The Double Expansion in gravitational EFTs

Alvaro Herraez

Based on:

[arXiv:2310.07708] with A. Castellano, L.E. Ibáñez

[arXiv:2501.14880] with J. Calderón-Infante, A. Castellano

MAX-PLANCK-INSTITUT FÜR PHYSIK

Workshop on Quantum Gravity and Strings, Corfu Summer Institute 2025

Sept 10, 2025

What is the Structure of Gravitational EFTs?

Can we exploit the multi-scale structure to bound gravitational Wilson Coefficients?

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[Calderón-Infante, Castellano, AH ´25]

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Quantum Gravity cutoff

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Mass of a (tower of) state(s) outside original EFT

[Calderón-Infante, Castellano, AH ´25]

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- Reaching the scale where genuine Quantum-Gravitational effects kick-in ⟶ Intrinsic non-localities, no local EFT framework

[van de Heisteeg, Vafa, Wiesner,(Wu) '22-'24]
[Castellano, AH, Ibáñez ´23]
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⟶ Particularly interesting in the *Asymptotic Regime:* $M_{\mathrm{Pl},d} \gg \Lambda_{\mathrm{QG}} \gtrsim M_{\mathrm{t}}$

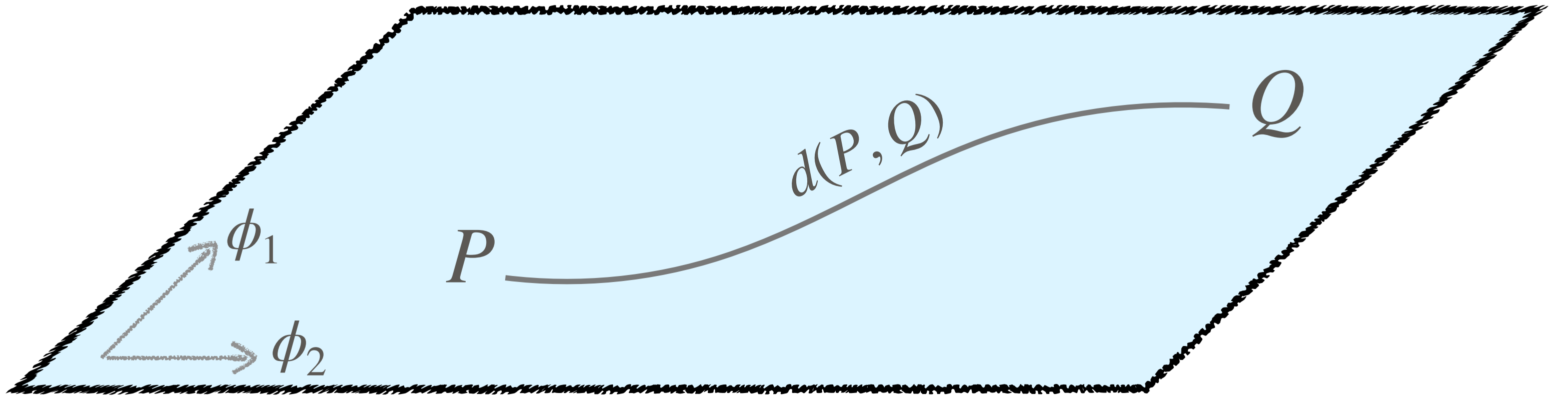
The Asymptotic regime I:
The Distance Conjecture

In a theory of QG, moving in moduli space from a point P to a point Q an infinite distance away, an infinite tower of states become light **exponentially in the geodesic distance**

$$M(Q) \sim M(P)\ e^{-\alpha\,\Delta_\phi(P,Q)}$$

[Ooguri, Vafa ´06]

