# From Gauge Theory to Gravity: Classical Double Copy as a Path to Higher-Derivative Corrections

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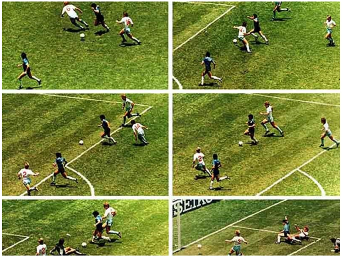
# From Greece to Argentina

Gauge theory $\rightarrow$ Gravitational theory

For many Argentinians, the most unforgettable memory of Greece is Athens 2004, the Olympic Games. That year, Argentina proudly won two gold medals, a kind of double copy glory, an achievement that still makes our hearts race today.

# Introduction: On the construction of higher-derivative terms

In this talk, I will show how to use T-duality invariant theories to produce the higher-derivative structure of string theory.



Scattering amplitudes
$\beta-$functions

$\mathrm{T}-\mathrm{duality}$

# Introduction: On the construction of higher-derivative terms

While using manifestly T-duality invariant geometries has been a successful program to construct $O(\alpha')$- and $\mathcal{O}(\alpha'^2)$-corrections in the last years, some terms in the $O(\alpha'^3)$ Lagrangian indicate that it is not possible to use them to construct the full Lagrangian at this order:

$$S_3 = -\frac{\zeta(3)}{2^5}\int d^{10}x\sqrt{-g}e^{-2\phi}R_{\alpha\beta}{}^{\lambda\epsilon}R^{\alpha\beta\gamma\delta}R_{\gamma}{}^{\mu}{}_{\lambda}{}^{\xi}R_{\delta\xi\epsilon\mu}+\dots$$

# Introduction: On the construction of higher-derivative terms

Open problem:[Hronek-Wulff] and [Hsia-Kamal-Wulff] showed that the terms proportional to $\zeta(3)$ cannot be constructed from a T-duality invariant formulation.

One recent proposal [E.L-J.A.Rodriguez] is the use of Classical Double copy as a program to produce $\alpha'$-corrections with the ambition to attack this problem from a different angle.

One major success of the double copy program [Bern, Carrasco, Johansson] is the relation between two copies of the Yang-Mills amplitudes, which produces the amplitudes of the NS-NS sector of the low energy limit of string theory:

$$\boxed{\text{YM} \times \text{YM} \to \text{NS-NS SUgra.}}$$

In its original manifestation [Kawai, Lewellen, Tye], closed string tree-level amplitudes can be written in terms of open-string amplitudes giving the more general

$$\boxed{\text{gauge}_1 \times \text{gauge}_2 \to \text{gravity.}}$$

# Introduction: Double copy and Higher-derivative terms

Recently, a Classical Double Copy procedure [Hohm, Jaramillo, Plefka] was constructed giving the following map up to cubic order in fields,

$$\begin{array}{ccc} \text{YM} & \to \text{pert. Double Field Theory} \to & \text{pert. SUgra} \\ \text{(flat space)} & \text{(non-manifest T-dual form)} & \text{(flat space)} \end{array}$$

The idea of the authors is to consider an off-shell realization of the double copy map. This formalism allows one to start from the (quadratic, cubic) YM Lagrangian and recover the (quadratic, cubic) NS-NS supergravity Lagrangian.

# Introduction: Double copy and Higher-derivative terms

Extending the previous idea one has,

$$\underset{\text{(flat space)}}{\text{YM + higher der.}} \rightarrow \underset{\text{(non-manifest T-dual form)}}{\text{pert. DFT + higher der.}} \rightarrow \underset{\text{(flat space)}}{\text{pert. SUgra}} + O(\alpha')$$

Therefore, the idea is to consider a higher-derivative gauge theory and perform [Hohm-Jaramillo-Plefka] procedure to obtain $\alpha'$-corrections.

