Modular completions and RR sector couplings in Type II at higher derivatives

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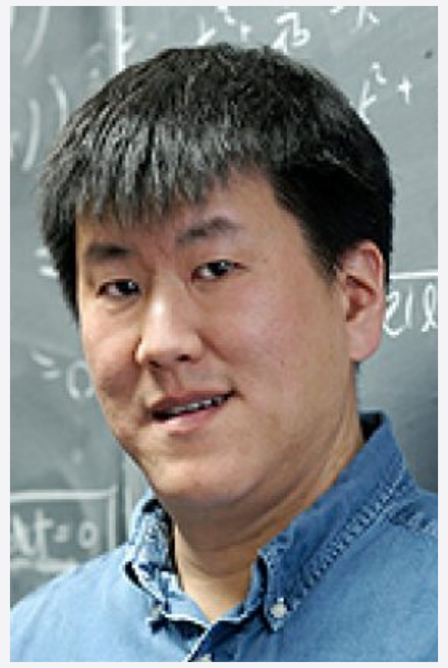
Upshot

We compute **new Ramond-Ramond (RR) couplings** in the 10D Type II effective actions from 5-point string scattering and demonstrate **consistency** with **modular invariance, supersymmetry, T-duality and M-theory**!

2205.11530: Type IIB at eight derivatives: insights from superstrings, superfields and superparticles

2507.07934: Type IIB at eight derivatives: five-point axio-dilaton couplings



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Outline

1 Introduction

2 Five-point kinematics for 3-forms in Type IIB

2205.11530

3 The scalar-graviton sector at five points

2507.07934

4 M-theory vs. Type IIA vs. Type IIB

2507.07934

5 Conclusions

Introduction

The α' expansion in Type IIB supergravity

Schematic α' expansion of Type IIB supergravity

$$\mathcal{L} = \mathcal{L}^{\text{tree}} + (\alpha')^3\, \mathcal{L}^{(3)} + \sum_{n=4}^{\infty} (\alpha')^n\, \mathcal{L}^{(n)}$$

The tree level supergravity action for these fields is

$$\mathcal{L}^{\text{tree}} = R - 2\left|\mathcal{P}\right|^2 - \frac{1}{2\cdot 3!}\left|G_3\right|^2 - \frac{1}{4\cdot 5!}F_5^2$$

Massless bosonic fields

$$\tau = \chi + ie^{-\phi}\,, \quad \mathcal{P}_m = \frac{i}{2\mathrm{Im}(\tau)}\nabla_m\tau\,, \quad G_3 = \frac{1}{\mathrm{Im}(\tau)^{1/2}}(F_3 - \tau H_3)$$

Operators in $\mathcal{L}^{(3)}$ correspond to 8-derivative terms

$$\mathcal{L}^{(3)} \sim \left\{R^4 + R^3\left(G_3^2 + \left|G_3\right|^2 + \overline{G}_3^2 + \left|\mathcal{P}\right|^2\right) + \ldots\right\}$$

The full action is invariant under $\mathrm{SL}(2,\mathbb{Z})$ transformations

$$\tau \to \frac{a\tau + b}{c\tau + d}\,, \quad \mathcal{P} \to \frac{c\overline{\tau} + d}{c\tau + d}\mathcal{P}\,, \quad G_3 \to \left(\frac{c\overline{\tau} + d}{c\tau + d}\right)^{1/2} G_3 \longrightarrow$$

MODULAR INVARIANCE OF TERMS IN $\mathcal{L}^{(3)}$

Introduction

The quartic action of Type IIB

Policastro, Tsimpis hep-th/0603165, 0812.3138

Quartic action completely determined using pure spinor formalism

$$\mathscr{L}^{(3)} \supset f_0(\tau,\bar{\tau})\left(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10}\right)\left\{R^4 + 6R^2\left(\left|\nabla\mathscr{P}\right|^2 + \left|\nabla G_3\right|^2\right) + \dots\right\} + \dots$$

$SL(2,\mathbb{Z})$-invariance through **Eisenstein series of weight 3/2** [Green hep-th/9706175]

$$f_0(\tau,\bar{\tau}) = 2\zeta(3)\,\mathrm{Im}(\tau)^{3/2} + \frac{2\pi^2}{3}\,\mathrm{Im}(\tau)^{-1/2} + \mathcal{O}\left(e^{-\mathrm{Im}(\tau)}\right)$$

Kinematics (mostly) fixed by **index structures** t_8 **and** ϵ_{10}

$$t_8 M^4 = 6\left(4\mathrm{tr}(M^4) - \mathrm{tr}(M^2)^2\right) \qquad \epsilon_{\alpha_1\dots\alpha_m\mu_1\dots\mu_n}\,\epsilon^{\alpha_1\dots\alpha_m\nu_1\dots\nu_n} = -(m!)\,(n!)\,\delta^{[\nu_1}_{\mu_1}\dots\delta^{\nu_n]}_{\mu_n}$$

Introduction

The role of $U(1)$ violation — Part 1

Fields are charged under $U(1)$ R-symmetry

$$Q_R(\mathscr{P}) = 2 \quad , \quad Q_R(G_3) = 1 \quad , \quad Q_R(g_{MN}) = Q_R(F_5) = 0$$

P-POINT INTERACTIONS CARRY A MAXIMAL $U(1)$ CHARGE $\left|Q_{U(1)}\right| \leq 2(P-4)$

Green hep-th/9903124

Maximally $U(1)$-violating (MUV) couplings $\left|Q_{U(1)}\right| \equiv 2(P-4)$ are special:

Green hep-th/9903124
Boels 1204.4208
Green et al. 1904.13394

- unique modular completion,
- protected by supersymmetry (1/2-BPS), and
- kinematic information determined by a linearised superfield.

