MAX-PLANCK-INSTITUT
FÜR PHYSIK

Emergence of CY Triple Intersection Numbers

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in collaboration with A. Gligovic

[arXiv: 2506.20725]

(also with M. Artime, N. Cribiori, A. Paraskevopoulou)

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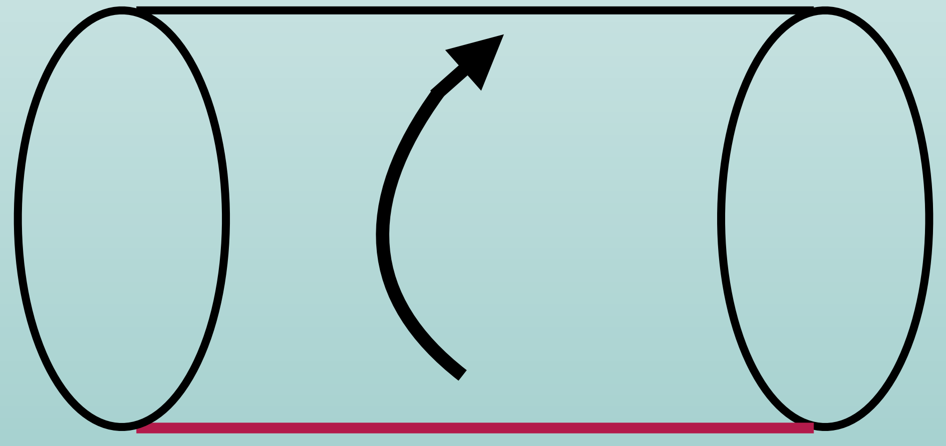
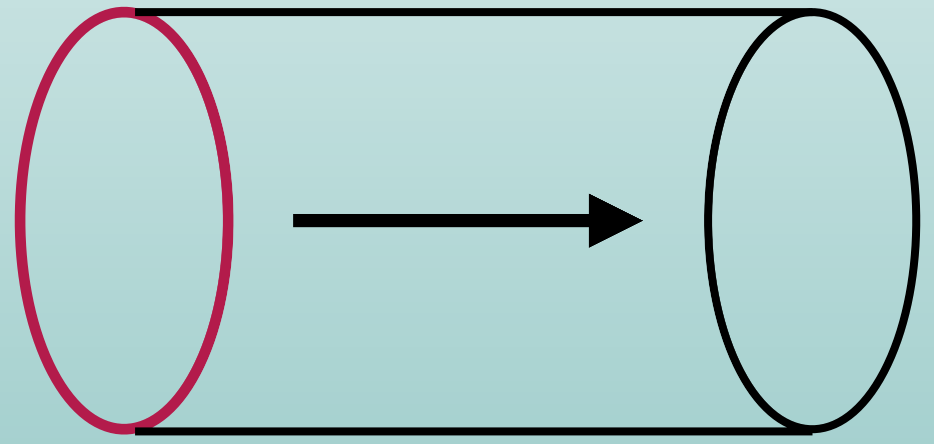
Heuristic reflection on Emergence

Appearance of properties of a system that are novel with respect to other (more fundamental) descriptions of the same system. [Butterfield, (2011)]

Example: 1-loop annulus amplitude for D-branes → tree-level graviton exchange





a.) $g_s \ll 1$ regime: no open strings without closed strings → no emergence

b.) emergence of gravity: consistent QG theory with light D-branes and decoupled open/closed strings, where gravity is solely a quantum effect

(reminiscent: BFSS matrix model [review: W.Taylor. (2001)])

Swampland Distance Conjecture + Species Scale

Moduli space of QG contains infinite distance limits: $\phi \to \infty$

- SDC: in such a limit a tower of states becomes exponentially light

$$m \sim m_0\, e^{-c\phi}$$

(in Planck units!)

