Geometry of supergravity and the Batalin–Vilkovisky formulation of the $\mathcal{N} = 1$ theory in 10 dimensions

Fridrich Valach

Charles University, Prague

w/ Julian Kupka and Charles Strickland-Constable

2410.16046, 2412.04968, 2501.18008, 2508.06398

Overview of the talk

Idea 1: simplify the treatment of $\mathcal{N}=1$, $D=10$ supergravity (including fermions)

Overview of the talk

Idea 1: simplify the treatment of $\mathcal{N} = 1$, $D = 10$ supergravity (including fermions)

Idea 2: build its Batalin–Vilkovisky (BV) formulation

Overview of the talk

Idea 1: simplify the treatment of $\mathcal{N} = 1$, $D = 10$ supergravity (including fermions)

Idea 2: build its Batalin–Vilkovisky (BV) formulation

First simplification = generalised geometry

- efficient repackaging of fields, action, and symmetries
- inclusion of the Yang–Mills supermultiplets for free
- fully geometric

Overview of the talk

Idea 1: simplify the treatment of $\mathcal{N} = 1$, $D = 10$ supergravity (including fermions)

Idea 2: build its Batalin–Vilkovisky (BV) formulation

First simplification = generalised geometry

- efficient repackaging of fields, action, and symmetries
- inclusion of the Yang–Mills supermultiplets for free
- fully geometric

Second simplification = metric formulation

- direct and geometric (including the diffeomorphism action on fermions)
- avoids vielbeine and Lorentz symmetry

  $\Longrightarrow$ considerably simplifies the symmetry structure
- equally applicable to supergravities in any dimension

$\mathcal{N} = 1$ supergravity in 10 dimensions

- Lorentzian metric $g_{\mu\nu}$ (graviton)
- 2-form $B_{\mu\nu}$ (Kalb–Ramond field)
- 1-form A_μ valued in $\mathfrak{g}$ (gauge field)
- function φ (dilaton)

- vector-spinor ψ^μ (gravitino)
- spinor ρ (dilatino; $\rho = \gamma_\mu \psi^\mu - \lambda$)
- $\mathfrak{g}$-valued spinor χ (gaugino)

$\mathcal{N} = 1$ supergravity in 10 dimensions

- Lorentzian metric $g_{\mu\nu}$ (graviton)
- 2-form $B_{\mu\nu}$ (Kalb–Ramond field)
- 1-form A_μ valued in $\mathfrak{g}$ (gauge field)
- function φ (dilaton)

- vector-spinor ψ^μ (gravitino)
- spinor ρ (dilatino; $\rho = \gamma_\mu \psi^\mu - \lambda$)
- $\mathfrak{g}$-valued spinor χ (gaugino)

$$S = \int_M \sqrt{|g|} e^{-2\varphi} \Big(R + 4|\nabla\varphi|^2 - \tfrac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \tfrac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}^\mu \not{\nabla} \psi_\mu + \rho \not{\nabla} \rho + \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{\nabla}_A \chi$$
$$- 2\bar{\psi}^\mu \nabla_\mu \rho + \tfrac{1}{4} \bar{\psi}^\mu \not{H} \psi_\mu - \tfrac{1}{4} \bar{\rho} \not{H} \rho - \tfrac{1}{8} \operatorname{Tr} \bar{\chi} \not{H} \chi + \tfrac{1}{2} H_{\mu\nu\rho} \bar{\psi}^\mu \gamma^\nu \psi^\rho + \tfrac{1}{4} \bar{\psi}^\mu H_{\mu\nu\rho} \gamma^{\nu\rho} \rho$$
$$+ \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{F} \rho + \operatorname{Tr} F_{\mu\nu} \bar{\psi}^\mu \gamma^\nu \chi + \dots \Big)$$

$\mathcal{N} = 1$ supergravity in 10 dimensions

- Lorentzian metric $g_{\mu\nu}$ (graviton)
- 2-form $B_{\mu\nu}$ (Kalb–Ramond field)
- 1-form A_μ valued in $\mathfrak{g}$ (gauge field)
- function φ (dilaton)