Scalar manifold with metric $g_{ij}(\phi_i)$ from kinetic terms



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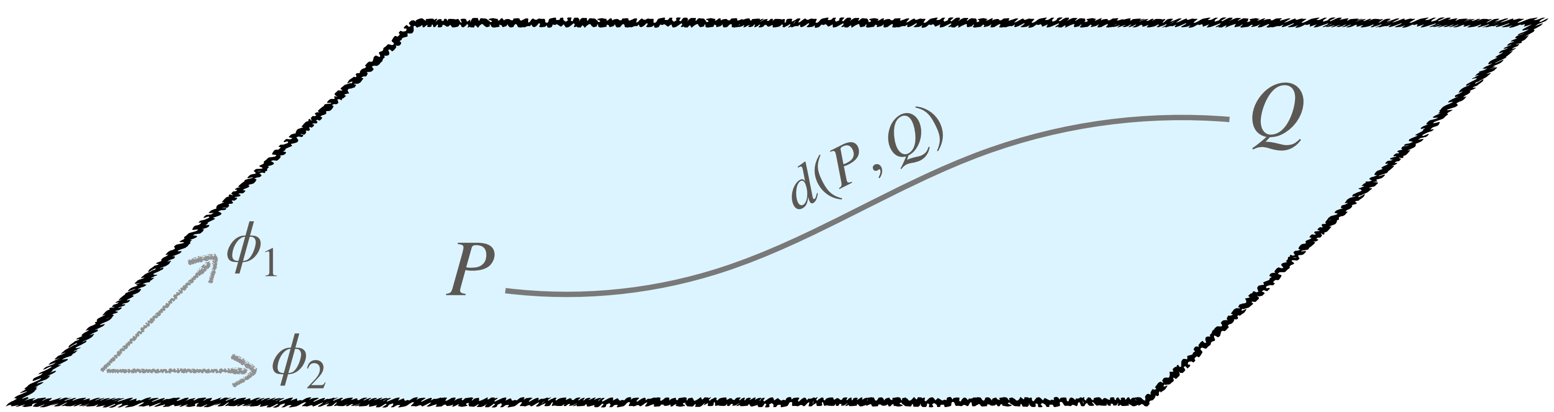

<u>Emergent String Conjecture:</u> The tower becoming light is either:

- (Dual to) Kaluza-Klein tower
- Excitation modes of weakly coupled string

[Lee, Lerche, Weigand ´19]

The Asymptotic regime II: The Species Scale

- Maximum UV cut-off in QG in the presence of N light species [Dvali '07] [Dvali, Reedi '08] [Dvali, Lüst '10] [Dvali, Gómez '10]

$$M_{\rm Pl,d} \longrightarrow \Lambda_{\rm QG} = \frac{M_{\rm Pl,d}}{N_{\rm sp}^{\frac{1}{d-2}}} = N_{\rm sp}^{1/p} M_t$$

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$$\Lambda_{\mathrm{QG}} \simeq M_t \left(\frac{M_{\mathrm{Pl},d}}{M_t}\right)^{\frac{d-2}{d+p-2}} \longrightarrow M_s \qquad N_{\mathrm{sp}} \simeq \left(\frac{M_{\mathrm{Pl},d}}{M_t}\right)^{\frac{p(d-2)}{d+p-2}}$$

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Species Scale of the d-dim EFT $\begin{cases} M_{\text{Pl},\, d+p} & \text{(decompactification of } p \text{ dimensions)} \\ M_{\text{str}} & \text{(weakly coupled string limits)} \end{cases}$

The double EFT expansion: Leading Contributions

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(QG) (FT)

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What is this scale in the gravitational sector? ⟶ Which term dominates in general?

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EXAMPLE: Maximal 10d supergravity (type IIA) ⟶ 4 graviton amplitude

- ✳ Decompactification limit ($g_s \to \infty$)
- ✳ Weak string coupling limit ($g_s \to 0$)

$$\mathcal{A}_4 = \mathcal{A}_4^{\mathrm{sugra}}\; f(s,t)\,, \qquad \mathcal{A}_4^{\mathrm{sugra}} = -\,\hat{K}\,\ell_{\mathrm{Pl},10}^8\, g_s^{-4}\frac{64}{\alpha'^3 stu}\,,$$

$$f(s,t) = 1 + \ell_{\mathrm{Pl},10}^6\, f_{R^4}(s,t) + \ell_{\mathrm{Pl},10}^{10}\, f_{D^4R^4}(s,t) + \ell_{\mathrm{Pl},10}^{12}\, f_{D^6R^4}(s,t) + \ldots\,,$$

[Green, Gutperle, Kwan, Russo, Tsytelin, Vanhove´97-´07]

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(Duals of) Decompactification limits

