New consistent truncations and applications

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Based on : [to appear] Inverso & CS

Motivations and key results

Why consistent truncations?

Low-d SUGRA solution

- Black holes
- Solitons
- Domain-walls

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Type II/M-theory solution

KK-like solution

String theory interpretation

AdS/CFT, probe branes, ….

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Key results

- Algorithm for classifying embeddings of $(3 <)d$-dimensional gSUGRA in 10/11D SUGRA.
- Constraints on the internal manifold.

- **Example:** Uplift of $N = 4$ $D = 4$ gauged SUGRA in
  - Type IIB on $S^2 \times S^2 \times \Sigma$ — Dual to D3-D5-NS5 brane setups [Gaiotto-Witten]; Include Janus/S-folds/… [D'Hoker, Estes, Gutperle]

  - Not shown: M-theory embedding on $S^3 \times S^3 \times I$

Consistent truncations

State of the art

Consistent truncations
Definition

A *truncation* where the equations of motion of the reduced theory imply those of the full theory.

Non-examples:

- KK-compactification without the dilaton (*not* consistent)
- **Effective descriptions**: keep modes with $m \leq \Lambda_{cut-off}$

E.g.: Calabi-Yau compactifications $\Rightarrow$ CY-moduli ($\mathbb{C}$-structure, Khäler, …)

Consistent truncations

Examples

Classical examples

- Supergravities on group manifold (Scherk-Schwarz reduction)

Using Exceptional Field Theory (ExFT)

- Truncations on product of spheres/hyperboloids

  $\Rightarrow$4D/5D *maximal* gSUGRA

- 5D N=2 [Cassani, Josse, Malek, Petrini, Waldram]
- …

A 4d $\mathcal{N} = 4$ example

[D'Hoker, Estes, Gutperle]

Uplifts are rarely unique

M-theory:

Type IIB:

[Guarino, Trigiante, CS]

[Lu, Cvetic, Pope]

A 4d $\mathcal{N} = 4$ example

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Uplifts are rarely unique

M-theory: On $S^3 \times S^3 \to M_{int} = S^7$

$$\downarrow$$
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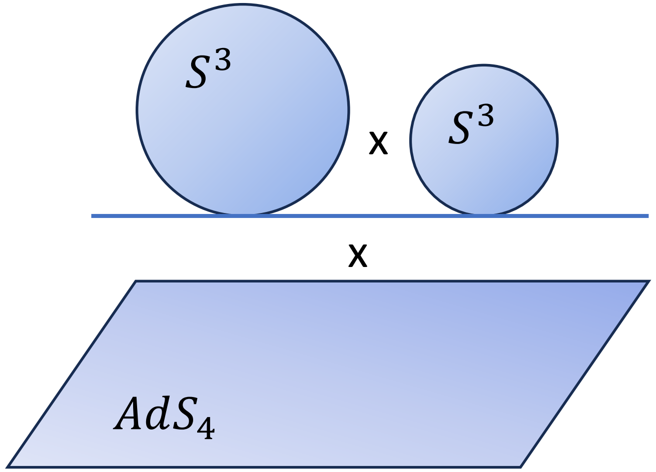

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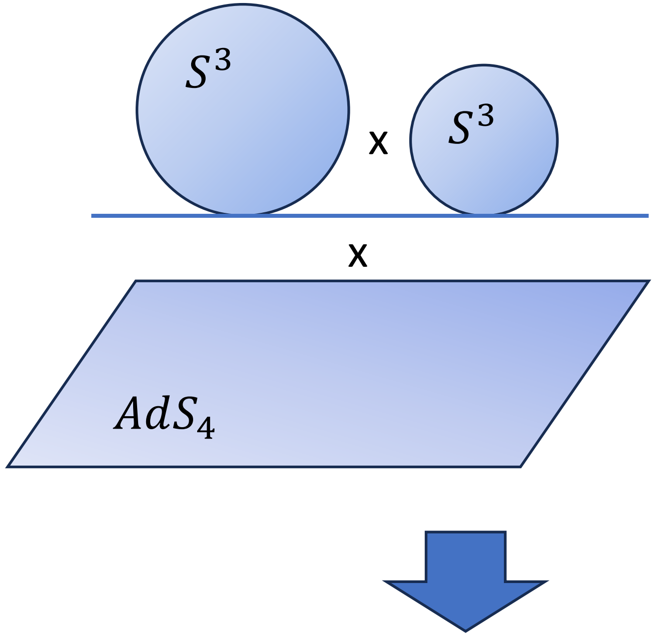