- vector-spinor ψ^μ (gravitino)
- spinor ρ (dilatino; $\rho = \gamma_\mu \psi^\mu - \lambda$)
- $\mathfrak{g}$-valued spinor χ (gaugino)

$$S = \int_M \sqrt{|g|} e^{-2\varphi} \Big(R + 4|\nabla\varphi|^2 - \tfrac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \tfrac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}^\mu \not{\nabla} \psi_\mu + \rho \not{\nabla} \rho + \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{\nabla}_A \chi$$
$$- 2\bar{\psi}^\mu \nabla_\mu \rho + \tfrac{1}{4} \bar{\psi}^\mu \not{H} \psi_\mu - \tfrac{1}{4} \bar{\rho} \not{H} \rho - \tfrac{1}{8} \operatorname{Tr} \bar{\chi} \not{H} \chi + \tfrac{1}{2} H_{\mu\nu\rho} \bar{\psi}^\mu \gamma^\nu \psi^\rho + \tfrac{1}{4} \bar{\psi}^\mu H_{\mu\nu\rho} \gamma^{\nu\rho} \rho$$
$$+ \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{F} \rho + \operatorname{Tr} F_{\mu\nu} \bar{\psi}^\mu \gamma^\nu \chi + \ldots \Big)$$

$$\delta g_{\mu\nu} = \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)} \qquad \delta B_{\mu\nu} = \bar{\epsilon} \gamma_{[\mu} \psi_{\nu]} - \operatorname{Tr} A_{[\mu} \bar{\epsilon} \gamma_{\nu]} \chi \qquad \delta\rho = -\not{\nabla}\epsilon + (\nabla_\mu \varphi) \gamma^\mu \epsilon + \tfrac{1}{4} \not{H} \epsilon + \ldots$$

$$\delta A_\mu = -\tfrac{1}{2} \bar{\epsilon} \gamma_\mu \chi \qquad \delta\varphi = \tfrac{1}{4} \bar{\rho} \epsilon - \tfrac{1}{4} \bar{\psi}^\mu \gamma_\mu \epsilon \qquad \delta\psi_\mu = \nabla_\mu \epsilon - \tfrac{1}{4} \not{H}_\mu \epsilon + \ldots \qquad \delta\chi = \tfrac{1}{2} \not{F} \epsilon + \ldots$$

[Bergshoeff–de Roo–de Wit–van Niewenhuizen '82] [Chapline–Manton '83] [Dine–Rohm–Seiberg–Witten '85]

Generalised geometry crashcourse [Coimbra–Strickland-Constable–Waldram '11] [Coimbra–Minasian–Triendl–Waldram '14]

Generalised geometry crashcourse [Coimbra–Strickland-Constable–Waldram '11] [Coimbra–Minasian–Triendl–Waldram '14]

Transitive Courant algebroid (local picture) [Ševera '90s]:

- data: manifold $M = \mathbb{R}^{10}$, Lie algebra $\mathfrak{g}$ with an invariant pairing Tr
- vector bundle: $E = TM \oplus T^*M \oplus (\mathfrak{g} \times M)$, $\langle \cdot\,,\,\cdot \rangle$, $E \to TM$
- bracket: $[x + \alpha + s, y + \beta + t] = L_x y + (L_x\beta - i_y d\alpha + \mathrm{Tr}\, t\, ds) + (L_x t - L_y s + [s, t]_{\mathfrak{g}})$

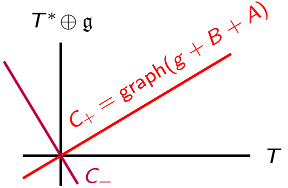
Generalised geometry crashcourse [Coimbra–Strickland-Constable–Waldram '11] [Coimbra–Minasian–Triendl–Waldram '14]

Transitive Courant algebroid (local picture) [Ševera '90s]:

- data: manifold $M = \mathbb{R}^{10}$, Lie algebra $\mathfrak{g}$ with an invariant pairing Tr
- vector bundle: $E = TM \oplus T^*M \oplus (\mathfrak{g} \times M)$, $\langle \cdot\,,\,\cdot \rangle$, $E \to TM$
- bracket: $[x + \alpha + s, y + \beta + t] = L_x y + (L_x \beta - i_y d\alpha + \mathrm{Tr}\, t\, ds) + (L_x t - L_y s + [s, t]_{\mathfrak{g}})$

Bosonic fields

- generalised metric = symmetric map $\mathcal{G} \colon E \to E$ s.t. $\mathcal{G}^2 = 1$
  $\rightsquigarrow$ orthogonal splitting $E = C_+ \oplus C_-$
- nonzero half-density $\sigma \in \Gamma(H)$ $\quad (\sigma^2 = \sqrt{|g|}e^{-2\varphi})$



