FIELD STRENGTHS IN POISSON ELECTRODYNAMICS

CALISTA GENERAL MEETING

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OUTLINE OF THE TALK

- Introduction
- Poisson Electrodynamics
- A Technical Interlude
  - (Local) Symplectic Groupoid
  - Bisections & Exonential Map
- Relation between Field Strength
- Conclusion & Outlooks

Introduction

- Following the standard approach to non-commutative U(1)-gauge theories:
  - $(\mathbb{M}, A)$ $\quad \underset{(\mathbb{C}\otimes A)}{\mathbb{M}}$ is a module over the NC-algebra $(A, \ast)$
  - $\nabla : \mathrm{Der}\,A \times \mathbb{M} \longrightarrow \mathbb{M}$ Covariant derivative
  - $\left([\nabla_X, \nabla_Y] - \nabla_{[X,Y]}\right) m = F(X,Y)\, m$ Curvature
  - $A(X) := \nabla_X(\mathbb{1})$ Gauge Potential
  - $U(\mathbb{M})$ = group of gauge transformations

Via gauge transformations:

- $\nabla_X^g : \mathbb{M} \to \mathbb{M}$ $\quad \nabla_X^g = g^{-1} \circ \nabla_X \circ g$
- $F(X,Y)^g : \mathbb{M} \to \mathbb{M}$ $\quad F(X,Y)^g = g^{-1} \circ F(X,Y) \circ g$

An example: the Moyal plane $\mathbb{R}^2_\theta$ (constant non-commutative θ)

- $F_{\mu\nu} = F(\partial_\mu, \partial_\nu) = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_*$ $\quad A_\mu = i \nabla_\mu(\mathbb{1})$
- $A_\mu^{g_f} = g_f * A_\mu * g_f^\dagger - i\, \partial_\mu g_f * g_f^\dagger$ and $F_{\mu\nu}^{g_f} = g_f * F_{\mu\nu} * g_f^\dagger$

where $g_f := \quad \exp_*(if) = \sum_{n=0}^{\infty} \frac{(i)^n}{n!} \underbrace{f * \cdots * f}_{n\text{-times}}$

$\longrightarrow$ Infinitesimal $\delta A_\mu = \partial_\mu f + i[f, A_\mu]_* \xrightarrow{\theta \to 0} \partial_\mu f$

$$\delta F_{\mu\nu} = i[f, F_{\mu\nu}]_*$$

Constant non-commutativity is an exceptional case

- $*$-Derivations do not reduce to standard derivatives when $\theta \to 0$

QUESTION : Can we suitably modify the definition of gauge fields and gauge transformations in order to make them compatible with space-time non-commutativity and reproduce the correct commutative limit?

$\longrightarrow$ POISSON ELECTRODYNAMICS

A First Look at Poisson Electrodynamics

- (M, Θ) $<$ M space-time; Θ non-commutative parameter (Poisson bivector field)
- Standard $U(1)$-gauge theory $\delta^0_f A = \partial f$ and $[\delta^0_f, \delta^0_g] = 0$
- The presence of a non-trivial Θ modifies the algebra as follows

$$[\delta_f, \delta_g] A = \delta_{\{f,g\}} A$$

- A solution

$$\delta_f A_a = \gamma_a^k(A)\, \partial_k f + \{A_a, f\}_\Theta$$

was obtained using a bootstrap procedure $\longrightarrow$ L_∞-algebras

$$\gamma_a^k(A) = \sum_{n=0}^{+\infty} \gamma_a^{k(n)}(A) = \delta_a^k - \frac{1}{2}(\partial_a \Theta^{kb}) A_b + O(\Theta^2)$$

- $\delta_f A_a = \{f, p_a - A_a\}_\Lambda \big|_{p=A}$ where $(T^*M, \{\cdot,\cdot\}_\Lambda)$ is a symplectic realization of (M, Θ)

