"

BPS solutions

- general 1/2 BPS solution
- IIB and D=12 uplift

IKKT matrix model

IKKT matrix model

[Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996]

SYM reduced to D=0 dimensions (global SO(10), global SU(N) symmetry)

$$S_{\rm IKKT} = -\,\mathrm{Tr}\left[\,\frac{1}{4}[X_a, X_b]\,[X^a, X^b] - \frac{1}{2}\Psi^\alpha(\mathscr{C}\Gamma^a)_{\alpha\beta}[X_a, \Psi^\beta]\,\right]$$

with $\mathrm{SU}(N)$ matrices X^a : vector in D=10 $a = 1,\ldots,10$

Ψ^α : spinor in D=10 $\alpha = 1,\ldots,32$

Euclidean: $\mathbf{10}$, $\mathbf{16}_s$ of SO(10), complex chiral spinors $(\Gamma_*\Psi)^\alpha = \Psi^\alpha \longrightarrow \Psi^\alpha = \begin{pmatrix}\Psi^I \\ 0\end{pmatrix}$

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supersymmetry $\delta_\epsilon X^a = \epsilon^\alpha\,(\mathscr{C}\Gamma^a)_{\alpha\beta}\,\Psi^\beta$ $\delta_\epsilon \Psi^\alpha = \frac{1}{2}(\Gamma^{ab})^\alpha{}_\beta\,\epsilon^\beta\,[X_a, X_b]$

algebra $\left[\,\delta_{\epsilon_1}, \delta_{\epsilon_2}\,\right] \;=\; \delta_\lambda^{SU(N)}$ (no translations)

$$\lambda = 2\,\epsilon_2\,\mathscr{C}\Gamma^b\,\epsilon_1 X_b = 2\,\bar\epsilon_2\,\Gamma^b\,\epsilon_1 X_b$$

'gauge' invariant single trace operators

$$\mathcal{O} = \mathrm{Tr}\big[\,\Phi_1\,\ldots\,\Phi_n\big] \qquad\qquad \Phi_i \in \{\,X^a, \Psi^\alpha\,\}$$

spectrum: cyclic words in the alphabet $\{X^a, \Psi^\alpha\}$ "minus field equations" (Polya counting) ‘gauge’ invariant single trace operators

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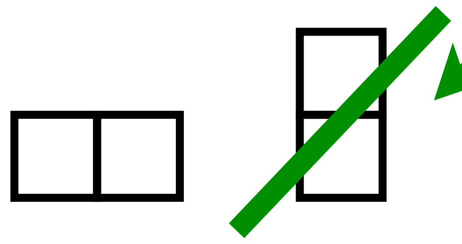
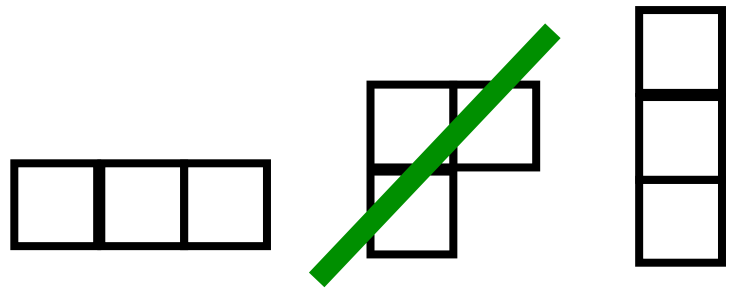
forbidden by trace

2-letters :
$\mathrm{Tr}(\mathcal{O}\mathcal{O})$



minus quadratic field equations $\Gamma^a\big[\Psi, X_a\big] = 0$

3-letters :
$\mathrm{Tr}(\mathcal{O}\mathcal{O}\mathcal{O})$



minus cubic field equations $\big[X_b, [X^a, X^b]\big] + \frac{1}{2}\{\bar\Psi, \Gamma^a\Psi\} = 0$

organizes in supermultiplets

(on gauge-invariant operators, supersymmetry is nilpotent!) $\big[\,\delta_{\epsilon_1}, \delta_{\epsilon_2}\,\big] = \delta_\lambda^{SU(N)}$

IKKT matrix model: gauge invariant operators

spectrum: $$\mathscr{Z}_{\rm IKKT} = \mathscr{Z}_{\rm long} \oplus \sum_{n=2}^{\infty} {\rm BPS}_n$$ [Morales, HS, 2005]

with an infinite tower of BPS multiplets ${\rm BPS}_n$

$$[n,0000]_n \oplus [n-1,0001]_{n+\frac{1}{2}} \oplus [n-2,0100]_{n+1} \oplus [n-3,1010]_{n+\frac{3}{2}} \oplus [n-3,0020]_{n+2}$$
$$\oplus [n-4,2000]_{n+2} \oplus [n-4,1010]_{n+\frac{5}{2}} \oplus [n-4,0100]_{n+3} \oplus [n-4,0001]_{n+\frac{7}{2}} \oplus [n-4,0000]_{n+4}$$

