Gravitational Solitons and Non-relativistic String Theory

Troels Harmark, Niels Bohr Institute

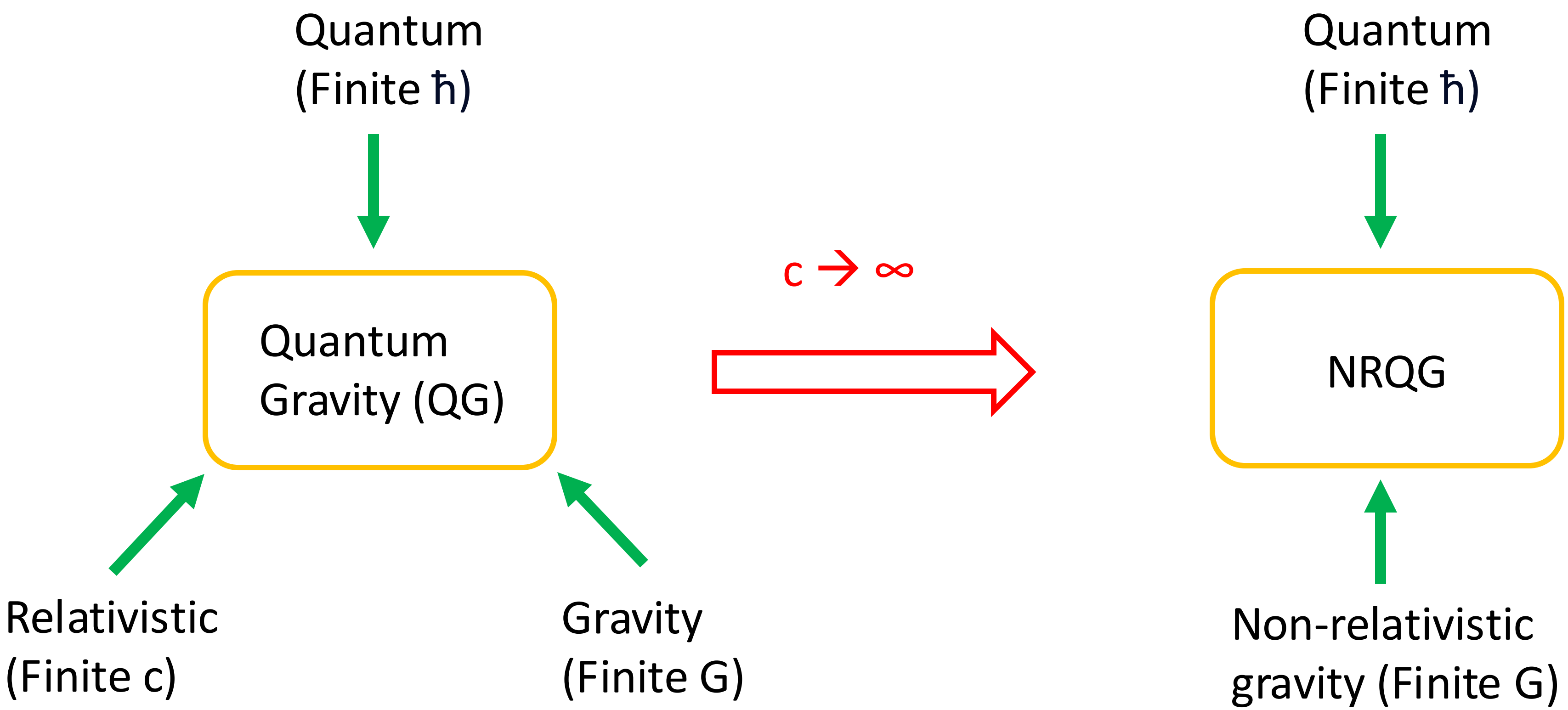
September 8, 2025
Workshop on Quantum Gravity and Strings / Corfu Summer Institute

Based primarily on: 2501.10178 (JHEP) with J. Lahnsteiner and N. Obers

Also based on the previous works:
2107.00642 and 2309.14467 with Bidussi, Hartong, Obers and Oling
2011.02539 with Hartong, Obers and Oling
1907.01663 with Hartong, Menculini, Obers and Oling
1810.05560 with Hartong, Obers and Yan
1705.03535 with Hartong and Obers