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$$f_{D^6R^4} = \frac{(4\pi)^{3/2}}{(2\pi)^{12}}\frac{stu(s^3+t^3+u^3)}{2^{12}}\left(\frac{2\zeta(3)^2}{3}g_s^{-3} + \frac{4\zeta(2)\zeta(3)}{3}g_s^{-1} + \frac{8\zeta(2)^3}{5}g_s + \frac{4\zeta(6)}{27}g_s^{3}\right).$$

The double EFT expansion: Leading Contributions

(Duals of) Decompactification limits

$\Lambda_{\rm QG} \simeq M_t^{\frac{p}{d+p-2}}$

- $\dfrac{M_{{\rm Pl},d}^{d-2}}{\Lambda_{\rm QG}^{n-2}}$ if $n < d + p$ ⟶ UV divergent (Relevant in the decomp. th)
- $\dfrac{1}{M_{\rm t}^{n-d}}$ if $n > d + p$ ⟶ UV convergent (Irrelevant in the decomp. th)

$$f(s,t) = 1 + \ell_{{\rm Pl},10}^6 f_{R^4}(s,t) + \ell_{{\rm Pl},10}^{10} f_{D^4R^4}(s,t) + \ell_{{\rm Pl},10}^{12} f_{D^6R^4}(s,t) + \dots\,, \qquad g_s \to \infty$$

$$f_{R^4} = \frac{(4\pi)^{3/4}}{(2\pi)^6}\frac{stu}{64}\left(2\zeta(3)g_s^{-3/2} + \frac{2\pi^2}{3}g_s^{1/2}\right) \longrightarrow \ell_{{\rm Pl},10}^6\, g_s^{1/2} = \ell_{{\rm Pl},11}^6 = \Lambda_{\rm QG}^{-6}$$

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$$\longrightarrow \ell_{{\rm Pl},10}^{12}\, g_s^{3} = \ell_{{\rm Pl},10}^{8} M_{KK}^{-4} \neq \Lambda_{\rm QG}^{-12}$$

The double EFT expansion: Leading Contributions

Weakly coupled (emergent) string limits

$$\Lambda_{\rm QG} \simeq M_{\rm tower} = M_s$$

$$\ell_{\rm Pl}^{d-2} = g_s^2 \, \ell_s^{d-2}$$

- $\frac{M_{{\rm Pl},d}^{d-2}}{\Lambda_{\rm QG}^{n-2}} \simeq g_s^{-2}$ ⟶ Tree-level string perturbation theory, unles vanishing
- $\frac{1}{M_{\rm tower}^{n-2}} \simeq g_s^{0}$ ⟶ 1-loop contribution in string perturbation theory

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The double EFT expansion: General Lessons

(QG) (FT)

$$S_{\text{EFT},\,d} = \int d^d x \sqrt{-g}\,\frac{M_{\text{Pl},d}^{d-2}}{2}\left(R + \sum_{n>2} \frac{\mathcal{O}_n(R)}{\Lambda_{\text{QG}}^{n-2}}\right) + \int d^d x \sqrt{-g} \sum_{n\geq d} \frac{\mathcal{O}_n(R)}{M_{\text{t}}^{n-d}}$$

- Decompactification limits: Non-vanishing contribution to an infinite tower of UV convergent operators of the form $\dfrac{D^{2\ell}R^4}{M_t^{8+2\ell-d}}$ $\longrightarrow$ Field-theoretic series diverges as the scale M_t is approached $\longrightarrow$ Resummed into higher dim. 2-derivative with more dof

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- Emergent string limits: 4-graviton amplitude in string perturbation theory ⟶ Tree-level contribution to an infinite tower of operators of the form $\frac{D^{2\ell}R^4}{M_s^{(2\ell+2)-2}}$