Introduction

The role of $U(1)$ violation — Part 2

Green et al. hep-th/9808061, 1904.13394

Total $U(1)$ charge → organisational principle at 8-derivatives

$$\mathscr{L}^{(3)} = f_{12}\,\Lambda^{16} + \ldots + f_4\,G_3^8 + \ldots + f_1\,G_3^2 R^3 + f_0\left(R^4 + \left|G_3\right|^2 R^3 + \ldots\right) + f_{-1}\,\overline{G}_3^2 R^3 + \ldots + f_{-12}\,(\Lambda^*)^{16}$$

For example, the dilatino Λ^{16} terms were obtained in [Green et al. hep-th/9710151]

matching supersymmetry approach of [Green, Sethi hep-th/9808061]

$\mathrm{SL}(2,\mathbb{Z})$-invariance through suitable $\mathrm{SL}(2,\mathbb{Z})$-covariant modular forms

$$f_w(\tau,\bar\tau) = \sum_{(\hat l_1,\hat l_2)\neq(0,0)} \frac{\mathrm{Im}(\tau)^{3/2}}{(\hat l_1 + \tau \hat l_2)^{3/2+w}(\hat l_1 + \bar\tau \hat l_2)^{3/2-w}} \qquad f_w\Big(\frac{a\tau+b}{c\tau+d}, \frac{a\bar\tau+b}{c\bar\tau+d}\Big) = \Big(\frac{c\tau+d}{c\bar\tau+d}\Big)^w f_w(\tau,\bar\tau)$$

Introduction

Schwinger-time computations in M-theory

Green et al. hep-th/9706175, hep-th/9710151, hep-th/9907155

How can we compute $(\alpha')^3$ corrections to the Type IIB effective action beyond four points?

We compute **Schwinger-like** amplitudes in M-theory compactified on **2-torus** T^2

Integral over non-compact momenta

Trace over fermion operators

Vertex operators of 9D M-theory

$$\mathcal{A}_P(h, C_3) = \int \frac{\mathrm{d}t}{t} \int \mathrm{d}^9 p \, \mathrm{e}^{-t\,\mathbf{p}^2} \sum_{l_1, l_2} \mathrm{e}^{-t\, g^{ab} l_a l_b} \, \mathrm{Tr}_S \left\langle \prod_{r=1}^{P} \int_0^t \mathrm{d}t^{(r)} \, V_r(h, C_3; t^{(r)}) \right\rangle \xrightarrow{v_0 \to 0} \textbf{Effective Vertices in 10D type IIB action}$$

Sum of KK/winding states along T^2

Integration over points on the worldline

$\mathrm{Vol}(T^2) = v_0$

Reviews and introductions:

[Strassler hep-ph/9205205, Russo, Tseytlin: hep-th/9707134, Schubert hep-th/0101036]

Introduction

Example: 4-graviton amplitude — schematics

Green, Gutperle, Vanhove: hep-th/9706175

Graviton vertex operator

$$V_h \supset 2h_{ij}\,(\theta\Gamma^{il}\theta)(\theta\Gamma^{jm}\theta)k_l k_m$$

$$\mathscr{A}_4(h^4) \sim \int \frac{\mathrm{d}t}{t^{3/2}} \sum_{l_1,l_2} \mathrm{e}^{-t\,g^{ab}\,l_a l_b}\,\mathrm{Tr}\langle V_h^4\rangle \xrightarrow{v_0\to 0} t_8 t_8 R^4 \Big(\underbrace{2\zeta(3)\,\mathrm{e}^{-3\phi/2}}_{\text{tree}} + \underbrace{\frac{2\pi^2}{3}\,\mathrm{e}^{\phi/2}}_{1-\text{loop}}\Big) + \ldots$$

Green, Gutperle, Vanhove [hep-th/9706175] showed that this amplitude matches closed superstring scattering at tree and 1-loop [Gross, Witten 1986, Sasaki et al. 1986] …

… **but there is more!**

Introduction

Example: 4-graviton amplitude — details

Green, Gutperle, Vanhove: hep-th/9706175

Kinematics from trace over fermion zero modes

$$t_{16}\,R^4 = \mathrm{Tr}(\,\theta\,\Gamma^{i_1 i_2}\,\theta\,\ldots\,\theta\,\Gamma^{i_{15} i_{16}}\,\theta\,)\,R_{i_1 i_2 i_3 i_4}\,\ldots\,R_{i_{13} i_{14} i_{15} i_{16}} \supset t_8 t_8 R^4$$

$$\mathscr{A}_4(h^4) = t_{16}\,R^4 \left(C v_0 + \frac{f_0(\tau,\bar{\tau})}{\sqrt{v_0}} \right)$$

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M-theory R^4-term in decompactification limit $v_0 \to \infty$

Divergence $C = \infty$ in this formalism

$\downarrow$

regularisation scheme proposed in [Blumenhagen, Cribiori, Gligovic, Paraskevopoulou: 2404.01371]

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Contribution in Type IIB limit $v_0 \to 0$

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Contribution in Type IIB limit $v_0 \to 0$

