AXION-SCALAR DYNAMICS & NON-GEODESIC DISTANCES

Workshop on Quantum Gravity and Strings, Corfu

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2510.XXXX with Thomas Grimm, Damian van de Heisteeg

PART 1

DYNAMICS & THE DISTANCE CONJECTURE

MOTIVATION – THE DISTANCE CONJECTURE

Swampland Distance Conjecture (SDC)

Infinite distance points in moduli space → *towers of light states*

[Ooguri, Vafa '06]
[Ooguri,Palti,Shiu,Vafa '19]

Invalidate EFT

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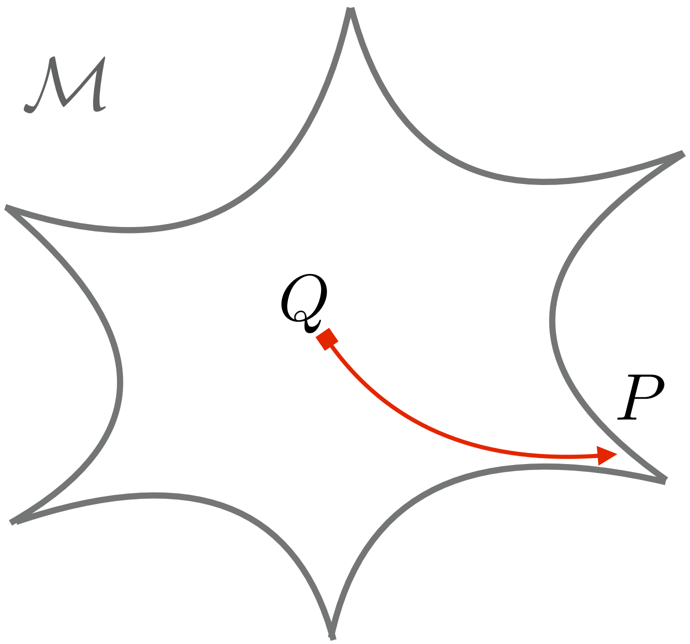


$$m(P) = m(Q)e^{-\lambda d(P,Q)}$$

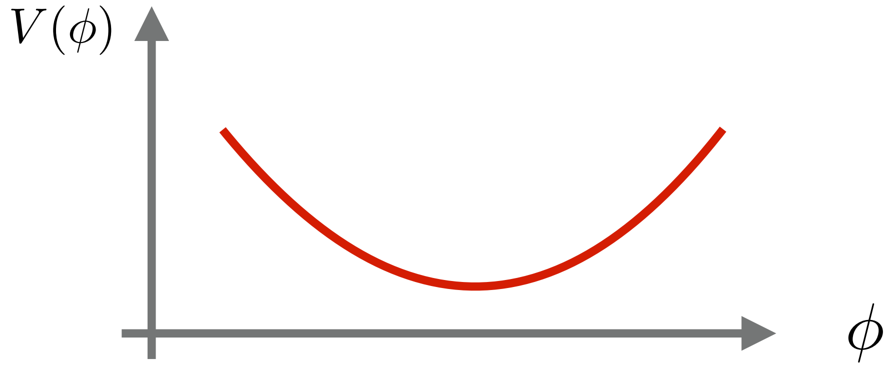
↓ *geodesic distance*

*Best established for **exact** moduli spaces*

SDC WITH A POTENTIAL

*Moduli **must** be stabilised*

SDC still thought to hold,
far <u>less evidence</u>



[Klaewer,Palti '16] [Calderon,Uranga,Valenzuela '20] + …

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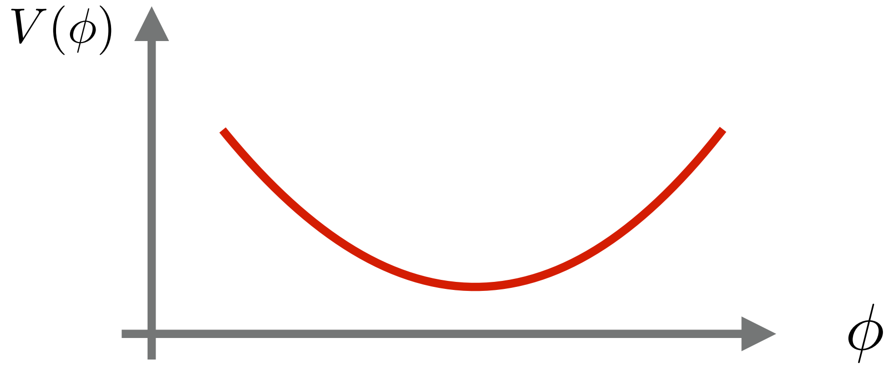