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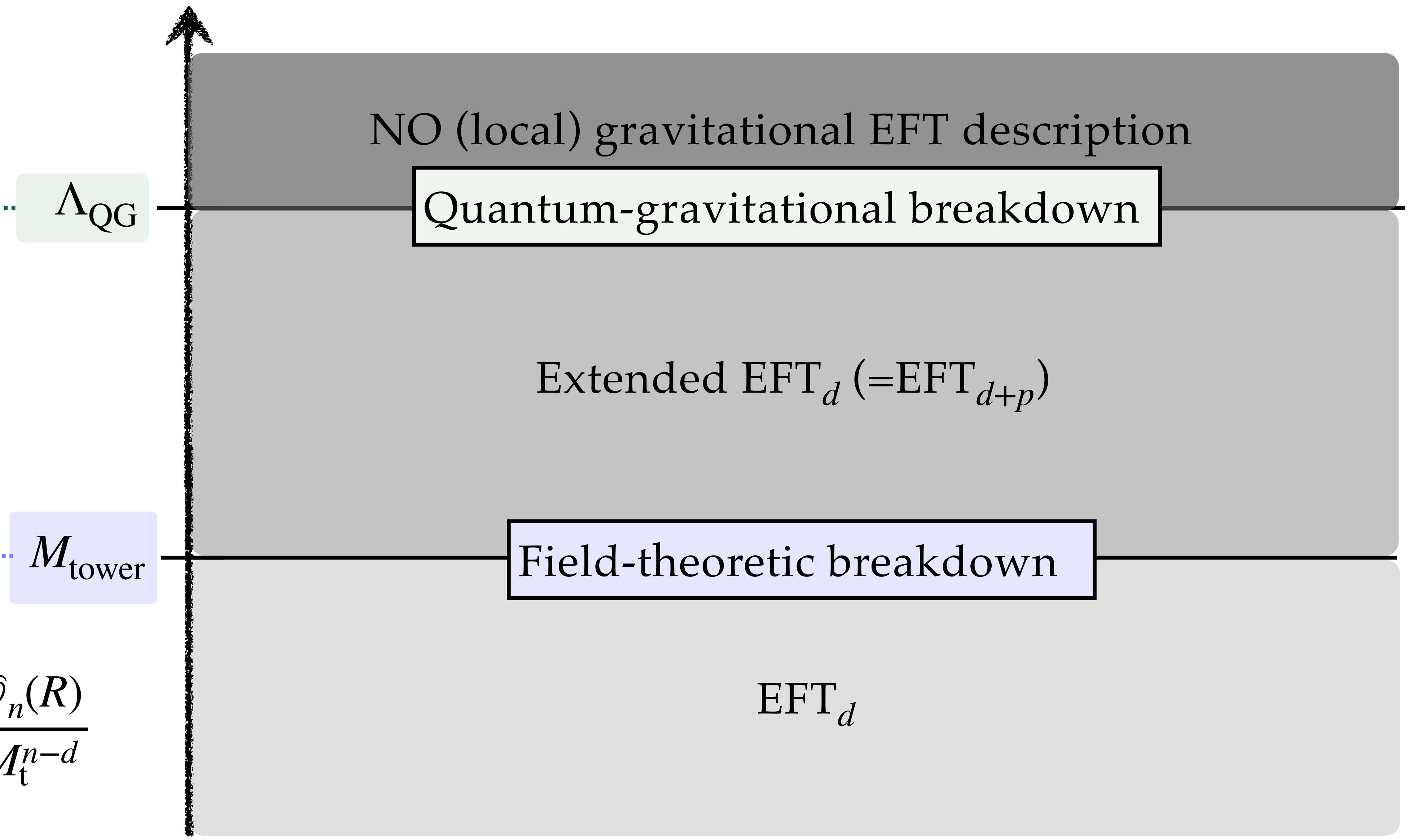
$$\alpha_n = \frac{a_n}{\Lambda_{\mathrm{QG}}^{n-2}} + \frac{b_n}{M_t^{n-d}} + \dots$$

In all observed examples, the double EFT expansion captures the leading contribution for every (non-vanishing) Wilson Coefficient

The double EFT expansion: General Lessons

$$M_{\text{Pl},d+p}^{d+p-2}\sum_{n>2}\frac{\mathcal{O}_n(R)}{\Lambda_{\text{QG}}^{n-2}}$$

$$M_{\text{Pl},d}^{d-2}\sum_{n>2}\frac{\mathcal{O}_n(R)}{\Lambda_{\text{QG}}^{n-2}}+\sum_{n\geq d}\frac{\mathcal{O}_n(R)}{M_{\text{t}}^{n-d}}$$



[Calderón-Infante, Castellano, AH '25]

Can we exploit this multi-scale structure to bound gravitational Wilson Coefficients?