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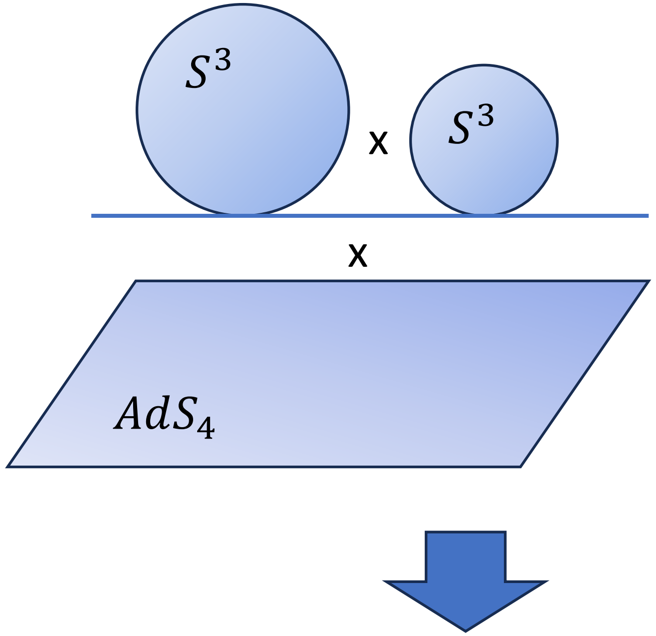

Type IIB:

$D = 4\ \mathcal{N} = 4$ SO(4)-gauged supergravity

[Guarino, Trigiante, CS]

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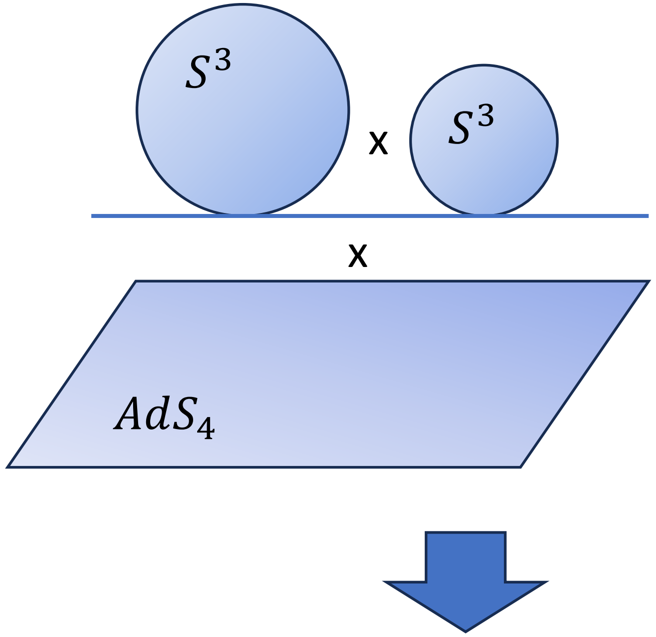

Type IIB: On $S^2 \times S^2 \to M_{int} = S^5 \times S^1$, $\downarrow \Sigma$

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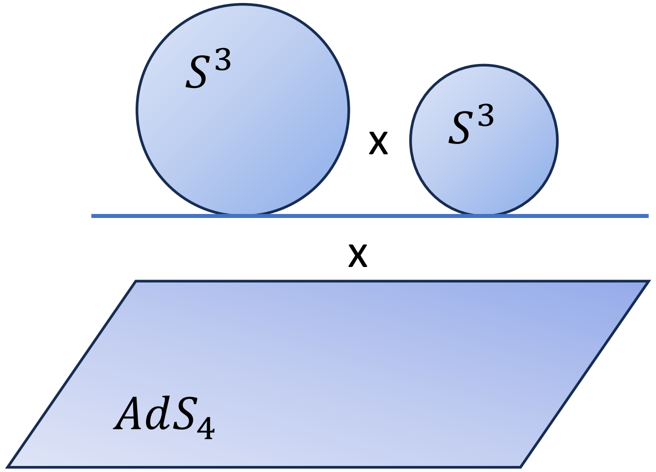