Introduction

Idea of non-relativistic quantum gravity (NRQG)



Non-relativistic quantum gravity still contains the combination of quantum and gravity but should be simpler

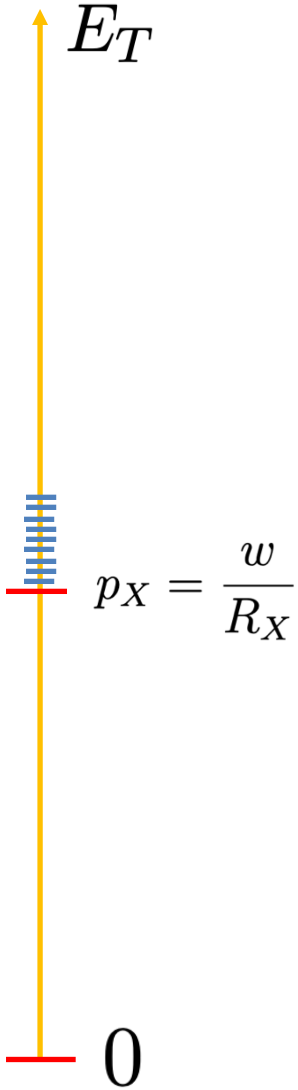
Perhaps we can learn aspects of QG that so far eluded us?

A near-BPS limit of strings

Gomis & Ooguri '00;
Danielsson, Guijosa & Kruczenski '00

Flat space with X being a circle: $ds^2 = -dT^2 + dX^2 + \sum_{i=1}^{8} dr^i dr^i$

BPS state - momentum along a circle: $p_X = \frac{w}{R_X}$



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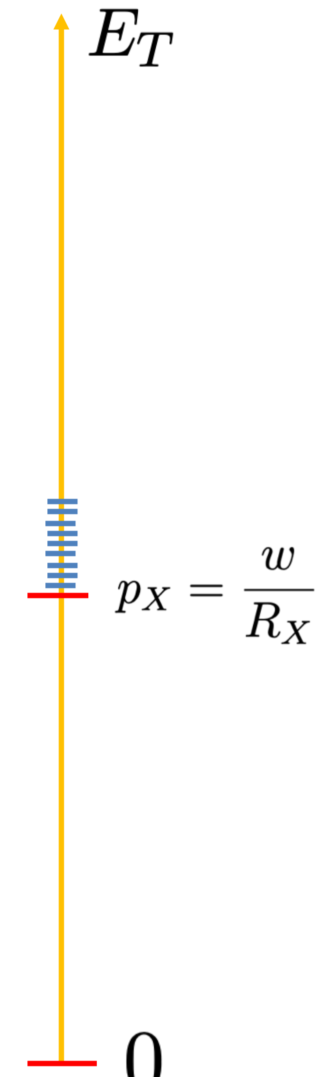
Focus on modes close to BPS state using coordinates

$$T = c\,t\;,\quad X = c\,t + \frac{1}{c}u \quad \Rightarrow \quad E_t = c\,(E_T - p_X)\;,\quad p_u = \frac{1}{c}p_X$$

c → ∞ approaches BPS bound $E_T \geq p_X$ → A DLCQ limit



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$$T = ct \ , \quad X = ct + \frac{1}{c}u \quad \Rightarrow \quad E_t = c\,(E_T - p_X) \ , \quad p_u = \frac{1}{c} p_X$$

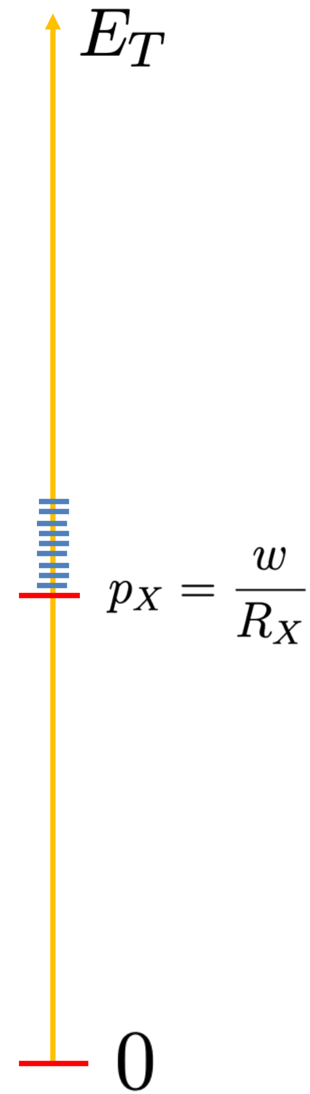
c → ∞ approaches BPS bound $E_T \geq p_X$ → A DLCQ limit