The Bottom-up Perspective: Constraints on Wilson Coefficients

- Assuming the Double EFT expansion, with the leading piece for a (non-vanishing) gravitational Wilson coefficient being given by the FT or QG pieces

$$\alpha_n = \frac{a_n}{\Lambda_{\rm QG}^{n-2}} + \frac{b_n}{M_t^{n-d}} + \ldots \qquad \text{(In Planck Units)}$$

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(In Planck Units)

- GOAL: Obtain bounds for Wilson coefficients in *asymptotic Regime* ($M_{\mathrm{Pl},d} \gg \Lambda_{\rm QG} \gtrsim M_{\rm t}$) and compare with S-Matrix bootstrap

Double EFT Expansion

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Double EFT Expansion

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Emergent String Conjecture

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Double EFT Expansion

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Double EFT Expansion ← Inform — S-Matrix Bootstrap

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Emergent String Conjecture

The Bottom-up Perspective: Constraints on Wilson Coefficients

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Double EFT Expansion → Provide Targets for → S-Matrix Bootstrap

S-Matrix Bootstrap → Inform → Double EFT Expansion

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Emergent String Conjecture

The Bottom-up Perspective:
Scaling relations between Wilson Coefficients

$$\alpha_n = \frac{a_n}{\Lambda_{\rm QG}^{n-2}} + \frac{b_n}{M^{n-d}} + \ldots$$

(In Planck Units)

Asymptotic Regime: $M_{\mathrm{Pl},d} \gg \Lambda_{\rm QG} \gtrsim M$

- We have two scales, $\Lambda_{\rm QG}$ and M and an infinite number of Wilson Coefficients

$\longrightarrow$ Solve for the scales in terms of Wilson coefficients and substitute back

$$\Lambda_{\rm QG} \simeq M^{\gamma} \qquad 0 \leq \gamma \leq 1$$

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ESC: $\gamma = \dfrac{p}{d+p-2} \longrightarrow \dfrac{1}{d-1} \leq \gamma \leq 1$

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ESC: $\gamma = \frac{p}{d+p-2} \longrightarrow \frac{1}{9} \leq \gamma \leq 1$

- Consider **10d maximal SUGRA**:

$$\alpha_{R^4} = a_{R^4}\, M^{-6\gamma} + b_{R_4}\, M^2 + \dots$$

Assume $a_{R^4} \neq 0$

$$\alpha_{R^4} \sim M^{-6\gamma}$$

The Bottom-up Perspective:
Scaling relations between Wilson Coefficients

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[Guerrieri, (Murali,) Penedones, Vieira '21 '23]

S-Matrix Bootstrap $a_{R^4} \neq 0$ $\qquad \alpha_{R^4} \sim M^{-6\gamma}$

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Bounds on $\alpha_{D^4R^4} = a_{D^4R^4}\, M^{-10\gamma} + b_{D^4R^4}\, M^{-2}$ as $\alpha_{R^4} \to \infty$

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Scaling relations between Wilson Coefficients

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(In Planck Units)

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[Guerrieri, (Murali,) Penedones, Vieira '21 '23]

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Bounds on $\alpha_{D^4R^4} = a_{D^4R^4}\, M^{-10\gamma} + b_{D^4R^4}\, M^{-2}$ as $\alpha_{R^4} \to \infty$

<u>Upper bounds:</u> If $\gamma \geq \dfrac{1}{5}$: $|\alpha_{D^4R^4}| \leq \alpha_{R^4}^{5/3}$

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The Bottom-up Perspective: Scaling relations between Wilson Coefficients

$$\alpha_n = \frac{a_n}{\Lambda_{\rm QG}^{n-2}} + \frac{b_n}{M^{n-d}} + \dots$$

(In Planck Units)

Asymptotic Regime: $M_{\text{Pl},d} \gg \Lambda_{\rm QG} \gtrsim M$

- We have two scales, $\Lambda_{\rm QG}$ and M and an infinite number of Wilson Coefficients
  $\longrightarrow$ Solve for the scales in terms of Wilson coefficients and substitute back

$$\Lambda_{\rm QG} \simeq M^{\gamma} \qquad 0 \le \gamma \le 1$$

ESC: $\gamma = \dfrac{p}{d+p-2} \longrightarrow \dfrac{1}{9} \le \gamma \le 1$

- Consider **10d maximal SUGRA**:

[Guerrieri, (Murali,) Penedones, Vieira '21 '23]

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[Albert, Knop, Rastelli '24]

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Stronger than S-Matrix Bootstrap $|\tilde{\alpha}_{D^4R^4}| \leq \tilde{\alpha}_{R^4}$

[Albert, Knop, Rastelli '24]

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What is the Structure of Gravitational EFTs?

