Near-extremal Quantum Field Theories in 2d

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Motivation: Extremal and near-extremal black holes

Extremal black holes

- Definition 1: Extremal black holes have vanishing surface gravity on the horizon, i.e., their temperature is zero.
- Definition 2: There is a maximum charge and angular momentum for a given mass. When this bound is saturated, we have an extremal black hole.
- Generically, both definitions are equivalent but there are counterexamples. [Dias, Horowitz, Santos '21]
- We will only focus on cases where temperature goes to zero.

Universal features of extremal black holes

- Extremal black holes develop an infinitely long throat in the near-horizon region. The proper distance from horizon to any point outside the horizon is infinite.
- For a large class of black holes, near-horizon region contains an AdS_2 factor. [Kunduri, Lucietti,Reall, Figueras, Rangamani '07 '08]
- This universal behavior doesn't survive addition of any finite energy excitation. [Maldacena,Michelson, Strominger '98]
- There are several ways to see this: 1. 2d gravity Lagrangian is topological. So, stress-energy tensor vanishes.
  2. Consider 2d dilaton gravity models. Any non-zero stress-energy tensor implies dilaton diverges near the boundary destroying the AdS_2 asymptotics.
  3. Even going slightly away from extremality, near-horizon region is no-longer decoupled from the remaining spacetime. Proper distance from horizon to any point outside becomes finite.

Universal features of near-extremal black holes

- If this were the whole story, it would be of limited interest since it would be like studying just the ground state of a quantum mechanical system without any finite energy excitations.
- However, for black holes with small deviations away from extremality, a universal description also emerges by keeping leading order effect of backreaction. [Almheiri, Polchinski '14]
- It is obtainted by correcting Einstein-Hilbert action by Jackiw-Teitelboim (JT) gravity action

$$I_{JT} = C_{JT} \int d^2x \sqrt{-g}\Phi \left(R + \frac{2}{\ell_2^2} \right)$$

- The onshell JT action is given by Schwarzian action [Maldacena, Stanford, Yang '16]

$$I_{\mathrm{Sch}} = C_{\mathrm{Sch}} \int \mathrm{d}\tau \{f(\tau), \tau\} \ , \quad \{f(u), u\} = \frac{f'''}{f''} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 \ .$$

  $f(\tau)$ represents the reparametrizations of boundary AdS_2 given by

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 - \left(e^{\rho/\ell_2} + \frac{\ell_2}{2}\{f(\tau), \tau\}e^{-\rho/\ell_2}\right)\mathrm{d}\tau^2$$

- It also famously captures the low-energy regime of SYK model.
- The Schwarzian action describes a quantum mechanical model that is exactly solvable. The partition function is one-loop exact [Stanford, Witten '17]

$$Z_{\mathrm{Schw}} = \left(\frac{\pi}{\tilde{\beta}}\right)^{3/2} e^{\pi^2/\tilde{\beta}} \ , \quad \tilde{\beta} = \frac{\beta}{2C_{\mathrm{Sch}}} \ .$$

Near-extremal QFTs

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- We will refer to this regime as near-extremal/ Schwarzian regime of the dual field theory. In this regime, the field theory computation of quantities like partition function and correlation functions should be consistent with Schwarzian theory.
- This regime should involve studying thermal field theory close to zero temperature amongst other limits.

- We will address the question of existence of near-extremal limit of QFTs by looking at QFTs in two dimensions.
- We will consider CFTs, warped CFTs and Carrollian CFTs.
- A common characteristic of these theories is that there is a Virasoro factor in their symmetry algebra.

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- A common characteristic of these theories is that there is a Virasoro factor in their symmetry algebra.
- **Near-extremal CFTs** For a large class of 2d CFTs with large central charge, there exists a regime of parameters, namely, low temperature and large angular momentum where partition function and correlation functions are determined by Schwarzian theory. [Ghosh, Maxfield, Turiaci '19]
- These results are in line with the bulk computations of near-extremal BTZ.