A first approach to Field strenght

$$\mathcal{F}_{ab} = \{p_a - A_a, p_b - A_b\}\big|_{p=A}$$

$$\lim_{\Theta \to 0} \mathcal{F}_{ab} = \partial_a A_b - \partial_b A_a$$ CLASSICAL LIMIT

$$\delta_f \mathcal{F}_{ab} \neq \{f, \mathcal{F}_{ab}\}$$ NOT GAUGE COVARIANT

SOLUTION $$F_{ab} = \rho_a^c(x,A)\,\{p_c - A_c, p_d - A_d\}\big|_{p=A}\,\rho_b^d(x,A)$$

$$\rho(x,A) = \rho(x,p)\big|_{p=A}$$

$$\gamma_b^j\,\partial_p^b \rho_a^i + \rho_a^b\,\partial_p^i \gamma_b^j + \Theta^{jb}\partial_b \rho_a^i = 0$$

Symplectic Realizations

- $(M, \Theta) \rightsquigarrow$ A symplectic realization (S, ω) is
  - $S \xrightarrow{\pi} M$ a surjective submersion
  - ω a symplectic 2-form such that $\{\pi^* f, \pi^* g\}_\Lambda = \pi^* \{f, g\}_\Theta$

Consider:

* $\mathcal{U} \subset T^*M$, $\omega_0 = d\lambda$ canonical symplectic structure
* $V^\Theta \in \mathfrak{X}(T^*M)$ a Poisson spray
  - $\forall \xi \in T^*M$, $\pi_*(V^\Theta_\xi) = \Theta(\xi, \cdot)$
  - $(m_\nu)_*(V^\Theta) = \nu V^\Theta$ (homogeneous of degree 1)

Theorem: The pair $(\mathcal{U}, \omega)$, where $\left.\begin{aligned} &\mathcal{U} \xrightarrow{\pi} M \\ &\omega = \int_0^1 (\varphi_t^\Theta)^* \omega_0 \, dt \end{aligned}\right\}$ is a symplectic realization of (M, Θ).

In a local chart,

$$\omega = \bar{\gamma}_\mu^\nu \, dx^\mu \wedge dp_\nu + \frac{1}{2} \bar{\gamma}_\mu^\alpha \Theta^{\mu\nu} \bar{\gamma}_\nu^\beta \, dp_\alpha \wedge dp_\beta$$

REMARK: $(M,\Theta) \xleftarrow{\pi} (T^*M, \Lambda) \xrightarrow{\pi_1} (M,-\Theta)$

$$\pi_1 = \pi \circ \varphi_1^\Theta$$

There are two foliations $\mathcal{F}(\pi)$ and $\mathcal{F}(\pi_1)$ that are symplectic orthogonal (DUAL PAIRS)

EXAMPLE: $\mathfrak{g}^* \xleftarrow{\pi} T^*\mathfrak{g}^* \xrightarrow{\pi_1} \mathfrak{g}^*$ $\quad T^*\mathfrak{g}^* = \mathfrak{g} \times \mathfrak{g}^*$ $\quad \Theta = c_{jk}^{\ell}\, x_\ell \frac{\partial}{\partial x_j} \wedge \frac{\partial}{\partial x_k}$

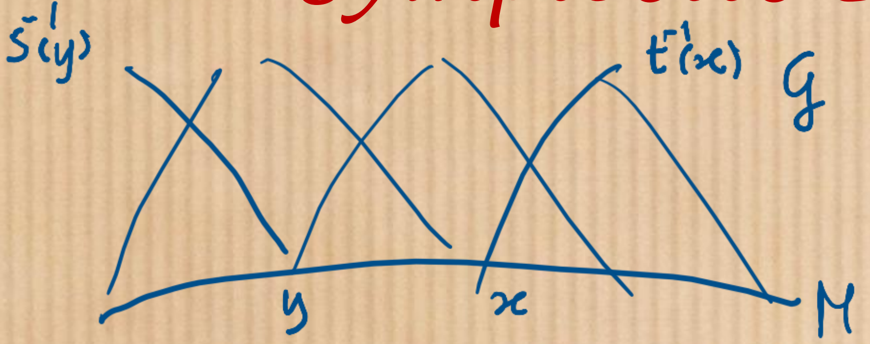
$$V^\Theta_{(\xi,x)} = c_\ell^{jk}\, x^\ell p_j \frac{\partial}{\partial x^k} \Rightarrow \begin{cases} y_t = y_t(x,p) \\ \eta_t = \eta_t(x,p) \end{cases} \longrightarrow \begin{array}{l} y_t = Ad^*_{e^{-tp}}(x) \\ \eta_t = p \end{array} \longrightarrow \pi_1(x,p) = Ad^*_{e^{-p}}(x)$$