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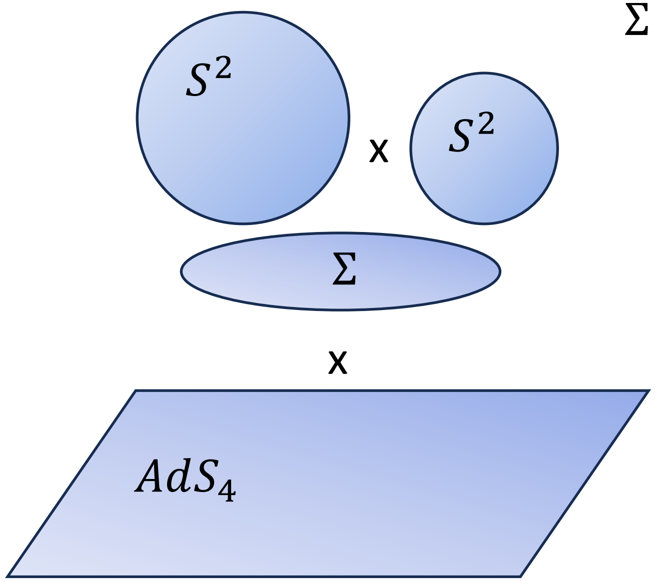

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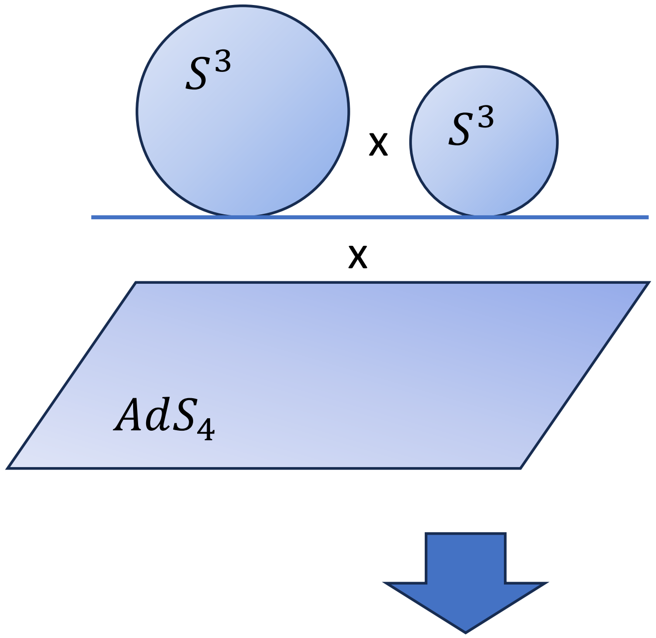

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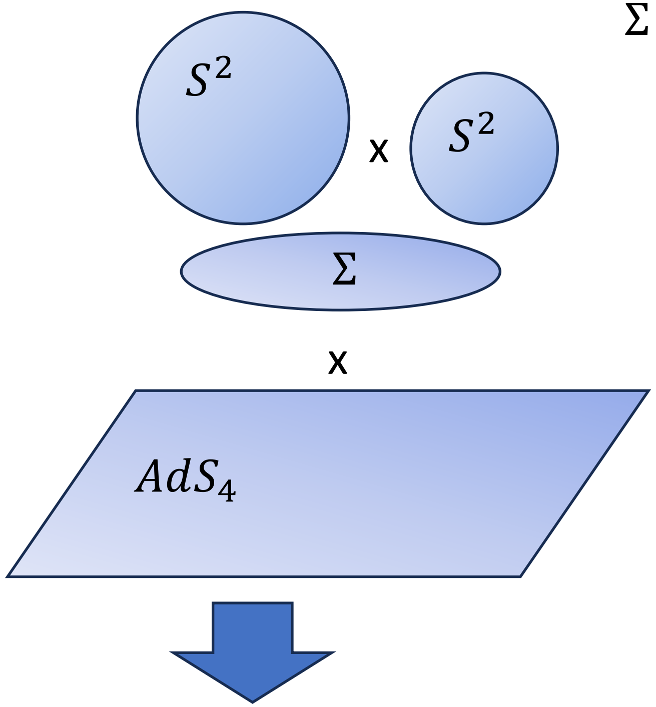

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2) Inverse problem?

