A new open string "tiling" and the closed string spectrum

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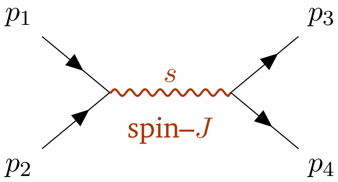
A theory of ∞–many states

- let's consider $2 \to 2$ scalar field scattering (s–channel):



diverges at high energies!

- Veneziano amplitude (s–channel):

$$\mathcal{A}_4 \sim \frac{\Gamma\big(-\alpha(s)\big)\Gamma\big(-\alpha(t)\big)}{\Gamma\big(-\alpha(s)-\alpha(t)\big)}\,, \quad \alpha(s) = 1 + \alpha' s$$

$\Rightarrow$ *cures* divergences via *infinitely many* simple poles

e.g. @ $s = -\frac{1}{\alpha'}, 0, \frac{1}{\alpha'}, \frac{2}{\alpha'}, \ldots$

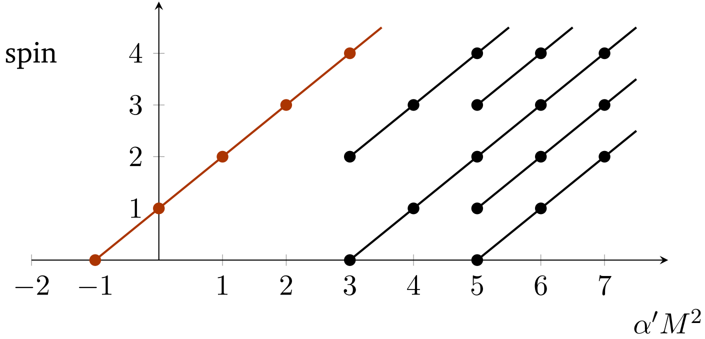
A theory of ∞–many states

- result: *string* theory with *infinitely many physical states*

$$M^2 = \text{integer} \times \frac{1}{\alpha'}$$

- TT polarisations $\Rightarrow$ irreps of $\mathfrak{so}(D-1)$ or $\mathfrak{so}(D-2)$
- all allowed tensor symmetries possible!

  e.g. open bosonic string, symmetric tensors:



A theory of ∞–many states

(one box per spacetime index, symmetric rows)

N	decomposition in physical states (open bosonic string)
0	$\bullet$
1	$\ydiagram{1}_{\mathfrak{so}(D-2)}$
2	$\ydiagram{2}$
3	$\ydiagram{3} \oplus \ydiagram{1,1}$
4	$\ydiagram{4} \oplus \ydiagram{2,1} \oplus \ydiagram{2} \oplus \bullet$
5	$\ydiagram{5} \oplus \ydiagram{3,1} \oplus \ydiagram{3} \oplus \ydiagram{2,1} \oplus \ydiagram{1,1} \oplus \ydiagram{1}$
6	$\ydiagram{6} \oplus \ydiagram{4,1} \oplus \ydiagram{4} \oplus \ydiagram{3,1} \oplus \ydiagram{2,1} \oplus \ydiagram{3} \oplus \ydiagram{2,2} \oplus 2\,\ydiagram{2} \oplus \ldots$

let's call
$w = N - N_{\text{min}}$
the "*depth*"
CM, Skvortsov '23

see e.g. Weinberg 1985, Mañes, Vozmediano 1989

beyond the leading Regge, the spectrum is *repetitive*

∞–many massive higher spins

$$\Downarrow$$

crucial to string UV softness

it is thus important to understand string higher spins

A new technology

- define $\mathfrak{sp}(2K)$ algebra $\qquad (k, \ell = 1, 2, \ldots, K)$

$$T^{k}{}_{\ell} := \tfrac{1}{k}\, \alpha_{-k} \cdot \alpha_{\ell}\,, \quad T_{k\ell} := \alpha_k \cdot \alpha_\ell\,, \quad T^{k\ell} = \tfrac{1}{k\ell}\, \alpha_{-k} \cdot \alpha_{-\ell}$$

- can *rewrite* (transverse) Virasoro constraints as

$$L_{n>0} F = \Bigg[\sum_{m=1}^{n-1} T_{m,n-m} + 2 \sum_{m=1}^{\infty} m\, T^{m}{}_{n+m} \Bigg] F = 0$$