[Ooguri, Vafa (2006)]

Examples: weak coupling limit , decompactification limit

- Species scale UV cutoff of quantum gravity: $\tilde{\Lambda} < M_{\rm pl}^{(d)}$

$$\boxed{\tilde{\Lambda} \simeq \frac{M_{\rm pl}}{N_{\rm sp}^{\frac{1}{d-2}}}}$$

Weak coupling limit: $\tilde{\Lambda} \sim M_s$

Decompact. limit: $\tilde{\Lambda} \sim M_{\rm pl}^{(d+k)}$

Perturbative QG Theories

The SDC and $\tilde{\Lambda}$ are usually interpreted as limitations on validity of an EFT

Working assumption: They also reflect the structure of full perturbative QG theories arising in infinite distance, $t \to \infty$, limits in moduli space

QG in infinite distance limits:

perturbation theory in small parameter $\boxed{g \sim 1/\langle t \rangle \ll 1}$

Hierarchy of towers of states

- Light towers $\boxed{m_{\text{pert}}(n) \sim g^{\alpha}\, n^{\beta}\ \tilde{\Lambda}}$ (fundamental dof)
- Heavy towers $\boxed{m_{\text{NP}}(n) \sim n^{\gamma} \dfrac{\tilde{\Lambda}}{g^{\delta}}}$ (classical soliton-like=coherent states)

$(\alpha, \ldots, \delta > 0)$

Perturbative QG Theories

Sort of a prerequisite of the Emergence Proposal: [Heidenreich, Reece, Rudelius (2018)],[Grimm, Palti, Valenzuela (2018)]

see also [Marchesano, Melotti (2022)] [Castellano,Herráez,Ibáñez (2022)] [Bhg,Gligovic,Paraskevopoulou (2023)]

The dynamics (kinetic terms) for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale, which is below the Planck scale.

EP0: Integrate out only the full light towers in the infinite distance regime

(yesterdays talk by Antonia Paraskevopoulou)

Infinite distance limits

Perturbative fundamental string

- Lightest towers are strings, mass scale M_s, string coupling $g_s \ll 1$
- Accompanied by particle like states of mass $M \sim M_s$, KK + winding
- Species scale $\tilde{\Lambda} \sim M_s$
- All other towers are non-perturbative:

  (classical = coherent quantum states) $m_{Dp} \simeq \dfrac{\tilde{\Lambda}}{g_s}$, $m_{NS5} \simeq \dfrac{\tilde{\Lambda}}{g_s^2}$

M-theory limit (special decompactification limit)

$$R_{11} \to \lambda R_{11}\,, \qquad M_* \to \frac{M_*}{\lambda^{\frac{1}{d-1}}}\,, \qquad R_I \to \lambda^{\frac{1}{d-1}} R_I,$$

(compactified type IIA: $g_s \to \lambda^{\frac{3(d-2)}{2(d-1)}} g_s\,, \qquad M_s \to \lambda^{\frac{d-4}{2(d-1)}} M_s\,, \qquad R_I \to \lambda^{\frac{1}{d-1}} R_I\,.$)

Light BPS Towers

- Lightest towers of states: D0-branes with [Bhg, Cribiori, Gligovic, Paraskevopoulou, 2309.11554]

$$\boxed{M_{D0}} \sim M_s/g_s \sim M_{\rm pl}^{(d)}/\lambda$$

- For such a KK-like tower, the species scale is

$$\tilde{\Lambda} \sim M_{\rm pl}^{(d)}/\lambda^{1/(d-1)} \sim M_{\rm pl}^{(d+1)} \sim \boxed{M_*}$$

- Room for additional light towers

$$\boxed{M_{D2,NS5}} \sim M_s/g_s^{1/3} \sim M_{\rm pl}^{(d)}/\lambda^{1/(d-1)} \sim \tilde{\Lambda}$$

M-theory: transverse M2 and M5 branes with KK momentum

M-theoretic Emergence Proposal

EP: In the infinite distance M-theory limit $M_* R_{11} \gg 1$ with the Planck scale kept fixed, a perturbative QG theory arises whose low energy effective description emerges via **quantum effects** by integrating out the full **infinite towers** of states with a mass scale parametrically not larger than the 11D Planck scale.

Problems:
- in general requires quantization of M-theory (UV-finite)
- gravity, i.e. space-time itself has to emerge

Approach:
- collect evidence from 1/2 BPS saturated amplitudes
- technically, we evaluate 1/2 BPS saturated one-loop integrals providing a working regularization of the UV divergences