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with the lowest member (~ "sugra multiplet")

$$\begin{aligned} {\rm BPS}_2 \;=\; & \mathbf{54}_{+2} \\ & \oplus \mathbf{144}_{+\frac{5}{2}} \\ & \oplus \mathbf{120}_{+3} \\ & \ominus \mathbf{45}_{+4} \\ & \ominus \mathbf{16}_{+\frac{9}{2}} \end{aligned}$$

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$$\begin{aligned}
{\rm BPS}_2 \;=\; & \mathbf{54}_{+2} && \mathcal{O}^{ab} = {\rm Tr}\left[X^{(\!(a}X^{b)\!)}\right] \\
& \oplus \mathbf{144}_{+\frac{5}{2}} && \mathcal{O}^{a\alpha} = {\rm Tr}\left[X^{a}\,\Psi^{\alpha}\right] - \text{trace} \\
& \oplus \mathbf{120}_{+3} && \mathcal{O}^{abc} = {\rm Tr}\left[X^{[a}X^{b}X^{c]}\right] + \text{ferm}^2 \\
& \ominus \mathbf{45}_{+4} && - SO(10) \\
& \ominus \mathbf{16}_{+\frac{9}{2}} && - \text{susy}
\end{aligned}$$

$\#\,{\rm bos} = 54 + 120 - 45 = 129$

$\#\,{\rm ferm} = 144 - 16 = 128$

all BPS$_n$: $\#\,{\rm bos} = \#\,{\rm ferm} + 1$

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on the dual gravity side:

⟶ the BPS tower $\sum_{n=2}^{\infty} \text{BPS}_n$ should be realized as IIB supergravity KK fluctuations

⟶ the lowest BPS multiplet BPS_2 should be realized as a "D=1 maximal supergravity"

IIB supergravity

and consistent truncation on S^9

dual gravity side: S^9 compactification of (Euclidean) IIB

Kaluza-Klein spectrum [Salam, Strathdee, 1981]
organizes into representations of the SO(10) isometry group

harmonic analysis on $\frac{\mathrm{SO}(10)}{\mathrm{SO}(9)}$ $\longrightarrow$ group theory exercise

KK spectrum of fluctuations around S^9 : $\underbrace{}_{\text{linear}}$ (under "fluctuations": linear)

$$\mathscr{Z}_{\text{sugra}} = \sum_{n=2}^{\infty} \mathrm{BPS}_n$$ [Morales, HS, 2005]

c.f. IKKT spectrum $\mathscr{Z}_{\text{long}} \oplus \sum_{n=2}^{\infty} \mathrm{BPS}_n$

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(linear)

c.f. IKKT spectrum $\mathcal{Z}_{\text{long}} \oplus \sum_{n=2}^{\infty} \mathrm{BPS}_n$

full non-linear embedding of the lowest BPS multiplet BPS_2 as “D=1 maximal supergravity” ?

$\longrightarrow$ consistent sphere truncations to D=1 ?

consistent sphere truncations to D=1: Einstein-dilaton-axion on S^9 $\mu^i\mu^i = 1\,,\quad i = 1,\dots,10$

$$\mathscr{L}_{10} \;=\; R \;-\; \tfrac{1}{2}\partial_M\Phi\,\partial^M\Phi \;+\; \tfrac{1}{2}e^{2\phi}\,\partial_M\chi\,\partial^M\chi$$

subsector of Euclidean IIB

D(−1) instanton background

$$ds_{10}^2 = dt^2 + t^2\,d\Omega_9^2\,,\qquad \chi = e^{-\Phi} = g^8\,t^8$$

[Gibbons, Green, Perry, 1996]
[Ooguri, Skenderis, 1998]
[Bergshoeff, Behrndt, 1998]

flat metric (in Einstein frame)

half supersymmetric

consistent sphere truncations to D=1: Einstein-dilaton-axion on S^9 $\mu^i\mu^i = 1\,,\quad i=1,\dots,10$

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subsector of Euclidean IIB

reduction ansatz inspired by [Cvetic, Lu, Pope, 2000] [Ciceri, HS, 2023]

$$ds_d^2 = Y^{9/40}\,\Delta\,ds_1^2 + g^{-2}\,Y^{1/(40)}\,T^{-1}_{ij}\,\mathcal{D}\mu^i\,\mathcal{D}\mu^j$$

$$e^\Phi = Y^{-1/10}\,\Delta^{-1}$$

$$d\chi = -\,g\,Y^{1/5}\,U\,\omega_1 - g^{-1}\,T^{-1}_{ij}({*\mathcal{D}T_{jk}})\,\mu^k\,\mathcal{D}\mu^i$$

$$\mathcal{D}\mu^i = d\mu^i + gA^{ij}\mu^j$$

$$\mathcal{D}T_{ij} = dT_{ij} + gA^{ik}\,T_{kj} + gA^{jk}\,T_{ik}$$

$$\Delta = T_{ij}\,\mu^i\mu^j$$

$$U = 2\,T_{ik}T_{jk}\mu^i\mu^j - T_{ii}\Delta$$

D=1 scalars $T_{ij} \in \frac{\mathrm{SL}(10)}{\mathrm{SO}(10)}$: $T_{ij} = T_{ji}\,,\;\; \det T_{ij} = 1$