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Winding modes on T^2 give rise to modular function

$$f_0(\tau,\bar\tau) = \sum_{(\hat{l}_1,\hat{l}_2)\neq(0,0)} \frac{\mathrm{Im}(\tau)^{\frac{3}{2}}}{\hat{l}_1 + \tau\hat{l}_2^{\;\;3}} = 2\zeta(3)\,\mathrm{Im}(\tau)^{3/2} + \frac{2\pi^2}{3}\,\mathrm{Im}(\tau)^{-1/2} + \mathcal{O}(e^{-\mathrm{Im}(\tau)})$$

VERIFIES EXPECTED $\mathrm{SL}(2,\mathbb{Z})$ MODULAR COMPLETION OF R^4 IN TYPE IIB

Green, Gutperle hep-th/9612127, hep-th/9701093, Kiritsis, Pioline hep-th/9707018

Five-point Kinematics for 3-forms in Type IIB

MUV contact terms from M-theory

Liu, Minasian, Savelli, **AS**: 2205.11530

Compute MUV amplitudes for 3-forms and gravitons in M-theory on T^2

Graviton vertex operator	3-form vertex operator	KK-momenta
$V_h \supset 2(k_l k_m h_{ij})\,(\theta\Gamma^{il}\theta)(\theta\Gamma^{jm}\theta)$	$V_{G_3} \supset -\sqrt{2}\,G_{lmn}\,P_z\,(\theta\Gamma^{lmn}\theta)$	$P_z = \frac{1}{\sqrt{v_0\tau_2}}(l_1 - \tau l_2)$

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MUV P-point amplitudes of $\mathrm{U}(1)$ charge $2w$

$$\mathscr{A}(G_3^{2w}\,h^{4-w}) = \frac{2^{4-w}\,2^w}{2^6\,\Gamma(3/2)}\,I(P,w)\,t_{16+2w}G_3^{2w}R^{4-w}$$

$$t_{16+2w} = \mathrm{Tr}(\underbrace{\theta\,\Gamma^{(2)}\,\theta\,\ldots}_{2(4-w)}\;\underbrace{\theta\,\Gamma^{(3)}\,\theta\ldots}_{2w})$$

where upon Poisson resummation

$$I(P,w) = \int_0^\infty \frac{\mathrm{d}t}{t^{\frac{11}{2}-P}} \sum_{l_1,l_2\in\mathbb{Z}} P_z^{2w}\,\mathrm{e}^{-t\,g^{ab}_{T^2}\,l_a l_b} = 4v_0^{-1/2}\,\Gamma\left(\tfrac{3}{2}+w\right)f_w(\tau,\bar\tau)$$

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U(1) NON-EXISTENT IN M-THEORY
→ NO TERM $\sim v_0$

Green, Gutperle, Kwon: hep-th/9907155

Five-point Kinematics for 3-forms in Type IIB

MUV contact terms and supersymmetry

Liu, Minasian, Savelli, **AS**: 2205.11530

In the limit $v_0 \to 0$, we obtain MUV contact terms in 10D Type IIB effective action

$$\mathscr{L}^{MUV}(G_3, \overline{G}_3, R) = \sum_{w=0}^{4} c_w\, f_w(\tau,\bar\tau)\, t_{16+2w}\, G_3^{2w}\, R^{4-w} + \mathrm{c\,.\,c\,.} \quad , \qquad c_w = \frac{2}{\sqrt{\pi}}\,\Gamma\!\left(\tfrac{3}{2}+w\right)$$

Coefficients c_w are consistent with SUSY [Green et al. hep-th/9808061, 1904.13394]

$$c_w f_w = 2^w\, \mathscr{D}_{w-1} \,\dots\, \mathscr{D}_0\, f_0 \qquad\qquad \mathscr{D}_w f_w = i\left(\tau_2 \frac{\partial}{\partial\tau} - \frac{iw}{2}\right) f_w = \frac{3+2w}{4} f_{w+1}$$

Index structures expected from linearised superspace description [Howe, West 1983]

$$\Phi(x,\Theta) = \tau + \Theta^2 G_3 + \Theta^4 R + \dots \qquad \Rightarrow \qquad \int \mathrm{d}^{10}x\, \mathrm{d}^{16}\Theta\; e\, \Phi^{4+w} \supset t_{16+2w}\, G_3^{2w}\, R^{4-w}$$

Five-point Kinematics for 3-forms in Type IIB

Superparticles vs. Superstrings

So far, effective supergravity amplitudes in compactified M-theory.

How does this compare against fully fledged closed string amplitudes?

Five-point Kinematics for 3-forms in Type IIB

Superparticles vs. Superstrings

So far, effective supergravity amplitudes in compactified M-theory.

How does this compare against fully fledged closed string amplitudes?

Let us make a comparison at the **5-point function** level:

$$G_3^2 R^3 \qquad \left|G_3\right|^2 R^3 \qquad \overline{G}_3^2 R^3$$

Closed string 5-point amplitudes

1-loop: [Richards 0807.2421, 0807.3453]

tree: [Schlotterer et al. 1205.1516, 1307.3534]

The corresponding effective actions obtained in

[Liu, Minasian 1304.3137, 1912.10974]