We can resolve limit by T-duality along u (at finite c)

$$ds^2 = c^2(-dt^2 + dv^2) + dr^i dr^i \ , \quad B = -c^2 dt \wedge dv \ , \quad g_s e^{\Phi} = g_s c$$

Limit c → ∞ of critical electric B-field, with winding w along v:

Turns out to be Non-Relativistic String theory (NRST) limit
of flat space



NRST limit is a near-BPS limit

Gomis & Ooguri ’00;
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String mass-formula in original frame:

$$E_T^2 - \frac{w^2}{R_X^2} - \delta^{ij} p_i p_j = n^2 \frac{R_X^2}{(\alpha')^2} + \frac{2}{\alpha'}(N + \bar{N} - a)$$

Energy | Momentum | Transverse momenta | Winding | String excitations

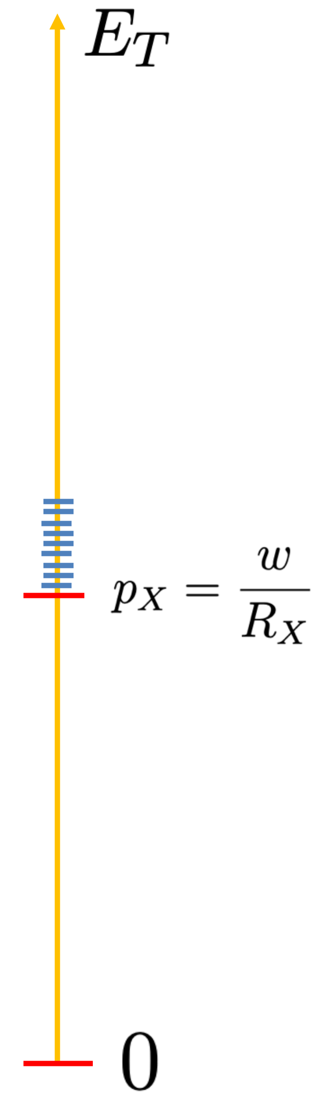
After transformation and T-duality:

$$\frac{1}{c^2} E_t^2 + 2E_t \frac{wR_v}{\alpha'} - \frac{1}{c^2}\frac{n^2}{R_v^2} - \delta^{ij} p_i p_j = \frac{2}{\alpha'}(N + \tilde{N} - a)$$

Taking c → ∞ limit:

$$E_t = \frac{\alpha'}{2wR_v}\left[\delta^{ij} p_i p_j + \frac{2}{\alpha'}(N + \tilde{N} - a)\right]$$

NRST is in a sector of fixed non-zero w



General NRST limit

TH, Hartong & Obers ’17; Bergshoeff, Gomis & Yan ‘18;
TH, Hartong, Menculini, Obers & Yan ’18
Review: Oling & Yan ‘22

Relativistic string: $$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det\left(\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}\right)} + \frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}\right)$$

General formulation of (near-)critical B-field (A,B=0,1):

$$g_{\mu\nu} = c^2 \eta_{AB} \tau_\mu^A \tau_\nu^B + h_{\mu\nu}\ , \quad B_{\mu\nu} = c^2 \epsilon_{AB} \tau_\mu^A \tau_\nu^B + m_{\mu\nu}$$

longitudinal transverse longitudinal transverse

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NRST limit c → ∞ gives: ($\det(\tau) = \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \tau_\mu^0 \tau_\nu^1$)

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[\det(\tau)\,\eta^{AB}\tau_A^\alpha\tau_B^\beta\partial_\alpha X^\mu \partial_\beta X^\nu h_{\mu\nu} + \frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu m_{\mu\nu}\right]$$