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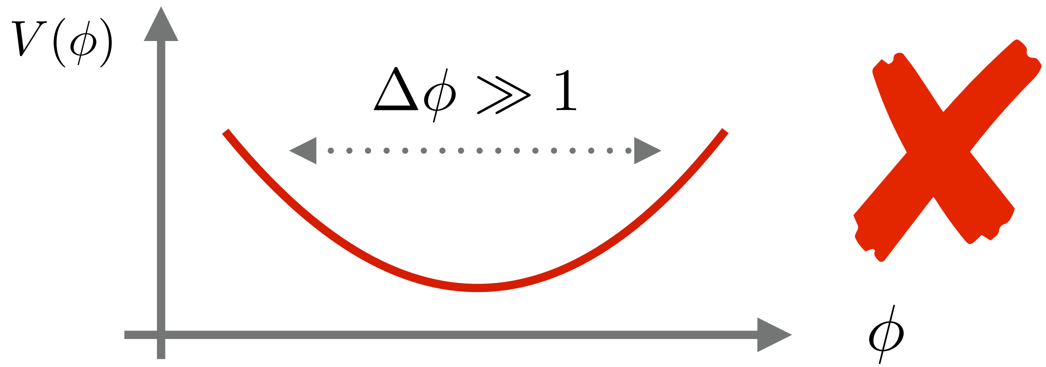
Typical application:
Rule out large field inflation

Compact directions (axions) important:
e.g. monodromy inflation



More recent:
generalised notion of distance, including V?

[Tonioni, Van Riet '24]
[Montero,Mohseini,Vafa,Valenzuela '24]

DYNAMICS

Related issue: SDC applies to ***adiabatic field variations***

$V = 0$	$V \neq 0$
adiabatic: $\dot{\phi} \to 0$	
geodesic trajectories	

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$$\ddot\phi^i+\Gamma^i_{jk}\dot\phi^j\dot\phi^k+(d-1)H\dot\phi^i+\partial^i V=0$$

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What becomes of the SDC in a cosmological setting?

Some (sparse) comments appear in [Conlon,FR '22][Tonioni,Tran,Shiu' 23][Tonioni,Van Riet '24]

Similar issues for species scale discussed in [Bedroya, Lu, Steinhardt '24][Bedroya, Lee, Steinhardt '25]

A POSSIBLE GENERALISATION*

Question:

For trajectories approaching the boundary of moduli space, do towers of states become exponentially light in the dynamical distance ?

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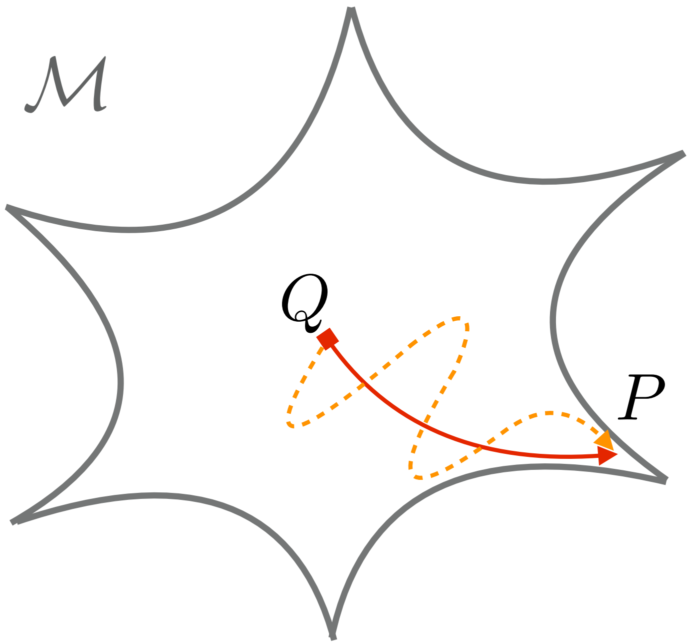

$$m(P) = m(Q) e^{-\lambda \Delta(P,Q)}$$

$$\Delta = \int_{t_1}^{t_2} \mathrm{d}\tau \sqrt{G_{I\bar{J}} \dot{\Phi}^I \dot{\bar{\Phi}}^{\bar{J}}}$$

along trajectory

From usual SDC, equivalent to relationship between length of trajectories and geodesics