- **Near-extremal Warped CFTs (WCFTs)** For a large class of non-unitary WCFTs with large central charge, there exists a regime of parameters, where partition function is determined by warped-Schwarzian theory. It matches the low energy behavior of complex SYK model. [AA, Castro, Detournay, Mühlmann '22]
- These results are also in line with the bulk near-extremal limit of warped black holes. Based on this we conjectured that only non-unitary WCFTs have interesting holographic duals. [AA, Castro, Detournay, Mühlmann '23]

- **Near-extremal Carrrollian CFTs (CCFTs):** CCFTs also contain a universal "near-extremal" sector. Partition function is dominated by vacuum character and looks similar to the Schwarzian partition function. [AA, Bagchi, Detournay, Grumiller, Riegler, Simon '25]
- One might wonder what is the bulk interpretation of this "near-extremal" regime of CCFTs.
- The putative bulk is 3d asymptotically flat spacetime. There are no black holes in 3d in absence of cosmological constant.
- There are flat space cosmologies but they only have one horizon.
- We argue that the bulk interpretation is given in terms of $O-$plane orbifolds which are certain solutions of 3d flat gravity. [Cornalba, Costa '05]

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- In all three cases, there is another universal regime, i.e., Cardy regime.

Near- extremal Carroll CFT$_2$

Carroll Symmetries

- Carroll symmetries arise as the $c \to 0$ limit of Poincare symmetries, making space absolute and time relative. [Lévy-Leblond '65, Sen Gupta '66]

- Carroll symmetries are associated to null hypersurfaces and are thus relevant for flat space holography.

- $\mathfrak{ccar}_d$, is isomorphic to the $(d+1)$-dimensional Bondi–van der Burgh–Metzner–Sachs (BMS) algebra, $\mathfrak{bms}_{d+1}$, which is the algebra of $d+1$ dimensional asymptotically flat spacetimes. [Duval, Gibbons, Horvathy '14].

Carroll CFT$_2$

- We are interested in $d = 2$, i.e., $\mathfrak{bms}_3$ or $\mathfrak{ccar}_2$. It consists of semidirect sum of Virasoro and an abelian algebra. Expanding the generators in Fourier modes

$$
\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} + \frac{c_{\mathrm{L}}}{12}(n^3-n)\,\delta_{n+m,\,0} \\
[L_n,\, M_m] &= (n-m)\, M_{n+m} + \frac{c_{\mathrm{M}}}{12}(n^3-n)\,\delta_{n+m,\,0} \\
[M_n,\, M_m] &= 0
\end{aligned}
$$

- L_ns are superrotations and M_ns are supertranslations. $L_0, L\pm 1, M_0, M_{\pm 1}$ generate global subalgebra $\mathfrak{isl}(2,\mathbb{R})$ coresponding to the isometries of 3d Minkowski. $c_L = 0$ and $c_M \neq 0$ for Einstein gravity.
- The 2d QFTs with these symmetries are Carroll CFT$_2$ (CCFT$_2$)—natural holographic duals to 3d asymptotically flat gravity.

Carroll Partition Function, modular transformations

- We define the partition function of a Carroll CFT_2 as

$$Z_{\mathfrak{ccar}}(\beta_{\mathfrak{car}}, \theta_{\mathfrak{car}}) = \mathrm{Tr}\, e^{-\beta_{\mathfrak{car}} H + i\theta_{\mathfrak{car}} J},$$

where $\beta_{\mathfrak{car}}$ is the inverse Carroll temperature, $\theta_{\mathfrak{car}}$ is the angular potential and

$$H = M_0, \quad J = L_0.$$

- One can obtain Carroll CFT_2 from a Lorentzian CFT_2 in the limit of vanishing speed of light. This limiting procedure also provides a way to obtain Carroll modular transformations

$$\sigma \to \frac{a\sigma + b}{c\sigma + d} \qquad \rho \to \frac{\rho}{(c\sigma + d)^2}, \qquad \sigma \equiv i\frac{\beta_{\mathfrak{car}}}{2\pi}, \rho \equiv \frac{\theta_{\mathfrak{car}}}{2\pi}.$$

- If σ is thought of as coordinate on the base manifold, $\mathcal{H}$, on which Carroll modular transformations act, ρ transforms like a vector in $\mathcal{T}_\sigma\mathcal{H}$. Thus, it is useful to think about thermal Carroll CFT$_2$ to be defined on the complex upper half plane and the corresponding tangent space.