Example: 1-loop diagrams in string theory

Higher derivative term $$S_{R^4} \simeq M_s^{d-8}\, V_k \int d^d x \sqrt{-g}\, a_d\, t_8 t_8\, R^4,$$

with the one-loop contribution

$$a^{(1)}_{d,\text{string}} \simeq \frac{2\pi}{V_k} \overline{\sum_{m_i, n^i \in \mathbb{Z}}} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^{\frac{d-6}{2}}}\, e^{-\pi\tau_2 M^2 - 2\pi i \tau_1 m_i n^i}$$

$$M^2 = m_i G^{ij} m_j + n^i G_{ij} n^j$$

1/2 BPS: $m_i n^i = 0$

undo integral τ_1: $$a^{(1)}_d \simeq \frac{2\pi}{V_k} \overline{\sum_{m_i, n^i \in \mathbb{Z}}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\text{BPS})\, e^{-\pi t M^2}$$

UV divergence → regularization

(minimal substraction + zeta function)

similar integrals in M-theory

What happened so far

Collected evidence for M-theoretic emergence for various couplings

- Higher derivative R^4-terms in theories with maximal supersymmetry

[Bhg, Cribiori, Gligovic, Paraskevopoulou, 2404.01371]

pert. string theory | desert | pert. M-theory

$g_s \ll 1$ $\quad$ $g_s = O(1)$ $\quad$ $g_s \gg 1$

$$a_d = \frac{c_0}{g_s^2} + \underbrace{\left(c_1 + \mathcal{O}\left(e^{-S_{\rm ws}}\right)\right)}_{1-\text{loop}} + \mathcal{O}\left(e^{-S_{\rm st}}\right) \overset{\text{EP}}{=} \mathcal{E}^{E_{k+1(k+1)}}_{\Lambda_{E_{k+1}},s=\frac{k}{2}-1} \overset{\text{EP0}}{=} \mathcal{E}^{E_{k(k)}}_{\Lambda_{E_k}\oplus 1,s=\frac{k}{2}-1}$$

- Higher derivative F^4-term for type IIA on K3 and its heterotic duals

[Artime, Bhg, Paraskevopoulou, 2504.05392]

$$a_d = \frac{1}{g_s^2}\left(c_0 + \mathcal{O}\left(e^{-S_{\rm ws}}\right)\right) = \mathcal{E}^{SO(4,4)}_{V,s=1} = \mathcal{E}^{SO(3,3)}_{V\oplus 1,s=1}$$

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Type IIA on CY

Type IIA compactified on a CY to 4D with N=2 susy:

[Bhg, Cribiori, Gligovic, Paraskevopoulou (2023), 2309.11551]
[Hattab, Palti 2312.15440] [Hattab, Palti 2404.05176]
[Bhg, Gligovic (2025), 2506.20725] (discussions with N.Cribiori)

Prepotential is 1/2 BPS saturated and enjoys an expansion

$$\mathcal{F}_0(t) = \frac{(2\pi i)^3}{g_s^2}\left[\frac{1}{3!}\kappa_{ijk}t^i t^j t^k + \frac{\zeta(3)}{2(2\pi i)^3}\chi(X) + \frac{1}{(2\pi i)^3}\sum_{\beta\in H_2(X,\mathbb{Z})}\alpha_0^\beta \; \mathrm{Li}_3\left(e^{2\pi i\beta\cdot t}\right)\right],$$

$$(t_i = b_i + i\tau_i)$$

- determines kinetic terms for vector-multiplets
- Gopakumar-Vafa invariants $\alpha_0^\beta \in \mathbb{Z}$

Gopakumar/Vafa: in the M-theory limit given by integrals over D2-D0 bound states

$$\mathcal{F}_0 = \sum_\beta \alpha_0^\beta \sum_{n\in\mathbb{Z}} \int_0^\infty \frac{ds}{s^3} e^{sZ_n(\beta)},$$

with central charge

$$Z_n(\beta) = \frac{2\pi i}{g_s}\Big(\beta\cdot t - n\Big)$$

Emergence of Yukawa couplings

For the Yukawa coupling one gets

weak coupling: $g_s \ll 1$

tree-level?

world-sheet instantons

- for $h_{11} = 1$

$$Y_{ttt} = \frac{g_s^2}{(2\pi i)^3}\partial_t^3 \mathcal{F}_0(t) = \sum_{\beta>0} \alpha_0^\beta\, \beta^3 \underbrace{\left(\frac{1}{2} + \frac{e^{2\pi i\beta t}}{1-e^{2\pi i \beta t}}\right)}_{\text{resolved conifold}}$$