$$\omega = d\left(\langle x, \int_0^1 e^{-t\, ad_p} dt\,(dp)\rangle\right) = -\bar{\gamma}_a^j(p)\, dx^a \wedge dp_j + \frac{1}{2}\bar{\gamma}_a^j\, c_\ell^{\ b}\, x^\ell\, \bar{\gamma}_b^k\, dp_j \wedge dp_k$$

$\mathcal{F}(\pi) = \{\bar{\rho}^j = \bar{\gamma}_a^j(p)\,\partial_p^a\}$ right-invariant vector fields on G

$\mathcal{F}(\pi_1) = \{X^\ell + c_{\ell m}^{\ m}\, x_m \frac{\partial}{\partial x_m}\}$ $\quad X^\ell = \gamma_a^\ell\, \partial_p^a$ left-invariant vector fields on G

Symplectic Local Lie Groupoids



$\mathcal{G} \overset{s}{\underset{t}{\rightrightarrows}} M$ is a Lie groupoid

- s, t smooth surjective submersions

$\circ : \{(\beta,\alpha) \mid s(\beta)=t(\alpha)\} =: \mathcal{G}^{(2)} \to \mathcal{G}$ composition law

ASSOCIATIVE
UNITS
INVERSES

EXAMPLE: $G \times \mathfrak{g}^* \overset{s}{\underset{t}{\rightrightarrows}} \mathfrak{g}^*$

$s(g,x) = x$

$t(g,x) = Ad^*_{g^{-1}}(x)$

$(h, Ad^*_{g^{-1}}(x)) \circ (g,x) = (hg, x)$

$1_x = (e, x)$

$(g,x)^{-1} = (g^{-1}, Ad^*_{g^{-1}}(x))$

If the composition is only defined for a subset $\mathcal{G}_m \subset \mathcal{G}^{(2)}$ we have a LOCAL LIE GROUPOID

A LOCAL SYMPLECTIC GROUPOID is a pair $(\mathcal{G}, \omega)$

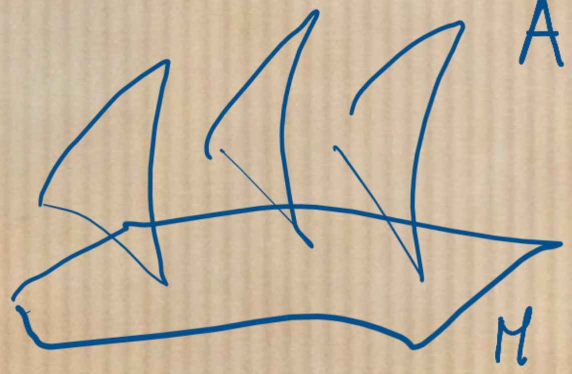
* $\mathcal{G}$ is a local Lie groupoid
* ω is a symplectic structure and $\Gamma \subset \bar{\mathcal{G}} \times \mathcal{G} \times \mathcal{G}$ is a Lagrangian submanifold

Γ is the graph of the composition law

LOCAL SYMPLECTIC GROUPOID $\longrightarrow$ SYMPLECTIC REALIZATIONS

Th. 1 (Coste-Dazord-Weinstein) Given a paracompact Poisson manifold (M,Θ) there is a local symplectic groupoid $(\mathcal{G},\omega)$ having M as space of units.