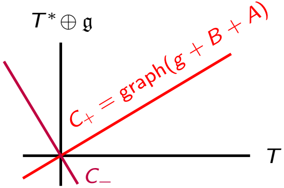
Generalised geometry crashcourse [Coimbra–Strickland-Constable–Waldram '11] [Coimbra–Minasian–Triendl–Waldram '14]

Transitive Courant algebroid (local picture) [Ševera '90s]:

- data: manifold $M = \mathbb{R}^{10}$, Lie algebra $\mathfrak{g}$ with an invariant pairing Tr
- vector bundle: $E = TM \oplus T^*M \oplus (\mathfrak{g} \times M)$, $\langle \cdot\,,\,\cdot \rangle$, $E \to TM$
- bracket: $[x + \alpha + s, y + \beta + t] = L_x y + (L_x\beta - i_y d\alpha + \mathrm{Tr}\, t\, ds) + (L_x t - L_y s + [s,t]_{\mathfrak{g}})$

Bosonic fields

- generalised metric = symmetric map $\mathcal{G} \colon E \to E$ s.t. $\mathcal{G}^2 = 1$
  $\rightsquigarrow$ orthogonal splitting $E = C_+ \oplus C_-$
- nonzero half-density $\sigma \in \Gamma(H)$ $\quad (\sigma^2 = \sqrt{|g|}e^{-2\varphi})$



Fermionic fields

- assume signature$(C_+) = (9,1)$ $\rightsquigarrow$ Majorana–Weyl spinor bundles $S_\pm^{C_+}$
- $\rho \in \Gamma(\Pi S_+^{C_+} \otimes H)$, $\quad \psi \in \Gamma(\Pi S_-^{C_+} \otimes C_- \otimes H)$ $\quad$ (SuSy parameter $\epsilon \in \Gamma(\Pi S_-^{C_+} \otimes H)$)

Generalised geometry crashcourse

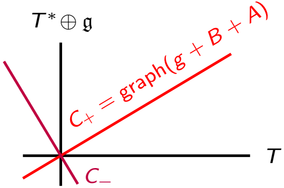
[Coimbra–Strickland-Constable–Waldram '11]
[Coimbra–Minasian–Triendl–Waldram '14]

Transitive Courant algebroid (local picture) [Ševera '90s]:

- data: manifold $M = \mathbb{R}^{10}$, Lie algebra $\mathfrak{g}$ with an invariant pairing Tr
- vector bundle: $E = TM \oplus T^*M \oplus (\mathfrak{g} \times M)$, $\langle \cdot\,,\,\cdot \rangle$, $E \to TM$
- bracket: $[x + \alpha + s, y + \beta + t] = L_x y + (L_x\beta - i_y d\alpha + \mathrm{Tr}\, t\, ds) + (L_x t - L_y s + [s,t]_{\mathfrak{g}})$

Bosonic fields

- generalised metric = symmetric map $\mathcal{G}\colon E \to E$ s.t. $\mathcal{G}^2 = 1$
  $\rightsquigarrow$ orthogonal splitting $E = C_+ \oplus C_-$
- nonzero half-density $\sigma \in \Gamma(H)$ $\quad (\sigma^2 = \sqrt{|g|}e^{-2\varphi})$



Fermionic fields

- assume signature$(C_+) = (9,1) \rightsquigarrow$ Majorana–Weyl spinor bundles $S_\pm^{C_+}$
- $\rho \in \Gamma(\Pi S_+^{C_+} \otimes H)$, $\quad \psi \in \Gamma(\Pi S_-^{C_+} \otimes C_- \otimes H)$ $\quad$ (SuSy parameter $\epsilon \in \Gamma(\Pi S_-^{C_+} \otimes H)$)

Unique operators (only depending on $\mathcal{G}$ and σ): $\mathcal{R}$, $\not{D}\psi^\alpha$, $D_\alpha\rho$, $\ldots$ $\quad (e_a, e_\alpha \leftrightarrow C_+, C_-)$