Given a gauged SUGRA in d-dimension, what are its uplifts ?

- With maximal supersymmetry: solved! [Inverso][Inverso, Rovere]
- Non-pure/Non-maximal sugra: Open

Today: How to uplift non-maximal gauged SUGRA ?

Systematics of consistent truncations

Exceptional field theories

Frames, sections and torsion

Exceptional field theories

[Hull] [Pacheco, Waldram] [Hohm, Samtleben][…]

Type IIA/B/ 11D —Dictionary→ $E_{d(d)}$-Exceptional field theory on M_{ext} of dimension $11-d$

Exceptional field theories

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  $y^m \to Y^M \in R_1$ a rep of $E_{d(d)}$

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- Extend internal coordinates:

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  $V^M \to$ (vector) $\bigoplus$ even/odd p-forms under $GL(d') \subset E_{d(d)}$

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Diffeomorphisms (→ vector)

Gauge transformations (→ even/odd p-forms)

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Diffeomorphisms — Gauge transformations

- Redefine the Lie derivative:

$$(L_\Lambda V)^M := \Lambda^N \partial_N V^M - \mathbb{P}^{M}{}_{N}{}^{P}{}_{Q} \partial_P \Lambda^Q V^N$$

$$:= \mathcal{L}_{\Lambda^m} V^M + \text{correction in } \partial_m \Lambda^M$$

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- Field content:

$\mathrm{E} \in E_{d(d)}/K_d$ "gen. metric" , $A_\mu{}^M$ "gen. KK modes", p-forms

Consistent truncations

Methods

[Hohm Samtleben]
[Cassani, Josse, Petrini, Waldram]

Type IIA/B/ 11D

Dictionary

$E_{d(d)}$-Exceptional field theory on M_{ext} of dimension $11 - d$

Truncation

"G_S" – invariant subsector

$(11 - d)$ supergravity
$\mathcal{N}$, G_g, matter multiplets depend on G_S

Statement

In ExFT, consistent truncations are equivalent to generalised G_S-structure with constant and G_S-invariant intrinsic torsion.

[Cassani, Josse, Petrini, Waldram]

Generalised G-structure

In terms of sections

In terms of frame

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Generalised G-structure

In terms of sections

- ${K_A}^M$ s.t. $Stab\big(K_A(p)\big) = G_S$

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In terms of frame

- $E \in E_{7(7)}/G_S$ which is *globally* well-defined

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$$\nabla_{E_M} E_N = 0$$

Then the *intrinsic torsion* of ∇ must be a constant G_S singlet.

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$${K_A}^N = {\mathbb{P}_A}^M {E_M}^N$$

The projector on G_S singlets

Classification of uplift

Given a gauged SUGRA, what are the generalised G_S-structure with singlet constant torsion equal to a given embedding tensor?

$X_{AB}{}^{C}$
specifying the gauging

Truncation to non-maximal SUGRA

Start from D-dim SUGRA with gauge group G_g

We show that:

1) The internal manifold is locally:

$$M_{int} \cong \frac{G_g}{H} \times B.$$

H specifies the type of uplift (IIB/M-theory)

2) When $H^{1,3,5}(G_g/H) = 0$ (IIB), then ∃ a gauge s.t.

$$L_{K_A} W = \mathcal{L}_{k_A} W.$$

⇒Simplifies the torsion condition!

Truncation to non-maximal SUGRA
The double ansatz

We build the most generic sections:

$$K_A = \theta_A{}^{\alpha} k_{\alpha}{}^{m} + \; K_A{}^{(p)}$$

Equivariant and closed polyforms
on $M_{int} = G_g/H \times B$.