- The Carroll modular group is generated by composing S and T transformations

$$S:\ \sigma \to -\frac{1}{\sigma} \qquad \rho \to \frac{\rho}{\sigma^2} \qquad\qquad T:\ \sigma \to \sigma + 1 \qquad \rho \to \rho\,.$$

- They satisfy the usual identities

$$S^2 = \mathbb{1} \qquad (ST)^3 = \mathbb{1}$$

Carroll Characters

- The states in a 2d Carrollian CFT are labelled with the eigenvalues of L_0 and M_0:

$$L_0|\Delta, \xi\rangle = \Delta|\Delta, \xi\rangle \qquad\qquad M_0|\Delta, \xi\rangle = \xi|\Delta, \xi\rangle \,.$$

- One can construct highest weight representations by defining primary states as

$$L_n|\Delta, \xi\rangle_p = M_n|\Delta, \xi\rangle_p = 0 \qquad \forall n > 0$$

- A generic descendant takes the form

$$|\Psi\rangle = L_{-n_1} L_{-n_2} \dots L_{-n_q} M_{-m_1} M_{-m_2} \dots M_{-m_r} |\Delta, \xi\rangle_p \qquad n_i, m_j > 0$$

Characters for highest weight representations

- For non-vacuum states, the Carroll characters are given by

$$\chi_{(c_{\rm M},c_{\rm L},\xi,\Delta)}(\rho,\sigma) = \frac{e^{-2\pi i(\sigma\frac{c_{\rm L}-2}{24}+\rho\frac{c_{\rm M}}{24})}e^{2\pi i(\sigma\Delta+\xi\rho)}}{\eta(\sigma)^2}$$

where $\eta(\sigma)$ is the Dedekind eta-function.

- For the vacuum $(\Delta = 0, \xi = 0)$, we have

$$\chi_{(c_L,c_M,0,0)}(\sigma,\rho) = \frac{e^{-2\pi i(\sigma\frac{c_{\rm L}-2}{24}+\rho\frac{c_{\rm M}}{24})}}{\eta(\sigma)^2}\left(1-e^{2\pi i\sigma}\right)^2 .$$

- The Carroll partition function is then the sum of Carroll characters

$$Z_{\mathfrak{ccar}}(\sigma,\rho) = \sum_{\rm primaries} D(\Delta,\xi)\,\chi_{(c_{\rm L},c_{\rm M},\Delta,\xi)}(\sigma,\rho) \ .$$

where $D(\Delta,\xi)$ is multiplicity of the primaries with weight (Δ,ξ).

Vacuum Dominance and Universal Carroll Sectors

Are there any universal sectors present in a generic class of 2d Carroll CFTs?

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- Modular invariance of the 2d Carrollian partition function under the S- transformation implies

$$Z_{\mathfrak{ccar}}(\sigma,\rho)=Z_{\mathfrak{ccar}}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)=\sum_{\text{primaries}}\chi_{(c_{\rm L},c_{\rm M},\Delta,\xi)}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$$

  where we wrote the partition function in terms of characters in the S-dual channel.