Supergravity in generalised geometry

$$S = \int_M \mathcal{R}\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha$$
$$- \tfrac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{cde}\psi^\alpha)(\bar{\rho}\gamma^{cde}\rho) - \tfrac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{cde}\psi^\alpha)(\bar{\psi}_\beta\gamma^{cde}\psi^\beta)$$

$$\delta_\epsilon \mathcal{G}_{a\beta} = \tfrac{1}{2}\sigma^{-2}(\bar{\epsilon}\gamma_a\psi_\beta)$$
$$\delta_\epsilon\sigma = \tfrac{1}{8}\sigma^{-1}(\bar{\rho}\epsilon)$$
$$\delta_\epsilon\rho = \not{D}\epsilon + \tfrac{1}{192}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{cde}\psi^\alpha)\gamma^{cde}\epsilon$$
$$\delta_\epsilon\psi_\alpha = D_\alpha\epsilon + \tfrac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\epsilon + \tfrac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\gamma_c\epsilon)\gamma^c\rho$$

[Siegel '93] [Coimbra–Strickland-Constable–Waldram '11] [Coimbra–Minasian–Triendl–Waldram '14] [Kupka–Strickland-Constable–FV '24]

Going back

$$S = \int_M \sqrt{|g|} e^{-2\varphi} \Big(R + 4|\nabla\varphi|^2 - \tfrac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \tfrac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}^\mu \not{\nabla} \psi_\mu + \rho \not{\nabla} \rho$$
$$+ \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{\nabla}_A \chi - 2\bar{\psi}^\mu \nabla_\mu \rho + \tfrac{1}{4} \bar{\psi}^\mu \not{H} \psi_\mu - \tfrac{1}{4} \bar{\rho} \not{H} \rho - \tfrac{1}{8} \operatorname{Tr} \bar{\chi} \not{H} \chi$$
$$+ \tfrac{1}{2} H_{\mu\nu\rho} \bar{\psi}^\mu \gamma^\nu \psi^\rho + \tfrac{1}{4} \bar{\psi}^\mu H_{\mu\nu\rho} \gamma^{\nu\rho} \rho + \tfrac{1}{2} \operatorname{Tr} \bar{\chi} \not{F} \rho + \operatorname{Tr} F_{\mu\nu} \bar{\psi}^\mu \gamma^\nu \chi$$
$$+ \tfrac{1}{384} (\bar{\psi}_\mu \gamma_{\nu\rho\sigma} \psi^\mu)(\bar{\rho} \gamma^{\nu\rho\sigma} \rho) - \tfrac{1}{768} (\bar{\rho} \gamma^{\mu\nu\rho} \rho) \operatorname{Tr}(\bar{\chi} \gamma_{\mu\nu\rho} \chi)$$
$$- \tfrac{1}{192} (\bar{\psi}_\mu \gamma_{\rho\sigma\tau} \psi^\mu)(\bar{\psi}_\nu \gamma^{\rho\sigma\tau} \psi^\nu) + \tfrac{1}{192} (\bar{\psi}_\mu \gamma_{\nu\rho\sigma} \psi^\mu) \operatorname{Tr}(\bar{\chi} \gamma^{\nu\rho\sigma} \chi)$$
$$- \tfrac{1}{768} \operatorname{Tr}(\bar{\chi} \gamma_{\mu\nu\rho} \chi) \operatorname{Tr}(\bar{\chi} \gamma^{\mu\nu\rho} \chi) \Big)$$

$$\delta g_{\mu\nu} = \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)} \qquad \delta B_{\mu\nu} = \bar{\epsilon} \gamma_{[\mu} \psi_{\nu]} - \operatorname{Tr} A_{[\mu} \bar{\epsilon} \gamma_{\nu]} \chi$$

$$\delta A_\mu = -\tfrac{1}{2} \bar{\epsilon} \gamma_\mu \chi \qquad \delta \varphi = \tfrac{1}{4} \bar{\rho} \epsilon - \tfrac{1}{4} \bar{\psi}^\mu \gamma_\mu \epsilon$$

$$\delta \rho = -\not{\nabla} \epsilon + (\nabla_\mu \varphi) \gamma^\mu \epsilon + \tfrac{1}{4} \not{H} \epsilon + \tfrac{1}{96} (\bar{\psi}_\mu \gamma_{\nu\rho\sigma} \psi^\mu) \gamma^{\nu\rho\sigma} \epsilon + \tfrac{1}{4} (\bar{\rho} \epsilon) \rho - \tfrac{1}{192} \operatorname{Tr}(\bar{\chi} \gamma_{\mu\nu\rho} \chi) \gamma^{\mu\nu\rho} \epsilon$$