**Outline of the talk:**

- Part 0: Brief intro to DFT (manifestly T-duality invariant formulation).
- Part 1: Deforming DFT towards the $\alpha'$- and $\alpha'^2$-corrections.
- Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

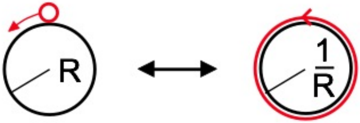
## Part 0: Review of Double Field Theory

Based on G. Aldazabal, D. Marques and C. Nunez, ‘Double Field Theory: A Pedagogical Review’, Class. Quantum Grav. 30 (2013) 163001, [hep-th/1305.1907]..

# Motivation: T-duality as an organizing principle

Let's focus on the LEEA for the bosonic string compactified on a $d$-dimensional torus, so that the dimension of the external space is $N = D - d$.

The resulting theory is invariant under $O(d, d, Z)$. In its simplest form, this symmetry indicates that string theory compactified on a circle of radius $R$ and momentum $p = n$ ($n \in Z$) is equivalent to a theory compactified on a circle of radius $R' = 1/R$ and winding momentum $\omega = n$.




On the left, a string with momentum p. On the right, a string wrapping over the compactified dimension.

# Motivation: T-duality as an organizing principle

The main point of Double Field Theory is to accomplish $O(D, D, R)$ as a global symmetry group. In doing so, the coefficients of the SUgra Lagrangian are fixed

$$\int d^D x e^{-2d} R(E, d) \leftrightarrow \int d^D x \sqrt{-g} e^{-2\phi} (R(g) + 4(\partial\phi)^2 - \frac{1}{12} H^2) \, .$$

DFT can be understood from (at least) two philosophies:

- We are rewriting SUgra $\leftarrow$.
- We are constructing an $O(D, D, R)$ invariant theory, which reduces to SUgra $\rightarrow$.

**Double geometry:**
The duality group is $O(D,D)$. We consider a $2D$ space with coordinates $X^M = (\tilde{x}_\mu, x^\mu)$, and $M = 0,\ldots,2D-1$.

The $O(D,D)$ invariant metric is

$$\eta_{MN} = \begin{pmatrix} \eta^{\mu\nu} & \eta^{\mu}{}_{\nu} \\ \eta_{\mu}{}^{\nu} & \eta_{\mu\nu} \end{pmatrix} = \begin{pmatrix} 0 & \delta^{\mu}{}_{\nu} \\ \delta_{\mu}{}^{\nu} & 0 \end{pmatrix} ,$$

with $\mu,\nu = 0,\ldots,D-1$.

**Symmetries:** The generalized diffeomorphisms are given by a generalized Lie derivative,

$$\delta_{\hat{\xi}} V^M = \mathcal{L}_{\hat{\xi}} V^M = \hat{\xi}^N \partial_N V^M + (\partial^M \hat{\xi}_P - \partial_P \hat{\xi}^M) V^P + \omega \partial_N \hat{\xi}^N V^M \,,$$

where $V^M$ is an arbitrary double vector.

# Introduction to heteroric DFT

The closure of the transformations

$$\left[\delta_{\hat{\xi}_1}, \delta_{\hat{\xi}_2}\right] V^M = \delta_{\hat{\xi}_{21}} V^M \,,$$

is provided by a C-bracket,

$$\hat{\xi}^M_{12} = \hat{\xi}^P_1 \frac{\partial \hat{\xi}^M_2}{\partial X^P} - \frac{1}{2} \hat{\xi}^P_1 \frac{\partial \hat{\xi}_{2P}}{\partial X_M} - (1 \leftrightarrow 2) \,.$$

The closure is satisfied if we impose

$$\partial_M \star \partial^M \star \;\; = \;\; \partial_M \partial^M \star = 0 \,.$$

We solve this constraint considering $\tilde{\partial}^\mu = 0$.