Torsional string Newton-Cartan geometry: $\tau_\mu^A\ ,\quad h_{\mu\nu}\ ,\quad m_{\mu\nu}$

A non-relativistic geometry: Infinite speed of light in transverse space

Galilei-type boosts mixing time and space: $h_{\mu\nu} = \delta_{ab}e_\mu^a e_\nu^b$

$$\delta h_{\mu\nu} = -\eta_{AB}\lambda_b^B(\tau_\mu^A e_\nu^b + \tau_\nu^A e_\mu^b)\ , \quad \delta m_{\mu\nu} = -\epsilon_{AB}\lambda_b^B(\tau_\mu^A e_\nu^b - \tau_\nu^A e_\mu^b)$$

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Above limit is geometric, not based on how string theory backgrounds behave

So can one actually take this limit of gravitational solitons in string theory?

Gravitational solitons

Gravitational solitons in NRST

TH, Lahnsteiner & Obers ‘25

Importance of gravitational solitons:

- Consistent target space background
- Non-perturbative physics
- Holography
- Thermal physics

How can one find grav. solitons in NRST?

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How can one find grav. solitons in NRST?

Use the finite-c T-duality argument as a soliton solution generator

Soliton with ISO(1,1) symmetry along T,X directions

⇩ $T = c\,t\,, \quad X = c\,t - \frac{1}{c}u$

Transformed "finite-c soliton" with t,u coordinates

⇩ T-duality along u

Finite-c soliton with correct scaling in c

We can perform c → ∞ limit and get gravitational soliton of NRST

Gravitational solitons in NRST

TH, Lahnsteiner & Obers '25

Example: D(p+1)-brane

$$ds^2 = H^{-\frac{1}{2}}\left(-dT^2 + dX^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\sum_{i=1}^{8-p} dr^i dr^i \ , \quad H = 1 + \frac{(2\pi\sqrt{\alpha'})^{6-p}\bar{g}_s N}{(6-p)\Omega_{7-p}r^{6-p}}$$

$$A^{(p+1)} = (H^{-1}-1)dT \wedge dX \wedge dy^1 \wedge \cdots \wedge dy^p \ , \quad \bar{g}_s e^{\Phi} = \bar{g}_s H^{\frac{2-p}{4}}$$

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NRST geometry:

$$\tau_{\mu\nu} dx^\mu dx^\nu = -H^{-\frac{1}{2}} dt^2 + H^{\frac{1}{2}} dv^2 \ , \quad h_{\mu\nu} dx^\mu dx^\nu = H^{-\frac{1}{2}} \sum_{a=1}^{p}(dy^a)^2 + H^{\frac{1}{2}} \sum_{i=1}^{8-p} dr^i dr^i$$

$$a_{(p+1)} = (H^{-1} - 1) dt \wedge dy^1 \wedge \cdots \wedge dy^p \ , \quad g_s e^{\varphi} = g_s H^{\frac{3-p}{4}}$$

$$H = 1 + \frac{(2\pi\sqrt{\alpha'})^{7-p} g_s N}{(6-p)\Omega_{7-p} R_v r^{6-p}}$$

Interpretation: Dp-brane in NRST smeared along v-direction (transverse direction)

- Agrees with open string POV Gomis, Yan & Yu ‘20; Hartong & Have ‘24
- Consistent with torsion conditions for ½-BPS solutions in NRST Blair ‘25

Gravitational solitons in NRST

TH, Lahnsteiner & Obers '25

We can take near-horizon limit before applying solution generator
→ Gives near-horizon limit of NRST Dp-brane solution:

$$\tau_{\mu\nu}dx^\mu dx^\nu = -\left(\frac{r}{\ell}\right)^{(6-p)/2} dt^2 + \left(\frac{r}{\ell}\right)^{-(6-p)/2} dv^2$$

$$h_{\mu\nu}dx^\mu dx^\nu = \left(\frac{r}{\ell}\right)^{(6-p)/2} \sum_{a=1}^{p} (dy^a)^2 + \left(\frac{r}{\ell}\right)^{-(6-p)/2} \sum_{i=1}^{8-p} dr^i dr^i$$

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For p=3: Dilaton is constant!

Could this give a new holographic correspondence?