CLASSIFICATION OF 1-MODULUS ASYMPTOTIC COSMOLOGIES

SETTING

Cosmology of asymptotic limits in type F-theory flux compactifications

[See also Calderon-Infante,Ruiz,Valenzuela '22,FR '23]

$$S = \frac{M_{P,d}^2}{2} \int d^d x \sqrt{-g} \left\{ \mathcal{R} + \frac{1}{2} G_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \bar{\Phi}^{\bar{J}} + V(\Phi, \bar{\Phi}) \right\}$$

Complex Structure moduli

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Complex Structure moduli

EFTs classified with Asymptotic Hodge Theory

[Grana,Grimm,Herraez,Monnee,Plauschinn,Palti,Lanza,Li,Schlechter,Valenzuela,van de Heisteeg… '19-24]

$$V \sim \sum_{\ell \in \mathcal{E}} \left(\frac{s^1}{s^2} \right)^{\ell_1 - 4} \cdots \left(\frac{s^{\hat{n}-1}}{s^{\hat{n}}} \right)^{\ell_{\hat{n}-1} - 4} \left(s^{\hat{n}}\right)^{\ell_{\hat{n}} - 4} \|\rho_\ell\left(G_4, a_i\right)\|_\infty^2$$

[Bonus]

Complete classification of F-theory scalar potentials with $h^{3,1} = 1$

$\Phi = s + ia$ *Including leading non-perturbative corrections*

EQUATIONS OF MOTION

Solve coupled EOMs of scalar fields, on FLRW background

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0$$

$$\frac{(d-1)(d-2)}{2}H^2 = \frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V\left(\phi^i\right)$$

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Polynomials

$$G_{ij} = \frac{C}{s^2}\delta_{ij} \qquad V(s,a) = \frac{1}{s^\lambda}\sum_{n=0}^{N}\frac{1}{s^n}P_n\left(\frac{a}{s}\right)$$

Hyperbolic metric *Flux scalar potential, positive definite*

Complete classification for $V \geq 0, \quad V(a,s) \underset{s\to+\infty}{\to} 0$

DYNAMICAL SYSTEM FORMULATION

Simpler case $V(s,a) = \frac{P_n(w)}{s^\lambda}$ $\left(\quad V(s,a) = \frac{1}{s^\lambda}\sum_{i=0}^{n}\frac{P_i(w)}{s^i} \quad \right)$

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$$x = \frac{\dot{s}}{\alpha H s} \quad y = \frac{\dot{a}}{\alpha H s} \quad w = \frac{a}{s}$$

x^2, y^2 normalized kinetic terms

$$\begin{cases} \frac{dx}{dN} = -\alpha y^2 - (1 - x^2 - y^2)\left[(d-1)x - \frac{\alpha}{2}\left(\lambda + \frac{w\partial_w P_n(w)}{P_n(w)}\right)\right] \\ \frac{dy}{dN} = \alpha x y - (1 - x^2 - y^2)\left[(d-1)y + \frac{\alpha}{2}\frac{\partial_w P_n(w)}{P_n(w)}\right] \\ \frac{dw}{dN} = \alpha(y - wx) \end{cases}$$

[Copeland,Liddle,Wands '97][Russo,Townsend '06-'19][(Brinkmann),Cicoli, Dibitetto,Pedro '20-'22][FR '23] [Tonioni,Tran,Shiu '23-'24][Andriot,Parameswaran,Tsimpis,Wrase,Zavala '24][Licciardello, Rahimy, Zavala'25]+…

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Asymptotic behaviour → *1) Kination*

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$$V(s,a) = \frac{P_n(w)}{s^\lambda} \longrightarrow T \to \frac{\alpha\lambda}{2(d-1)}$$

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2) *Fixed point* - *easy*

[(Brinkmann),Cicoli, Dibitetto,Pedro '20-'22][FR '23]