We define flat indices as $A = (\underline{a}, \bar{a})$ where $\underline{a} = 1, \ldots, D$ and $\bar{a} = 1, \ldots, D$. The double Lorentz transformations are given by

$$\delta_\Lambda V^A = V^B \Lambda_B{}^A \,,$$

with $V^A$ an arbitrary flat vector. We use the following parametrizations for the DFT symmetry parameters:

$$\begin{aligned} \hat{\xi}^M &= (\zeta_\mu, \xi^\mu)\,, \\ \Lambda_{\bar{a}\bar{b}} &= \Lambda_{ab}\delta^{ab}_{\bar{a}\bar{b}}\,, \quad \Lambda_{\underline{ab}} = -\Lambda_{ab}\delta^{ab}_{\underline{ab}} \end{aligned}$$

The double Lorentz invariant metrics are

$$\eta_{AB} = \begin{pmatrix} -\eta_{\underline{ab}} & 0 \\ 0 & \eta_{\overline{ab}} \end{pmatrix} \ , \quad H_{AB} = \begin{pmatrix} \eta_{\underline{ab}} & 0 \\ 0 & \eta_{\overline{ab}} \end{pmatrix} \ ,$$

where $\eta_{\underline{ab}}$ and $\eta_{\overline{ab}}$ can be identified with a flat and constant metric $\eta_{ab}$

$$\eta_{\bar{a}\bar{b}}\delta^{\bar{a}\bar{b}}_{ab} = \eta_{\underline{ab}}\delta^{\underline{ab}}_{ab} = \eta_{ab} \, .$$

# Introduction to DFT

**Fundamental fields:**
A generalized frame $E_{MA}$ and a generalized dilaton $d$. The former is equivalent to a double vielbein and satisfies

$$
\begin{aligned}
E_{MA}H^{AB}E_{NB} &= H_{MN}\,, \\
E_{MA}\eta^{AB}E_{NB} &= \eta_{MN}\,.
\end{aligned}
$$

While the generalized frame is a vector under generalized diffeos and double Lorentz transformations, the generalized dilaton is a double Lorentz invariant, and $e^{-2d}$ is a scalar with $\omega(e^{-2d}) = 1$.

The generalized frame is parametrized as

$$E^M{}_A = \begin{pmatrix} E_{\mu\underline{a}} & E^\mu{}_{\underline{a}} \\ E_{\mu\bar{a}} & E^\mu{}_{\bar{a}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -e_{\mu\underline{a}} - b_{\rho\mu} e^\rho{}_{\underline{a}} & e^\mu{}_{\underline{a}} \\ e_{\mu\bar{a}} - b_{\rho\mu} e^\rho{}_{\bar{a}} & e^\mu{}_{\bar{a}} \end{pmatrix}.$$

The generalized dilaton is parametrized as $e^{-2d} = \sqrt{-g}e^{-2\phi}$. The vielbeins $e_{\mu\underline{a}}$, $e_{\mu\bar{a}}$ produce the same metric tensor, $g_{\mu\nu}$. We identify each vielbein (and their inverses) considering the following gauge fixing,

$$\begin{aligned} e_{\mu\underline{a}}\delta^{\underline{a}}_a &= e_{\mu\bar{a}}\delta^{\bar{a}}_a = e_{\mu a} \\ e^\mu{}_{\underline{a}}\delta^{\underline{a}}_a &= e^\mu{}_{\bar{a}}\delta^{\bar{a}}_a = e^\mu{}_a\,. \end{aligned}$$

The generalized fluxes are defined as

$$F_{ABC} = 3E_{[A}E^{M}{}_{B}E_{MC]}\,,$$

with $E_A = \sqrt{2}E^{M}{}_{A}\partial_M$. Typically we project different components of the fluxes considering the projectors $P_{AB} = \frac{1}{2}(\eta_{AB} - H_{AB})$ and $\overline{P}_{AB} = \frac{1}{2}(\eta_{AB} + H_{AB})$.

# Introduction to DFT

Using the fluxes we can construct a generalized Ricci scalar,

$$R(E,d) \;=\; 2E_{\underline{A}}F^{\underline{A}} + F_{\underline{A}}F^{\underline{A}} - \frac{1}{6}F_{\underline{ABC}}F^{\underline{ABC}} - \frac{1}{2}F_{\overline{A}\underline{BC}}F^{\overline{A}\underline{BC}}\,,$$

where $F_A = \sqrt{2}\partial^M E_{MA} - 2E_A d\,.$

It is easy to prove that,

$$e^{-2d}R(E,d) \to \sqrt{-g}\,e^{-2\phi}\big(R(g) + 4(\partial\phi)^2 - \frac{1}{12}H^2\big)\,.$$

Part 1: Using DFT towards the $\alpha'$- and $\alpha'^2$-corrections.