String/gravity dual: Above NH D3-brane in NRST (non-relativistic gravity!)

Field theory dual: Field theory living on an F1 – D3 (perpendicular) in multi-critical limit (MM3T limit)
Field theory still to be written down…

Blair, Lahnsteiner, Obers & Yan ´23 and ´24
Gomis & Yan '23

Non-extremal Dp-brane soliton in NRST?

Can one have a gravitational soliton with an event horizon in NRST?

$$ds^2 = H^{-\frac{1}{2}}\left(-f dT^2 + dX^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\sum_{i=1}^{8-p}(dr^i)^2 \quad \text{with} \quad f = 1 - \frac{r_0^{6-p}}{r^{6-p}}$$

$$T = c\,t\,, \quad X = c\,t + \frac{1}{c}u$$

$$ds^2 = H^{-\frac{1}{2}}\left((1-f)c^2 dt^2 + 2dtdu + \frac{1}{c^2}du^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\sum_{i=1}^{8-p}(dr^i)^2$$

T-duality along u

$$ds^2 = H^{-\frac{1}{2}}\left(-c^2 f\, dt^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\left(c^2 dv^2 + \sum_{i=1}^{8-p} dr^i dr^i\right)\,, \quad B = -c^2 dt \wedge dv$$

Problem: B-field is not critical in limit

Critical B-field means $B_{tv} = \sqrt{g_{vt}^2 - g_{tt}g_{vv}}$

TH, Lahnsteiner & Obers '25

Perhaps one cannot have a non-extremal brane in NRST?

NS5-brane gravitational solitons in NRST

TH, Lahnsteiner & Obers '25

Example: NS5-brane

$$ds^2 = -dT^2 + dX^2 + \sum_{m=1}^{4} dy_m^2 + H \sum_{i=1}^{4} dr_i^2 \qquad H = 1 + \frac{N\alpha'}{r^2}$$

$$(dB)_{ijk} = \varepsilon_{ijk}{}^{l} H \partial_l H \ , \quad \bar{g}_s e^{\Phi} = \bar{g}_s H^{1/2}$$

NRST geometry:

$$\tau_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dv^2 \ , \quad h_{\mu\nu} dx^\mu dx^\nu = \sum_{m=1}^{4} dy_m^2 + H \sum_{i=1}^{4} dr_i^2 \ ,$$

$$(dm)_{ijk} = \varepsilon_{ijk}{}^{l} H \partial_l H \ , \quad g_s e^{\varphi} = g_s H^{1/2} \ , \quad H = 1 + \frac{N\alpha'}{r^2} \ .$$

Reproduces solution of Bergshoeff, Lahnsteiner, Romano & Rosseel found within NRST by demanding ½ BPS SUSY (corresponds to 1/4 BPS in relativistic theory)

Interpretation of winding mode soliton

NRST winding mode treated as a probe:

$$ds^2 = c^2(-dt^2 + dv^2) + dr^i dr^i \ , \quad B = -c^2 dt \wedge dv \ , \quad g_s e^{\Phi} = g_s c$$

What if w is so high one has backreaction?

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F-string soliton solution with charge = winding w

$$ds^2 = c^2 H^{-1}(-dt^2 + dv^2) + dr^2 + r^2 d\Omega_7^2 \ , \quad B = -c^2 H^{-1} dt \wedge dv \ , \quad g_s e^{\Phi} = \frac{g_s c}{\sqrt{H}}$$

$$H = 1 + c^2 \frac{L^6}{r^6} \ , \quad L^6 = 32\pi^2 w \, g_s^2 \alpha'^3$$

Couples to gravity as: $\text{Tension} \times \text{Gravitational coupling} \sim c^2$

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Couples to gravity as: $\text{Tension} \times \text{Gravitational coupling} \sim c^2$

Coupling goes to infinity for c → ∞ → NRST limit = Near-horizon limit of F-string

$$ds^2 = \frac{r^6}{L^6}(-dt^2 + dv^2) + dr^2 + r^2 d\Omega_7^2 \ , \quad B = -\frac{r^6}{L^6} dt \wedge dv \ , \quad g_s e^{\Phi} = g_s \frac{r^3}{L^3}$$

Ávila, Guijosa & Olmedo ‘23; TH, Lahnsteiner & Obers ‘25

A soliton in relativistic string theory instead of NRST - What is the interpretation?