We build the most generic frame:

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$$\mathbb{P}_A{}^{M} Z_M{}^{N} \mathcal{E}_N{}^{m} = \theta_A{}^{m}$$

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$$\mathbb{P}_A{}^M Z_M{}^N \mathcal{E}_N{}^m = \theta_A{}^m$$

Always solves the compatibility constraints $(Stab(K_A) = G_S)$

Final torsion constraint

$$\mathbb{P} Z \mathcal{E} = \Theta$$

Projector on G_S-singlets (matter content)

Section constraints tensor (IIA/B/M)

Embedding tensor (gauging)

$$d(L \cdot Z \cdot e)_{(p)} = 0$$

PDE on B only

An application

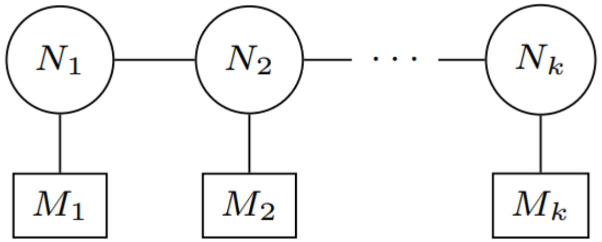
[D'Hoker, Estes, Gutperle]

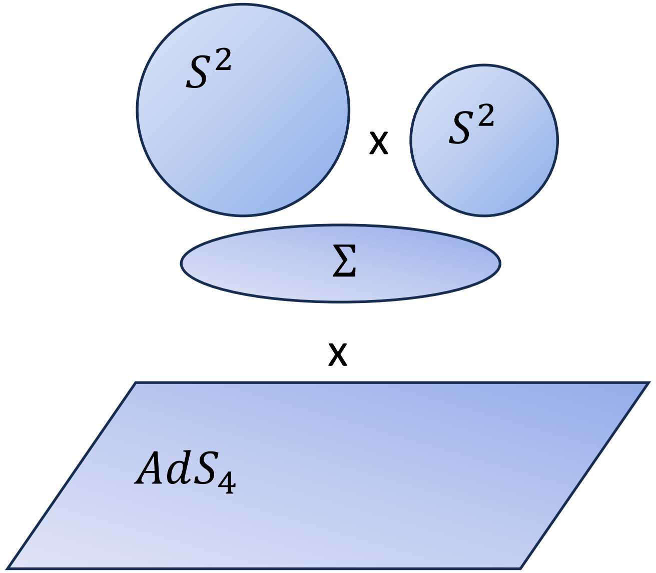
Example:
Consistent truncation of the DEG solutions

Solutions:

Depend on two holomorphic functions on Σ

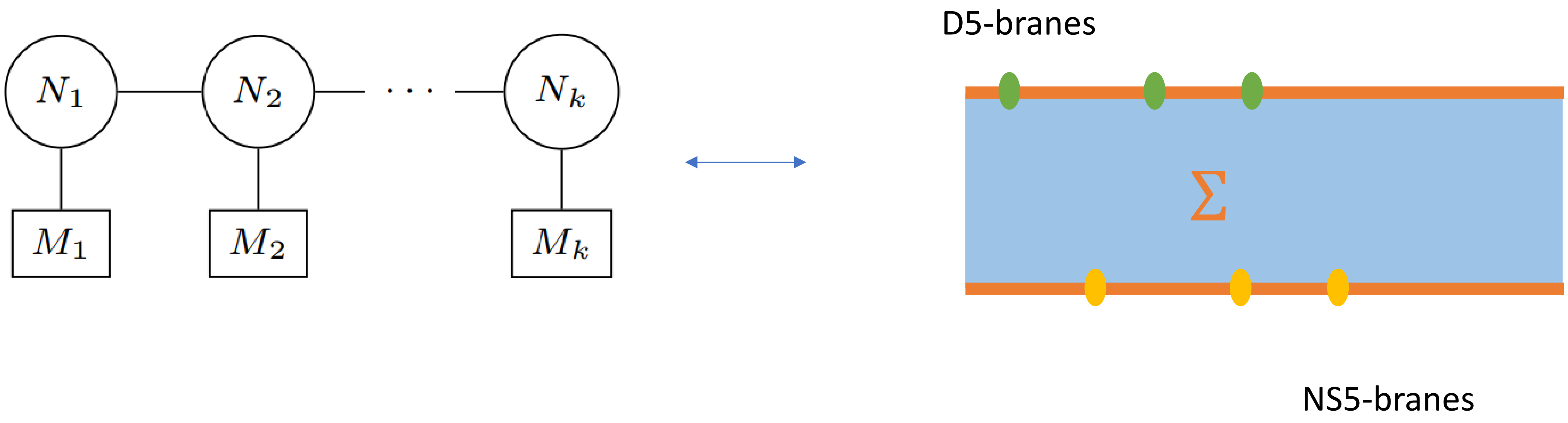
$\mathcal{A}_1 = h_1 + i\, h_1^D$ and $\mathcal{A}_2 = h_2 + i\, h_2^D$