Based on E.L, '$\alpha'$-corrections and their double formulation', J.Phys.A 55 (2022) 5, 053002. Online lectures available on Youtube.

The four-derivative corrections were historically computed considering three- and four-point scattering amplitudes for the massless states. The effective action, originally computed by Metsaev and Tseytlin, takes the form

$$S_{MT} = \int d^D x \sqrt{-g} e^{-2\phi} (R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + L^{(1)}_{MT}) \, ,$$

$$\begin{aligned} L^{(1)}_{\rm MT} &= -\frac{a+b}{8}\Big[R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{1}{2}H^{\mu\nu\rho}H_{\mu\sigma\lambda}R_{\nu\rho}{}^{\sigma\lambda} + \frac{1}{24}H^4 - \frac{1}{8}H^2_{\mu\nu}H^{2\mu\nu}\Big] \\ &\quad + \frac{a-b}{4}H^{\mu\nu\rho}C_{\mu\nu\rho} \end{aligned}$$

where $a = b = -\alpha'$ for bosonic string theory, and $a = -\alpha', b = 0$ for heterotic string theory.

# Gravitational 4-derivative corrections

The action

$$\int d^D x(-g)^{\frac{1}{2}} e^{-2\phi}\Big(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H^2 + L^{(1)}_{MT}\Big)\,,$$

requires a first-order Lorentz transformation for the b-field since $C_{\mu\nu\rho}$ does not transform covariantly,

$$\delta C_{\mu\nu\rho} = -\partial_{[\mu}\Lambda^{ab}\partial_\nu w_{\rho]ab}\,.$$

# b) Gravitational 4-derivative corrections

The deformation is given by the Green-Schwarz mechanism for the b-field,

$$\delta^{(1)}_{\Lambda} b_{\mu\nu} = \frac{1}{2}(a-b)\partial_{[\mu}\Lambda^{ab} w_{\nu]ab} \, .$$

The corrected action is Lorentz invariant since the previous transformation compensates the non-covariant transformation of $L^{(1)}_{MT}$.

# Part 1: Using DFT towards the $\alpha'$- and $\alpha'^2$-corrections

The bi-parametric extension of DFT can be easily constructed considering a generalized GS mechanism [Marques-Nunez],

$$\delta_\Lambda E_M{}^A = E_M{}^B \Lambda_B{}^A + a\partial_{[\overline{P}}\Lambda^{\overline{BC}} F_{\underline{M}]\overline{BC}} E^{PA} + b\partial_{[\underline{P}}\Lambda^{\underline{BC}} F_{\overline{M}]\underline{BC}} E^{PA} \, .$$

Here the parameters are related to (super)gravity formulations when

$$(a,b) = \begin{cases} (-1,-1) & \text{bosonic DFT} \quad , \\ (-1,0) & \text{heterotic DFT} \quad , \\ (-1,1) & \text{HSZ} \quad . \end{cases}$$

# The generalized Green-Schwarz mechanism

We still need a recipe to construct the four-derivative action at the DFT level,

$$S_{DFT} = \int d^D x e^{-2d} (\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)}) \,.$$

Since $\delta_\Lambda^{(1)}\mathcal{R} \neq 0$, then $\mathcal{R}^{(\pm)} \neq 0$ to ensure the invariance of the action. This procedure was constructed by [Baron, E.L, Marques] and we called it "the generalized Bergshoeff-de Roo" identification. This procedure is systematic and can include all the $\alpha'$ and $\alpha'^2$ corrections for both heterotic and bosonic strings.

