Theoretical Motivations and Experimental Confirms of the Quantum Statistical Parton Distribution Functions.

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Summary

1) QUARKS : FROM MATHEMATICAL TOOL TO DESCRIBE SYMMETRIES TO QUANTUM FIELDS OF QCD .

2) PHENOMENOLOGICAL MOTIVATIONS TO INTRODUCE THE QUANTUM STATISTICAL PARTON MODEL

3) CONSEQUENCES OF THE PARTON MODEL SUM RULES, OF THE QCD EQUILIBRIUM CONDITIONS AND OF THE DIFFRACTIVE CONTRIBUTION

4) THE PARTON DISTRIBUTIONS PROPOSED IN 2002 BY CLAUDE BOURRELY, JACQUES SOFFER AND F. B. AND ITS EXTENSION TO THE TRANSVERSE DEGREES OF FREEDHOM

5) EXPERIMENTAL CONFIRMS FROM THE GLUON DISTRIBUTIONS FOUND BY THE ATLAS COLLABORATION, FROM THE PARTON DISTRIBUTIONS IN THE CHARGED KAONS AND FROM THE ANALYSIS OF HERA DATA .

6) COMPARISON WITH MARATHON,SEA-QUEST AND STAR EXPERIMENTS

7) CONCLUSION

Scale Invariance in the Deep Inelastic Scattering at SLAC.

For a decade quarks have been considered as a mathematical tool to describe symmetries, but experiments at SLAC and ADONE lead to consider them as the fundamental fields of the quantum theory of strong interactions . In fact in the deep inelastic interaction of electron on a proton target :

electron + proton

$$\rightarrow$$

electron + hadrons

$$k + p \rightarrow k' + (p + k - k')$$

at large values of

$$Q^2 = -(k - k')^2$$

the structure functions,which describe the data. are approximately scale invariant, since they depend mainly on the ratio of the two invariants ,

$$\frac{Q^2}{2p \times q}$$

,

where q = k - k'

The Parton Model.

Feynman gave a brilliant explanation of the scale invariace, assuming that at large Q^2 the proton behaves as an uncoherent set of point-like charged objects, called partons, with a probability to carry the percentage

$$x = \frac{Q^2}{2p.q}$$

,

which interact elastically with the incident electron . In fact by requiring :

$$x^2 p^2 = (xp + q)^2$$

$$x = \frac{-q^2}{2p \times q}$$

is the scale invariant variable .

By comparing the deep inelastic scattering with incident electrons or neutrinos LLewynn-Smith identified the partons with the quarks .

The Parton Model Sum Rules.

For the distributions in the proton the parton model sum rules hold

$$\int_0^1 x p_i(x) dx = 1$$

$$\int_0^1 [u(x) - \bar{u}(x)] dx = u - \bar{u} = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = d - \bar{d} = 1$$

$$\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} = \frac{G_A}{G_V} = 1.26$$

where

$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

Asymptotic Fredhom and Confinement of the Gauge Theories .

The behaviour of the quarks of appearing free at short distances (high scales) and being confined in the hadrons may be explained, assuming a gauge theory based on $SU(3)$, which implies asymptotic freedhom at large scales and confinement due to the sign opposite to quantum electrodynamics of the beta function, which describes the scale dependance of the coupling constant .

Hadron Production at ADONE and QCD

At ADONE the ratio of the cross-sections to produce hadrons or muons has been found to be around 2, which can be explained by the producion of point-like objects, for which the sum of the squares of the charges is 2 . The sum of square of the charges of the three quarks produced, u, d and s is $\frac{2}{3}$ and the factor 3 is given by the color degree of freedhom of the quarks and the color antisimmetry of the three quarks restores the antisymmetry expected for the fermions . All these facts induced Gell-Mann, Fritsch and Leutwyler to propose quantum cromodynamics, a gauge theory based on $SU(3)$ of color, as the theory of strong interactions .

The Altarelli Parisi Equations and the Standard Formulas Assumed for their Boundary Conditions.