TH, Lahnsteiner & Obers ‘25

Validity of NH F-string requires curvature length scale should be larger than string length

Ithzaki, Maldacena, Sonnenschein & Yankielowicz ‘98

$$r \gg \sqrt{\alpha'}$$

TH, Lahnsteiner & Obers '25

Validity of NH F-string requires curvature length scale should be larger than string length

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$$r \gg \sqrt{\alpha'}$$

We can translate this to a requirement of the energy of the worldsheet theory of the F-string:

$$E \gg \frac{1}{\sqrt{w} g_s \sqrt{\alpha'}}$$

Our interpretation: This is when the worldsheet theory is *strongly coupled*

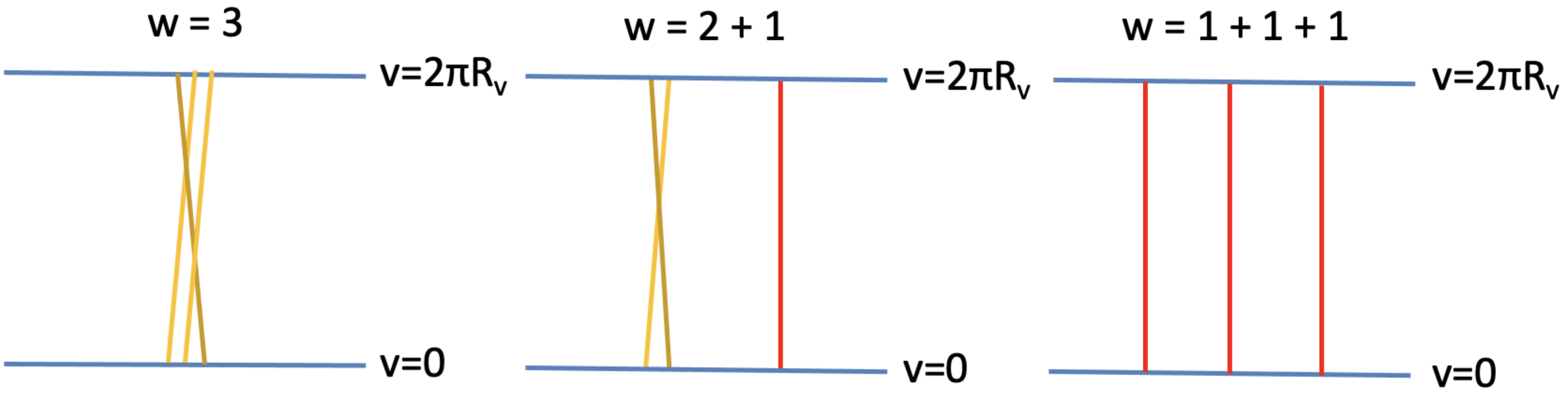
To argue for this, we need to consider 2nd quantization of NRST

2nd quantization of NRST

Total winding along v always fixed (and positive)

→ Multi-string states are partitions of total winding number

Example with winding w = 3:



Multistrings from matrices: Assign 3 x 3 matrix X^i to each transverse direction

Closed string b.c.: $$X^i(\sigma + 2\pi) = U X^i(\sigma) U^{-1}$$

If U=I: w = 1+1+1

If U exchanges two eigenvalues: w = 2 + 1

If U exchanges three eigenvalues: w = 3

→ Matrix description of 2nd quantized NRST

For flat-space type IIB string theory: **Matrix String theory**

Motl '97; Dijkgraaf, Verlinde & Verlinde '97

Matrix String theory is revealed to be Non-Relativistic String theory!