Lie Algebroids



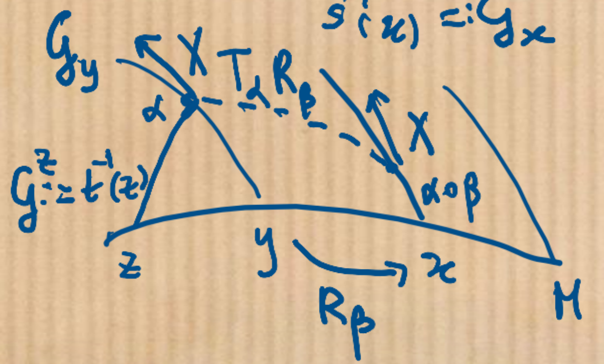
$A \xrightarrow{\pi} M$ is a Lie algebroid

- $A \to M$ smooth vector bundle
- $a : A \to TM$ anchor map
- $[\cdot,\cdot]_A : \Gamma(A)\times\Gamma(A) \to \Gamma(A)$ Lie bracket

$$[X, fY]_A \to f[X,Y]_A + [a(X)(f)]\,Y$$

EXAMPLE : $TM \xrightarrow{\pi} M \qquad a = id \quad , \quad [\cdot,\cdot]_A = [\cdot,\cdot]$

Every Lie groupoid determines a Lie algebroid



$\mathcal{G} \rightrightarrows M \qquad \beta : x \to y \qquad R_\beta : \mathcal{G}_{t(\beta)} \to \mathcal{G}_{s(\beta)}$

A right-invariant vector field X on $\mathcal{G}$ $\quad (X \in \mathfrak{X}^{RI}(\mathcal{G}))$

- $X(\alpha) \in T_\alpha \mathcal{G}_y \qquad (\alpha : y \to z)$
- $(T_\alpha R_\beta)(X(\alpha)) = X(\alpha \circ \beta) \qquad T_\alpha R_\beta : T_\alpha \mathcal{G}_{t(\beta)} \to T_{\alpha\circ\beta}\mathcal{G}_{s(\beta)}$

$$\left.\begin{array}{l} X(\alpha) = (T_{1_{t(\alpha)}} R_\alpha)(X(1_{t(\alpha)})) \\ X_1, X_2 \in \mathfrak{X}^{RI}(\mathcal{G}) \ , \ [X_1, X_2] \in \mathfrak{X}^{RI}(\mathcal{G}) \end{array}\right\} \rightarrow \begin{array}{l} A = \bigcup_{x \in M} T_{1_x} \mathcal{G}_x \xrightarrow{\pi} M \\ [\xi_1, \xi_2]_A^{RI} = [\xi_1^{RI}, \xi_2^{RI}] \end{array} \quad \begin{array}{l} a : A \to TM \\ a := (T_{1_x} t)|_{A_x} \end{array}$$

EXAMPLE : $\mathcal{G} = G \times \mathfrak{g}^* \rightrightarrows \mathfrak{g}^* \qquad s^{-1}(x) \simeq G \qquad T_{1_x}\mathcal{G}_x \simeq \mathfrak{g} \qquad A \simeq T^*\mathfrak{g}^* = \mathfrak{g} \times \mathfrak{g}^*$

$$X(g,x) = \left(T_{1_{Ad^*_{g^{-1}}(x)}} R_{(g,x)}\right)(X(x),0) = (X(Ad^*_{g^{-1}}(x))g, 0) \qquad a(X) = c^l_{jk}\, x_l\, X^j \frac{\partial}{\partial x_k}$$

Bisections & Exponential Map

Given a Lie groupoid $\mathcal{G} \rightrightarrows M$, a bisection $\Sigma \subset \mathcal{G}$ is a submanifold such that $s|_\Sigma$ and $t|_\Sigma$ are bijective maps. Therefore we have:

$\Sigma_s : M \to \mathcal{G}$

- $s \circ \Sigma_s = id_M$
- $t \circ \Sigma_s = l_\Sigma : M \to M$ is a diffeomorphism

$\Sigma_t : M \to \mathcal{G}$

- $t \circ \Sigma_t = id_M$
- $s \circ \Sigma_t = r_\Sigma : M \to M$ is a diffeomorphism

$$l_\Sigma \circ r_\Sigma = id_M$$

$\Sigma \to L_\Sigma : \mathcal{G} \to \mathcal{G}$ left-translation $\quad L_\Sigma(\alpha) = \Sigma_s(t(\alpha)) \circ \alpha$

(φ, φ_0) $\quad \varphi : \mathcal{G} \to \mathcal{G}$, $\varphi_0 : M \to M$ such that $t \circ \varphi = \varphi_0 \circ t$, $s \circ \varphi = s$

$\varphi^x : \mathcal{G}^x \to \mathcal{G}^{\varphi_0(x)} \rightsquigarrow \exists!\, \beta : x \to \varphi_0(x)$, $\varphi^x = L_\beta$

Analogously we have right-translation

THEOREM Let $\mathcal{G} \rightrightarrows M$ be a Lie groupoid, $W \subset M$ an open subset of M and $X \in \Gamma_W(A)$. Then, $\forall x_0 \in M$, $\exists\, \mathcal{U} \subset W$, an $\varepsilon > 0$ and a unique smooth family of local bisections $\operatorname{Exp} tX$, $-\varepsilon < t < \varepsilon$, such that

- $\frac{d}{dt} \operatorname{Exp} tX \big|_{t=0} = X$
- $\operatorname{Exp}(0X) = 1_U$
- $\operatorname{Exp}(t+s)X = \operatorname{Exp} tX \star \operatorname{Exp} sX$
- $\operatorname{Exp}(-tX) = (\operatorname{Exp} tX)^{-1}$

($\star$ is the product between bisections)

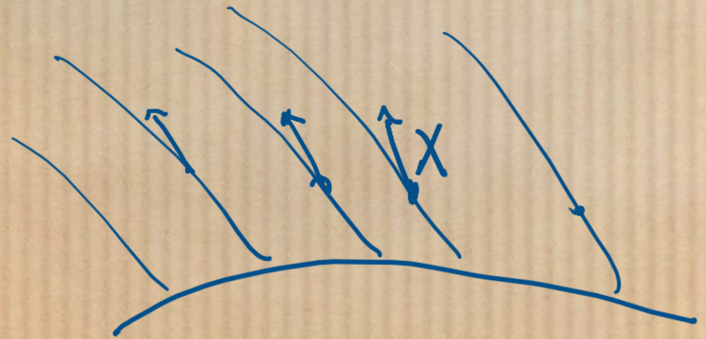
$$(\rho \star \sigma)(x) = \rho(t(\sigma(x))) \circ \sigma(x)$$

Therefore, we have the exponential map

$$\operatorname{Exp} : \Gamma(A) \to \mathcal{B}_W(\mathcal{G})$$

$$\operatorname{Exp}(X) = \operatorname{Exp}(1 \cdot X) = \sigma \in \mathcal{B}_W(\mathcal{G})$$

Given $X \in \Gamma(A) \rightsquigarrow X^{RI}$ is a right-invariant vector field



$\Phi_X(t, g)$ is the flow of X^{RI}

- $s(\Phi_X(t, \cdot)) = s(\cdot)$
- $R_\beta \circ \Phi_X(t, \cdot) = \Phi_X(t, \cdot) \circ R_\beta$

$$\begin{tikzcd} \mathcal{G} \arrow[r, "\Phi_X(t,\cdot)"] \arrow[d] & \mathcal{G} \arrow[d] \\ M \arrow[r, "\psi(t,\cdot)"] & M \end{tikzcd}$$