Altarelli and Parisi derived the dependance of the parton distribution on Q^2, the DGLAP equations, which hold in the perturbative regime of QCD and allow to deduce them from their values at a sufficiently high value of

$$Q^2$$

For the boundary condition is usually assumed the standard form $Ax^B(1-x)^C P(x)$ with A, B, C and P(x) depending on the distribution considered . For the proton one may expect $u(x) = 2d(x)$ and $\bar{d}(x) = \bar{u}(x)$, since there are two u valence quarks in the proton and one d in the proton and the antiquarks are produced by the gluons, which are isoscalars .

The Phenomenological Information on the Parton Distributions Suggests a role of Quantum Statistical Mechanics

Instead the ratio

$$\frac{u(x)}{d(x)}$$

is an increasing function of x from 1 at

$$x = 0$$

to values near to 4 . As long for the antiquarks the ratio $\frac{\bar{d}(x)}{\bar{u}(x)}$ is an increasing function of x starting from 1 at $x = 0$ up to values larger than 2 , while the isospin asymmetry

$$x[\bar{d}(x) - \bar{u}(x)]$$

is well described by

$$A_{Asy} x^{\bar{b}} \exp \frac{-x}{\bar{x}}$$

Niegawa and Sisiki and Feynman and Field interpreted the isospin asymmetry as a consequence of the Pauli principle, which implies as a logical consequence that the parton distributions are described by Fermi-Dirac functions for the quarks and Bose-Einstein for the gluons of the variable x, which appears in the sum rules of the parton model .

Quantum Statistical Parton Distributions

In 2002 Claude Bourrely, Jacques Soffer and F. B. proposed the following parametrization for the light quarks and the gluons :

$$xq(x) = \frac{AX_q^h x^b}{(\text{exp}\,\frac{x-X_q^h}{\bar{x}}+1)} + \frac{\tilde{A}x^{b_G-1}}{(\text{exp}\,\frac{x}{\bar{x}}+1)}$$

$$x\bar{q}(x) = \frac{\bar{A}x^{\bar{b}}}{X_q^h(\text{exp}\,\frac{x+X_q^{-h}}{\bar{x}}+1)} + \frac{\tilde{A}x^{b_G-1}}{(\text{exp}\,\frac{x}{\bar{x}}+1)}$$

$$xG(x) = \frac{A_G x^{b_G}}{(\text{exp}\,\frac{x}{\bar{x}}-1)}$$

For the fermions the first term depends on the helicity, h, and the flavor, q, while the second term is isoscalar, unpolarized and equal for the quarks and antiquarks .

The Equilibrium Conditions Proposed by Baleraho

The conditions for the potentials :

$$X_G = 0$$

$$X_q^{\uparrow} + X_{\bar{q}}^{\downarrow} = 0$$

$$X_q^{\downarrow} + X_{\bar{q}}^{\uparrow} = 0$$

fpllow from the equilibrium condition at

$$Q_0^2$$

with respect to the QCD processes, which modify the parton distributions according to the DGLAP equartions, the emission of a gluon by a quark and the conversion of a gluon into a quark and an antiquark with opposite chirality . The equilibrium conditions imply that at

$$Q_0^2$$

,

which should be chosen as the separation between the non perturbative and perturbative evolutions of the parton distributions, the entropy is maximum . Indeed according to the third principle of the thermodynamics in the limit $\bar{x} = 0$ the entropy vanishes .

The Values of the Parameters Found in 2002

From the comparison with the experimental information available in 2002 one got the following values for the "temperature", $\bar{x}$, and the "potentials" of the valence partons :

$$\bar{x} = 0.099$$

$$X_u^{\uparrow} = 0.461$$

$$X_d^{\downarrow} = 0.301$$

$$X_u^{\downarrow} = 0.298$$

$$X_d^{\uparrow} = 0.228$$

and the values of these parameters did not change so much keeping into account the experiments after 2002.