D=1 vectors $A_{ij} \in \mathfrak{so}(10)$: $A_{ij} = -\,A_{ji}$

consistent truncation: field equations reduce to D=1 field equations consistent sphere truncations to D=1: Einstein-dilaton-axion on S^9 $\mu^i\mu^i = 1\,,\quad i = 1,\ldots,10$

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subsector of Euclidean IIB

D=1 Lagrangian

$$\mathscr{L}_1 \;=\; \frac{1}{160}\,e^{-1}\,Y^{-2}\dot{Y}^2 \;+\; \frac{1}{4}\,e^{-1}\,\mathscr{D}T^{-1}_{ij}\,\mathscr{D}\,T_{ij} \;-\; \frac{1}{2}\,e\,g^2\,Y^{2/d}\,\underbrace{\left(2\,T_{ij}T_{ij} - T_{ii}^2\,\right)}_{\text{typical sphere potential}}$$

D=1 einbein and dilaton $e\,,\,Y$

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not yet the full answer: $\mathrm{BPS}_2 = \mathbf{54}_{+2} \oplus \mathbf{144}_{+\frac{5}{2}} \oplus \mathbf{120}_{+3} \ominus \mathbf{45}_{+4} \ominus \mathbf{16}_{+\frac{9}{2}}$

→ for the full multiplet, we need 120 scalars and the fermions

D=1 maximal supergravity

D=1 maximal supergravity (32 supercharges)

1	D=1 einbein and dilaton	$e\,,\,Y$	
45	D=1 vectors	$A_{ij} \in \mathfrak{so}(10)$	
54	D=1 scalars	$T_{ij} = \mathscr{V}_i{}^a \mathscr{V}_j{}^a \in \mathrm{SL}(10)/\mathrm{SO}(10)$	
120	D=1 axions	a_{ijk}	
16	dilatino	λ^α	
16	gravitino	ψ^α	
144	matter fermions	$\chi^{a\alpha}$	

D=1 Lagrangian (to quadratic order in fermions)

$$\mathscr{L}_1 = \frac{1}{160}\,e^{-1}\,Y^{-2}\dot{Y}^2 + \frac{1}{4}\,e^{-1}\,\mathscr{D}T^{ij}\,\mathscr{D}\,T_{ij} - \frac{1}{12}\,e^{-1}\,Y^{-1/20}\,\mathscr{D}a_{ijk}\,\mathscr{D}a_{lmn}\,T^{im}T^{jn}T^{kn} + \frac{1}{80}\bar{\lambda}\,\mathscr{D}_t\lambda - 2\,\bar{\chi}^a\,\Gamma_{ab}\,\mathscr{D}_t\chi^b$$

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Noether & Yukawa

$$\begin{cases} -\frac{1}{80} e^{-1} \frac{\dot{Y}}{Y} \bar{\psi} \Gamma_* \lambda + 2 e^{-1} \bar{\psi} \Gamma^b \chi^a \mathscr{P}^{ab} - \frac{1}{2} e^{-1} Y^{-1/40} \bar{\chi}^a \Gamma^{bc} \psi\, p_{abc} - \frac{1}{240} e^{-1} Y^{-1/40} \bar{\lambda} \Gamma^{abc} \Gamma_* \psi\, p_{abc} - \frac{1}{1600} Y^{-1/40} \bar{\lambda} \Gamma^{abc} \lambda\, p_{abc} - \frac{1}{40} Y^{-1/40} \bar{\chi}^c \Gamma^{ab} \Gamma_* \lambda\, p_{abc} - \frac{1}{12} Y^{-1/40} \bar{\chi}^a \Gamma^{bcd} \chi^a\, p_{bcd} - Y^{-1/40} \bar{\chi}^a \Gamma^b \chi^c\, p_{abc} \\ -\frac{3}{80} g\, Y^{3/40} \bar{\lambda} \Gamma^{abc} \psi B^{abc} - \frac{1}{160} g\, Y^{1/20} \bar{\lambda} \Gamma^{abcd} \psi B^{abcd} - \frac{7}{1600} ge\, Y^{3/40} \bar{\lambda} \Gamma^{abc} \Gamma_* \lambda B^{abc} - \frac{9}{12800} ge\, Y^{1/20} \bar{\lambda} \Gamma^{abcd} \Gamma_* \lambda B^{abcd} + \frac{1}{6} g\, Y^{3/40} \bar{\chi}^a \Gamma^{bc} \Gamma_* \psi (3 B^{abc} - 4 C^{a,bc}) - \frac{1}{2} g\, Y^{1/20} \bar{\chi}^d \Gamma^{abc} \Gamma_* \psi B^{abcd} \\ +\frac{1}{40} ge\, Y^{3/40} \bar{\chi}^a \Gamma^{bc} \lambda (3 B^{abc} - 4 C^{a,bc}) - \frac{1}{20} ge\, Y^{1/20} \bar{\chi}^d \Gamma^{abc} \lambda B^{abcd} - \frac{1}{4} ge\, Y^{3/40} \bar{\chi}^a \Gamma^{bcd} \Gamma_* \chi^a B^{bcd} + \frac{1}{16} ge\, Y^{1/20} \bar{\chi}^a \Gamma^{bcde} \Gamma_* \chi^a B^{bcde} + \frac{1}{3} ge\, Y^{3/40} \bar{\chi}^a \Gamma^b \Gamma_* \chi^c (3 B^{abc} - 4 C^{b,ac}) - \frac{3}{2} ge\, Y^{1/20} \bar{\chi}^a \Gamma^{bc} \Gamma_* \chi^d B^{bcde} \end{cases}$$