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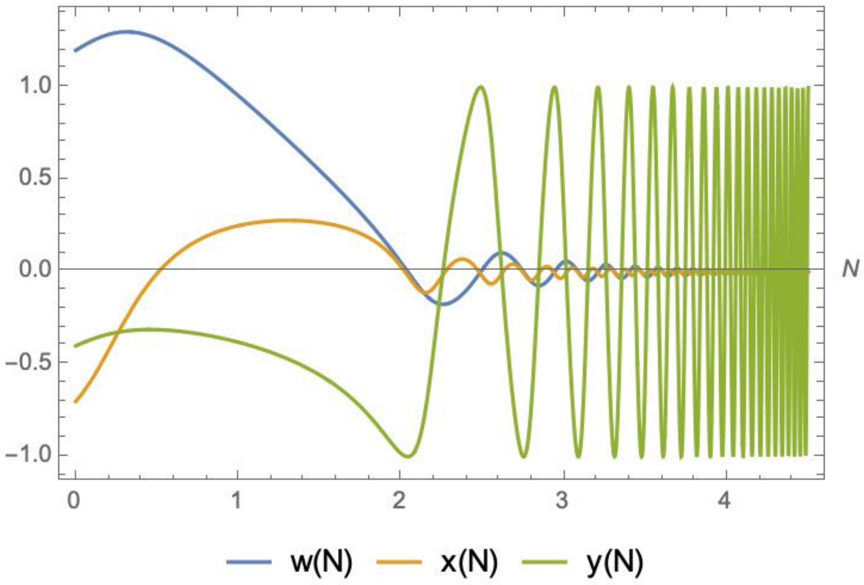
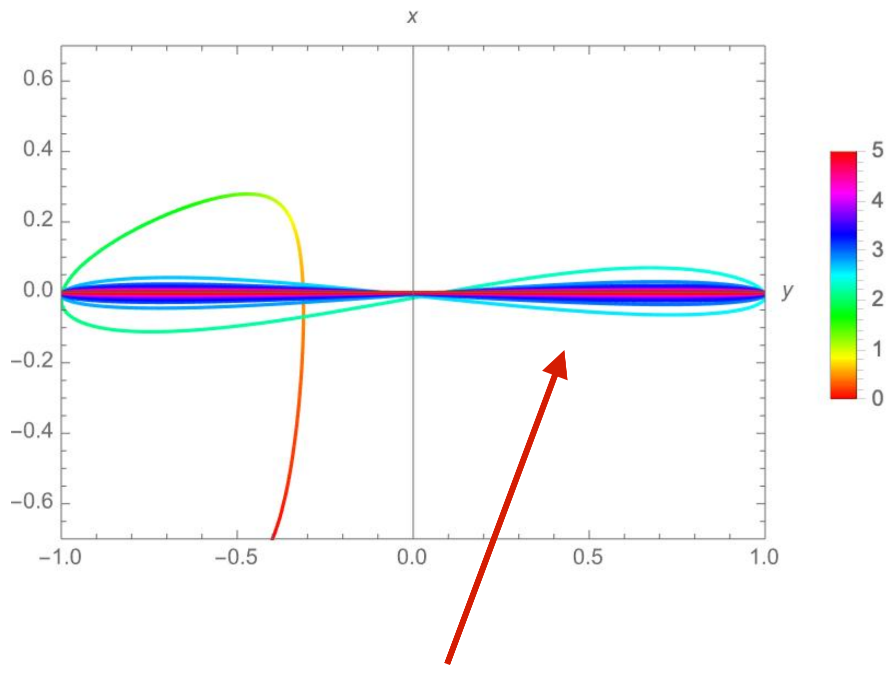
2) Fixed point - easy $w \to w_0$ | $P_0(w_0) = 0$ → ***3) New oscillating solutions***

[(Brinkmann),Cicoli, Dibitetto,Pedro '20-'22][FR '23]