# The generalized Bergshoeff-de Roo identification

In order to construct the higher-derivative action principle we need some components of the fluxes $\left\{\mathcal{F}_{\mathcal{A}}, \mathcal{F}_{\mathcal{ABC}}\right\}$. The $O(D, D+K)$ Lagrangian is given by,

$$\mathcal{R} \;=\; 2\mathcal{E}_{\underline{\mathcal{A}}}\mathcal{F}^{\underline{\mathcal{A}}} + \mathcal{F}_{\underline{\mathcal{A}}}\mathcal{F}^{\underline{\mathcal{A}}} - \frac{1}{6}\mathcal{F}_{\underline{\mathcal{ABC}}}\mathcal{F}^{\underline{\mathcal{ABC}}} - \frac{1}{2}\mathcal{F}_{\overline{\mathcal{A}}\underline{\mathcal{BC}}}\mathcal{F}^{\overline{\mathcal{A}}\underline{\mathcal{BC}}} \, .$$

# The generalized Bergshoeff-de Roo identification

The different projections of $\mathcal{F}_{\mathcal{ABC}}$ can be written as,

$$
\begin{aligned}
\mathcal{F}_{\underline{ABC}} &= F_{\underline{ABC}} + \frac{3a}{4}\left(E_{[\underline{A}} F^{\overline{CD}}{}_{\underline{B}} - \frac{1}{2} F_{\underline{D}[\underline{AB}} F^{\underline{D}\overline{CD}} - \frac{2}{3} F^{\overline{C}}{}_{\overline{E}[\underline{A}} F_{\underline{B}}{}^{\overline{ED}}\right) F_{\underline{C}]\overline{CD}}\,, \\
\mathcal{F}_{\overline{A}\underline{BC}} &= F_{\overline{A}\underline{BC}} + \frac{a}{4}\left(E_{\overline{A}} F^{\overline{CD}}{}_{[\underline{B}} + F^{\underline{E}\overline{CD}} F_{\overline{A}\underline{E}[\underline{B}}\right) F_{\underline{C}]\overline{CD}}\,, \\
\mathcal{F}_{\underline{A}\overline{BC}} &= F_{\underline{A}\overline{BC}} - \frac{a}{8} F_{\underline{D}\overline{EF}} F^{\overline{EF}}{}_{\underline{A}} F^{\underline{D}}{}_{\overline{BC}}\,, \\
\mathcal{F}_{\underline{A}} &= F_{\underline{A}} - \frac{a}{8} F^{\underline{B}} F_{\underline{B}}{}^{\overline{CD}} F_{\underline{A}\overline{CD}} - \frac{a}{8} E^{\underline{B}} (F_{\underline{B}}{}^{\overline{CD}} F_{\underline{A}\overline{CD}})\,.
\end{aligned}
$$

# The generalized Bergshoeff-de Roo identification

The (bosonic) four-derivative contributions to the heterotic DFT are $\int d^{2D+K}Xe^{-2d}\mathcal{R}(\mathcal{E},d) = \int d^Dxe^{-2d}\Big(\mathcal{R}(E,d) + a\mathcal{R}^{(-)}\Big)$,