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$$\mathcal{L}_1 = \frac{1}{160} e^{-1} Y^{-2} \dot{Y}^2 + \frac{1}{4} e^{-1} \mathcal{D}T^{ij} \mathcal{D} T_{ij} - \frac{1}{12} e^{-1} Y^{-1/20} \mathcal{D}a_{ijk} \mathcal{D}a_{lmn} T^{im}T^{jn}T^{kn} + \frac{1}{80} \bar{\lambda} \mathcal{D}_t \lambda - 2 \bar{\chi}^a \Gamma_{ab} \mathcal{D}_t \chi^b$$

Noether & Yukawa

$$\begin{cases} -\frac{1}{80} e^{-1} \frac{\dot{Y}}{Y} \bar{\psi} \Gamma_* \lambda + 2 e^{-1} \bar{\psi} \Gamma^b \chi^a \mathscr{P}^{ab} - \frac{1}{2} e^{-1} Y^{-1/40} \bar{\chi}^a \Gamma^{bc} \psi\, p_{abc} - \frac{1}{240} e^{-1} Y^{-1/40} \bar{\lambda} \Gamma^{abc} \Gamma_* \psi\, p_{abc} - \frac{1}{1600} Y^{-1/40} \bar{\lambda} \Gamma^{abc} \lambda\, p_{abc} - \frac{1}{40} Y^{-1/40} \bar{\chi}^c \Gamma^{ab} \Gamma_* \lambda\, p_{abc} - \frac{1}{12} Y^{-1/40} \bar{\chi}^a \Gamma^{bcd} \chi^a\, p_{bcd} - Y^{-1/40} \bar{\chi}^a \Gamma^b \chi^c\, p_{abc} \\ -\frac{3}{80} g Y^{3/40} \bar{\lambda} \Gamma^{abc} \psi B^{abc} - \frac{1}{160} g Y^{1/20} \bar{\lambda} \Gamma^{abcd} \psi B^{abcd} - \frac{7}{1600} ge Y^{3/40} \bar{\lambda} \Gamma^{abc} \Gamma_* \lambda B^{abc} - \frac{9}{12800} ge Y^{1/20} \bar{\lambda} \Gamma^{abcd} \Gamma_* \lambda B^{abcd} + \frac{1}{6} g Y^{3/40} \bar{\chi}^a \Gamma^{bc} \Gamma_* \psi (3 B^{abc} - 4 C^{a,bc}) - \frac{1}{2} g Y^{1/20} \bar{\chi}^d \Gamma^{abc} \Gamma_* \psi B^{abcd} \\ +\frac{1}{40} ge Y^{3/40} \bar{\chi}^a \Gamma^{bc} \lambda (3 B^{abc} - 4 C^{a,bc}) - \frac{1}{20} ge Y^{1/20} \bar{\chi}^d \Gamma^{abc} \lambda B^{abcd} - \frac{1}{4} ge Y^{3/40} \bar{\chi}^a \Gamma^{bcd} \Gamma_* \chi^a B^{bcd} + \frac{1}{16} ge Y^{1/20} \bar{\chi}^a \Gamma^{bcde} \Gamma_* \chi^a B^{bcde} + \frac{1}{3} ge Y^{3/40} \bar{\chi}^a \Gamma^b \Gamma_* \chi^c (3 B^{abc} - 4 C^{b,ac}) - \frac{3}{2} ge Y^{1/20} \bar{\chi}^a \Gamma^{bc} \Gamma_* \chi^d B^{bcde} \end{cases}$$

$$-\frac{1}{2} e\, g^2 Y^{1/5} \left(2\, T_{ij} T_{ij} - (T_{ii})^2\right) + \frac{1}{4} e\, g^2 Y^{3/20} \left(2\, a_{ijk} a_{ijp} T^{kp} - a_{ijk} a_{mnp} T^{im} T^{jn} T_{kp}\right) + \mathcal{O}(a^4)$$

typical sphere potential — typical axion potential

D=1 maximal supergravity (32 supercharges)