→ Matrix description of 2nd quantized NRST

For flat-space type IIB string theory: **Matrix String theory**

Motl '97; Dijkgraaf, Verlinde & Verlinde '97

Splitting/joining interaction:
Twist field operators with coupling $\lambda \sim g_s \sqrt{\alpha'}$

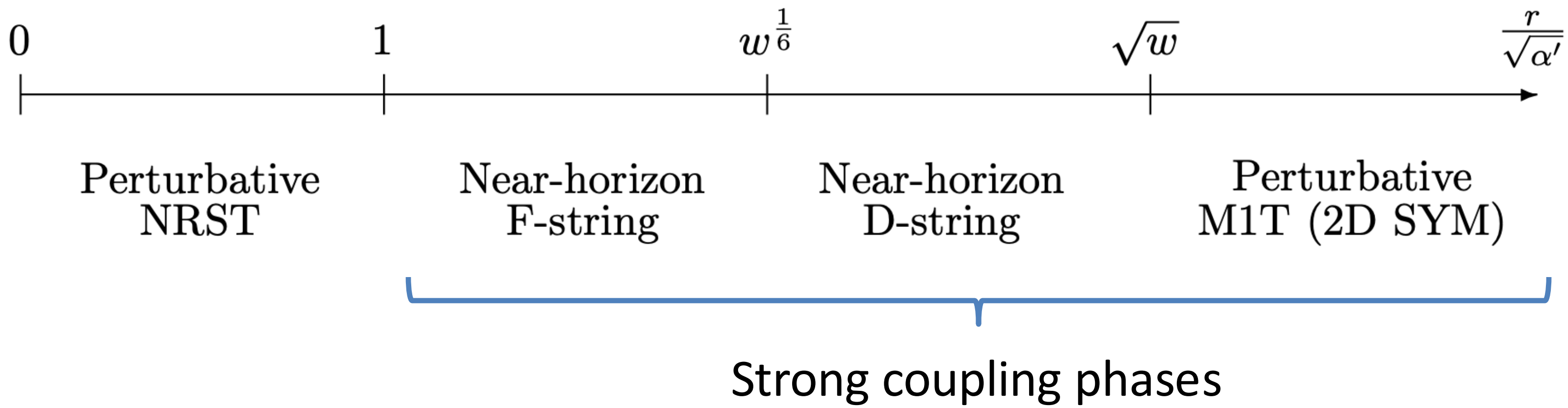
Dijkgraaf, Verlinde & Verlinde '97

NRST worldsheet theory weakly coupled: $$E \ll \frac{1}{g_s \sqrt{w\alpha'}}$$

This fits with our interpretation of $E \gg \frac{1}{\sqrt{w} g_s \sqrt{\alpha'}}$ being strong coupling

Analysis à la IMSY (Itzhaki, Maldacena, Sonnenschein, Yankielowicz) '98

Type IIB



Type IIA

TH, Lahnsteiner & Obers '25

Outlook

Gravitational solitons

We found non-perturbative states in NRST that at low energy can be described geometrically as gravitational solitons