Can we exploit this multi-scale structure to bound gravitational Wilson Coefficients?

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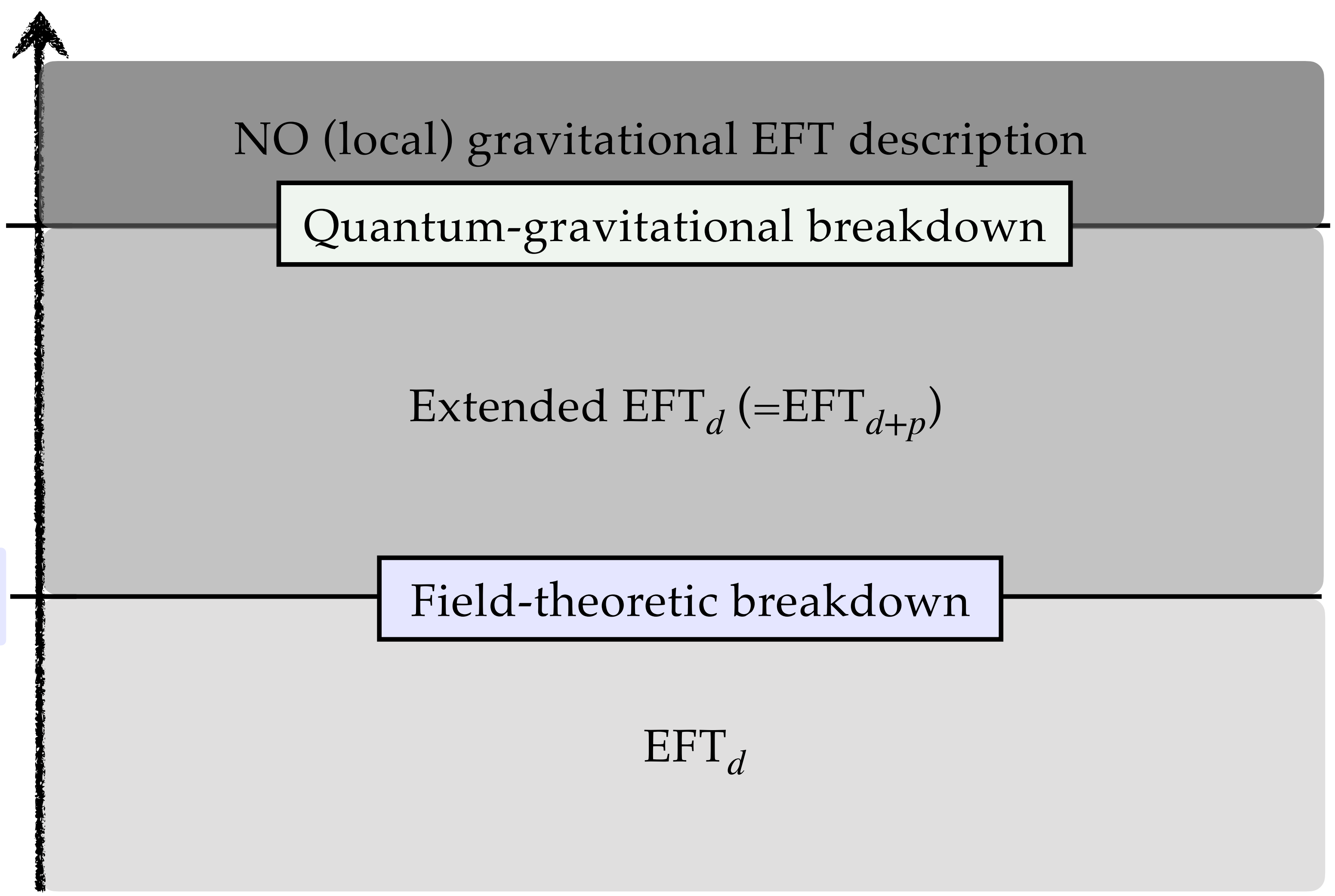
$$S_{\mathrm{EFT},\,d} = \int d^dx\sqrt{-g}\,\frac{M_{\mathrm{Pl},d}^{d-2}}{2}\left(R + \sum_{n>2}\frac{\mathcal{O}_n(R)}{\Lambda_{\mathrm{QG}}^{n-2}}\right) + \int d^dx\sqrt{-g}\,\sum_{n\geq d}\frac{\mathcal{O}_n(R)}{M_{\mathrm{t}}^{n-d}}$$