$$\delta \psi_\mu = \nabla_\mu \epsilon - \tfrac{1}{8} H_{\mu\nu\rho} \gamma^{\nu\rho} \epsilon - \tfrac{1}{4} (\bar{\psi}_\mu \rho) \epsilon - \tfrac{1}{4} (\bar{\psi}_\mu \gamma_\nu \epsilon) \gamma^\nu \rho + \tfrac{1}{4} (\bar{\rho} \epsilon) \psi_\mu$$

$$\delta \chi = \tfrac{1}{2} \not{F} \epsilon - \tfrac{1}{4} (\bar{\chi} \rho) \epsilon - \tfrac{1}{4} (\bar{\chi} \gamma_\mu \epsilon) \gamma^\mu \rho + \tfrac{1}{4} (\bar{\rho} \epsilon) \chi,$$

Supersymmetry algebra [Kupka–Strickland-Constable–FV ’25]

Generalised diffeomorphisms: $\zeta \in \Gamma(E)$, $\quad \delta_\zeta = \mathcal{L}_\zeta \quad$ (diffeo + 1-form + gauge)

Supersymmetry algebra [Kupka–Strickland-Constable–FV '25]

Generalised diffeomorphisms: $\zeta \in \Gamma(E)$, $\quad \delta_\zeta = \mathcal{L}_\zeta \quad$ (diffeo + 1-form + gauge)

Reducible transformations: $\mathcal{L}_{Df} = 0$, $\quad f \in C^\infty(M)$

Supersymmetry algebra [Kupka–Strickland-Constable–FV '25]

Generalised diffeomorphisms: $\zeta \in \Gamma(E)$, $\quad \delta_\zeta = \mathcal{L}_\zeta \quad$ (diffeo + 1-form + gauge)

Reducible transformations: $\mathcal{L}_{Df} = 0, \quad f \in C^\infty(M)$

Algebra: $[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{\mathcal{L}_{\zeta_1}\zeta_2}, \quad [\delta_\zeta, \delta_\epsilon] = \delta_{\mathcal{L}_\zeta \epsilon}, \quad [\delta_{\epsilon_1}, \delta_{\epsilon_2}]\Phi = (\delta_\epsilon + \delta_\zeta)\Phi + \mathcal{O}_\Phi \cdot \mathcal{EOM}_\Phi$

$$\zeta^a := \tfrac{1}{4}\sigma^{-2}(\bar\epsilon_2 \gamma^a \epsilon_1), \quad \epsilon := -\tfrac{1}{2}\not{\zeta}\rho, \qquad \mathcal{O}_\rho = -\tfrac{1}{4}\not{\zeta}, \quad \mathcal{O}_\psi = \tfrac{1}{8}\epsilon_{[2}\bar\epsilon_{1]} - \tfrac{1}{4}\not{\zeta}, \quad \mathcal{O}_\sigma = \mathcal{O}_{\mathcal{G}} = 0$$

Supersymmetry algebra [Kupka–Strickland-Constable–FV '25]

Generalised diffeomorphisms: $\zeta \in \Gamma(E)$, $\quad \delta_\zeta = \mathcal{L}_\zeta \quad$ (diffeo + 1-form + gauge)

Reducible transformations: $\mathcal{L}_{Df} = 0, \quad f \in C^\infty(M)$

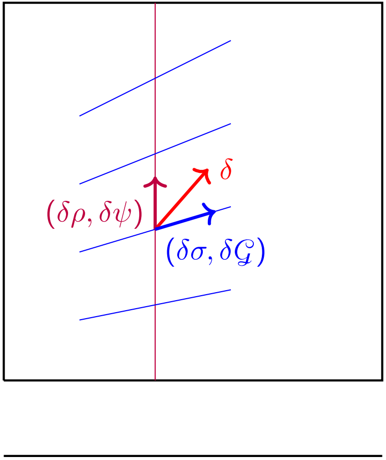
Algebra: $[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{\mathcal{L}_{\zeta_1}\zeta_2}, \quad [\delta_\zeta, \delta_\epsilon] = \delta_{\mathcal{L}_\zeta \epsilon}, \quad [\delta_{\epsilon_1}, \delta_{\epsilon_2}]\Phi = (\delta_\epsilon + \delta_\zeta)\Phi + \mathcal{O}_\Phi \cdot \mathcal{EOM}_\Phi$

$$\zeta^a := \tfrac{1}{4}\sigma^{-2}(\bar\epsilon_2\gamma^a\epsilon_1), \quad \epsilon := -\tfrac{1}{2}\not{\zeta}\rho, \qquad \mathcal{O}_\rho = -\tfrac{1}{4}\not{\zeta}, \quad \mathcal{O}_\psi = \tfrac{1}{8}\epsilon_{[2}\bar\epsilon_{1]} - \tfrac{1}{4}\not{\zeta}, \quad \mathcal{O}_\sigma = \mathcal{O}_{\mathcal{G}} = 0$$