- problem: regularization of diverging sum over $\beta \in H_2(X,\mathbb{Z})$

$$Y_{ttt}^{(0)} := \frac{1}{2}\sum_{\beta=1}^{\infty} \beta^3\, \alpha_0^\beta \Big|_{\text{reg.}} = \kappa_{111} \;\; !? \qquad \text{with} \qquad \alpha_0^\beta \sim \exp(\gamma\beta)$$

zero point Yukawa coupling

Simple Example

Regularization via modular forms

$$\sum_{k=1}^{\infty} k^{2n-1} = -2 \lim_{\Lambda\to\infty} \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} k^{2n-1} q^{lk}$$

with

$$q = \exp(-2\pi/\Lambda)$$

$$\zeta(0) = \sum_{l=1}^{\infty} 1 = -1/2$$

$$= -2 \lim_{\Lambda\to\infty} \sum_{m=1}^{\infty} \sigma_{2n-1}(m)\, q^m$$

$$= \lim_{\Lambda\to\infty} \left[-\frac{2}{c_{2n}} \left(E_{2n}\left(\frac{i}{\Lambda}\right) - 1 \right) \right]$$

Eisenstein series $E_{2n}(\tau) = 1 + c_{2n} \sum_{m=1}^{\infty} \sigma_{2n-1}(m)\, q^m$ with $c_{2n} = \dfrac{2}{\zeta(1-2n)}$

Modular form of weight 2n:

$$E_{2n}\left(\frac{i}{\Lambda}\right) = (-1)^n \Lambda^{2n} + \mathcal{O}(e^{-2\pi\Lambda})$$

divergence vanishing

Minimally subtracting the isolated divergence:

$$\sum_{k=1}^{\infty} k^{2n-1} = \frac{2}{c_{2n}} = \zeta(1-2n)$$

Aim: generalize this procedure to zero point Yukawa couplings

Elliptically fibered CY

Consider $CP_{1,1,1,6,9}[18]$ with $(h_{21}, h_{11}) = (272{,}2)$

Decompactification limit to 6D: $\tau_b \to \lambda\tau_b,\ \ g_s \to \lambda g_s,\ \ \tau_f = \text{const} \quad (\, M_{\text{pl}}^{(4)} = \text{const}\,)$

Only D2-branes wrapping the elliptic fibre are among the light towers of states

GV invariants: $\alpha_0^{(n_1,0)} = 540$

Integrating out this tower of D2-D0 bound states:

$$Y_{t_1t_1t_1} = \sum_{k=1}^{\infty} 540\, k^3 \left(\frac{1}{2} + \frac{q^k}{(1-q^k)} \right) = \frac{9}{4} E_4(t_1) \qquad (\, q = \exp(2\pi i t)\,)$$

$= 270\,\zeta(-3) = 9/4 \;\Rightarrow$ the zero point Yuk. completes $E_4(t_1)$

Elliptically fibered CY

$$Y_{t_1 t_1 t_1} = 540 \left(\sum_{k=1}^{\infty} k^3 \frac{q^k}{(1-q^k)} + \frac{1}{2} \right)$$

$$= \sum_{k,l=1}^{\infty} k^3 q^{kl}$$

this expression was appearing in the regularization of

$$\frac{1}{2} \sum_{k=1}^{\infty} k^3 = - \lim_{\mathrm{Im}(t_1) \to 0} \sum_{k,l=1}^{\infty} k^3 q^{lk}$$

Lesson: the exponential terms in the Yukawa couplings can serve as a regularization of the zero point Yukawas $Y^{(0)}_{t_1 t_1 t_1}$.

(here it seems to be not more than a consistency check but will prove to be useful)

M-theory limit for CYs

Now, we consider the actual case of interest: M-theory limit of IIA on CY

$$\tau_i \to \lambda \tau_i, \quad g_s \to \lambda^{\frac{3}{2}} g_s, \qquad\qquad (M_{\rm pl}^{(4)} = \text{const})$$

Emergence: all TINs κ_{ijk} should arise from regularized $Y^{(0)}_{t_i t_j t_k}$

For CY with one Kähler modulus we would like to define

$$Y^{(0)}_{ttt} := -\lim_{t \to 0} \left[\sum_{n=1}^{\infty} \alpha_0^n \, n^3 \frac{q^n}{1-q^n} - \text{Div} \right]$$

GVs grow $\sim e^{\gamma n}$

$\text{Im}(t) = 0$ is not in the moduli space!