$$\begin{aligned}
\mathcal{R}^{(-)} \;=\; & -\frac{1}{4}\Big[(E_{\underline{A}}E_{\underline{B}}F^{\underline{B}}{}_{\overline{CD}})F^{\underline{A}\overline{CD}} + (E_{\underline{A}}E_{\underline{B}}F^{\underline{A}}{}_{\overline{CD}})F^{\underline{B}\overline{CD}} + 2(E_{\underline{A}}F_{\underline{B}}{}^{\overline{CD}})F^{\underline{A}}{}_{\overline{CD}}F^{\underline{B}} \\
& +(E_{\underline{A}}F^{\underline{A}\overline{CD}})(E_{\underline{B}}F^{\underline{B}}{}_{\overline{CD}}) + (E_{\underline{A}}F_{\underline{B}}{}^{\overline{CD}})(E^{\underline{A}}F^{\underline{B}}{}_{\overline{CD}}) + 2(E_{\underline{A}}F_{\underline{B}})F^{\underline{B}}{}_{\overline{CD}}F^{\underline{A}\overline{CD}} \\
& +(E_{\overline{A}}F_{\underline{B}\overline{CD}})F_{\underline{C}}{}^{\overline{CD}}F^{\overline{A}\underline{BC}} - (E_{\underline{A}}F_{\underline{B}\overline{CD}})F_{\underline{C}}{}^{\overline{CD}}F^{\underline{ABC}} + 2(E_{\underline{A}}F^{\underline{A}}{}_{\overline{CD}})F_{\underline{B}}{}^{\overline{CD}}F^{\underline{B}} \\
& -4(E_{\underline{A}}F_{\underline{B}}{}^{\overline{CD}})F^{\underline{A}}{}_{\overline{CE}}F^{\underline{B}\overline{E}}{}_{\overline{D}} + \frac{4}{3}F^{\overline{E}}{}_{\underline{A}\overline{C}}F_{\underline{B}\overline{ED}}F_{\underline{C}}{}^{\overline{CD}}F^{\underline{ABC}} + F^{\underline{B}}{}_{\overline{CD}}F_{\underline{A}}{}^{\overline{CD}}F_{\underline{B}}F^{\underline{A}} \\
& +F_{\underline{A}}{}^{\overline{CE}}F_{\underline{B}\overline{ED}}F^{\underline{A}}{}_{\overline{CG}}F^{\underline{B}\overline{GD}} - F_{\underline{B}}{}^{\overline{CE}}F_{\underline{A}\overline{ED}}F^{\underline{A}}{}_{\overline{CG}}F^{\underline{B}\overline{GD}} - F_{\overline{A}\underline{BD}}F^{\underline{D}}{}_{\overline{CD}}F_{\underline{C}}{}^{\overline{CD}}F^{\overline{A}\underline{BC}}\Big]
\end{aligned}$$

which reproduces the BdR approach upon parametrization and field-redefinitions.

# The generalized Bergshoeff-de Roo identification

The systematic procedure used to construct $\mathcal{R}^{(-)}$ can be easily adapted to construct $\mathcal{R}^{(+)}$. This other correction to $\mathcal{R}(E, d)$, that we will call it $\mathcal{R}^{(+)}$, has the form of $\mathcal{R}^{(-)}$ but exchanging the projections of the different fields. Then, the bi-parametric action principle is

$$S_{DFT} = \int d^D x e^{-2d}(\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)}) \, .$$

Part 2:

Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

Based on E.L and J.A.Rodriguez, "Quadratic Curvature Corrections in Double Field Theory via Double Copy", Phys.Rev.D 112 (2025) 2, 026004 and 2409.05628

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

Let's start reviewing the classical and off-shell double copy procedure.

**Quadratic order**

$$S^{(2)}_{\rm YM} = -\frac{1}{2}\int_k \kappa_{ab}\, k^2\, \Pi^{\mu\nu}(k) A_\mu{}^a(-k) A_\nu{}^b(k)\ ,$$

where $\Pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$. The classical double copy prescription consists on replacing the color indices by a second set of space-time indices $(a \to \bar\mu)$ corresponding to a second set of space-time momenta $\bar k^{\bar\mu}$,

$$A_\mu{}^a(k) \ \to\ e_{\mu\bar\mu}(k,\bar k)\ ,$$

$$\kappa_{ab} \ \to\ \frac{1}{2}\,\bar\Pi^{\bar\mu\bar\nu}(\bar k)\,.$$

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

Using the previous rules the quadratic YM action becomes,

$$S^{(2)}_{\rm DC} = -\frac{1}{4}\int_{k,\bar{k}} k^2\, \Pi^{\mu\nu}(k)\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k}) e_{\mu\bar{\mu}}(-k,-\bar{k}) e_{\nu\bar{\nu}}(k,\bar{k})\,.$$

The combination $\Pi^{\mu\nu}(k)\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k})$ provides a YM $\times$ YM effective construction.