Λ_{QG}

$$M_{\mathrm{Pl},d+p}^{d+p-2}\sum_{n>2}\frac{\mathcal{O}_n(R)}{\Lambda_{\mathrm{QG}}^{n-2}}$$

M_{tower}

$$M_{\mathrm{Pl},d}^{d-2}\sum_{n>2}\frac{\mathcal{O}_n(R)}{\Lambda_{\mathrm{QG}}^{n-2}} + \sum_{n\geq d}\frac{\mathcal{O}_n(R)}{M_{\mathrm{t}}^{n-d}}$$



Summary

Can we exploit this multi-scale structure to bound gravitational Wilson Coefficients?

$$\alpha_n = \frac{a_n}{\Lambda_{\mathrm{QG}}^{n-2}} + \frac{b_n}{M^{n-d}} + \ldots$$

Double EFT Expansion + Emergent String Conjecture → Provide Targets for → S-Matrix Bootstrap

S-Matrix Bootstrap → Inform → Double EFT Expansion

Explore upper/lower bounds (in combination with emergent string conjecture) and **relative bounds** in the asymptotic regime

Outlook

- Study subleading terms and multi-scale structure
- Investigate further top-down constructions
- Explore connections with S-Matrix Bootstrap constraints
- Generalize understanding of corrections to *minimal* Black Holes in neutral case
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Thank you!

Back-up slides

The double EFT expansion: Decompactification Limits

- 4-graviton amplitude: Non-vanishing contribution to an infinite tower of UV convergent operators of the form $\frac{D^{2\ell}R^4}{M_t^{8+2\ell-d}}$ $\longrightarrow$ Change EFT at scales around $M_{\rm t} \leq \Lambda_{\rm QG}$

- Massive vs. massless thresholds and different EFTs: $\mathscr{A}_4(s,t) = \mathscr{A}_4^{\rm non-an}(s,t) + \mathscr{A}_4^{\rm an}(s,t)$

  [Green, Gutperle, Kwan, Russo, Tsytelin, Vanhove´97-´07]
  [Calderón-Infante, Castellano, AH ´25}

$$d\text{-dim EFT:} \quad \underbrace{\text{d-dimensional Massless-threshold}}_{n=0} + \underbrace{\sum b_\ell \frac{D^{2\ell}R^4}{M_t^{2\ell+8-d}}}_{n\neq 0,\ \text{Massive threshold}}\ (\textbf{FT}) + \sum a_\ell M_{{\rm Pl},d}^{d-2}\frac{D^{2\ell}R^4}{\Lambda_{\rm QG}^{2\ell+6}}\ (\textbf{QG})$$

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$D=(d+1)$-dim EFT: D-dimensional Massless-threshold

$R \to \infty$

$n=0$ $\quad$ $n \neq 0$

d-dim EFT: d-dimensional Massless-threshold $+$ $\sum b_\ell \dfrac{D^{2\ell}R^4}{M_t^{2\ell+8-d}}$ (FT) Massive threshold $+$ $\sum a_\ell M_{\rm Pl,d}^{d-2}\dfrac{D^{2\ell}R^4}{\Lambda_{\rm QG}^{2\ell+6}}$ (QG)