OSCILLATING SOLUTIONS

Potential of the form $$P(w) = f^2(w - w_0)^{2m}$$





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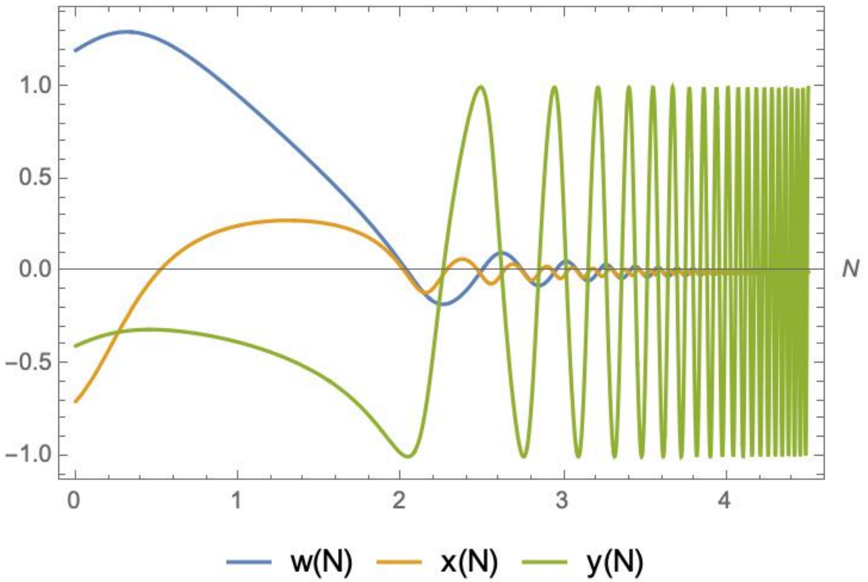

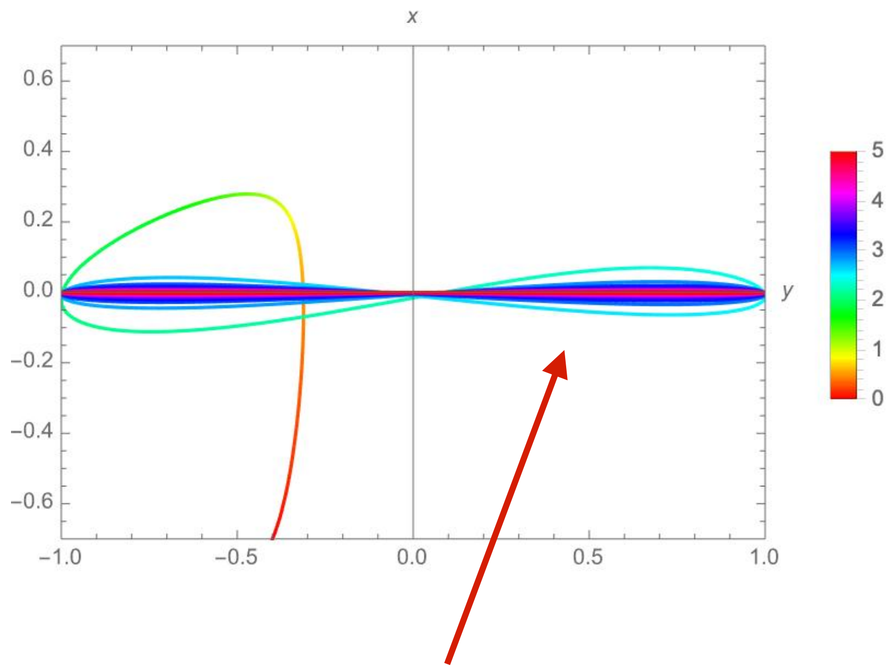

Fixed line

Asymptotic analytical solution: generalised trig. functions

E.g. $S \equiv x^2 + y^2 \sim C_1 \sin_{2,2m}\left(e^{-\gamma N}\right) + C_2$

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$\bar{x} \quad | \quad f(\bar{x}) = 0$ *Deduce local (in)stability from linearisation*

TECHNIQUES FOR DYNAMICAL SYSTEMS

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Lyapunov-like theorems: **Global**

Find Lyapunov function $\mathcal{L}(x(t))$ *s.t.* $\dot{\mathcal{L}}(x(t)) \leq 0$ *everywhere*

If $\mathcal{L}(x(t)) > 0, \mathcal{L}(\bar{x}) = 0$ → $x \to \bar{x}$

On compact set $x \to \left\{ y \quad | \quad \dot{\mathcal{L}}(y) = 0 \right\}$ *Not only points!*

Trivial example: $\dot{x} = -x, \quad \mathcal{L} = x^2 \quad \dot{\mathcal{L}} = 2x\dot{x} = -x^2 \leq 0$

IMPLICATIONS FOR THE DISTANCE CONJECTURE

Distance conjecture tower: $m_t \sim s^{-\alpha} \overset{?}{\sim} f(\Delta)$

How does $\boldsymbol{\Delta}$ *scale with* $\boldsymbol{s}$ *?*

$$\Delta = \sqrt{C}\int_{s(P)}^{s(Q)} \frac{\mathrm{d}s}{s}\sqrt{1+\left(\frac{\mathrm{d}a}{\mathrm{d}s}\right)^2}$$

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$\frac{y^2}{x^2}$ *bounded* → $m_t \sim \Delta^{-\beta}$

Kination: ✓ **Fixed points:** ✓ **Oscillating solutions:** ✓

if $w \nrightarrow 0$

$P(w) = w^{2m}Q(w)$ *leads to violation* ✗

A COUNTER-EXAMPLE?