Holographic duals

- Type IIB solutions are dual to 3d $\mathcal{N} = 4$ linear quivers



[D'Hoker, Estes, Gutperle]
WIP [Inverso, CS]

Example:

Consistent truncation of the DEG solutions

$$G_g = SO(3) \times SO(3)$$
$$H = U(1) \times U(1)$$

$$\Rightarrow \quad S^2 \times S^2 \to M_{int} \downarrow \Sigma$$

The sections:
Depend on two holomorphic functions

$\mathcal{A}_1 = h_1 + i\, h_1^D$ and $\mathcal{A}_2 = h_2 + i\, h_2^D$

$$\begin{aligned}
K_{i+} = \;& k_i \;\; +d(2h_2^D Y^i)^+ \\
& -d\big((2h_1 h_2 + 2h_1^D h_2^D - \lambda)\, \mathrm{vol}_2\, Y^i \big) \\
& - \star\, d\Big((2\,\bar\partial(\lambda\, h_2^D) + 4A_2(h_1\bar\partial h_2^D - h_2^D\bar\partial h_1))\epsilon_{ijk} Y^j dY^k \wedge \mathrm{vol}_2 \wedge dz \Big)^- \\
& +0 \\
K_{\underline{i}+} = \;& 0 \;\; +d(-2h_2 Y^{\underline{i}})^+ \\
& +d\Big(8(h_2\bar\partial h_1^D - h_1^D\bar\partial h_2)\epsilon_{\underline{ijk}} Y^{\underline{j}} dY^{\underline{k}} \wedge dz \Big) \\
& + \star\, d\Big((2\,\bar\partial(\lambda\, h_2) + 4\,i\,A_2(h_2\bar\partial h_1^D - h_1^D\bar\partial h_2))\epsilon_{\underline{ijk}} Y^{\underline{j}} dY^{\underline{k}} \wedge \mathrm{vol}_1 \wedge dz \Big)^- \\
& +4\sqrt{g_{S^2\times S^2}}\,\big(2h_1 h_2 + 2h_1^D h_2^D - \lambda\big)\,(\partial h_1\bar\partial h_2 + \bar\partial h_1 \partial h_2) k_i^* \\
K_{i-} = \;& 0 \;\; +d(-2h_1 Y^i)^- \\
& -d\Big(8(h_1\bar\partial h_2^D - h_2^D\bar\partial h_1)\epsilon_{ijk} Y^j dY^k \wedge dz \Big) \\
& + \star\, d\Big((2\,\bar\partial(\lambda\, h_1) + 4\,A_1(h_2^D\bar\partial h_1 - h_1\bar\partial h_2^D))\epsilon_{ijk} Y^j dY^k \wedge \mathrm{vol}_2 \wedge dz \Big)^+ \\
& -4\sqrt{g_{S^2\times S^2}}\,\big(2h_1 h_2 + 2h_1^D h_2^D + \lambda\big)\,(\partial h_1\bar\partial h_2 + \bar\partial h_1 \partial h_2) k_{\underline{i}}^* \\
K_{\underline{i}-} = \;& k_{\underline{i}} \;\; +d(-2h_1^D Y^{\underline{i}})^- \\
& +d\big((2h_1 h_2 + 2h_1^D h_2^D + \lambda)\, \mathrm{vol}_1\, Y^{\underline{i}} \big) \\
& + \star\, d\Big((2\,\bar\partial(\lambda\, h_1^D) + 4\,i\,A_1(h_2\bar\partial h_1^D - h_1^D\bar\partial h_2))\epsilon_{\underline{ijk}} Y^{\underline{j}} dY^{\underline{k}} \wedge \mathrm{vol}_1 \wedge dz \Big)^+ \\
& +0
\end{aligned}$$