- We look for regimes where the vacuum character is the dominant contribution to the partition function in the S-dual channel

$$\frac{\chi_{(c_{\rm L},c_{\rm M},\Delta,\xi)}}{\chi_{(c_{\rm L},c_{\rm M},0,0)}}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)\to 0,\quad \forall\Delta,\xi\neq 0\ .$$

- It turns out that there are six regimes/ sectors where this happens

Sector	β	Ω	θ	$\beta\Omega^2$	Δ	ξ
Cardy	0^-	< 0	$i \cdot 0$	≤ 0	≥ 0	> 0
hard	< 0	0^-	$i \cdot 0$	0^-	≥ 0	> 0
cold	$-\infty$	0^-	$\in i\,\mathbb{R}$	0^-	≥ 0	> 0
Schwarzian	$\boldsymbol{-\infty}$	$\boldsymbol{0^-}$	$\boldsymbol{i \cdot \infty}$	0^-	$\boldsymbol{\geq 0}$	$\boldsymbol{> 0}$
Boltzmann	> 0	$i \cdot 0^\pm$	$\in \mathbb{R}$	0^-	arbitrary	> 0
hot	0^+	$\in i\,\mathbb{R}$	0	0^-	arbitrary	> 0

Table 1: Six sectors of vacuum dominance grouped in three similar pairs

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hot	0^+	$\in i\,\mathbb{R}$	0	0^-	arbitrary	> 0

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- Carroll temperature is negative for all of the regimes except for the Boltzmann and the hot sectors.

- The partition function in the Cardy regime is well-approximated by

$$Z_{\mathrm{Cardy}}(\beta,\Omega)\approx\exp\left[\frac{\pi^2}{6}\left(\frac{c_{\mathrm{L}}}{|\beta\Omega|}+\frac{c_{\mathrm{M}}}{|\beta\Omega^2|}\right)\right].$$

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- In the Cold regime, the partition function is

$$Z_{\mathrm{cold}}(\beta,\Omega)\approx \exp\left[\frac{\pi^2}{6}\left(\frac{c_{\mathrm{L}}-2}{|\beta\Omega|}+\frac{c_{\mathrm{M}}}{|\beta\Omega^2|}\right)\right]\left(\frac{1-e^{-\frac{4\pi^2}{|\beta\Omega|}}}{\eta(\frac{2\pi i}{\beta\Omega})}\right)^2.$$

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- In both of the regimes regimes, one finds BMS-Cardy formula for the entropy to the leading order excluding logarithmic terms

$$S_{\text{Cardy}} \approx S_{\text{Cold}} \approx \frac{\pi^2}{3}\left(\frac{c_{\text{L}}}{|\beta\Omega|} + \frac{c_{\text{M}}}{|\beta\Omega^2|}\right) .$$

Schwarzian sector of $CCFT_2$

- The partition function in the Schwarizan sector is given by [AA, Bagchi, Detournay, Grumiller, Riegler, Simon '25]

$$\boxed{Z_{\text{Schwarzian}}(\beta,\Omega) \approx \frac{(2\pi)^5}{(\beta\Omega)^3}\,\exp\left[\frac{\beta\Omega}{12}+\frac{\pi^2}{6}\left(\frac{c_{\mathrm{L}}-2}{\beta\Omega}+\frac{c_{\mathrm{M}}}{|\beta\Omega^2|}\right)\right]}\,.$$

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- The prefactor indicates the contribution of six zero modes corresponding to six global generators $M_{0,\pm1}, L_{0,\pm1}$.
- This is in contrast to the 3 zero modes of the Schwarzian theory.

Holographic Interpretation

- There is a limit of BTZ metric

$$r_{\pm} = \sqrt{\ell}\hat{r}_{\pm}\ , \quad \ell \to \infty$$

which leads to $M \to 0$, $J = \frac{\hat{r}_+ \hat{r}_-}{4G}$.

- The geometry is an $O-$ plane orbifold ($O-$ fold) of flat space that has naked CTCs [Cornalba, Costa '03 '05]

$$ds^2 = -8GJ\,dt\,d\varphi + r^2\,d\varphi^2 + \frac{r^2}{(4GJ)^2}\,dr^2 \qquad \varphi \sim \varphi + 2\pi\,.$$

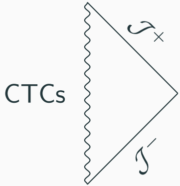

Figure 1: Penrose diagram of $O-$folds.