Confirms of the Statistical Parametrization

After 2002 experimentals confirms of the statistical parametrization have been found for the polarized distributions of the quarks and the antiquarks and for the isospin asymmetry $x[\bar{d}(x) - \bar{u}(x)]$. More recently the gluon distribution proposed by the ATLAS Collaboration for the gluons has been described by the Planck formula proposed in 2002 :

$$xG(x) = \frac{A_G x^{b_G}}{(\exp \frac{x}{\bar{x}} - 1)}$$

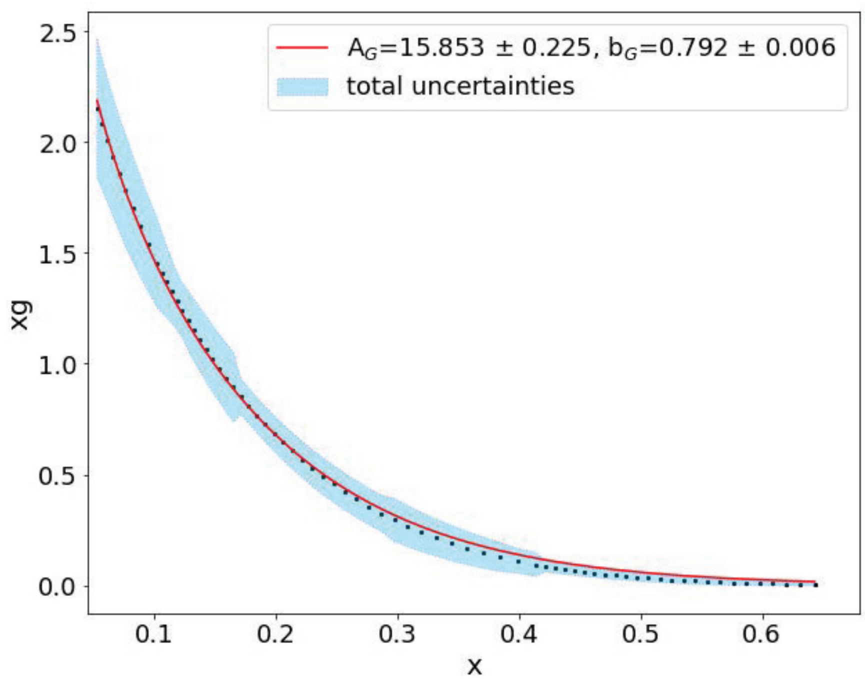
with

$$\bar{x} = 0.099$$

, while A_G and b_G are let free.

L. Bellantuono, R. Bellotti and F. B.

The figure



has been found with

$$A_G = 15.85$$

and

$$b_G = 0.79$$

rather near to the values found in 2002, 14.3 and 0.75, respectively

.

Statistical Parametrization of the Parton Distributions in the Charged Mesons

The statistical parametrization has been applied to the parton distributions of the charged pions and kaons .

For the charged pions isospin invariance implies the same potential for the two valence partons, which is found to be large, 0.75, accounting for the fact that the Drell-Yan muon pairs produced at large x by the incident $\bar{u}$ partons are produced more abundantly with incident negative pions than with antiprotons . The temperature, $\bar{x}$, is found very near , 0.102, to the value found for the proton, 0.099 . As long for the charged kaons the strange partons are expected to be rare at small x as a consequence of their mass . This implies that to obey the valence sum rule :

$$s - \bar{s} = 1$$

for the negative kaons

the strange partons should have a high second moment for the non diffractive contribution :

$$\int_0^1 x[s(x) - \bar{s}(x)]dx$$

for the negative kaon .

at the expenses of the other valence parton, $\bar{u}$.

This explains why the ratio for the Drell-Yan pairs and J/ψ particle production on nuclear targets with negative kaons or pions decreases at large x .

C. Bourrely, F. B., W. C. Chang and J. C. Peng

Statistical Parametrization of the Parton Distributions for Deep Inelastic Scattering Experiments at HERA

The HERA data on deep inelastic scattering have been described by M. Bonvini, F. Giuli, F. Silvetti and F. B. with the statistical parametrization with less parameters than the standard parametrization and a similar χ^2 . The distribution of the valence partons, which dominate at not small x, is similar to the one found in 2002 . The value of

$$\bar{x} = .097$$

found is very similar to the one found in 2002 and fixed to describe the gluons in 2023 .

Comparison Between the Standard and the Statistical Parametrizations for the Marathon Experiment

While the second term in the fermions is similar to the standard parametrization, the first term for the valence quarks is very different with the Fermi-Dirac denominator

$$(\exp \frac{x - X_q^h}{\bar{x}} + 1)$$

instead of the factor

$$(1 - x)^C$$

of the standard parametrization and their difference leads to different predictions for the recently measured ratio $\frac{F_n^2(x)}{F_p^2(x)}$ in Marathon experiment by studying the deep inelastic scattering of electrons with tritium and He_3 targets . In fact the ratio decreases and reaches a costant value at

$$x = 0.7$$

the behaviour expected by the statistical parametrization with values of the X_q^h smaller than 0.5, since the ratio

$$\frac{d(x)}{u(x)}$$

approaches a constant value larger than 0 and the ratio in the high x region dominated by the valence quarks is given by $\frac{F_n^2(x)}{F_p^2(x)} = \frac{u(x)+4d(x)}{4u(x)+d(x)}$.