$(T^*M, [\,,\,]_\Theta)$ Lie algebroid $\longleftarrow$ (M, Θ) $\longrightarrow$ $(\mathcal{G}, \omega)$ symplectic groupoid

$[a, b]_\Theta = L_{\Theta(a,\cdot)} b - L_{\Theta(b,\cdot)} a - d\,\Theta(a,b)$ Poisson Mfd

a is a 1-form $\rightsquigarrow$ $i_{Y_a^{(L)}}\omega = s^*(a)$ $\quad Y_a^{(L)}$ LEFT INVARIANT VECTOR FIELD

$i_{Y_a^{(R)}}\omega = -t^*(a)$ $\quad Y_a^{(R)}$ RIGHT INVARIANT VECTOR FIELD

$\Sigma_s^{(a)}(x) = \varphi_1^{(a)}(1_x)$ $\qquad R_{\Sigma_a^{-1}} : \mathcal{G} \to \mathcal{G}$ is the time-1 map of $Y_a^{(R)}$

$$\begin{tikzcd} \mathcal{G} \arrow[r, "R_{\Sigma_a^{-1}}"] \arrow[d] & \mathcal{G} \arrow[d] \\ M \arrow[r, "\tau_{\Sigma_a^{-1}}"] & M \end{tikzcd}$$

EXAMPLE: $\mathcal{G} = G \times \mathfrak{g}^*$ $\qquad A = \mathfrak{g} \times \mathfrak{g}^*$

$\Gamma(A) \ni a = a_j(x)\, t^j$

$Y_a^{(L)} = a_j(x)\gamma^j + c_e^{jc} x^e a_j(x)\partial_c$

$Y_a^{(R)} = -a_j(t(x,p))\,\bar{\rho}^j = -a_j(t(x,p))\,\bar{\rho}_a^j(p)\,\partial_p^a$

$\rightsquigarrow$ $\Sigma_s^{(a)}(x) = (x, \mathcal{A}(x)) = (x, \exp(t_j A^j(x)))$

$A_j(x) = \int_0^1 a_j(\xi(s))\, ds$

$R_{\Sigma_a^{-1}} : T^*G \to T^*G \quad \begin{cases} y^j = \tau_{\Sigma_a^{-1}}(x) = \mathcal{A}^{-1}(x)\, x\, \mathcal{A}(x) \\ \Pi_a = R_{\Sigma_a^{-1}}(x,p) = -A_j(x) \oplus P \end{cases}$

$\xi(s) = Fl_V(s; x) \qquad V = \Theta^{jk}(x)\, a_k(x)\, \partial_j$

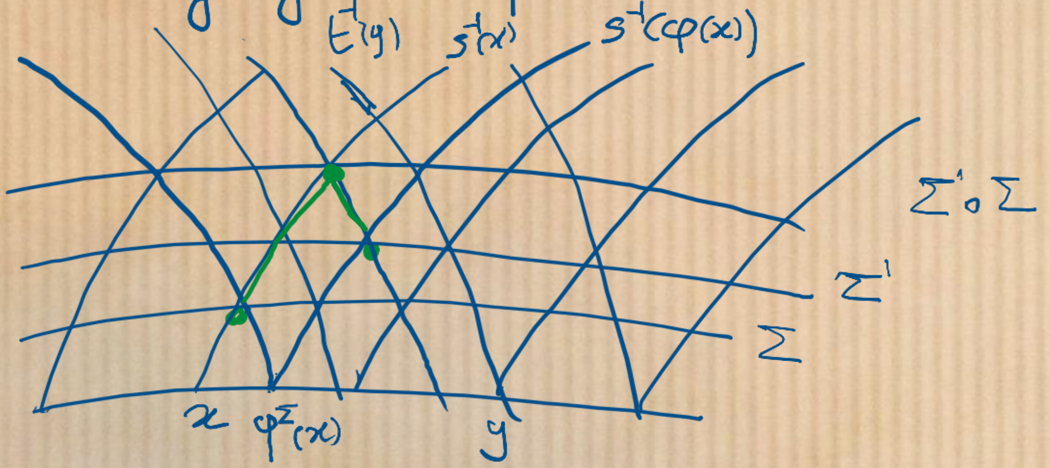
$e^{(A \oplus B)} := e^A e^B$

Relations among Field Strengths

Poisson Electrodynamics $\longleftrightarrow$ Symplectic groupoid

Gauge fields — Bisections

Gauge transformations — Right-action of Lagrangian bisection $\Sigma \to \Sigma' = \Sigma\circ\Lambda$