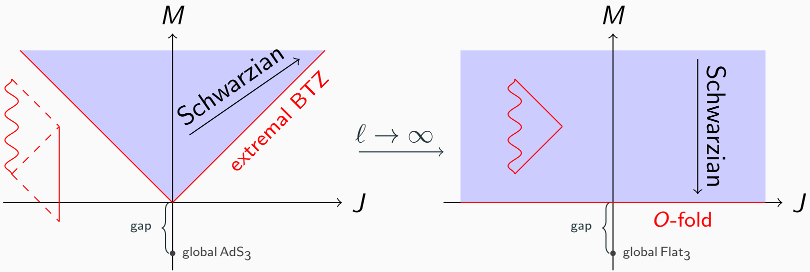

Figure 2: Gravity side of Schwarzian sectors in AdS_3/CFT_2 (left) and $Flat_3/CCFT_2$ (right)

Negativity of specific heat

- When the vacuum dominates in the modular dual channel, we have shown that the specific heat is negative for all sectors.
- If this also holds in higher dimensions, it would provide a field theory explanation for the negativity of specific heat of higher dimensional black holes like Schwarzschild and Kerr.
- Negative specific heat signals a thermodynamic instability.
- It also implies that one cannot use saddle point approximation to do the Laplace transform to go from GC ensemble to canonical or microcanonical ensemble.
- Nevertheless, we could do the integrals exactly, without using saddle point approximation, by choosing a suitable contour in the complex chemical potential (θ) plane.
- It would be interesting to understand these mysterious contours better.

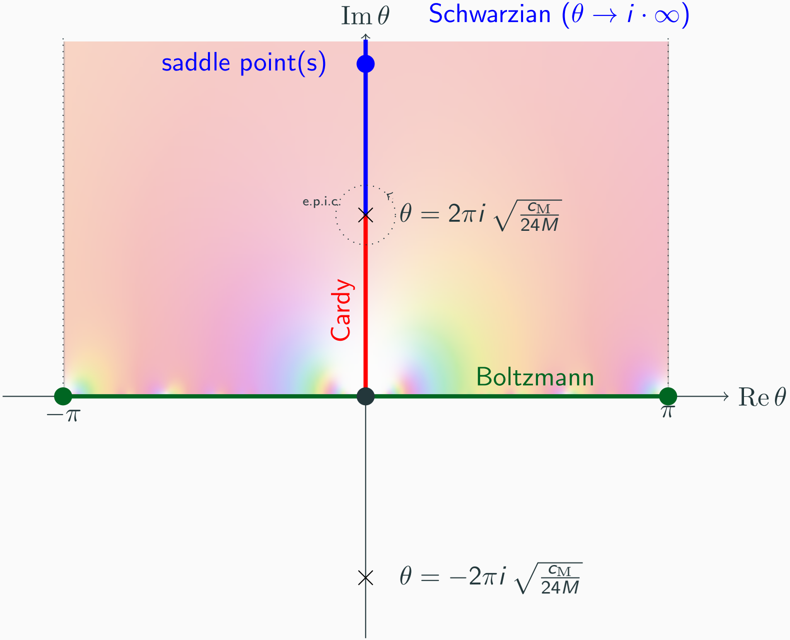

Figure 3: Analytic structure of partition function in complex θ-plane

Summary and future directions

- We observed that three different two-dimensional QFTs have a universal near-extremal/ Schwarzian-like sector; different from the universal Cardy sector of these theories.
- Common characteristics of these theories that lead to a Schwarzian-like sector: Modular symmetry, atleast one copy of Virsoro, and two dimensions.
- The holographic duals are near-extremal BTZ black holes for CFTs and warped balck holes for Warped CFTs while it is the $O-$plane orbifold for the Carrollian CFTs.
- Can one find such sectors in other QFTs, particularly in higher dimensions?
- There should be a mechanism for this to happen since higher dimensional near-extremal blackholes also show the Schwarzian behavior and their putative duals are higher dimensional QFTs.