M-theory limit for CYs

The CY has a conifold singularity at t_c (quantum corrected origin in the GLSM)

$$Y_{ttt} \sim \frac{1}{(t-t_c)\,\log^2(t-t_c)} + \dots\ ,$$

One needs a "CY demon" to determine the GV $\alpha_n^{(0)}$ by pure counting

But he is bound to find the known result from mirror symmetry, so that

$$Y_{ttt}^{(0)} := -\lim_{t\to t_c}\left[\sum_{n=1}^{\infty} \alpha_0^n\, n^3 \frac{q^n}{1-q^n} - \mathrm{Div}\right] = -\lim_{t\to t_c}\left[\left(\partial_t^3 \mathcal{F}_0\Big|_{\mathrm{weak}} - \kappa_{ttt}\right) - \mathrm{Div}\right]$$

should only give divergent and vanishing terms

The Procedure

To evaluate the expression

$$Y^{(0)}_{ttt} = -\lim_{t\to t_c}\left[\left(\partial_t^3 \mathcal{F}_0\Big|_{\text{weak}} - \kappa_{ttt}\right) - \text{Div}\right]$$

we observe

- starting with the periods in LCS regime, the conifold point t_c is at the boundary of convergence
- one cannot simply apply a modular transformation to get the behaviour close to t_c

follow the procedure

- determine the periods close to t_c by solving the Picard-Fuchs equations
- determine a symplectic basis and glue them continuously to the LCS chart
- take the limit $t \to t_c$

[Alvarez-Garcia, Mutchler, Qi, Rühle, to appear]

Example: the Quintic

Mirror quintic $\sum_{i=1}^{5} z_i^5 - (5\psi)\, z_1 z_2 z_3 z_5 z_5 = 0\,,$ conifold at $\psi = 1$

Carrying out the procedure, one gets the symplectic basis for small $u = 1 - \psi^{-5}$

$$
\begin{aligned}
X_0 &= -\,\rho_1(u)\frac{u}{2\pi i}\log u + \rho_2(u) \\
X_1 &= \rho_3(u) \\
F_1 &= \rho_4(u) \\
F_0 &= u\,\rho_1(u)\,.
\end{aligned}
$$

Then, the Kähler modulus reads

$$t = \frac{X_1}{X_0} = t_c + c\,u\log u + \ldots \qquad (t_c = i1.20812...)$$

Example: the Quintic

Using the expression for the Yukawa coupling

$$\partial_t^3 \mathscr{F}_0 \Big|_{\text{weak}} = \frac{1}{\omega_0^2} \kappa_{\psi\psi\psi} \frac{1}{(dt/d\psi)^3} \qquad \text{with} \qquad \kappa_{\psi\psi\psi} = \left(\frac{2\pi i}{5}\right)^3 \frac{5\psi^2}{(1-\psi^5)}$$

one obtains

$$\partial_t^3 \mathscr{F}_0 \Big|_{\text{weak}} = \frac{1}{u \log^3 u} \left(\sum_{n,k=0}^{\infty} \frac{(u \log u)^n}{\log^k u} a_{n,k}(u) \right)$$

$$\sim \underbrace{\boxed{\left(\frac{1}{u \log^3 u} + \dots \right)}}_{\text{divergent}} + \underbrace{\cancel{\mathbb{C}}}_{=0} + \underbrace{\boxed{\left(\frac{1}{\log^2 u} + \dots \right)}}_{\text{vanishing}}$$