$\mathfrak{sp}$ lowering operators

- *entire* spectrum splits in

$w = 0$: $\mathfrak{sp}$ lowest weight states (contract i–th row with α^{μ}_{-i})

$w > 0$: ∞–many clones *polynomials?*

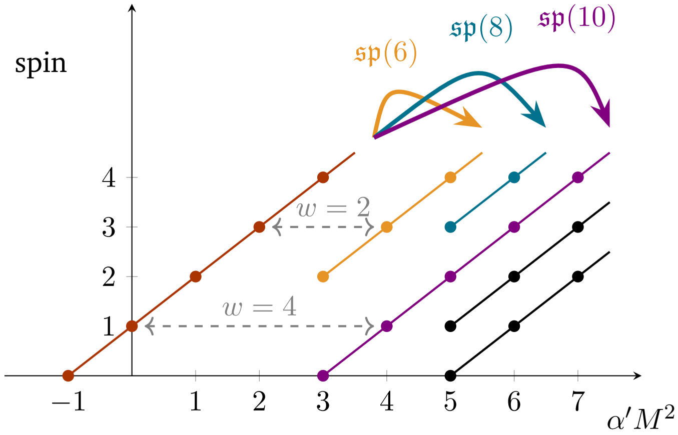
CM, Skvortsov '23

A new technology

- *cloning* possible thanks to *Howe duality*:

$$F^f_{w>0} = f_{w>0}(T^{mn}, T^{k>\ell}{}_{\ell})\, \boldsymbol{F_{w=0}} \quad \Rightarrow \quad \text{solve } L_1 \,\&\, L_2$$

$\Rightarrow$ excavate *entire* trajectories! (spin is a free parameter)



CM, Skvortsov '23

What about closed strings?

Closed string motivation

- graviton: universally a *closed* string state

$$\square_{\mathtt{L}} \otimes \square_{\mathtt{R}} = \left(\square\!\square \oplus \begin{smallmatrix}\square\\ \square\end{smallmatrix} \right) \oplus \bullet$$

e.g. bosonic string:

$$|\text{graviton}\rangle = \varepsilon^{\square\!\square}_{\mu\nu}\, \alpha^{\mu}_{-1} \bar{\alpha}^{\nu}_{-1}\ |0;p\rangle\ , \quad p^{\mu}\varepsilon_{\mu\nu} = 0 = \varepsilon^{\mu}{}_{\mu}$$

Scherk, Schwarz 1974, Yoneya 1974, . . .

- closed string spectrum: has a perturbative symmetry
  - generated by linear combinations of

$$\frac{1}{n}\left(A^{i}_{-n}A^{j}_{n} + \bar{A}^{i}_{-n}\bar{A}^{j}_{n}\right)$$

  - contains a Kac–Moody algebra

A, $\bar{A}$: DDF operators, "1–1" with oscillators

Del Giudice, Di Vecchia, Fubini 1972, Brower 1972

Gaberdiel, West 2002, . . .

Level–by–level construction?

- physicality conditions

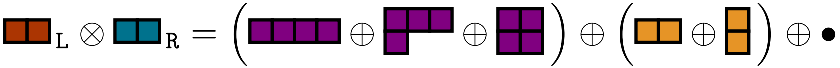
  $\mathfrak{sp}_{\mathrm{L}}$ $\mathfrak{sp}_{\mathrm{R}}$

  1. Virasoro constraints:

  $$\boldsymbol{L_1}\,|\text{phys}\rangle = 0 = \boldsymbol{\bar{L}_1}\,|\text{phys}\rangle$$
  $$\boldsymbol{L_2}\,|\text{phys}\rangle = 0 = \boldsymbol{\bar{L}_2}\,|\text{phys}\rangle$$

  2. level–matching: $$\sum_{k\geq 1} k\,\boldsymbol{T^{k}{}_{k}} = \sum_{k\geq 1} k\,\boldsymbol{\bar{T}^{k}{}_{k}}$$

- e.g. $N = 2 = \bar{N}$ (use Littlewood–Richardson rule):

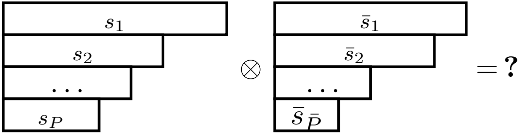


$$|\text{spin–2}\rangle = \varepsilon^{\square\square}_{\mu\nu}\left(T^{11}\bar{\alpha}^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} + \bar{T}^{11}\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} - 2(D-1)\,\boldsymbol{\tau^{11}}\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1}\right)|0\rangle$$

T. Basile, CM *to appear*

A bigger symmetry

- *trajectory* tensor product?