Thank you!

	induced	real Z	highest-weight	real Z
Cardy	✗	n.a.	✓	✓
hard	✗	n.a.	✓	✓
cold	✗	n.a.	✓	✓
Schwarzian	✗	n.a.	✓	✓
Boltzmann	✗✓	✓ if $c_{\mathrm{L}} = 0$	✗✓	✗
hot	✗✓	✓ if $c_{\mathrm{L}} = 0$	✗✓	✗

Table 2: CCFT sectors vs. CCFT representations

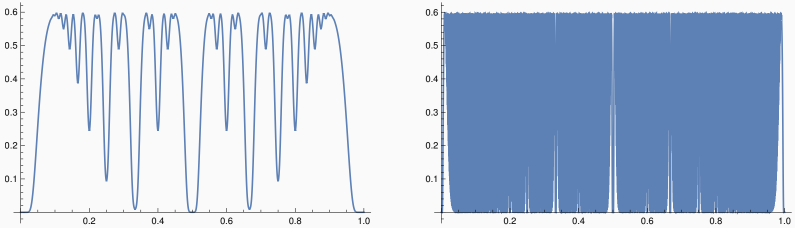

Figure 4: Plots of $\sqrt{\epsilon}\,|\eta(x+i\epsilon)|^2$ with $x \in [0,1]$. Left: $\epsilon = 10^{-2}$. Right: $\epsilon = 10^{-4}$.

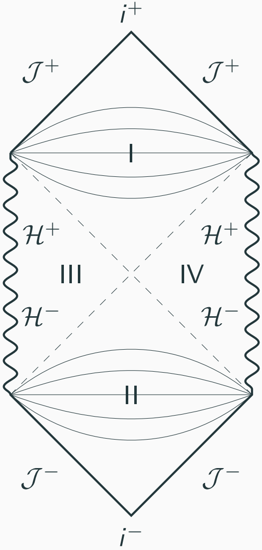

Figure 5: 2d slice of Penrose diagram for flat space cosmologies

CFT$_2$ rep.	Virasoro central charge	CCFT$_2$ rep.	BMS central charge
HW	$\boldsymbol{0 < \bar{c} = c < \infty}$	**Induced**	$\boldsymbol{c_{\mathrm{L}} = 0,\ c_{\mathrm{M}} = 0}$
HW	$0 < \bar{c} < c < \infty$	Induced	$c_{\mathrm{L}} = c - \bar{c},\ c_{\mathrm{M}} = 0$
HW	$c = \bar{c} \to +\infty$	*Induced*	$c_{\mathrm{L}} = 0,\ c_{\mathrm{M}} = \textit{finite}$
HW	$c = -\bar{c} \to +\infty$	Induced	$c_{\mathrm{L}} \to +\infty,\ c_{\mathrm{M}} = 0$
Flipped	$\boldsymbol{0 < \bar{c} = c < \infty}$	**HW**	$\boldsymbol{c_{\mathrm{L}} = 2c,\ c_{\mathrm{M}} = 0}$
Flipped	$0 < \bar{c} < c < \infty$	HW	$c_{\mathrm{L}} = c + \bar{c},\ c_{\mathrm{M}} = 0$
Flipped	$c = \bar{c} \to +\infty$	HW	$c_{\mathrm{L}} \to +\infty,\ c_{\mathrm{M}} = 0$
Flipped	$c = -\bar{c} \to +\infty$	HW	$c_{\mathrm{L}} = 0,\ c_{\mathrm{M}} = \text{finite}$
Flipped	$c - \bar{c} \to +\infty,\ c + \bar{c} = 2a$	*HW*	$c_{\mathrm{L}} = 2a,\ c_{\mathrm{M}} = \textit{finite}$

Table 3: Induced and highest-weight representations in CCFT$_2$ as limit of CFT$_2$. **Bold:** vanilla CFT$_2$ and its CCFT$_2$ limit. *Italics:* flat space Einstein gravity (with quantum corrected central charges)