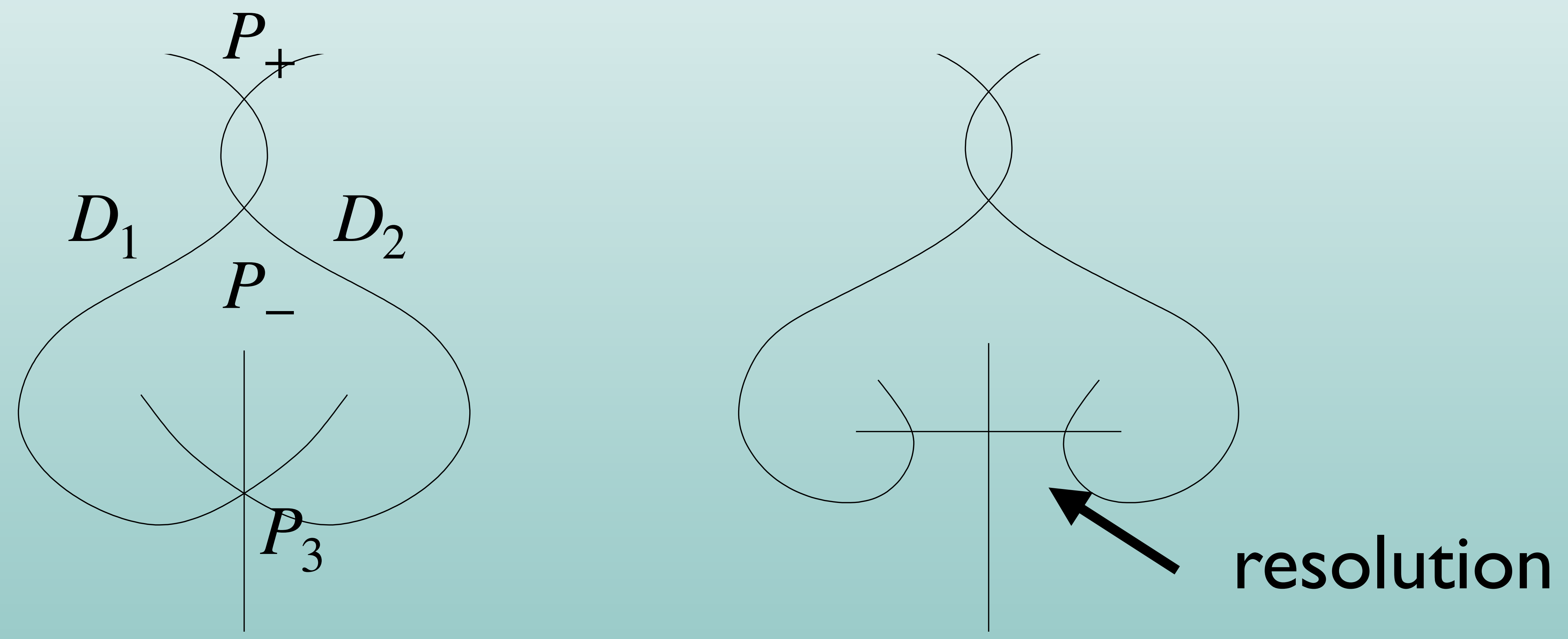
$$\Longrightarrow \boxed{Y_{ttt}^{(0)} = \kappa_{ttt} = 5}$$

Beyond the Quintic

- There are 14 CYs with $h_{11} = 1$ similar to the Quintic: expect analogous results

- We have analyzed also CYs with 2 Kähler moduli: $CP_{1,1,1,6,9}[18]$, $CP_{1,1,2,2,6}[12]$

  More involved singularity structure



but we have the methods to perform the analysis to evaluate

$$Y^{(0)}_{t_i t_j t_k} = \frac{1}{2} \sum_{\beta \in H_2(X,\mathbb{Z})}^{\infty} \alpha_0^{\beta}\, \beta_i \beta_j \beta_k \Big|_{\mathrm{reg}} = - \lim_{t_i \to t_{i,0}} \left[\left(\partial_{t_i} \partial_{t_j} \partial_{t_k} \mathcal{F}_0 \Big|_{\mathrm{weak}} - \kappa_{t_i t_j t_k} \right) - \mathrm{Div} \right]$$

(not yet fully conclusive, more evidence needed, consult [Bhg, Gligovic (2025), 2506.20725] for more details)

Conclusions

- Provided evidence for the M-theoretic Emergence of kinetic terms in 4D N=2 vacua (origin in 10D Einstein-Hilbert term)

- Pragmatic regularization method of the GV 1-loop integral (relation to singularities in the CY moduli space)

- CY moduli space "knows" about QG: it encodes the Emergent String Conjecture for infinite distance points and seems to encode Emergence via its singular loci.

- Generalization to non BPS amplitudes, like 10D kinetic terms, requires quantization of M-theory (relation to Fermi gas [Hattab, Palti 2404.05176])

- Emergence is consistent with the BFSS Matrix Model: graviton scattering at 1-loop

$$V = -\frac{15}{16}\frac{v^4}{r^7} + \dots \,, \qquad \text{(velocity } v \text{ breaks susy)}$$

(Note: Matrix Model limit is different from isotropic M-theory limit.)