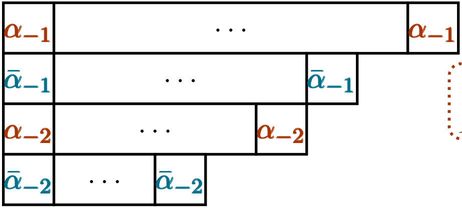
$\nexists$ general formula due to *complexity*

- now $\mathfrak{sp}(2K + 2\bar{K})$ generated by $\{T, \bar{T}, \tau\}$!

$$\tau^{m\,n} := \frac{1}{m\,n}\,\alpha_{-m}\cdot\bar{\alpha}_{-n}\,, \qquad \tau_{m\,n} := \alpha_m\cdot\bar{\alpha}_n\,,$$

$$\tau^m{}_n := \frac{1}{m}\,\alpha_{-m}\cdot\bar{\alpha}_n\,, \qquad \tau_m{}^n := \frac{1}{n}\,\alpha_m\cdot\bar{\alpha}_{-n}$$

lowest weight states:



generically ~~level-matching~~!

T. Basile, CM *to appear*

Idea: a new "tiling"

- juxtapose to superstring technology of T. Basile, CM '24 :
  - similarities:
    1. two types of oscillators, $(\alpha, \bar{\alpha})$ or (α, b)
    2. $\mathfrak{sp}(2K+2\bar{K})$ or Ramond $\mathfrak{osp}$: lowest weight states not sufficient for $w=0$ trajectories
  - disparity: $\nexists$ mixed generators $\boldsymbol{\tau}$ in physicality constraints

$$\Downarrow$$

need new, alternative approach ...

- realisation: *all* $\boldsymbol{\mathfrak{sp}}_{\mathtt{L}}$ (or $\mathfrak{sp}_{\mathtt{R}}$) lowest weight states pass half the closed string Virasoro

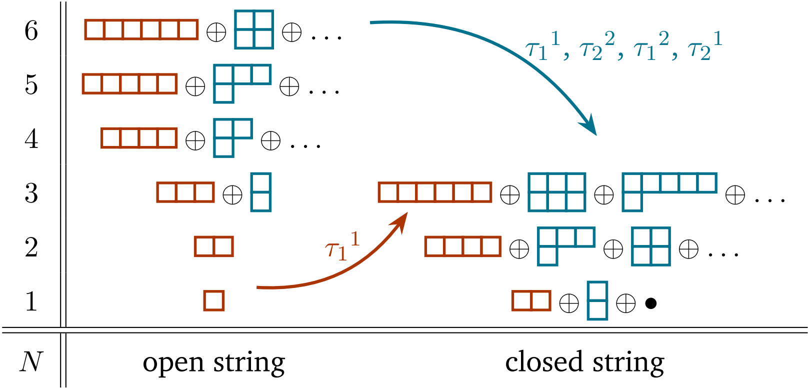
$$F_{\mathtt{L}}^{w=0} = \varepsilon_{\mu(s_1),\nu(s_2),\ldots,\lambda(s_P)}\, \alpha_{-1}^{\mu(s_1)} \alpha_{-2}^{\nu(s_2)} \ldots \alpha_{-P}^{\lambda(s_P)}$$

T. Basile, CM *to appear*

Idea: a new "tiling"

- **idea: dress open string trajectories** with $T^{k>\ell}{}_{\ell}\,, T^{k\ell}\,, \bar{T}^{k>\ell}{}_{\ell}\,, \bar{T}^{k\ell}\,, \tau^{k\ell}\,, \tau_{\ell}{}^{k}$
- example: *first* diagram apparitions

and impose Virasoro & level–matching



T. Basile, CM *to appear*

Conclusions

- $\exists$ technology for open bosonic, open superstring, closed bosonic
- future:
  - $\mathfrak{sp}(2K+2\bar{K})$ related to spectrum algebra of Gaberdiel, West 2002 ?
  - double copy from open string "tiling"?
  - ...
- ongoing:
  - black hole modelling from deep string scattering
  - technology around other vacua

How well do we understand the underlying principles of string theory?

The key to its deep mysteries may lie in its **massive spectrum**