Comparison Between the Standard and the Statistical Parametrizations for the Marathon Experiment

With the standard parametrization the ratio

$$\frac{d(x)}{u(x)}$$

should be proportional to

$$(1-x)^{C_d - C_u}$$

and vanish at

$$x = 1$$

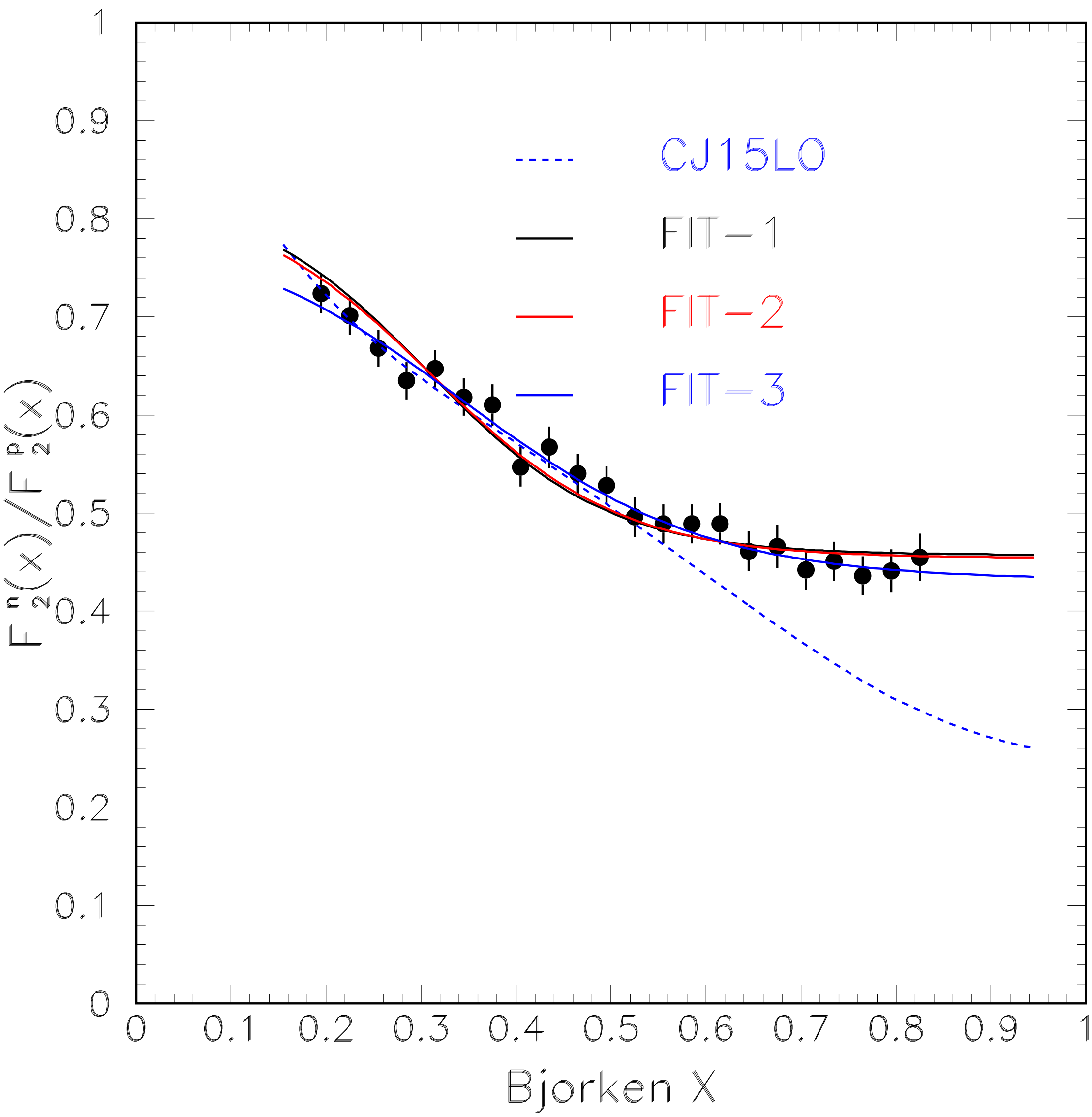
as C_d is larger than C_u . The comparison of Marathon data with the three parameters approximate formula