On the other hand, after imposing $\bar{k}^2 = k^2$ (DFT level-matching constraint) the action is symmetric under $k \leftrightarrow \bar{k}$. To complete the construction we have to Fourier transform the previous action to (double) position space.

# Review

After expanding the projectors and using the level-matching constraint we find,

$$S^{(2)}_{\rm DC} = \frac{1}{4}\int d^Dx\, d^D\bar{x}\Big(e^{\mu\bar{\nu}}\Box e_{\mu\bar{\nu}} + \partial^\mu e_{\mu\bar{\nu}}\,\partial^\rho e_\rho{}^{\bar{\nu}} \\ + \bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\,\bar{\partial}^{\bar{\sigma}} e^\mu{}_{\bar{\sigma}} - \Phi\Box\Phi + 2\Phi\partial^\mu\bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\Big)\,,$$

with $\Phi = \frac{1}{k^2}k^\mu \bar{k}_{\bar{\nu}} e_\mu{}^{\bar{\nu}}$ . This action reproduces the standard quadratic DFT action in a non-manifest T-dual form. To produce the NS-NS supergravity one needs to impose $x = \tilde{x}$ (level matching constraint), $e_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu}$ and $\Phi = \varphi - h$.

# Using classical double copy to produce $\alpha'$-corrections.

In this talk we will focus on the (quadratic) curvature four-derivative corrections for bosonic string theory,

$$S_{MT} = \int d^D x \sqrt{-g} e^{-2\phi} (R + \frac{\alpha'}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) \,.$$

The main goal is to construct a four-derivative gauge theory which allows us to access to the Riem$^2$ terms using the double copy program! Using covariant field redefinitions the previous action can be written as,

$$S_{\mathrm{R+CG}} = \int d^D x \, \sqrt{-g} (R + \frac{\alpha'}{4} C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda}) \,,$$

where the Weyl tensor is defined as

$$C_{\mu\nu\rho\lambda} = R_{\mu\nu\rho\lambda} - \frac{2}{D-2} \left( g_{\mu[\rho} R_{\lambda]\nu} - g_{\nu[\rho} R_{\lambda]\mu} \right) + \frac{2}{(D-1)(D-2)} R g_{\mu[\rho} g_{\lambda]\nu} \,.$$

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

We consider the following gauge Lagrangian

$$
\begin{aligned}
L &= a_1 \kappa_{ab} D_\mu F^{\mu\nu a} D_\rho F^{\rho}{}_{\nu}{}^{b} + a_2 \kappa^{\alpha\beta} D_\mu \phi_\alpha D^\mu \phi_\beta \\
&\quad + a_3 f_{abc} F_\mu{}^{\nu a} F_\nu{}^{\lambda b} F_\lambda{}^{\mu c} + a_4 C^\alpha{}_{ab} \phi_\alpha F_{\mu\nu}{}^{a} F^{\mu\nu b} \\
&\quad + a_5 d^{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \, ,
\end{aligned}
$$

where the $a_i$ are real coefficients to be determined and the indices $\alpha, \beta \ldots$ are in the fundamental representation.
This proposal is based on 1707.02965 [Johansson,Nohle] and 1806.05124 [Johansson, Mogull, Teng], where the authors obtained Weyl gravity amplitudes after imposing a double copy map for a particular choice of $a_i$ and $D = 6$.

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

**Quadratic order:** The higher-derivative gauge Lagrangian is

$$S^{(2)}_{\mathrm{DC}} = -\int d^D k \Big[ a_1 k^4 \kappa_{ab} \Pi^{\mu\nu}(k) A_\mu{}^a(k) A^b_\nu(-k) + a_2 k^2 \kappa^{\alpha\beta} \phi_\alpha(k) \phi_\beta(-k) \Big] ,$$

and we already know the identification for $\kappa^{ab}$ and $A_\mu{}^a$. Therefore we just need an identification for $\kappa^{\alpha\beta}$ and $\phi_\alpha$,

$$\phi_\alpha(k) \longrightarrow k_\mu e^{\mu\bar{\nu}}(k,\bar{k}) + 2\bar{k}^{\bar{\nu}} \Phi(k,\bar{k}) ,$$

$$\kappa^{\alpha\beta} \longrightarrow \frac{\bar{k}_{\bar{\mu}} \bar{k}_{\bar{\nu}}}{k^2} .$$