LCS point (type $V_{1,1}$ *):* $P(w) = 3g_3^2 w^2(1+w^2)$ $\lambda = 0$

Apparent counterexample to "dynamical" distance conjecture

A COUNTER-EXAMPLE, AND HOW TO RESOLVE IT

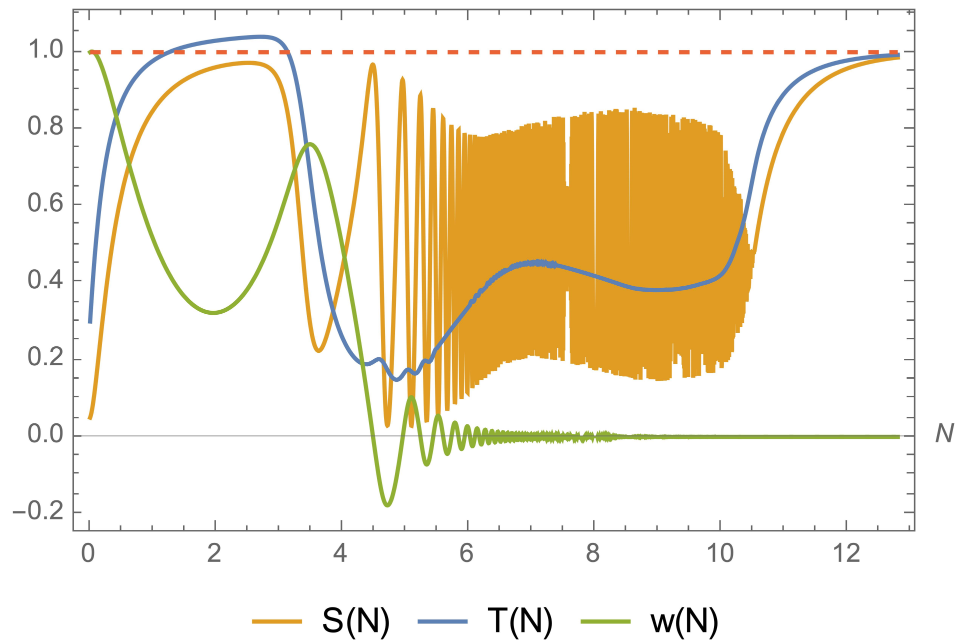
LCS point (type $V_{1,1}$): $P(w) = 3g_3^2 w^2(1+w^2)$ $\lambda = 0$

*Apparent **counterexample** to "dynamical" distance conjecture*

Way out: *subleading corrections*

$$K = -\log\left(\frac{2}{3}s^4 - 4s\xi\right)$$

$$V(s,a) \supset \frac{243 g_3^3 \xi^2}{s^6}$$



Additional arguments:
lack of explicit realisation (other singularity types), "physical" decays (as in in reheating)

COMMENTS ON PHENO

*Many caveats for **realistic** scenarios:*

Kahler + spectator moduli, moduli stabilisation…

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Runaway!

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Peculiar case: $\lambda = 0$ *Modulus "fixed" along runaway, thanks to axion + Hubble friction*

E.g. $V = \frac{a^2}{s^2}$

See also [Tonioni '24]

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General lessons?

SDC for traversed distance

Hard to realise axion monodromy inflation close to boundary

See also [Lanza, Westphal '24]

Don't forget the axions!

Often set to zero, but can have qualitative effects on the dynamics

Dynamical system techniques can be powerful !

OUTLOOK

Cosmology of 1-modulus asymptotic limits

Dynamical version of SDC

$$d(P,Q) \longleftrightarrow \Delta(P,Q)$$

More examples: finite distance singularities?

Universal patterns
& bottom up arguments $\left[G_{I\bar{J}}\frac{\mathrm{d}}{\mathrm{d}N}\left(\phi^I\bar{\phi}^{\bar{J}}\right) \to \frac{\lambda(d-2)}{2}\right]$

Connection to Tameness

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Big picture:

"Dynamical" Swampland constraints for cosmological backgrounds

THANK YOU FOR YOUR ATTENTION!