$$\frac{d(x)}{u(x)} = \frac{X'_d[\exp - \frac{-X'_u}{\bar{x}} + 1]}{X'_u[\exp - \frac{-X'_d}{\bar{x}} + 1}$$

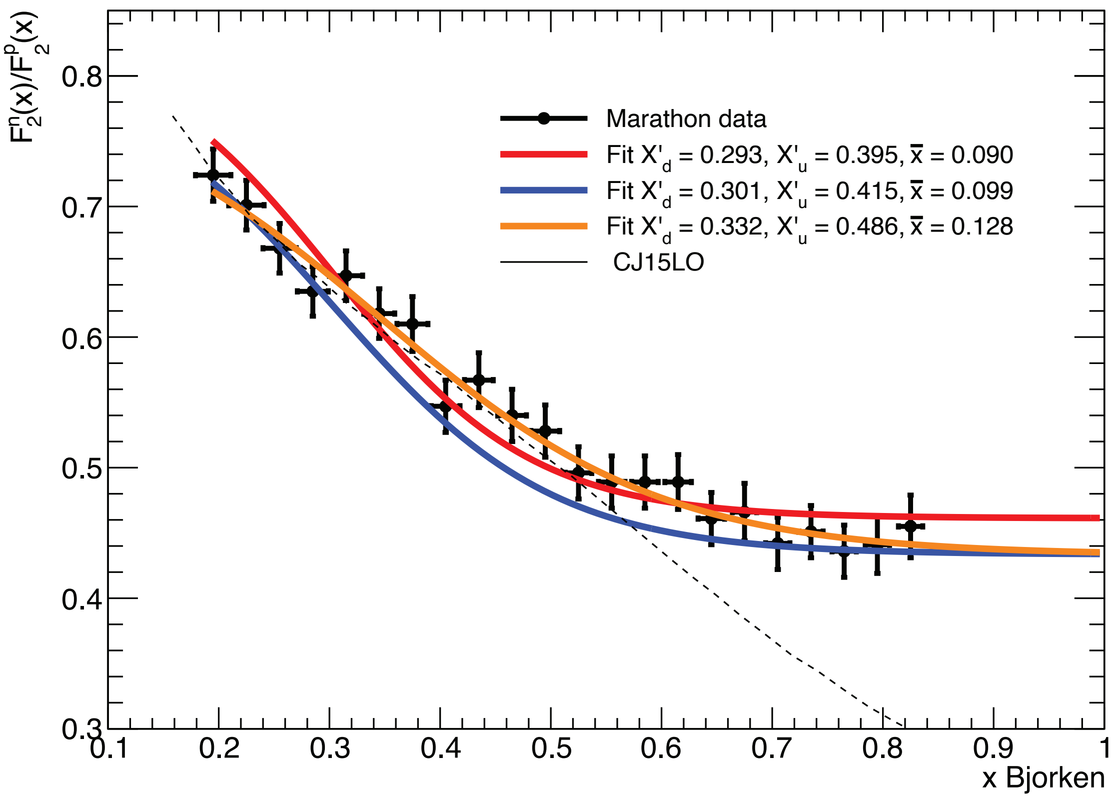
ispired by the statistical parametrization and with the standard distributions is very instructive .

