Holographic Baryons as Quantum Hall Droplets

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No Skyrmions with one flavor

Low-energy effective action for QCD for $N_f \geq 2$

Pion matrix

$$U = e^{i \sum_{a=1}^{N_f^2-1} \frac{\pi^a(x) T^a}{f_\pi}}$$

Chiral Lagrangian with Skyrme term (postulated)

$$\mathcal{L}_{\text{eff}} = -\frac{f_\pi^2}{4} Tr\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2} Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Low-energy description of baryons

Skyrme term allows for solitonic solutions, e.g. for $N_f = 2$

$$U = e^{i \frac{f(r)}{r} x^a \sigma^a}$$

(with known $f(r)$): a "Skyrmion".

If $f(0) = \pi k, \ k \in \mathbb{Z}$, then solution has winding number k for

$$\Pi_3(SU(N_f = 2)) = \mathbb{Z}$$

where the 3-sphere is in coordinate space.

This is interpreted as the baryon number

Low-energy description of baryons

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But

$$\Pi_3(U(N_f = 1)) = 0$$

$\Rightarrow$ No Skyrmions with one flavor

Is there a low-energy description of single flavor baryons?

A proposal for the solution

In single-flavor low-energy $SU(N \gg 1)$ QCD, baryons are charged sheets

[Komargodski 2018 and many others before]

η' (phase of quark condensate) light at $N \gg 1 \Rightarrow$ low energy Lagrangian is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2} m\Lambda_{QCD} \cos(\eta') - \frac{1}{2} m_{WV}^2 Min_{k\in\mathbb{Z}}(\eta' + \theta_{QCD} + 2\pi k)^2$$

m: quark mass, Λ_{QCD}: dynamical scale, m_{WV}: Witten-Veneziano mass, take $\theta_{QCD} = 0$.

Potential is multi-valued with cusp-singularity at $\eta' = \pi \Rightarrow$ effective action breaks down there, extra (gluonic) degrees of freedom: sheet.

- Infinite sheet similar to domain-wall but no related charge: unstable.
- η' has non-trivial monodromy through the sheet.
- Sheet can be stabilized by baryonic charge.

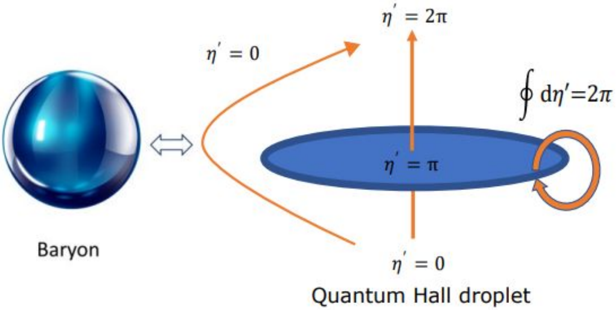
Proposal: sheet hosts a $U(1)_N$ Chern-Simons theory on its world-volume

"Quantum Hall Droplet"

If it has circular boundary $\Rightarrow$ can have chiral edge modes: baryonic charge!

Charge forbids shrinking of the sheet.

It has spin $J = N/2 \gg 1$, "pancake-shaped".



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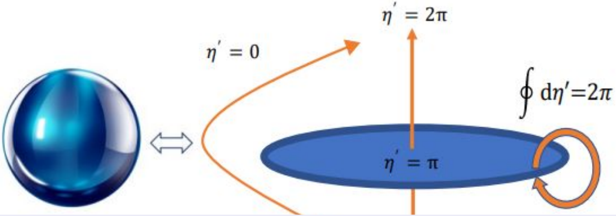


Effective field theory singular around the sheet,
how can we test the proposal and compute properties of this baryon?

The holographic model

Holographic QCD

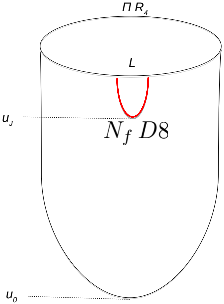
[Witten 1998, Sakai-Sugimoto 2004]

- Type IIA background from N D4-branes wrapped on circle S^1 + N_f D8/anti-D8-brane pairs in probe approximation (no backreaction).
- Low energy: dual to (non-susy) 4d YM + KK modes + chiral quarks.
- Confinement, mass gap, chiral symmetry breaking.
- Gravity description reliable if $N \gg 1, \;\; \lambda = g_{YM}^2 N \gg 1$

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(dx^\mu dx_\mu + f(u) dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2$$

- N units of flux of F_4 through S^4.

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(dx^\mu dx_\mu + f(u)dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2$$



Baryons [Hata et al. 2007]: instantonic configurations of gauge field on D8-branes, but only for $N_f \geq 2$!