D=1 Lagrangian (to quadratic order in fermions)

$$\mathscr{L}_1 = \frac{1}{160}\, e^{-1}\, Y^{-2}\dot{Y}^2 + \frac{1}{4}\, e^{-1}\, \mathscr{D}T^{ij}\, \mathscr{D}\, T_{ij} - \frac{1}{12}\, e^{-1}\, Y^{-1/20}\, \mathscr{D}a_{ijk}\, \mathscr{D}a_{lmn}\, T^{im}T^{jn}T^{kn} + \frac{1}{80}\bar{\lambda}\, \mathscr{D}_t\lambda - 2\,\bar{\chi}^a\,\Gamma_{ab}\,\mathscr{D}_t\chi^b$$

$$-\frac{1}{80}\, e^{-1}\frac{\dot{Y}}{Y}\,\bar{\psi}\,\Gamma_*\lambda + 2\, e^{-1}\,\bar{\psi}\,\Gamma^b\,\chi^a\mathscr{P}^{ab} - \frac{1}{2}e^{-1}\, Y^{-1/40}\bar{\chi}^a\,\Gamma^{bc}\;\psi\, p_{abc} - \frac{1}{240}\, e^{-1}\, Y^{-1/40}\bar{\lambda}\,\Gamma^{abc}\;\Gamma_*\psi\, p_{abc} - \frac{1}{1600}\, Y^{-1/40}\bar{\lambda}\,\Gamma^{abc}\;\lambda\, p_{abc} - \frac{1}{40}\, Y^{-1/40}\bar{\chi}^c\,\Gamma^{ab}\Gamma_*\,\lambda\, p_{abc} - \frac{1}{12}\, Y^{-1/40}\,\bar{\chi}^a\,\Gamma^{bcd}\,\chi^a\; p_{bcd} - Y^{-1/40}\,\bar{\chi}^a\,\Gamma^b\,\chi^c\; p_{abc}$$

$$-\frac{3}{80}\, g\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\,\psi\, B^{abc} - \frac{1}{160}\, g\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\,\psi\, B^{abcd} - \frac{7}{1600}\, ge\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\;\Gamma_*\lambda\, B^{abc} - \frac{9}{12800}\, ge\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\;\Gamma_*\lambda\, B^{abcd} + \frac{1}{6}\, g\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\;\Gamma_*\psi\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{2}\, g\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\Gamma_*\psi\, B^{abcd}$$

$$+\frac{1}{40}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\,\lambda\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{20}\, ge\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\lambda\, B^{abcd} - \frac{1}{4}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bcd}\,\Gamma_*\chi^a\, B^{bcd} + \frac{1}{16}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bcde}\,\Gamma_*\chi^a\, B^{bcde} + \frac{1}{3}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^b\,\Gamma_*\chi^c\,(3\, B^{abc} - 4\, C^{b,ac}) - \frac{3}{2}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bc}\,\Gamma_*\chi^d\, B^{bcde}$$

$$-\frac{1}{2}\, e\, g^2\, Y^{1/5}\left(2\, T_{ij}T_{ij} - (T_{ii})^2\right) + \frac{1}{4}\, e\, g^2\, Y^{3/20}\left(2\, a_{ijk}a_{ijp}T^{kp} - a_{ijk}a_{mnp}T^{im}T^{jn}T_{kp}\right) + \mathcal{O}(a^4)$$

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susy variation —> Killing spinor equations

$$\delta_\epsilon\psi = \mathscr{D}_t\,\epsilon + \frac{1}{24}\, Y^{-1/40}\,\Gamma_{abc}\;\epsilon\, p^{abc} + \frac{1}{8}\, g\, e\, Y^{3/40}\,\Gamma_{abc}\Gamma_*\,\epsilon\, B^{abc} - \frac{1}{32}\, g\, e\, Y^{1/20}\,\Gamma_{abcd}\,\Gamma_*\,\epsilon\, B^{abcd} - \frac{1}{4}\, g\, e\,\mathscr{T}\,\Gamma_*\,\epsilon$$

$$\delta_\epsilon\lambda = \frac{\dot{Y}}{2Y}e^{-1}\Gamma_*\,\epsilon + \frac{1}{6}\, e^{-1}\, Y^{-1/40}\,\Gamma_{abc}\Gamma_*\,\epsilon\, p^{abc} + \frac{3}{2}\, g\, Y^{3/40}\,\Gamma_{abc}\,\epsilon\, B^{abc} + \frac{1}{4}\, g\, Y^{1/20}\,\Gamma_{abcd}\,\epsilon\, B^{abcd} + 4\, g\,\mathscr{T}\,\epsilon$$

etc.