The invariant sections

$$
\begin{aligned}
K_{i+} = \quad & k_i \quad && +d(2h_2^D Y^i)^+ \\
& && -d\left((2h_1 h_2 + 2h_1^D h_2^D - \lambda)\, \mathrm{vol}_2\, Y^i \right) \\
& && - \star\, d\left((2\,\bar\partial(\lambda\, h_2^D) + 4A_2(h_1 \bar\partial h_2^D - h_2^D \bar\partial h_1))\epsilon_{ijk} Y^j dY^k \wedge \mathrm{vol}_2 \wedge dz \right)^- \\
& && +0
\end{aligned}
$$

$$
\begin{aligned}
K_{\underline{i}+} = \quad & 0 \quad && +d(-2h_2 Y^{\underline{i}})^+ \\
& && +d\left(8(h_2 \bar\partial h_1^D - h_1^D \bar\partial h_2)\epsilon_{\underline{ijk}} Y^{\underline{j}} dY^{\underline{k}} \wedge dz \right) \\
& && + \star\, d\left((2\,\bar\partial(\lambda\, h_2) + 4\, i\, A_2(h_2 \bar\partial h_1^D - h_1^D \bar\partial h_2))\epsilon_{\underline{ijk}} Y^{\underline{j}} dY^{\underline{k}} \wedge \mathrm{vol}_1 \wedge dz \right)^- \\
& && +4\sqrt{g_{S^2\times S^2}}\left(2h_1 h_2 + 2h_1^D h_2^D - \lambda \right)(\partial h_1 \bar\partial h_2 + \bar{\bar\partial} h_1 \partial h_2) k_i^*
\end{aligned}
$$

$$
\begin{aligned}
K_{i-} = \quad & 0 \quad && +d(-2h_1 Y^i)^- \\
& && -d\left(8(h_1 \bar\partial h_2^D - h_2^D \bar\partial h_1)\epsilon_{ijk} Y^j dY^k \wedge dz \right) \\
& && + \star\, d\left((2\,\bar\partial(\lambda\, h_1) + 4\, A_1(h_2^D \bar\partial h_1 - h_1 \bar\partial h_2^D))\epsilon_{ijk} Y^j dY^k \wedge \mathrm{vol}_2 \wedge dz \right)^+ \\
& && -4\sqrt{g_{S^2\times S^2}}\left(2h_1 h_2 + 2h_1^D h_2^D + \lambda \right)(\partial h_1 \bar\partial h_2 + \bar\partial h_1 \partial h_2) k_{\underline{i}}^*
\end{aligned}
$$

$$
K_{i} \;=\; k_{i} \quad +d(-2h_1^D Y^{\underline{i}})^-
$$

The SUGRA fields

From the sections/frame, we use the ExFT dictionary to obtain:

$$ds^2 = \Delta^{-1}\left(ds^2_{\text{ext}} - n_1^{-1} ds^2_{S^2_1} - n_2^{-1} ds^2_{S^2_2} - 2w\, ds^2_{\Sigma}\right)$$

$$B_2^{(0)} = -2\chi\, e^{\xi}\, \frac{h_1}{n_1} \text{vol}_{S^2_1} + 2h_1^D \text{vol}_{S^2_2} + 2h_1 \frac{(\log h_1 \wedge \log h_2)}{n_2\, w}\, \text{vol}_{S^2_2}$$

$$C_2^{(0)} = 2h_2^D \text{vol}_{S^2_1} - 2h_2 \frac{(\log h_1 \wedge \log h_2)}{n_1\, w}\, \text{vol}_{S^2_1} + 2\chi e^{\xi}\, \frac{h_2}{n_2} \text{vol}_{S^2_2}$$

$$\exp(\Phi) = e^{\xi} |\tau|^2\, \frac{h_1}{h_2}\, \frac{n_0}{(n_1 n_2)^{1/2}}$$

$$C_0 = \frac{\chi}{|\tau|^2}\, \frac{h_1 \wedge h_2}{h_1^2\, n_0\, w}$$

+ vector contributions,…

Thank you!

Summary

- Algorithm for classifying embeddings of $(3 <)d$-dimensional gSUGRA in 10/11D SUGRA.
- $M_{int} = \frac{G_g}{H} \times B$
- Classification reduces to a PDE on B

- **Example:** Uplift of $N = 4\ D = 4$ gauged SUGRA in
  - Type IIB on $S^2 \times S^2 \times \Sigma$

    Dual to D3-D5-NS5 brane setups [Hanany-Witten]

    Include Janus/S-folds/… [D'Hoker, Estes, Gutperle]