Holographic baryons as quantum Hall droplets

Dual of sheet identified as follows ($N_f = 1$):

- From potential

$$\frac{1}{2} m_{WV}^2 Min_{k\in\mathbb{Z}}(\eta' + \theta_{QCD} + 2\pi k)^2$$

  η' transforms as θ_{QCD}.

- On D4 in holographic QCD

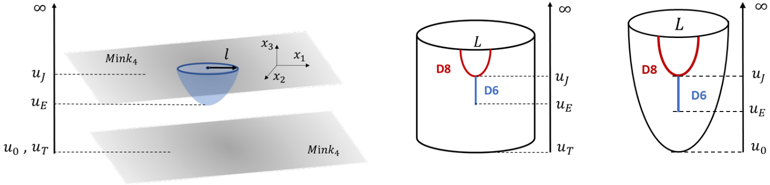
$$\int_{S^1\times\mathbb{R}^4} C_1 \wedge F \wedge F = \int_{\mathbb{R}^4} \theta_{YM} F \wedge F \quad \Rightarrow \quad \theta_{YM} = \int_{S^1} C_1$$

- η' non-trivial monodromy trough the sheet realized if sheet is a D6-brane (wrapped on S^4), since D6 magnetically charged under C_1. [Witten 1998, Dubovsky-Lawrence-Roberts 2011, Bigazzi-ALC-Olzi 2022].

- D6-brane hosts a $U(1)_N$ Chern-Simons theory on its world-volume
  [Acharya-Vafa 2001, Argurio-Bertolini-Bigazzi-ALC-Niro 2018]

$$\int_{S^4\times M_3} C_3 \wedge f \wedge f = \int_{S^4\times M_3} F_4 \wedge a \wedge f = N \int_{M_3} a \wedge f$$

- D6-brane can end on flavor D8-brane with circular boundary:
  [Bigazzi-ALC-Olzi 2024, Bigazzi-ALC-Olzi-Raymond 2025]



Set $f|_{\partial M_3} = 0$ for gauge invariance.

- D6 wants to shrink due to tension, but theory has $U(1)_B \Rightarrow$
  D6 could be stabilized by baryonic charge at its boundary.

Baryon charge:

- Baryon vertex is a D4-brane wrapped on S^4 [Witten 1998].
- D4 charged under C_5 RR-potential.
- On D6 world-volume (ψ angular coordinate):

$$\int_{D6} C_5 \wedge f_{\psi u}$$

$\Rightarrow$

Baryon number:

$$n_B = \int_{\psi u} f_{\psi u} \in \mathbb{Z}$$

Charge carried by D6 world-volume gauge field.

- Quark charge:

$$q_s = N\, n_B$$

Spin:

$$J = \int \sqrt{-g} g_{\psi\psi} T^{t\psi}$$

- Choose gauge $a_u = 0 \Rightarrow$

$$n_B = \frac{1}{2\pi} \int_{\psi u} f_{\psi u} = a_\psi(u_E) - a_\psi(u_J)$$

- Get from equations of motion $a_\psi(u_E) = n_B \ \Rightarrow \ a_\psi(u_J) = 0$, and

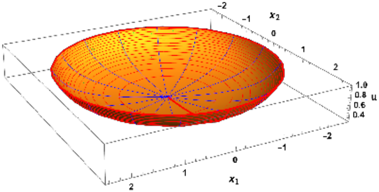
$$J = \frac{N}{2} n_B^2$$

- Curved-space analogs of “supertubes” [Mateos-Townsend 2001]: NF1s+D4 configuration blown-up into D6 and stabilized by angular momentum (Myers’ effect).

"Holographic baryon as quantum Hall droplet"

[Bigazzi-ALC-Olzi-Raymond 2025]

Solution of D6 e.o.m. ending perpendicularly to D8 ($\Rightarrow$ stability):



Can calculate exactly its properties, e.g.

- Radius $l \sim N^0$.
- Mass $M \sim N$.

D6-brane = gluonic core of the baryon (as string junction).

Note:

$$J = \frac{N}{2} n_B^2$$

hallmark of fractional quantum Hall effect, anyonic system:

- anyons $\Rightarrow$ quarks;
- electrons $\Rightarrow$ baryons;
- solitonic chiral edge mode $\Rightarrow$ χ s.t. $a|_{boundary} = d\chi$.

Conclusions

- Single-flavor baryons are quantum Hall droplets in holographic QCD.
- Holographic dual allows for precise investigation beyond effective theory.
- These baryons have "gluonic core" described by D6-brane, equivalent of baryon vertex = string junction in standard baryons.
- Baryons have "mesonic shell" described by D8-brane fields (ongoing work).
- Other related configurations: strings, loops, finite DWs, vortons, sandwich vortons, punctured domain walls.
  The model can have rich cosmological history.
  [Bigazzi-ALC-Olzi 2022, 2024, Bigazzi-ALC-Olzi-Raymond 2025]

Thank you for your time!