with Yukawa tensors $B^{abc} = \mathscr{V}_m{}^a\mathscr{V}^{bi}\mathscr{V}^{cj}\, a_{ijm}$ $\quad B^{abcd} = \mathscr{V}^{ai}\mathscr{V}^{bj}\mathscr{V}^{ck}\mathscr{V}^{dl}\, a_{m[ij}a_{kl]m}$

D=1 maximal supergravity (32 supercharges)

$$\mathscr{L}_1 = \frac{1}{160}\, e^{-1}\, Y^{-2}\dot{Y}^2 + \frac{1}{4}\, e^{-1}\, \mathscr{D}T^{ij}\, \mathscr{D}\, T_{ij} - \frac{1}{12}\, e^{-1}\, Y^{-1/20}\, \mathscr{D}a_{ijk}\, \mathscr{D}a_{lmn}\, T^{im}T^{jn}T^{kn} + \frac{1}{80}\bar{\lambda}\,\mathscr{D}_t\lambda - 2\bar{\chi}^a\,\Gamma_{ab}\,\mathscr{D}_t\chi^b$$

$$-\frac{1}{80}\, e^{-1}\frac{\dot{Y}}{Y}\,\bar{\psi}\,\Gamma_*\lambda + 2\, e^{-1}\,\bar{\psi}\,\Gamma^b\,\chi^a\,\mathscr{P}^{ab} - \frac{1}{2}e^{-1}\, Y^{-1/40}\bar{\chi}^a\,\Gamma^{bc}\,\psi\, p_{abc} - \frac{1}{240}\, e^{-1}\, Y^{-1/40}\bar{\lambda}\,\Gamma^{abc}\,\Gamma_*\psi\, p_{abc} - \frac{1}{1600}\, Y^{-1/40}\bar{\lambda}\,\Gamma^{abc}\,\lambda\, p_{abc} - \frac{1}{40}\, Y^{-1/40}\bar{\chi}^c\,\Gamma^{ab}\Gamma_*\,\lambda\, p_{abc} - \frac{1}{12}\, Y^{-1/40}\,\bar{\chi}^a\,\Gamma^{bcd}\,\chi^a\, p_{bcd} - Y^{-1/40}\,\bar{\chi}^a\,\Gamma^b\,\chi^c\, p_{abc}$$

$$-\frac{3}{80}\, g\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\,\psi\, B^{abc} - \frac{1}{160}\, g\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\,\psi\, B^{abcd} - \frac{7}{1600}\, ge\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\,\Gamma_*\lambda\, B^{abc} - \frac{9}{12800}\, ge\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\,\Gamma_*\lambda\, B^{abcd} + \frac{1}{6}\, g\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\,\Gamma_*\psi\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{2}\, g\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\Gamma_*\psi\, B^{abcd}$$

$$+\frac{1}{40}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\,\lambda\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{20}\, ge\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\lambda\, B^{abcd} - \frac{1}{4}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bcd}\,\Gamma_*\chi^a\, B^{bcd} + \frac{1}{16}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bcde}\,\Gamma_*\chi^a\, B^{bcde} + \frac{1}{3}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^b\,\Gamma_*\chi^c\,(3\, B^{abc} - 4\, C^{b,ac}) - \frac{3}{2}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bc}\,\Gamma_*\chi^d\, B^{bcde}$$

$$-\frac{1}{2}\, e\, g^2\, Y^{1/5}\left(2\, T_{ij}T_{ij} - (T_{ii})^2\right) + \frac{1}{4}\, e\, g^2\, Y^{3/20}\left(2\, a_{ijk}a_{ijp}T^{kp} - a_{ijk}a_{mnp}T^{im}T^{jn}T_{kp}\right) + \mathscr{O}(a^4)$$

"ground state" SO(10) invariant solution

no SO(10) invariant solution with constant dilaton —> $\mathrm{dW}_1 \times S^9$

solution $T_{ij} = \delta_{ij}\,,\ \ A_{ij} = 0\,,\ \ a_{ijk} = 0\,,\ \ Y = t^{80}\,,\ \ e = g^{-1}t^{-9}$

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$$\mathscr{L}_1 = \frac{1}{160}\, e^{-1}\, Y^{-2}\dot{Y}^2 + \frac{1}{4}\, e^{-1}\, \mathscr{D}T^{ij}\, \mathscr{D}\, T_{ij} - \frac{1}{12}\, e^{-1}\, Y^{-1/20}\, \mathscr{D}a_{ijk}\, \mathscr{D}a_{lmn}\, T^{im}T^{jn}T^{kn} + \frac{1}{80}\bar{\lambda}\,\mathscr{D}_t\lambda - 2\,\bar{\chi}^a\,\Gamma_{ab}\,\mathscr{D}_t\chi^b$$