C. Bourrely, W. C. Chang, D. Di Bari, P. H. Frampton, J. C. Peng and F. B.

Comparison Between the Standard and the Statistical Parametrizations for the Marathon Experiment



Comparison Between the Standard and the Statistical Parametrizations for the Marathon Experiment

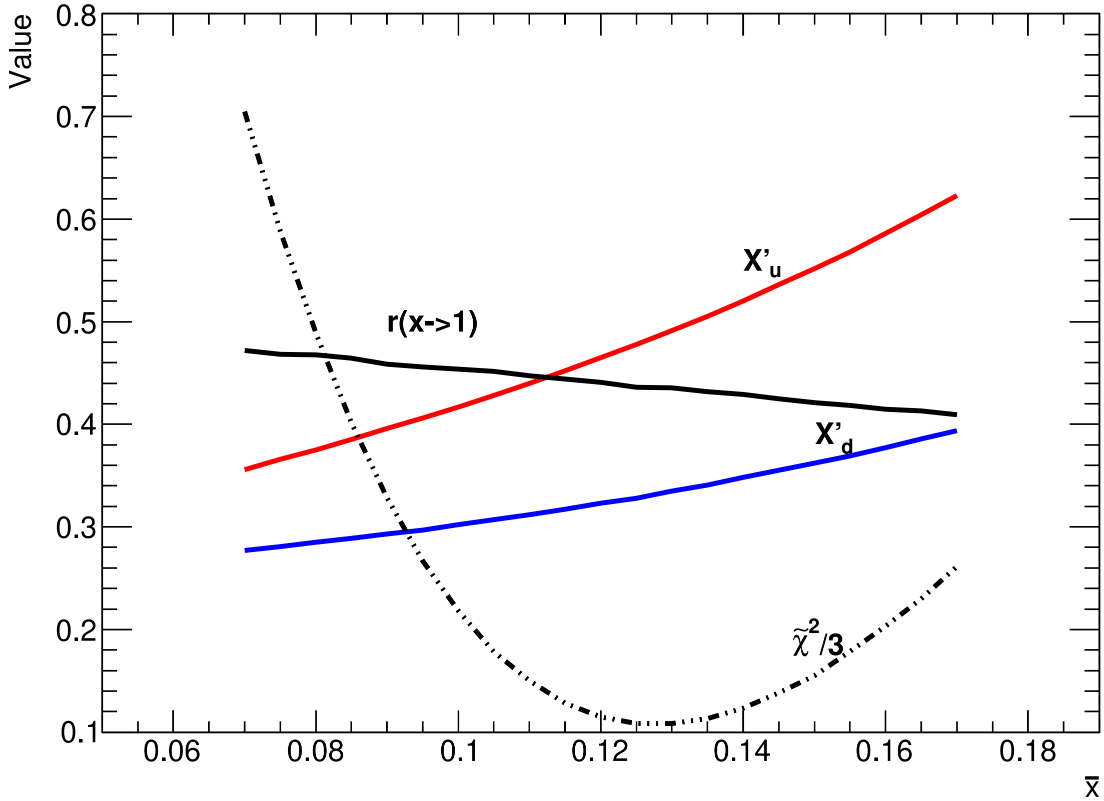


Comparison of the MARATHON data with calculations. The dashed curve is obtained using the CJ15LO proton PDFs. The solid curves are obtained from the statistical model using the three sets of parameters listed in the following table:

$\bar{x}$	X'_u	X'_d	$r(x \to 1)$	χ^2
0.090	0.395± 0.012	0.293± 0.009	0.460± 0.005	0.98
0.099	0.415± 0.013	0.301± 0.010	0.455± 0.006	0.68
0.128	0.486± 0.018	0.332± 0.013	0.436± 0.007	0.32

Comparison Between the Standard and the Statistical Parametrizations for the Marathon Experiment

A small $(\chi)^2$ is found with a large choice for $\bar{x}$ including the values found in previous works as it is shown in the figure, which gives the values of $(\chi)^2$ as a function of $\bar{x}$.



Isospin and Spin Asymmetries of the Proton Sea

The first term for the distributions of the antiparticles of the valence quarks, which depend on their potentials

$$X_{\bar{q}}^{h}$$

as a consequence of the equilibrium conditions, is responsible for the isospin and spin asymmetries in the proton sea . The negative potentials smaller than -0.2 imply that for x larger than 0.1 the spin asymmetries

$$\Delta \bar{u}(x)$$

and

$$\Delta \bar{d}(x)$$

are with a good approximation proportional to the to the isospin asymmetry

$$\bar{d}(x) - \bar{u}(x)$$

, for which in the Boltzmann approximation (neglecting one in the Fermi-Dirac denominator) and one can write the three parametersv formula

$$\bar{d}(x) - \bar{u}(x) = A_{Asy} x^{\bar{b}-1} \exp \frac{-x}{\bar{x}}$$

,

The proportionality between the spin and isospin asymmetries may be given in terms of