Five-point Kinematics for 3-forms in Type IIB

Effective actions from string scattering

Liu, Minasian 1304.3137, 1912.10974

From pure NSNS five-point scattering, one determines the string effective action

$$\mathcal{L}_{R(\Omega_+)^4} = \alpha f_0(\tau,\bar\tau)\left[t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right]\left(R^4 + 6\left|\nabla G_3\right|^2 R^2 + 2\left|G_3\right|^2 R^3\right)$$

$$\mathcal{L}_{G_3{}^2R^3} = \alpha f_0(\tau,\bar\tau)\left[-\frac{1}{2}t_8 t_8\left|G_3\right|^2 R^3 - \frac{7}{24}\epsilon_9\epsilon_9\left|G_3\right|^2 + 2\cdot 4!\sum_i d_i\, G_3^{\mu\nu\lambda}\,\overline{G}_3^{\rho\sigma\zeta}\,(R^3)^{(i)}_{\mu\nu\lambda\rho\sigma\zeta}\right]$$

$$\mathcal{L}_{G_3^2R^3} = \alpha f_1(\tau,\bar\tau)\left[\frac{3}{4}t_8 t_8 G_3^2 R^3 - \frac{1}{16}\epsilon_9\epsilon_9 G_3^2 R^3 - 3\cdot 4!\sum_i d_i\, G_3^{\mu\nu\lambda}\,G_3^{\rho\sigma\zeta}\,(R^3)^{(i)}_{\mu\nu\lambda\rho\sigma\zeta}\right]$$

$$\mathcal{L}_{\overline{G}_3^2R^3} = \alpha f_{-1}(\tau,\bar\tau)\left[\frac{3}{4}t_8 t_8 \overline{G}_3^2 R^3 - \frac{1}{16}\epsilon_9\epsilon_9 \overline{G}_3^2 R^3 - 3\cdot 4!\sum_i d_i\, \overline{G}_3^{\mu\nu\lambda}\,\overline{G}_3^{\rho\sigma\zeta}\,(R^3)^{(i)}_{\mu\nu\lambda\rho\sigma\zeta}\right]$$

YES, IT'S A MESS…

Five-point Kinematics for 3-forms in Type IIB

11D superparticles vs. 10D string scattering

Liu, Minasian, Savelli, **AS**: 2205.11530

Try to match MUV couplings derived independently from **string theory** vs. **M-theory**:

$$\alpha f_1(\tau,\bar\tau)\left[\frac{3}{4}t_8t_8G_3^2R^3-\frac{1}{16}\epsilon_9\epsilon_9G_3^2R^3-3\cdot 4!\sum_i d_i\,G_3^{\mu\nu\lambda}\,G_3^{\rho\sigma\zeta}\,(R^3)^{(i)}_{\mu\nu\lambda\rho\sigma\zeta}\right]\overset{???}{=}\frac{3\,\alpha}{2}\,f_1(\tau,\bar\tau)\,t_{18}G_3^2R^3$$

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Higher dimensional index structures:
[Green et al. hep-th/0506161,
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We compute the tensor structure $t_{18} = \mathrm{Tr}(\,(\theta\,\Gamma^{(2)}\,\theta)^6\,(\theta\,\Gamma^{(3)}\,\theta)^2\,)$ using group theory.

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We compute the tensor structure $t_{18}=\mathrm{Tr}(\,(\theta\,\Gamma^{(2)}\,\theta)^6\,(\theta\,\Gamma^{(3)}\,\theta)^2\,)$ using group theory.

We define 24-dimensional basis of R^3 and expand various expressions into this basis finding e.g.

	a_1	a_2	b_1	b_2	b_3	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
$\epsilon_9\epsilon_9G_3^2R^3$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{4}$	$\frac{-1}{4}$	$\frac{1}{2}$	$\frac{1}{32}$	$\frac{-1}{2}$	$\frac{1}{16}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	0	-1	$\frac{1}{4}$	$\frac{-1}{2}$
$t_{18}G_3^2R^3$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{4}$	$\frac{-1}{4}$	$\frac{1}{2}$	0	0	0	0	0	0	0	1	-2	$\frac{1}{2}$	$\frac{-1}{2}$

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
$\epsilon_9\epsilon_9G_3^2R^3$	0	$\frac{-1}{4}$	$\frac{-1}{2}$	$\frac{-1}{3}$	-1	$\frac{-1}{4}$	2	$\frac{-1}{8}$
$t_{18}G_3^2R^3$	1	$\frac{-1}{2}$	$\frac{-1}{2}$	0	0	0	0	0

Five-point Kinematics for 3-forms in Type IIB

11D superparticles vs. 10D string scattering

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$t_{18}G_3^2R^3$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{4}$	$\frac{-1}{4}$	$\frac{1}{2}$	0	0	0	0	0	0	0	1	-2	$\frac{1}{2}$	$\frac{-1}{2}$	1	$\frac{-1}{2}$	$\frac{-1}{2}$	0	0	0	0	0

Five-point Kinematics for 3-forms in Type IIB

5-point contact terms — Summary

Liu, Minasian, Savelli, **AS**: 2205.11530

Overall, the revised string effective action from 5-point scattering of two 3-forms can be written as

$$\mathscr{L}^{(3)} \supset f_0\, t_{16}\, R^4 + f_0 \left[2\, t_8\, t_8 - \tfrac{1}{2}\, \epsilon_8\, \epsilon_8 - \tfrac{1}{3}\, \epsilon_9\, \epsilon_9 - t_{18}\right] \left| G_3 \right|^2 R^3 + \tfrac{3}{2} \left(f_1\, t_{18} G_3^2 R^3 + f_{-1}\, t_{18} \overline{G}_3^2 R^3 \right)$$

Comments:

- huge simplification due to generalised index structure t_{18}
- modular forms $f_w(\tau, \bar{\tau})$ derived from first principles via M-theory
- MUV kinematics completely specified by t_{18} due to linearised superfield [Howe, West 1983]

The scalar-graviton sector at five points

Hide and Seek with the Dilaton

Liu, Minasian, Savelli, **AS**: 2507.07934

Let us look interactions involving dilatons ϕ and axions χ at the five-point level!