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$$-\frac{3}{80}\, g\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\,\psi\, B^{abc} - \frac{1}{160}\, g\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\,\psi\, B^{abcd} - \frac{7}{1600}\, ge\, Y^{3/40}\,\bar{\lambda}\,\Gamma^{abc}\,\Gamma_*\lambda\, B^{abc} - \frac{9}{12800}\, ge\, Y^{1/20}\,\bar{\lambda}\,\Gamma^{abcd}\,\Gamma_*\lambda\, B^{abcd} + \frac{1}{6}\, g\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\,\Gamma_*\psi\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{2}\, g\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\Gamma_*\psi\, B^{abcd}$$

$$+\frac{1}{40}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bc}\,\lambda\,(3\, B^{abc} - 4\, C^{a,bc}) - \frac{1}{20}\, ge\, Y^{1/20}\,\bar{\chi}^d\,\Gamma^{abc}\,\lambda\, B^{abcd} - \frac{1}{4}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^{bcd}\,\Gamma_*\chi^a\, B^{bcd} + \frac{1}{16}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bcde}\,\Gamma_*\chi^a\, B^{bcde} + \frac{1}{3}\, ge\, Y^{3/40}\,\bar{\chi}^a\,\Gamma^b\,\Gamma_*\chi^c\,(3\, B^{abc} - 4\, C^{b,ac}) - \frac{3}{2}\, ge\, Y^{1/20}\,\bar{\chi}^a\,\Gamma^{bc}\,\Gamma_*\chi^d\, B^{bcde}$$

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solution $T_{ij} = \delta_{ij}\,,\ A_{ij} = 0\,,\ a_{ijk} = 0\,,\ Y = t^{80}\,,\ e = g^{-1}t^{-9}$

uplift to IIB $ds^2_{10} = dt^2 + t^2\, d\Omega_9^2\,, \qquad \chi = e^{-\Phi} = g^8\, t^8$

D(−1) instanton background

half supersymmetric in D=1 and D=10

[Gibbons, Green, Perry, 1996]
[Ooguri, Skenderis, 1998]
[Bergshoeff, Behrndt, 1998]

further uplift to 12D pp-wave [Tseytlin, 1996]

BPS solutions, assume vanishing axions $a_{ijk} = 0$

gauge fixing: $T_{ij} = \delta_{ij}\, e^{\varphi_i}$ after $T \longrightarrow \mathcal{S} T \mathcal{S}^T$

$e = 1$ after diffeomorphisms

in this gauge, field equations imply $A_{ij} = 0$ and with $Y = e^{\varphi_0}$

$$\frac{1}{80}\,\ddot{\varphi}_0 - \frac{1}{2}\,\ddot{\varphi}_i = -\,e^{\varphi_0/5}\Big(2\,e^{2\varphi_i} - \sum_k e^{\varphi_i+\varphi_k}\Big) \qquad \frac{1}{80}\,(\dot{\varphi}_0)^2 - \frac{1}{2}\sum_k (\dot{\varphi}_k)^2 = -\,e^{\varphi_0/5}\Big(2\sum_k e^{2\varphi_k} - \Big(\sum_k e^{\varphi_k}\Big)^2\Big)$$

BPS solutions, assume vanishing axions $a_{ijk} = 0$

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$$\frac{1}{80}\,\ddot{\varphi}_0 - \frac{1}{2}\,\ddot{\varphi}_i = -\,e^{\varphi_0/5}\Big(2\,e^{2\varphi_i} - \sum_k e^{\varphi_i+\varphi_k}\Big) \qquad \frac{1}{80}\,(\dot{\varphi}_0)^2 - \frac{1}{2}\sum_k (\dot{\varphi}_k)^2 = -\,e^{\varphi_0/5}\Big(2\sum_k e^{2\varphi_k} - \Big(\sum_k e^{\varphi_k}\Big)^2\Big)$$

Killing spinor equations

$$0 = \delta_\epsilon \psi = \mathcal{D}_t \epsilon \;-\; \tfrac{1}{4}\, g\, e\, \mathcal{T}\, \Gamma_* \epsilon \qquad 0 = \delta_\epsilon \lambda = \frac{\dot{Y}}{2Y} e^{-1}\Gamma_* \epsilon \,+\, 4\, g\, \mathcal{T}\, \epsilon \qquad 0 = \delta_\epsilon \chi^a = \frac{1}{2} e^{-1}\Gamma^b\, \epsilon\, \mathcal{P}^{ab} \;-\; \frac{1}{2}\, g\, \mathcal{T}_{ab}\, \Gamma^b \Gamma_* \epsilon$$

imposes chiral spinor $(\Gamma_*\, \epsilon)^\alpha = \epsilon^\alpha \quad\longrightarrow\quad \epsilon^\alpha = \begin{pmatrix} \epsilon^I \\ 0 \end{pmatrix}$ solutions preserve half or no supersymmetry

in the above gauge: $\dot{\varphi}_i - \dfrac{1}{40}\,\dot{\varphi}_0 = 2\, e^{\varphi_i + \varphi_0/10}$