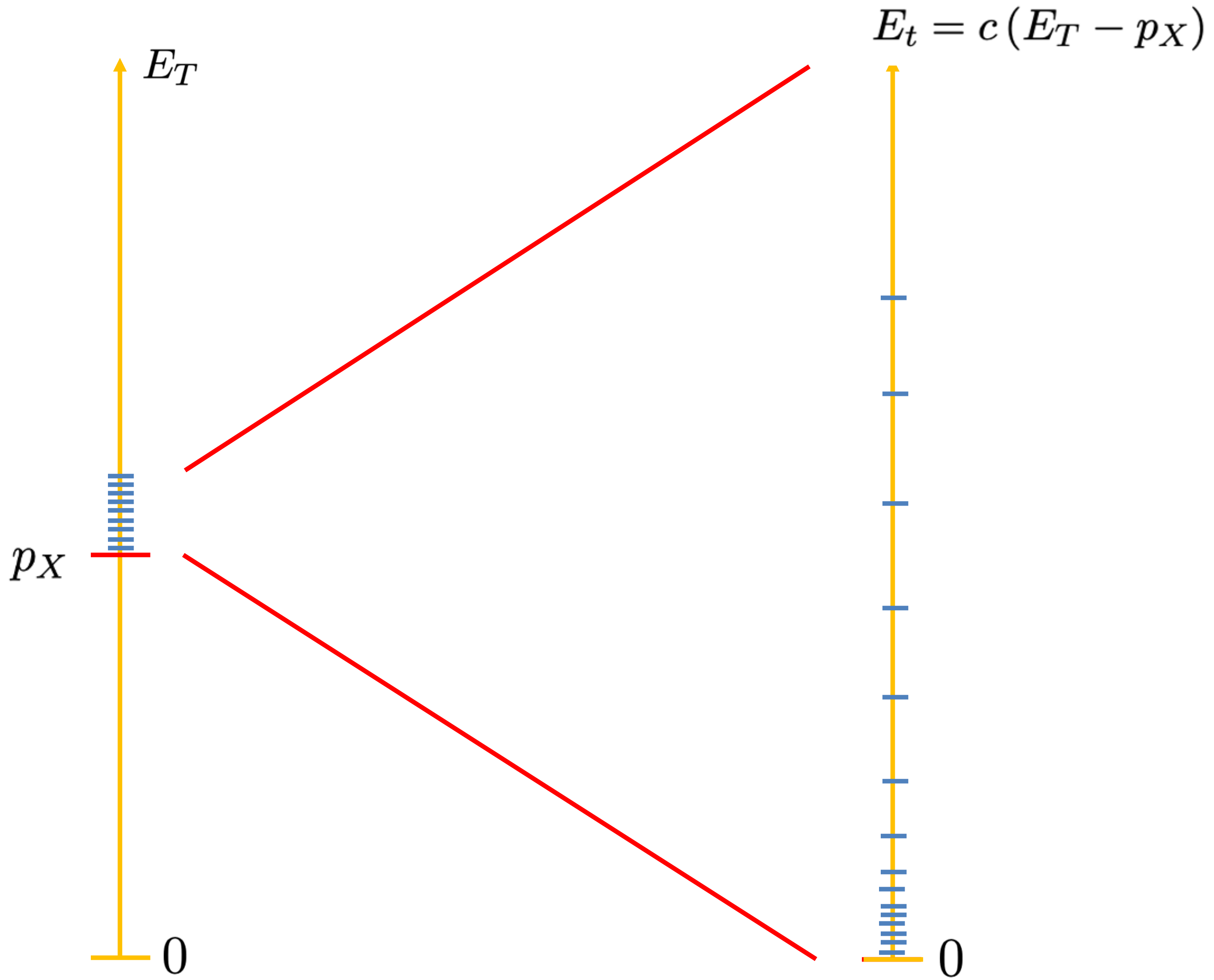
NS5-brane in NRST
Dp-brane in NRST
} From ¼ configurations in relativistic string theory

Near-horizon limit of D3-brane in NRST:
Could perhaps provide a consistent example of a holographic correspondence between a field theory and NRST in this background?

Is it possible to have thermal gravitational solitons in NRST?
DLCQ-type limit seems incompatible with this

NRST limit of longitudinal F-string soliton = Near-horizon limit
→ Strong coupling phase of NRST sigma-model

A corner of string theory with non-relativistic target space



2^{nd} quantized non-relativistic string theory?

Transverse part of the geometry would be non-commutative
→ A quantum geometry generated by multi-strings

One should find a matrix version of the NRST action:

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[\det(\tau)\,\eta^{AB}\tau_A^\alpha\tau_B^\beta\partial_\alpha X^\mu\partial_\beta X^\nu h_{\mu\nu} + \frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu m_{\mu\nu}\right]$$

Non-relativistic holography?

Interesting to combine matrix version of NRST with near-horizon NRST D3-brane holography
→ 2^{nd} quantized strings in a holographic background

How does holography work with infinite speed of light?

Part of a broader network of BPS decoupling limits

U-dual orbits of DLCQ limit

Ithzaki, Maldacena, Sonnenschein & Yankielowicz '98
de Boer, Dijkgraaf, TH, Obers '00 (unpublished)
Blair, Lahnsteiner, Obers & Yan '23 and '24

NRST (F1)
BFFS matrix theory (D0)
Maldacena's decoupling limit for AdS/CFT (D3)

Non-Lorentzian geometries appearing in all of them

Blair, Lahnsteiner, Obers & Yan '23 and '24

Systematic exploration in upcoming work with de Boer, Obers & Yan

For D3-brane case:

Global patch of AdS/CFT points to Spin Matrix Theory limits (TH & Orselli '14), that are also BPS decoupling limits

See upcoming work with Baiguera, Lei and Yan

The End