We need to compute **mixed RR/NSNS-sector amplitudes** to determine full modular completion!

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↓

Attempting to find $\mathrm{SL}(2,\mathbb{Z})$ completions **just from the structure of NSNS vertex operators** as in [Kehagias, Partouche hep-th/9710023] gives **incomplete result**:

- mixed RR/NSNS-sector couplings contribute new kinematics,
- bound on maximal $\mathrm{U}(1)$ charge has to be respected, and
- pole subtraction discriminates between RR- and NSNS-sector.

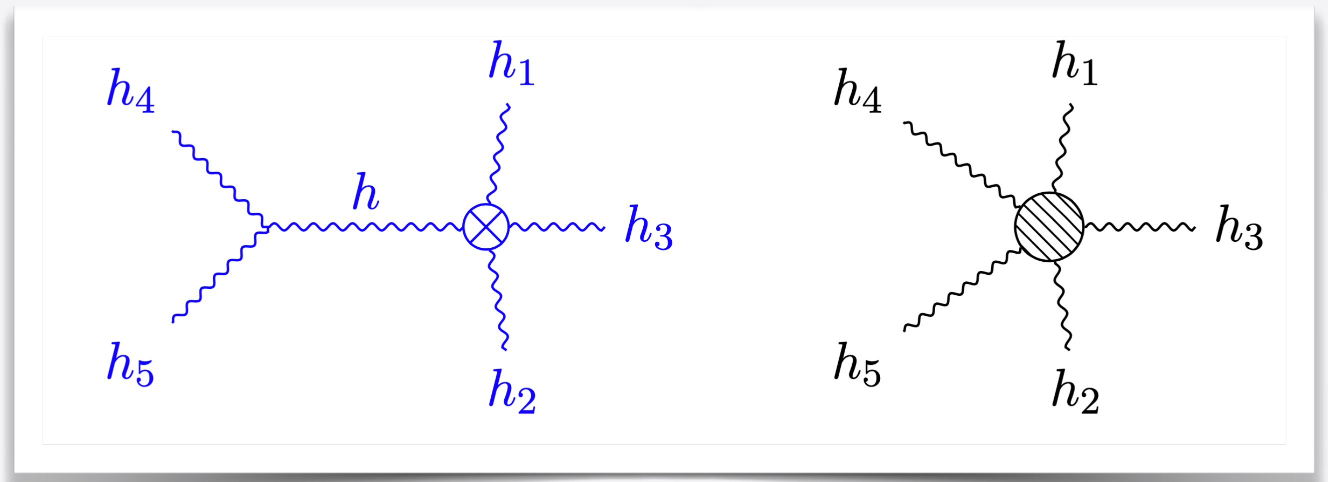
The scalar-graviton sector at five points

String scattering and pole subtraction in the NSNS-sector — Part 1

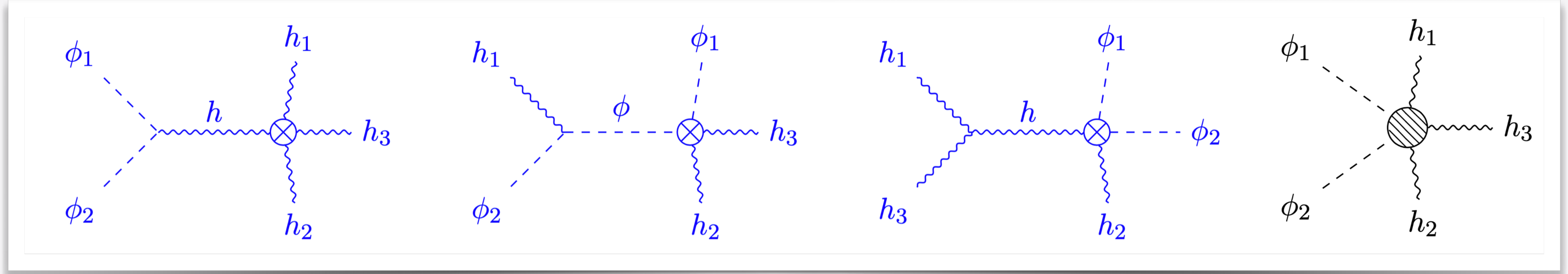
Liu, Minasian, Savelli, **AS**: 2507.07934

