Workshop on Quantum Gravity and Strings, Corfu 2025

SCALES IN MODULI SPACE

based on 2503.00107 with I. Basile

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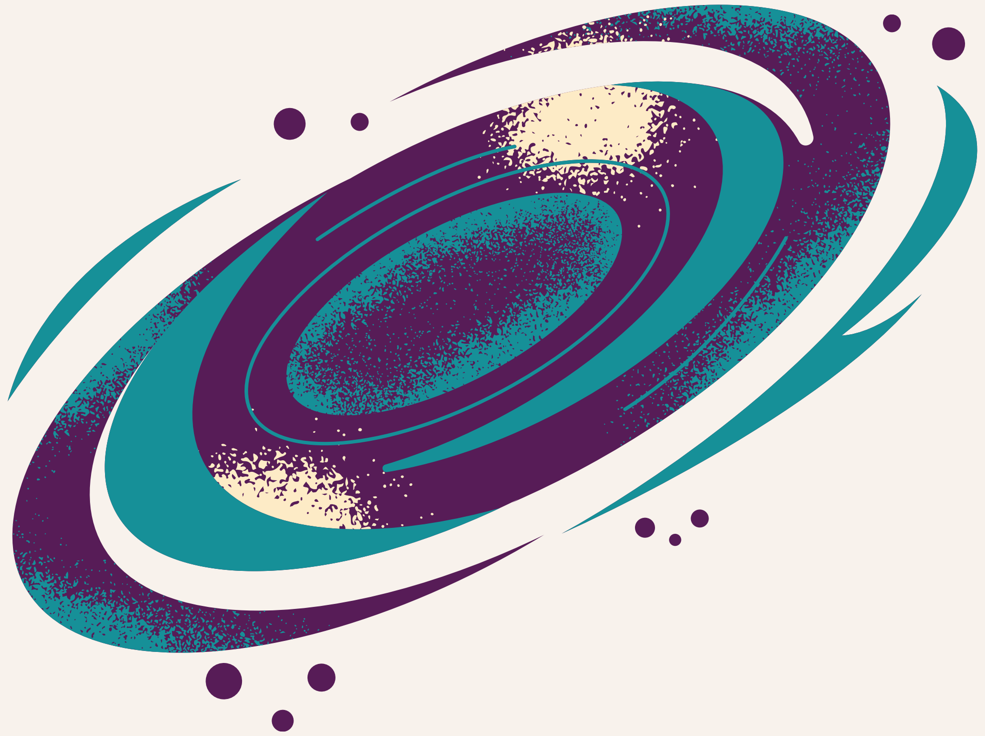

Agenda

- The species scale
- Species scale and moduli
- The CRV pattern
  - Corrections to the pattern
- The black hole scale
  - The pattern and the black hole scale
- Conclusion & outlook

The Species Scale

- the UV cutoff scale beyond which no EFT is reliable anymore
- the species scale is parametrically smaller than M_{Pl} if there are towers of many light degrees of freedom

$$\Lambda_{sp} = \frac{M_{pl}}{N^{\frac{1}{d-2}}}$$

[Aoufia, Basile, Bedroya, Castellano, Calderón-Infante, Cribiori, Dvali, Gomez, Herráez, Ibánez, Leone, Lüst, Montella, Montero, Valenzuela, van de Heisteeg, Vafa, Wiesner, Wu and many others]

Species scale and moduli

- moduli space is understood as a space of possible vacuum configurations determined by vacuum expectation values of massless scalar fields (=moduli)

$$\Lambda_{sp} = \frac{M_{pl}}{N^{\frac{1}{d-2}}}$$

How does the species scale depend on moduli in string compactifications? Are there universal patterns we can exploit to study conjectures & the moduli space?

The CRV pattern

- empirical pattern in moduli space which links the (logarithmic gradients of the) mass gaps of light towers to the (logarithmic gradients of the) species scale in infinite distance limits
- is it universal? can one study the interior of moduli space...?

$$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla \Lambda_{sp}}{\Lambda_{sp}} \to \frac{1}{d-2}$$

[Castellano, Ruiz, Valenzuela ‘23 x 2]

Corrections to the pattern

01 setup: isotropic torus $T^6/(\mathbb{Z}\times\mathbb{Z})$

$$\frac{\nabla m_t}{m_t}\cdot\frac{\nabla\Lambda_{sp}}{\Lambda_{sp}}\to\frac{1}{d-2}$$

$$\mathcal{K}=-\gamma\log[-i(T-\bar{T})]$$

$$F_1=-\gamma\log[(-i(T-\bar{T}))^{\beta}|\eta(T)|^4]$$

$$m_t=\frac{1}{[-i(T-\bar{T})]^{\frac{\gamma}{2}}}$$

[Basile, GS ‘25]

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[Basile, GS '25]
[van de Heisteeg, Vafa, Wiesner, Wu '23]

Corrections to the pattern

02 compute the corrections to the pattern

$$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla \Lambda_{sp}}{\Lambda_{sp}} \to \frac{1}{d-2}$$

$$\frac{1}{2}\left(1 - \beta \frac{3}{\pi y}(1 - \log(2y)) - 12 e^{-2\pi y}(e^{2\pi i x} + e^{-2\pi i x})\right)$$

- if $\beta = 1$, one approaches the pattern from above
- if $\beta = 0$, one approaches the pattern from above or below
- is there another scale that is the same at the boundary?