Field space:

$$\begin{array}{ccc} \{\text{all fields}\} & \ni & \{\mathcal{G}, \sigma, \rho, \psi\} \\ \downarrow & & \\ \{\text{bosons}\} & \ni & \{\mathcal{G}, \sigma\} \end{array} \quad \rightsquigarrow \quad \text{natural non-flat connection}$$

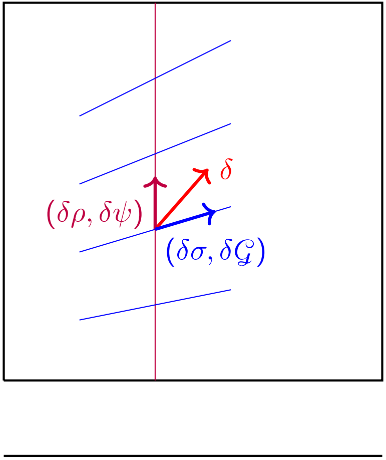
Interpretation of symmetry variations



all fields

bosonic fields

Interpretation of symmetry variations



all fields

bosonic fields

$\implies$ $[\delta_1, \delta_2]$ on fermions picks curvature contribution $\rightsquigarrow$ "cancellation" of Lorentz terms

BV formulation of supergravity [Kupka–Strickland-Constable–FV ’25]

Fields: physical $\mathcal{G}, \sigma, \rho, \psi$, ghosts e (SuSy), ξ (diffeo), ghost for ghost f $+$ antifields

BV formulation of supergravity [Kupka–Strickland-Constable–FV '25]

Fields: physical $\mathcal{G}, \sigma, \rho, \psi$, ghosts e (SuSy), ξ (diffeo), ghost for ghost f + antifields

$$
\begin{aligned}
S = \int_M & \mathcal{R}\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha \\
& - \tfrac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\rho}\gamma^{abc}\rho) - \tfrac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\psi}_\beta\gamma^{abc}\psi^\beta) \\
& + \sigma^*[\mathcal{L}_\xi\sigma - \tfrac{1}{8}\sigma^{-1}(\bar{\rho}e)] + \mathcal{G}^*_{a\beta}[(\mathcal{L}_\xi\mathcal{G})^{a\beta} + \tfrac{1}{2}\sigma^{-2}(\bar{e}\gamma^a\psi^\beta)] \\
& + \bar{\rho}^*[\mathcal{L}_\xi\rho + \not{D}e + \tfrac{1}{192}\sigma^{-2}(\bar{\psi}_\beta\gamma_{abc}\psi^\beta)\gamma^{abc}e] \\
& + \bar{\psi}^*_\beta[(\mathcal{L}_\xi\psi)^\beta + D^\beta e + \tfrac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\rho)e - \tfrac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\gamma_a e)\gamma^a\rho] \\
& + \bar{e}^*[\mathcal{L}_\xi e + \tfrac{1}{16}\sigma^{-2}(\bar{e}\gamma_a e)\gamma^a\rho] + \langle\xi^*, Df + \tfrac{1}{2}\mathcal{L}_\xi\xi\rangle - \tfrac{1}{8}\xi^*_a\sigma^{-2}(\bar{e}\gamma^a e) \\
& + \tfrac{1}{2}f^*(\mathcal{L}_\xi f + \tfrac{1}{8}\sigma^{-2}(\bar{e}\gamma_a e)\xi^a - \tfrac{1}{6}\langle\xi, \mathcal{L}_\xi\xi\rangle) \\
& - \tfrac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\psi}^*_\beta\gamma^a\psi^{*\beta}) - \tfrac{1}{32}\sigma^{-2}(\bar{e}\psi^*_\beta)(\bar{e}\psi^{*\beta}) - \tfrac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\rho}^*\gamma^a\rho^*)
\end{aligned}
$$

BV formulation of supergravity [Kupka–Strickland-Constable–FV '25]