Five-point scattering for **dilatons** ϕ and **gravitons** h

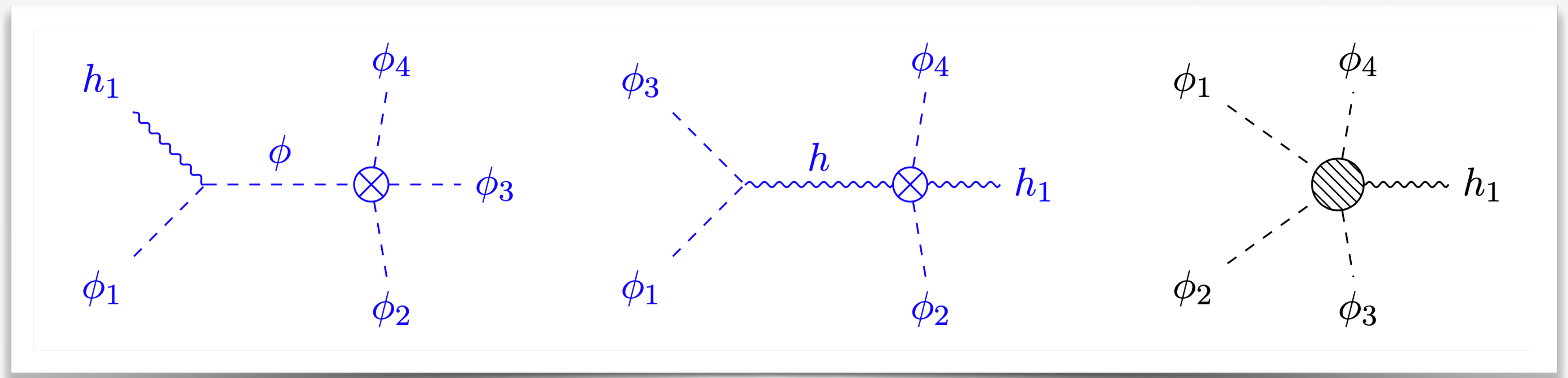
h^5



$\phi^2 h^3$



$\phi^4 h$



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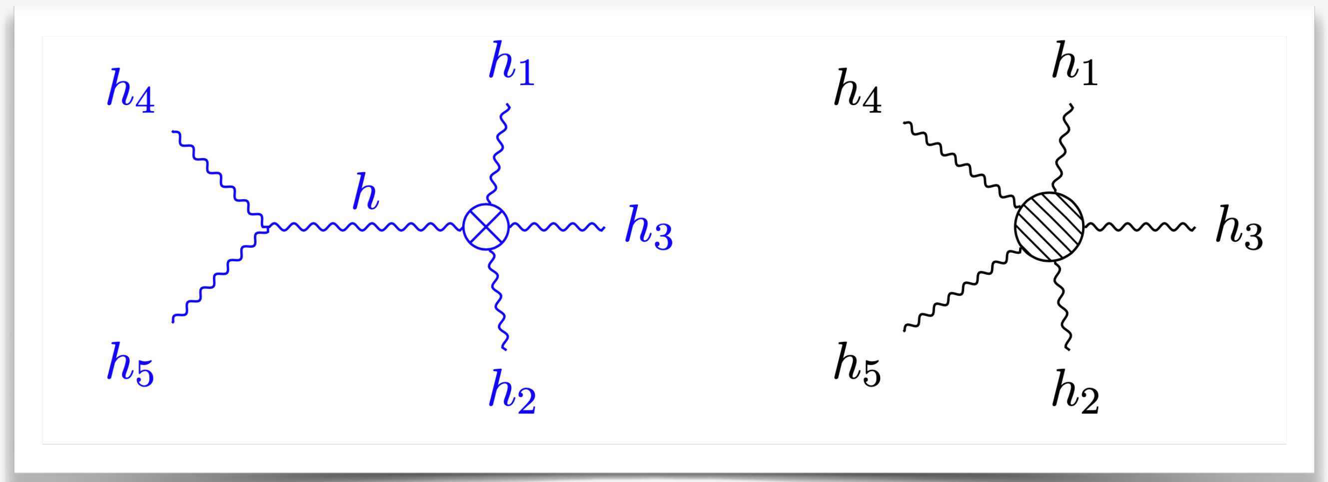

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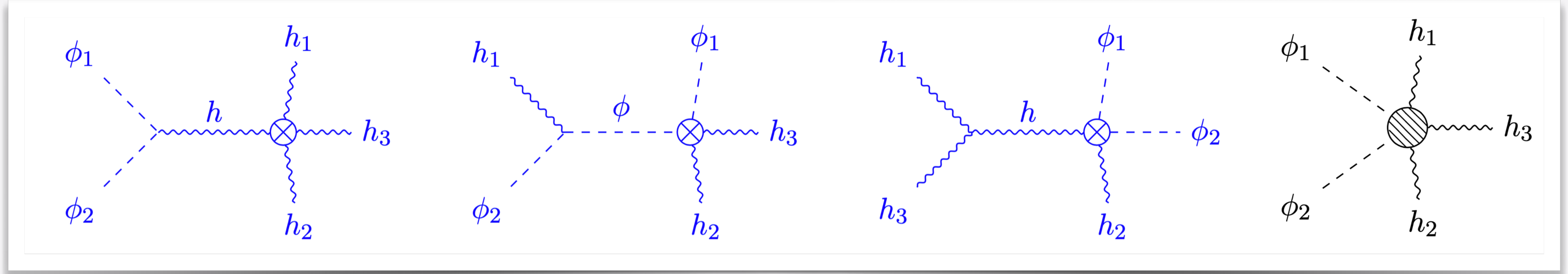