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

The quadratic higher-derivative Lagrangian, after performing all the identifications, is given by

$$
\begin{aligned}
S^{(2)}_{\rm DC} \;=\; & -\frac{1}{2}\int d^D x d^D \bar{x}\,\Big[a_1\big(\Box e^{\mu\bar{\nu}}\Box e_{\mu\bar{\nu}} - \Box e^{\mu\bar{\nu}}\partial_\mu\partial^\rho e_{\rho\bar{\nu}} \\
& - \Box e^{\mu\bar{\nu}}\bar{\partial}_{\bar{\nu}}\bar{\partial}^{\bar{\sigma}} e_{\mu\bar{\sigma}} + \partial^\mu\bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\partial^\rho\bar{\partial}^{\bar{\sigma}} e_{\rho\bar{\sigma}}\big) \\
& -2a_2\left(\partial_\mu\partial_{\bar{\nu}} e^{\mu\bar{\nu}} + 2\Box\Phi\right)^2\Big]\,.
\end{aligned}
$$

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

Now we consider the pure supergravity limit demanding $x = \bar{x}$.
The previous action becomes

$$S_{\rm DC}^{(2)} = -\frac{1}{2}\int d^D x\Big[a_1\left(\Box h^{\mu\nu}\Box h_{\mu\nu} - 2\Box h^{\mu\nu}\partial_\mu\partial^\rho h_{\rho\nu} + \partial^\mu\partial^\nu h_{\mu\nu}\partial^\rho\partial^\lambda h_{\rho\lambda}\right) - 2a_2\left(\Box h - \partial_\mu\partial_\nu h^{\mu\nu}\right)^2\Big] + L(b,\phi)\,,$$

which present the same structure as the quadratic contributions of the Weyl gravity action when $a_1 = -2\left(\frac{D-3}{D-2}\right)$ and $a_2 = -\frac{1}{(D-1)}\left(\frac{D-3}{D-2}\right)$

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

The b-field and dilaton contributions, to quadratic order, are given by

$$L(b,\phi) = \frac{a_1}{6}\Box\bar{h}^{\mu\nu\rho}\bar{h}_{\mu\nu\rho} - 8a_2(\Box h - \partial_\mu\partial_\nu h^{\mu\nu} + \Box\phi)\Box\phi$$

where these contributions can be eliminated by field redefinitions (see 2409.05628).

# Part 2: Using Classical Double Copy as an alternative method to capture $\alpha'$-corrections.

If we combine both the double copy procedure of [Hohm-Jaramillo-Plefka] and the previous one:

$$\boxed{YM + a_1(DF)^2 + a_2(D\phi)^2 \rightarrow \mathrm{DFT}+ \rightarrow R + \frac{\alpha'}{4}\mathrm{Weyl}^2\,.}$$

to quadratic order in fields.

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The formulation given by

$$S_{\rm DFT+} = \alpha' \int d^{2D} X e^{-2d} \Big( \frac{1}{\alpha'} R - R^{(-)} - R^{(+)} + \frac{1}{2(D-2)} R_{\overline{A}\underline{B}} R^{\overline{A}\underline{B}} - \frac{1}{2(D-2)(D-1)} R^2 \Big) .$$

only produces

$$S_{\rm R+CG} = \int d^D x \, \sqrt{-g} (R + \frac{\alpha'}{4} C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda}) ,$$

to quadratic order, while the b-field and dilaton contributions can be trivialized using field redefinitions.

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**Outlook and open questions:**

- Extension to cubic order (RiemHH term in the bosonic action plus interactions with the measure).
- Extension to $\alpha'^2$ in the pure gravitational limit ($\text{Riem}^3$ terms).
- Extensions beyond string theory: non-conmutativity! (wait for Larisa's talk!).

¡muchas gracias por su atención!