Fields: physical $\mathcal{G}, \sigma, \rho, \psi$, ghosts e (SuSy), ξ (diffeo), ghost for ghost f + antifields

$$
\begin{aligned}
S = \int_M & \mathcal{R}\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha \\
& - \tfrac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\rho}\gamma^{abc}\rho) - \tfrac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\psi}_\beta\gamma^{abc}\psi^\beta) \\
& + \sigma^*[\mathcal{L}_\xi\sigma - \tfrac{1}{8}\sigma^{-1}(\bar{\rho}e)] + \mathcal{G}^*_{a\beta}[(\mathcal{L}_\xi\mathcal{G})^{a\beta} + \tfrac{1}{2}\sigma^{-2}(\bar{e}\gamma^a\psi^\beta)] \\
& + \bar{\rho}^*[\mathcal{L}_\xi\rho + \not{D}e + \tfrac{1}{192}\sigma^{-2}(\bar{\psi}_\beta\gamma_{abc}\psi^\beta)\gamma^{abc}e] \\
& + \bar{\psi}^*_\beta[(\mathcal{L}_\xi\psi)^\beta + D^\beta e + \tfrac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\rho)e - \tfrac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\gamma_a e)\gamma^a\rho] \\
& + \bar{e}^*[\mathcal{L}_\xi e + \tfrac{1}{16}\sigma^{-2}(\bar{e}\gamma_a e)\gamma^a\rho] + \langle\xi^*, Df + \tfrac{1}{2}\mathcal{L}_\xi\xi\rangle - \tfrac{1}{8}\xi^*_a\sigma^{-2}(\bar{e}\gamma^a e) \\
& + \tfrac{1}{2}f^*(\mathcal{L}_\xi f + \tfrac{1}{8}\sigma^{-2}(\bar{e}\gamma_a e)\xi^a - \tfrac{1}{6}\langle\xi, \mathcal{L}_\xi\xi\rangle) \\
& - \tfrac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\psi}^*_\beta\gamma^a\psi^{*\beta}) - \tfrac{1}{32}\sigma^{-2}(\bar{e}\psi^*_\beta)(\bar{e}\psi^{*\beta}) - \tfrac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\rho}^*\gamma^a\rho^*)
\end{aligned}
$$

Claim: This satisfies the classical master equation $\{S, S\} = 0$.

Conclusions and remarks

- Completion of the program of simplifying $\mathcal{N} = 1$ supergravity via generalised geometry.
- Allows a (reasonably short) check of supersymmetry by hand.
- First BV formulation of higher-dimensional supergravity in a background independent way.
- Confirmation that gen. geometry provides natural variables for string massless sector.

Conclusions and remarks

- Completion of the program of simplifying $\mathcal{N} = 1$ supergravity via generalised geometry.
- Allows a (reasonably short) check of supersymmetry by hand.
- First BV formulation of higher-dimensional supergravity in a background independent way.
- Confirmation that gen. geometry provides natural variables for string massless sector.

- New approach: fibred structure of field space, avoids vielbeine & Lorentz transformations

Conclusions and remarks

- Completion of the program of simplifying $\mathcal{N} = 1$ supergravity via generalised geometry.
- Allows a (reasonably short) check of supersymmetry by hand.
- First BV formulation of higher-dimensional supergravity in a background independent way.
- Confirmation that gen. geometry provides natural variables for string massless sector.

- New approach: fibred structure of field space, avoids vielbeine & Lorentz transformations

- Case $\mathcal{G} = 1 \rightsquigarrow$ dilatonic supergravity (topological locally supersymmetric theory for (σ, ρ))
- Case $M = \{*\} \rightsquigarrow$ finite-dimensional toy model of supergravity

Conclusions and remarks

- Completion of the program of simplifying $\mathcal{N} = 1$ supergravity via generalised geometry.
- Allows a (reasonably short) check of supersymmetry by hand.
- First BV formulation of higher-dimensional supergravity in a background independent way.
- Confirmation that gen. geometry provides natural variables for string massless sector.

- New approach: fibred structure of field space, avoids vielbeine & Lorentz transformations

- Case $\mathcal{G} = 1 \rightsquigarrow$ dilatonic supergravity (topological locally supersymmetric theory for (σ, ρ))
- Case $M = \{*\} \rightsquigarrow$ finite-dimensional toy model of supergravity

- Outlook: type II, Costello–Li twist and link to Kodaira–Spencer