$\mathcal{B}(\mathcal{G})$ = group of bisections

$$(\Sigma'\circ\Sigma)_s(x) = \Sigma'_s(l_\Sigma(x))\circ\Sigma_s(x) = (R_\Sigma\circ\Sigma'_s\circ l_\Sigma)(x)$$

Several field strengths

$$\mathcal{F}_{ab} = \{P_a - A_a, P_b - A_b\}$$

$$F_{ab} = \rho_a^c\,\mathcal{F}_{cd}\,\rho_b^d$$

$$F^s(\Sigma) := \Sigma_s^*(\omega) \to (\Sigma\circ\Lambda)_s^*(\omega) = l_\Lambda^*(F^s(\Sigma))$$

COVARIANT FIELD STRENGTH

$$F^t(\Sigma) := \Sigma_t^*(\omega) \to (\Sigma\circ\Lambda)_t^*(\omega) = (R_\Lambda\circ\Sigma_t)^*(\omega) = \Sigma_t^*(\omega)$$

INVARIANT FIELD STRENGTH

Let Σ_a be a bisection of $(\mathcal{G},\omega)$ generated by $a \in \Gamma(T^*M)$

$\mathcal{G}$ a local symplectic groupoid

Local Description

$$T^*U \longrightarrow T^*U$$
$$(x^j, p_a) \qquad (y^j, \pi_a) = R_{\Sigma_a^{-1}}(x,p)$$

$$\begin{tikzcd}
(x,p)\ \mathcal{G} \arrow[r, "R_{\Sigma_a^{-1}}"] \arrow[d] & \mathcal{G}\ (y,\pi) \arrow[d] \\
(x)\ M \arrow[r, "\tau_{\Sigma_a^{-1}}"] & M\ (y)
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathcal{G} \arrow[d, shift left, "s"] & \mathcal{G} \arrow[l, "R_{\Sigma_a}"'] \arrow[d, shift left, "s"] \\
M \arrow[u, shift left, "\Sigma_a"] & M \arrow[l, "\tau_{\Sigma_a}"] \arrow[u, shift left, "i_0"]
\end{tikzcd}$$

$$R_{\Sigma_a} \circ i_0 = \Sigma_s^{(a)} \circ \tau_{\Sigma_a}$$

$$V_j^{(y)} = \frac{\partial}{\partial y^j} \qquad V^k_{(\pi)} = \frac{\partial}{\partial \pi_k}$$

$$E_j^{(\pi)} = d\pi_j \qquad E^k_{(y)} = s^*(dy^k)$$

$$\begin{tikzcd}
 & \hat{F}_{ab}(y) := \{\pi_a, \pi_b\}|_{\pi=0} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & F^t \arrow[d, leftrightarrow] \\
\mathcal{F}_{ab}(x) \arrow[r, leftrightarrow] & F_{ab}(x) = \ell^*_{\Sigma_a}(\hat{F}_{ab}) \arrow[r, leftrightarrow] & F^s = \ell^*_{\Sigma_a}(F^t)
\end{tikzcd}$$

STEP 1. $F^t(\partial_j, \partial_k) = i_0^*\left(\left(R_{\Sigma_a}^{*}\omega\right)\left(V_j^{(y)}, V_k^{(y)}\right)\right)$