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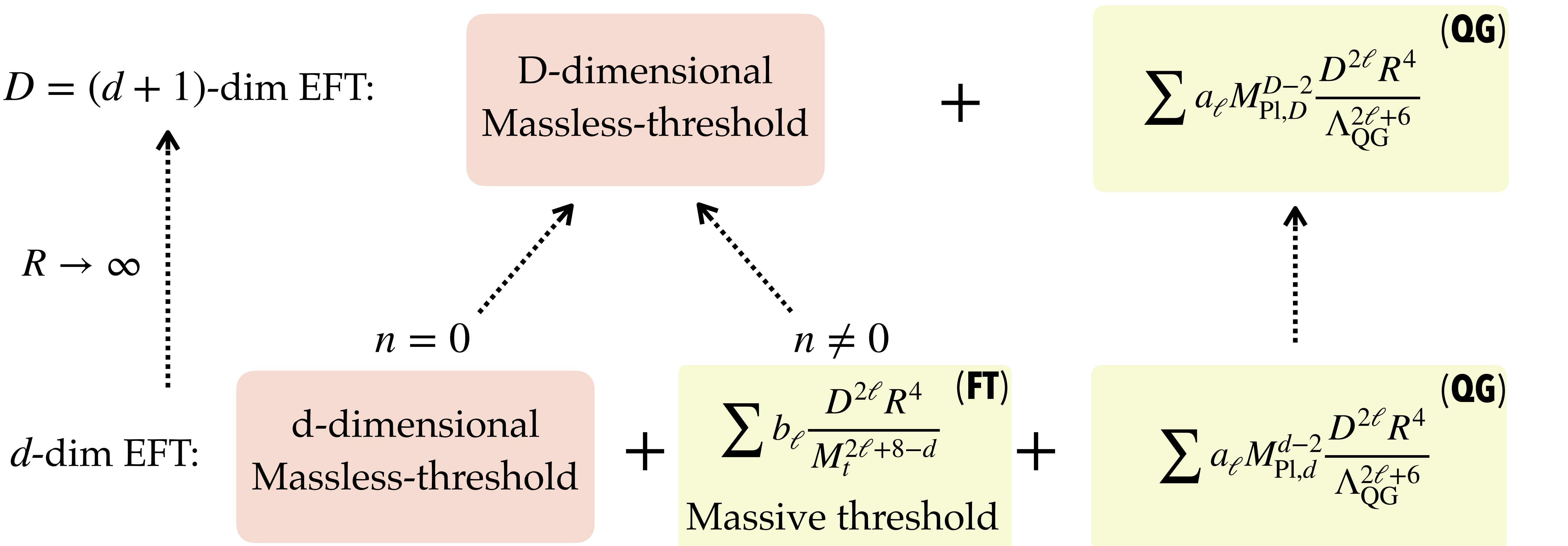

The Bottom-up Perspective: Absolute bounds in terms of EFT cutoff

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(In Planck Units)

Asymptotic Regime: $M_{\mathrm{Pl},d} \gg \Lambda_{\rm QG} \gtrsim M$

- Upper bounds in terms of M as $\alpha_n \to \infty$. Both terms bounded by:

$$|\alpha_n| \lesssim \frac{1}{M^{n-2}}$$

- Lower bounds in terms of M as $\alpha_n \to \infty$ (assuming non-vanishing coefficient):

For $n \leq d$

(FT) $|\alpha_n| \gtrsim \dfrac{1}{M^{n-d}}$

For $n > d$

(QG) $|\alpha_n| \gtrsim \dfrac{1}{\Lambda_{\rm QG}^{n-d}} \gg \mathcal{O}(1)$

\+ Emergent String Conjecture ($p = 1$):

$$M_t^{\frac{p}{d+p-2}} \gtrsim \Lambda_{\rm QG}$$

For $n \leq d+1$

(FT) $|\alpha_n| \gtrsim \dfrac{1}{M_t^{n-d}}$

For $n > d+1$

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- 1) <u>Absolute bounds in terms of EFT cutoff:</u>

[Caron-Huot, Mazac, Rastelli, Simons-Duffin '21] [Bern, Kosmopoulos, Zhiboedov '21]
2x[Caron-Huot, Li, Parra-Martinez, Simons-Duffin '22] [Albert, Knop, Rastelli '23] [Caron-Huot, Li '24]

Compatible with Double EFT bounds upon identification of EFT cutoff with M (or M_t)