BPS solutions, assume vanishing axions $a_{ijk} = 0$

gauge fixing: $T_{ij} = \delta_{ij}\, e^{\varphi_i}$ after $T \longrightarrow \mathcal{S} T \mathcal{S}^T$

$e = 1$ after diffeomorphisms

in this gauge, field equations imply $A_{ij} = 0$ and with $Y = e^{\varphi_0}$

$$\frac{1}{80}\,\ddot{\varphi}_0 - \frac{1}{2}\,\ddot{\varphi}_i = -\, e^{\varphi_0/5}\Big(2\, e^{2\varphi_i} - \sum_k e^{\varphi_i+\varphi_k}\Big) \qquad \frac{1}{80}\,(\dot{\varphi}_0)^2 - \frac{1}{2}\sum_k (\dot{\varphi}_k)^2 = -\, e^{\varphi_0/5}\Big(2\sum_k e^{2\varphi_k} - \Big(\sum_k e^{\varphi_k}\Big)^2\Big)$$

Killing spinor equations

$$0 = \delta_\epsilon \psi = \mathcal{D}_t\, \epsilon \;-\; \tfrac{1}{4}\, g\, e\, \mathcal{T}\, \Gamma_*\, \epsilon \qquad 0 = \delta_\epsilon \lambda = \frac{\dot{Y}}{2Y} e^{-1}\Gamma_*\, \epsilon \;+\; 4\, g\, \mathcal{T}\, \epsilon \qquad 0 = \delta_\epsilon \chi^a = \frac{1}{2} e^{-1}\Gamma^b\, \epsilon\, \mathcal{P}^{ab} \;-\; \frac{1}{2}\, g\, \mathcal{T}_{ab}\, \Gamma^b \Gamma_* \epsilon$$

imposes chiral spinor $(\Gamma_*\, \epsilon)^\alpha = \epsilon^\alpha \quad \longrightarrow \quad \epsilon^\alpha = \begin{pmatrix} \epsilon^I \\ 0 \end{pmatrix}$ solutions preserve half or no supersymmetry

in the above gauge: $\dot{\varphi}_i - \dfrac{1}{40}\,\dot{\varphi}_0 = 2\, e^{\varphi_i + \varphi_0/10}$

explicit solution:

$$e^{\phi_i} \;\equiv\; e^{\varphi_i - \varphi_0/40} \;=\; \frac{1}{u - c_i}$$

$\dot{u} = -\,2\, e^{\varphi_0/8}$

10 real constants c_i

general 1/2-BPS solutions

explicit solution:

$$e^{\phi_i} \equiv e^{\varphi_i - \varphi_0/40} = \frac{1}{u - c_i}$$

$\dot{u} = -\,2\,e^{\varphi_0/8}$

10 real constants c_i

uplift to IIB:

$$ds^2_{\rm IIB} = \tfrac{1}{4}\left(\mu^i\mu^i e^{\phi_i}\right) du^2 + d\mu^i\, d\mu^i\, e^{-\phi_i}$$

$$\chi = e^{-\Phi} = \left(\mu^i\mu^i\, e^{\phi_i}\right) \prod_k e^{-\phi_k/2}$$

$\mu^i\mu^i = 1\,, \quad i = 1,\dots,10$

the metric is still flat!

- for $c_i = C$, it reduces to the SO(10) invariant D(−1) instanton
- for generic $c_i = C$, SO(10) is completely broken

further uplift to D=12:

$$ds^2_{12} = -\,e^{-\Phi}d\tau^2 + e^{\Phi}\left(dy - e^{\Phi}\, d\tau\right)^2 + \tfrac{1}{4}\left(\mu^i\mu^i e^{\phi_i}\right) du^2 + d\mu^i d\mu^i\, e^{-\phi_i}$$

conclusions / outlook

- new **maximal D=1 supergravity** with 32 (complex) supercharges
- consistent truncation from Euclidean IIB on S^9
- matches the lowest BPS multiplet of IKKT operators
- general 1/2-BPS solution and uplift to IIB

- framework for holographic correlation functions
- holography for IKKT and deformations thereof

  polarised IKKT [Hartnoll, Liu, 2024] [Komatsu, Martina, Penedones, Vuignier, Zhao, 2024]
- relation to D(−1)/D7 holography and $AdS_1 \times S^1$ [Aguilar-Gutierrez, Parmentier, Van Riet, 2022]
- Euclidean vs Lorentzian versions [Chou, Nishimura, Tripathi, 2025]
- E10 in D=1 supergravity ..?
- flat space holography ..?