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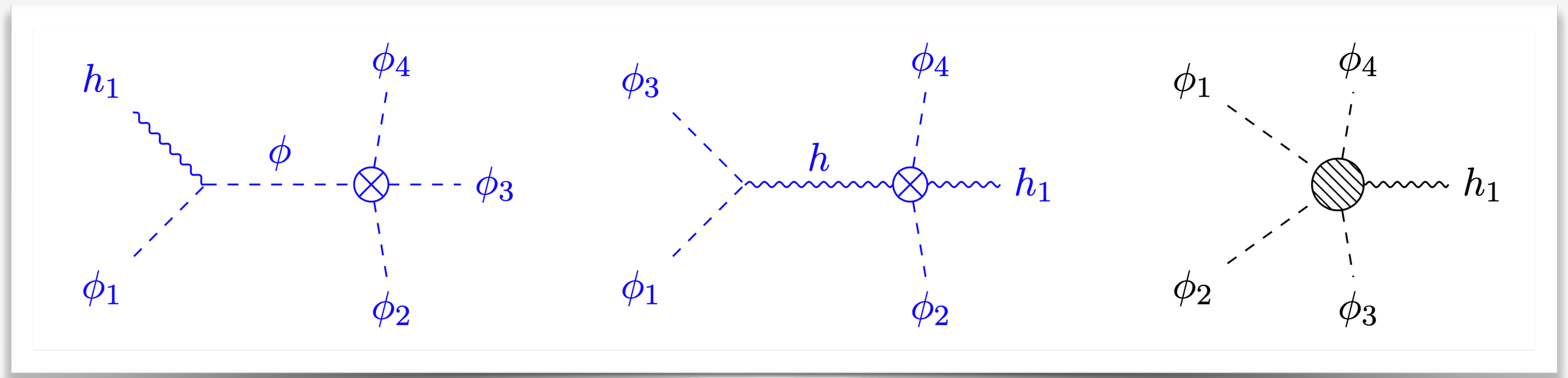
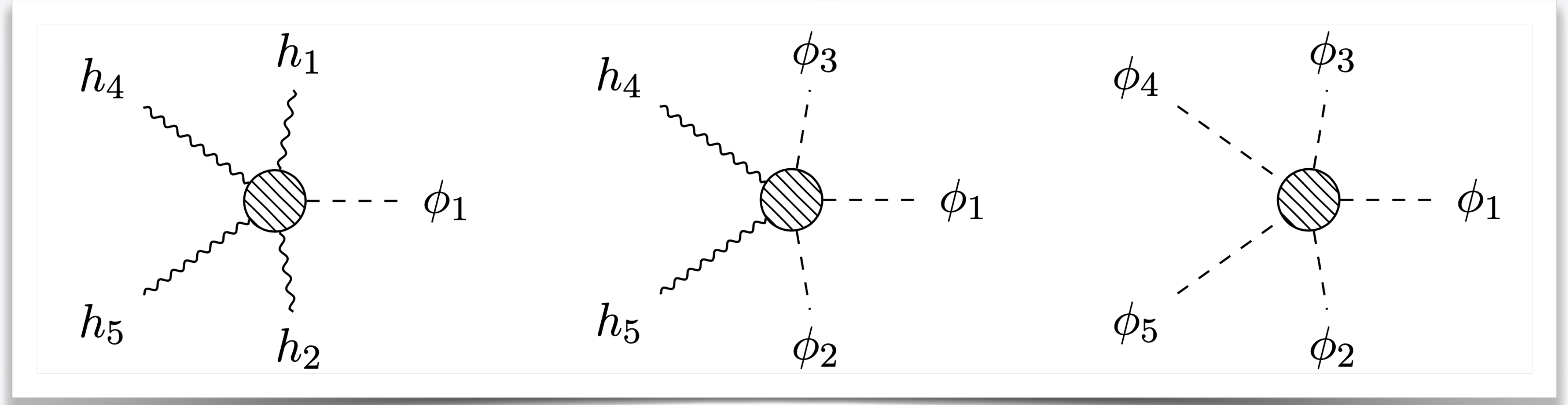


For odd number of ϕ, no pole subtraction



Five-point scattering amplitudes obtained from

- Tree level: Eq. (3.2) in [Sannan PRD34 (1986) 1749]
- 1-loop: Eq. (5.26) in [Richards 0807.2421]

The scalar-graviton sector at five points

String scattering and pole subtraction in the NSNS-sector — Part 2

Contact terms **after pole subtraction** (schematically)

<table>
<tr><th></th><th>Tree</th><th>Loop</th></tr>
<tr><td>h^5</td><td colspan="2">$\left(t_8t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right)R^4\|_{5\,\text{pts}}$</td></tr>
<tr><td>ϕh^4</td><td>$-\frac{3\phi}{2}\left(t_8t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right)R^4$</td><td>$\frac{\phi}{2}\left(t_8t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right)R^4$</td></tr>
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</table>

The scalar-graviton sector at five points

String scattering and pole subtraction in the NSNS-sector — Part 2

Contact terms **after pole subtraction** (schematically)

$$e^{-3\phi/2}R^4 \to R^4\left(1-\frac{3}{2}\phi+\ldots\right)$$

	Tree	
h^5	$\left(t_8t_8-\frac{1}{4}\epsilon_8\epsilon_8\right)R^4\vert_{5\,\text{pts}}$	
ϕh^4	$-\frac{3\phi}{2}\left(t_8t_8-\frac{1}{4}\epsilon_8\epsilon_8\right)R^4$	$\frac{\phi}{2}\left(t_8t_8-\frac{1}{4}\epsilon_8\epsilon_8\right)R^4$
ϕ^2h^3	$\left(t_8t_8-\frac{1}{4}\epsilon_8\epsilon_8\right)\left(6R^2(\nabla\nabla\phi)^2\vert_{5\,\text{pts}}+4R^3\left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\phi)(\partial^\lambda\phi)-\frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\phi)^2\right]\right)$	
ϕ^3h^2	$-\frac{3\phi}{2}\left(t_8t_8-\frac{1}{36}\epsilon_8\epsilon_8\right)6R^2(\nabla\nabla\phi)^2$	$\frac{\phi}{2}\left(t_8t_8-\frac{1}{36}\epsilon_8\epsilon_8\right)6R^2(\nabla\nabla\phi)^2$
ϕ^4h	$96((\nabla_\mu\nabla_\nu\phi)^2)^2\vert_{5\,\text{pts}}+24R^{\mu\nu\rho\sigma}\left(\nabla_\mu\nabla_\rho\phi\nabla_\nu\nabla_\sigma\phi(\partial\phi)^2-4\nabla_\mu\nabla_\alpha\phi\nabla_\rho\nabla^\alpha\phi\,\partial_\nu\phi\partial_\sigma\phi\right)$	
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BUT some terms arise from **4-point function** to non-linear order in the perturbations…