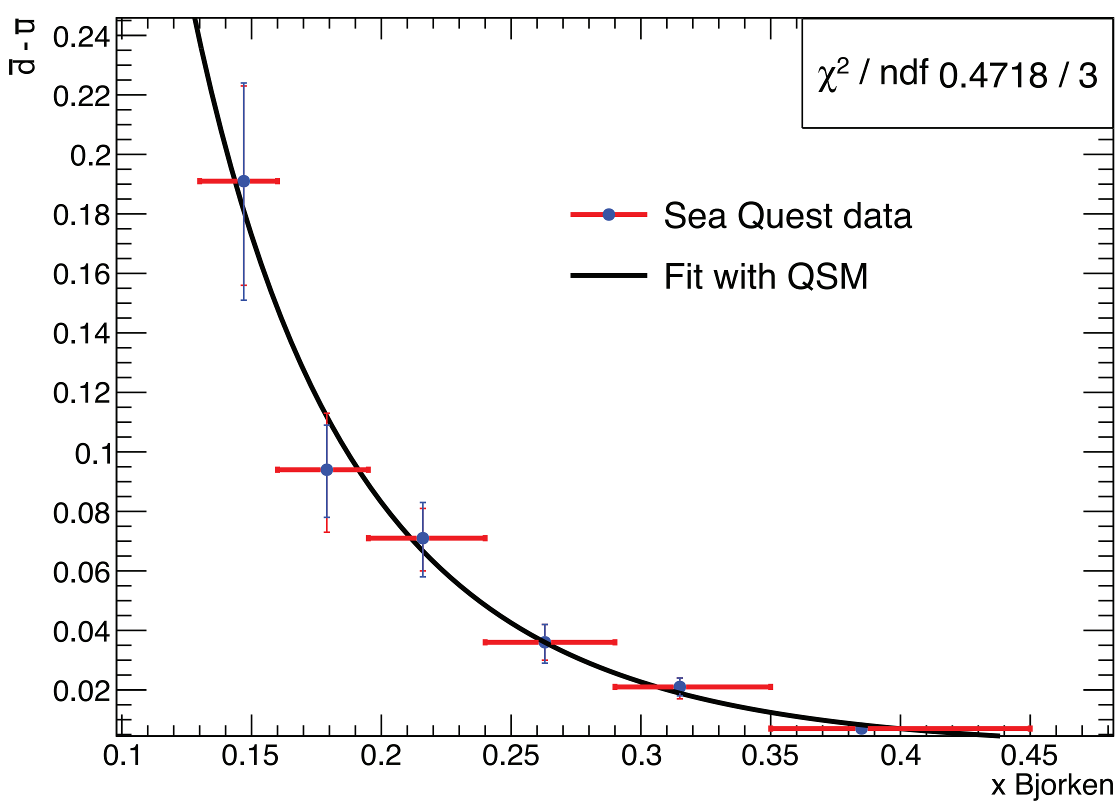
$$\bar{x}$$

and X_q, which are rather precisely fixed by the distributions of the valence quarks .

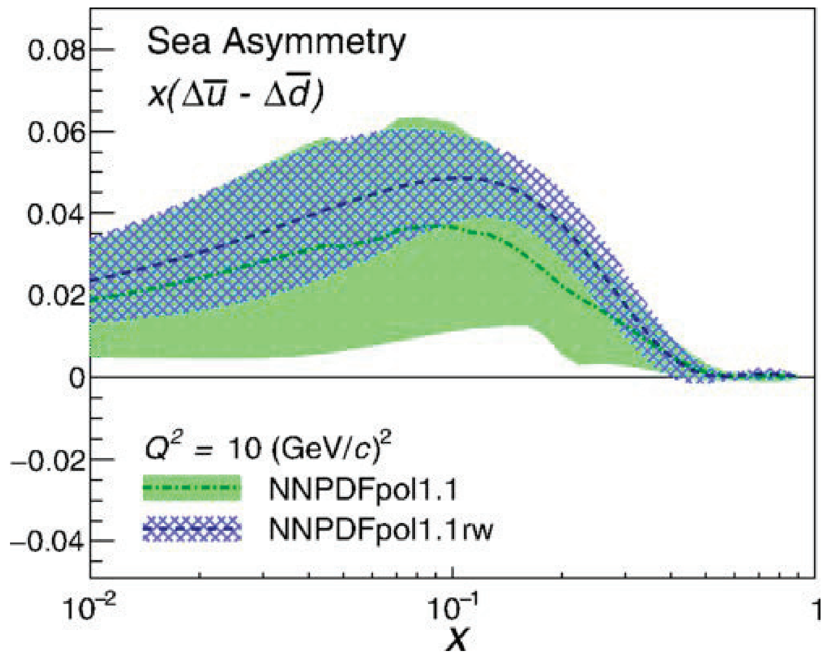
Comparison with Sea-Quest and STAR Experiments