STEP 2. $R_{\Sigma_a}^{*}\omega = \frac{1}{2} R_{\ell m}\, dy^\ell \wedge dy^m + \Gamma_\ell^m \left(d\pi_m \otimes dy^\ell - dy^\ell \otimes d\pi_m\right) + \frac{1}{2}\Xi^{\ell m} d\pi_\ell \wedge d\pi_m$

$$R_{\ell m}|_{\pi=0} = F^t(\partial_j, \partial_k) = \left(\tau_{\Sigma_a}^{*} F^s\right)(\partial_j, \partial_k)$$

STEP 3. $\hat{F}_{ab}(y) = \left(\left(\mathbb{I} + \bar{\Gamma} F^t \bar{\Gamma}\,\Xi\right)^{-1} \bar{\Gamma} F^t \bar{\Gamma}\right)_{ab}(y)$

Analogous relations holds between $\mathcal{F}_{ab}$ and F^s

STEP 4. $\mathcal{F}_{ab}(x) = \mathcal{F}_{jk}(x)\left(\frac{\partial \pi^j}{\partial p_a}\Big|_{p=\Sigma_a}\right)\left(\frac{\partial \pi^k}{\partial p_b}\Big|_{p=\Sigma_a}\right) = \rho_a^j\, \mathcal{F}_{jk}\, \rho_b^k$

$$\delta_f \mathcal{F}_{ab} = \{f, \hat{\mathcal{F}}_{ab}\}$$

All field strengths vanish simultaneously $\longleftrightarrow$ DEVIATION FROM BEING LAGRANGIAN

Poisson Chern-Simon Theory

(M,Θ) Poisson Manifold $\longrightarrow$ $(\mathcal{G}, \omega = d\vartheta)$ symplectic groupoid

$$S(\Sigma) = \int_M \vartheta(A) \wedge F^s \qquad \vartheta(A) = \Sigma_s^*(\vartheta)$$

Variations $\rightarrow$ $\Sigma = \exp(a) \rightsquigarrow \Sigma' = \exp(a+\delta a) = \Sigma \circ \delta\Sigma$

$$\delta\Sigma = \exp(-a) \circ \exp(a+\delta a)$$

$$\delta S(\delta\Sigma) = \frac{d}{d\varepsilon}\Big(S(\Sigma'_\varepsilon) - S(\Sigma)\Big) = \int_M \Sigma_s^*\big((L_{\delta\hat{a}}\vartheta) \wedge d\vartheta\big) = \int_M \big(\Sigma_s^*(L_{\delta\hat{a}}\vartheta)\big) \wedge F^s = 0$$

$F^s = 0 \rightsquigarrow$ Lagrangian bisections

Gauge Invariance $\rightsquigarrow$ $\Sigma' = \Sigma \circ \Lambda \qquad x \rightarrow \tau_\Lambda(x)$

$$S'(\Sigma\circ\Lambda) = \int_{\tau_\Lambda(M)} \ell_\Lambda^*(\vartheta(A) \wedge F^s) = \int_M \vartheta(A) \wedge F^s = S(\Sigma)$$

CONCLUSIONS

- Poisson Electrodynamics is an attempt of coinciling non-commutative space-time and gauge theories
- The gauge potentials have been interpreted as bisections of a symplectic realization S (Local symplectic groupoid) of the semiclassical space-time $\mathcal{M}$
- Lie Groupoids and Lie algebroids provide the geometrical framework where interpreting the gauge transformations of the model, previously derived via L_∞-algebras. The gauge transformations are associated with the action of the group of bisections of a local symplectic groupoid $\Sigma(\mathcal{M})$ over the semiclassical space-time
- Several Field strengths have been considered, and the relation among all of them has been shown. They all vanish simultaneously and measure the deviation of a bisection from being a Lagrangian submanifold.

PERSPECTIVES

- Dynamics of Test particles
- Introduction of metric & Ampere law
- Role of L_∞ - algebras (curved and flat)
- Minimal coupling with matter fields

THANKS FOR YOUR ATTENTION

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