[Basile, GS ‘25]

The black hole scale

- consider BH in $D = d + p$ dimensions, which correspond to black branes wrapped around compactified dimensions in the higher dimensional theory
- when the horizon radius becomes comparable to the radius of compactified dimensions, the BH decomposes into higher dimensional BH’s in the extra dimensions
- transition to class of new BH solutions
- this class is missed by lower dimensional EFT, thus its description is incomplete

$$\Lambda_{BH} \lesssim \Lambda_{sp} \lesssim M_{Pl}$$

[Gregory, Laflamme ‘93]
[Bedroya, Vafa, Wu ‘24]

The black hole scale

- Black hole scale is the transition scale between higher and lower dimensional BHs
- this scale agrees with m_t in weak coupling limits as well as in large volume decompactification limits
- thus a similar pattern might hold in the interior of moduli space

$$\Lambda_{BH} \lesssim \Lambda_{sp} \lesssim M_{Pl}$$

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \sim \frac{1}{d-2}$$

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$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

[Gregory, Laflamme '93]
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Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

01

equate entropies including Gauss-Bonnet correction α of higher and lower dimensional black holes to define a scale

$$S_d^{(1)} = c_d \left(\frac{M}{M_{Pl}} \right)^{\frac{d-2}{d-3}} \left(1 + \frac{2(d-2)(d-3)\alpha}{R_{BH}^2} \right)$$

$$R_{BH} \sim 2\pi \left(\frac{c_D}{c_d} \right)^{\frac{D-3}{p}} R \left(1 - \frac{2(d-2)(d-3)}{2\pi^2} \left(\frac{c_d}{c_D} \right)^{\frac{D-3}{p}} \frac{\alpha}{R^2} \right)$$

[Basile, GS ‘25]
[Bedroya, Vafa, Wu ‘24]
[Clunan, Ross, Smith ‘04]

Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

01

equate entropies including Gauss-Bonnet correction α of higher and lower dimensional black holes to define a scale

$$\Lambda_{BH} = R_{BH}^{-1} \sim m_t(1 + \alpha m_t^2)$$

CAREFUL: this correction is not small for $\gamma = 1$ (emergent string limit)

[Basile, GS ‘25]

Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

02

compute corrections to the black hole scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \sim \frac{1}{d-2}(1 + \epsilon + 2F_1 m^2 - m^2 \frac{|\nabla F_1|^2}{F_1})$$

- ϵ : terms calculated before in the isotropic torus case
- other terms proportional to $\frac{1}{y^2}$

[Basile, GS ‘25]

Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

03

look at the asymptotics

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \sim \frac{1}{d-2}\left(1+\epsilon+2F_1 m^2 - m^2\frac{|\nabla F_1|^2}{F_1}\right)$$

- $\epsilon = \frac{1}{2}(1-\beta\frac{3}{\pi y}(1-\log(2y)) - 12e^{-2\pi y}(e^{2\pi i x}+e^{-2\pi i x}))$
- other terms proportional to $\frac{1}{y^2}$
- $\beta = 1$: the red terms prevail, the pattern is violated

[Basile, GS ‘25]

Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

03

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- $\epsilon = \frac{1}{2}(1 - \beta\frac{3}{\pi y}(1 - \log(2y)) - 12e^{-2\pi y}(e^{2\pi i x} + e^{-2\pi i x}))$
- other terms proportional to $\frac{1}{y^2}$
- $\beta = 0$: red term is subleading

[Basile, GS '25]

Corrections to the BH scale

$$\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \leq \frac{1}{d-2}$$

03

look at the asymptotics

$$2F_1 m^2 - m^2 \frac{|\nabla F_1|^2}{F_1} \sim m^2 F_1 (2 - \frac{|\nabla F_1|^2}{F_1^2}) \sim \frac{2}{y^2}(1 - \frac{1}{\gamma})$$

- $\beta = 0$: red term is subleading
- the correction is always positive

[Basile, GS '25]

Summary

01 $$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla \Lambda_{sp}}{\Lambda_{sp}} \to \frac{1}{d-2}$$

	$\beta = 0$	$\beta = 1$
does the pattern hold?	✓	✓
$\leqslant \frac{1}{d-2}$?	not always	✗

[Basile, GS '25]

Summary

02 $\nabla \log \Lambda_{BH} \cdot \nabla \log \Lambda_{sp} \sim \dfrac{1}{d-2}$

	$\beta = 0$	$\beta = 1$
does the pattern hold?	✓	✓
$\leqslant \dfrac{1}{d-2}$?	✗	✗

[Basile, GS ‘25]

Conclusion & Outlook

- the species scale provides a rich playground for investigating swampland conjectures & the moduli space
- can the pattern be continued within moduli space by another scale?
- what about desert points of the species scale?

Thank you!