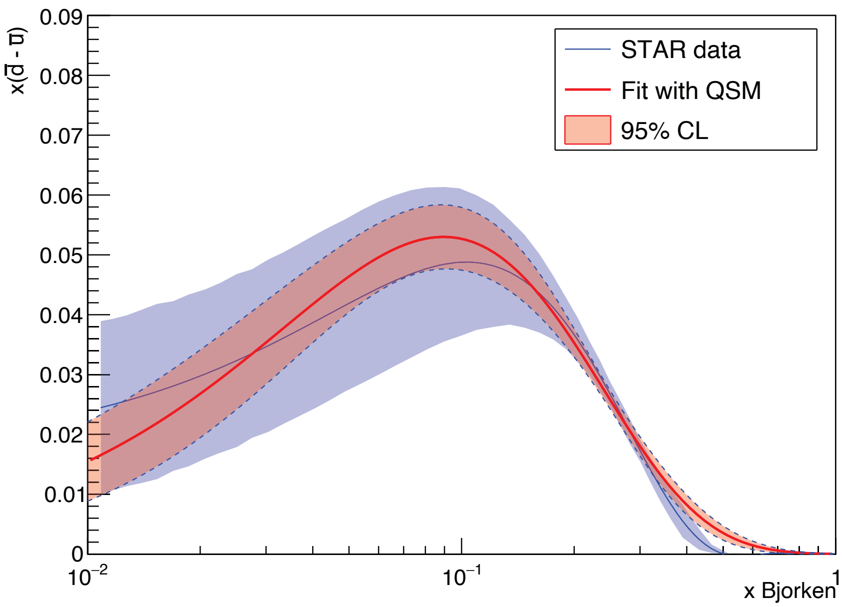
With the Botzmann approximation the isospin and spin asymmetries of the proton sea are described with three parameters formulas :

$$A_{Asy} x^{b_{Asy}} \exp \frac{-x}{\bar{x}}$$



Comparison with Sea-Quest and STAR Experiments





Conclusions

1) After several experimental confirms of the quantum statistical parton distributions, which follow logically from the validity of the conjecture by Feynman and other scientists that the isospin asymmetry for the antiquarks in the proton is a consequence of Pauli principle, the gluon distribution found by the ATLAS Collaboration has been well described by a Planck formula, the parton distribution in the charged mesons have been successfully studied and the unpolarized distribution of the valence quarks have been deduced from the deep inelastic scattering experiments at HERA with a NNLO analysis in good agreement at not small x with the ones found many years ago .

2) The quantum statistical denominators

$$-\frac{1}{\bar{x}}$$

about - 10

describe better the high x region than the standard factors

$$(1-x)^C$$

3) It is intriguing the role of $\bar{x}$, which describes the high x behavior and the variation of the valence quark distribution around their "potential" also for the stability of the determination of its value.

Conclusions

4) The isospin and spin asymmetry of the proton sea are with a good approximation proportional and described for x larger than 0.1 by a three parameter formula .

5) The NLO and NNLO contributions are unpolarized, isospin invariant and equal for the valence quarks and their antipaticles as the diffractive contribution .

6) The Marathon data on the ratio $\frac{F_2^n(x)}{F_2^p(x)}$ and the Sea-Quest data on the spin asymmetries $\bar{d}(x) - \bar{u}(x)$ are well described by three parameters formulas inspired bu the quantum statistical approach to parton distributions .