Liu, Minasian, Savelli, **AS**: 2507.07934

The scalar-graviton sector at five points

String scattering and pole subtraction in the NSNS-sector — Part 3

Contact terms **after subtracting 4-point function**

	Tree	Loop
h^5	0	
ϕh^4	0	0
$\phi^2 h^3$	$\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) 4R^3\left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\phi)(\partial^\lambda\phi) - \frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\phi)^2\right]$	
$\phi^3 h^2$	$2\epsilon_8\epsilon_8\, R^2(\partial\phi)^2(\nabla\nabla\phi)$	$-\frac{2}{3}\epsilon_8\epsilon_8\, R^2(\partial\phi)^2(\nabla\nabla\phi)$
$\phi^4 h$	$24R^{\mu\nu\rho\sigma}\left(\nabla_\mu\nabla_\rho\phi\nabla_\nu\nabla_\sigma\phi\,(\partial\phi)^2 - 4\nabla_\mu\nabla_\alpha\phi\nabla_\rho\nabla^\alpha\phi\,\partial_\nu\phi\partial_\sigma\phi\right)$	
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Tree level terms match T-duality results of [Garousi 2012.15091].

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Kinematic structures factorises at tree level and 1-loop!

Tree level terms match T-duality results of [Garousi 2012.15091].

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The scalar-graviton sector at five points

Towards modular completions

RESULTS CONSISTENT WITH $\mathrm{SL}(2,\mathbb{Z})$ UPON $\partial\phi \to \mathscr{P}$, $\partial\partial\phi \to \nabla\mathscr{P}$?

Even #dilatons → U(1)-preserving $\Rightarrow$ $(\partial\phi)^2 R^3 \to \left|\mathscr{P}\right|^2 R^3$

Odd #dilatons → MUV $\Rightarrow$ $(\partial\phi)^2(\partial\partial\phi)R^2 \to \left|\mathscr{P}\right|^2 \nabla\mathscr{P} R^3 + \ldots$

$$\tau = \chi + ie^{-\phi}\,, \quad \mathscr{P}_m = \frac{i}{2\mathrm{Im}(\tau)}\nabla_m\tau$$

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Even dilaton couplings

$$\frac{\text{tree}}{\text{loop}} = 1 \quad \Rightarrow \quad f_0(\tau,\bar\tau) = a_T + a_L + \dots$$

$$f_w(\tau,\bar\tau) = a_T + \frac{a_L}{1-4w^2} + \mathcal{O}(e^{-\mathrm{Im}(\tau)})$$

$$a_T = 2\zeta(3)\,e^{-3\phi/2}\,, \quad a_L = \frac{2\pi^2}{3}\,e^{\phi/2}$$

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The scalar-graviton sector at five points

Towards modular completions

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Even #dilatons → U(1)-preserving $\Rightarrow$ $(\partial\phi)^2 R^3 \to \left|\mathscr{P}\right|^2 R^3$

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	Tree	Loop
ϕ^2h^3	$\left(t_8t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right)4R^3\left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\phi)(\partial^\lambda\phi) - \frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\phi)^2\right]$	
ϕ^3h^2	$2\epsilon_8\epsilon_8\, R^2(\partial\phi)^2(\nabla\nabla\phi)$	$-\frac{2}{3}\epsilon_8\epsilon_8\, R^2(\partial\phi)^2(\nabla\nabla\phi)$
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Even dilaton couplings

$$\frac{\text{tree}}{\text{loop}} = 1 \quad \Rightarrow \quad f_0(\tau,\bar\tau) = a_T + a_L + \ldots$$

Odd dilaton couplings

$$\frac{\text{tree}}{\text{loop}} = -3 \quad \Rightarrow \quad f_{\pm1}(\tau,\bar\tau) = a_T - \frac{a_L}{3} + \ldots$$

$$f_w(\tau,\bar\tau) = a_T + \frac{a_L}{1-4w^2} + \mathcal{O}(e^{-\mathrm{Im}(\tau)})$$

$$a_T = 2\zeta(3)\,\mathrm{e}^{-3\phi/2}\,, \quad a_L = \frac{2\pi^2}{3}\,\mathrm{e}^{\phi/2}$$

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The scalar-graviton sector at five points

RR-sector couplings at five points

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To fully determine the $\mathrm{SL}(2,\mathbb{Z})$ invariant action, we need additional information about the RR-sector!

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To fully determine the $\mathrm{SL}(2,\mathbb{Z})$ invariant action, we need additional information about the RR-sector!

We work in **pure spinor formalism** [Berkovits hep-th/0001035] and repeat process from above (pole subtraction etc.) …

We compute closed tree amplitudes for RR zero-form χ by using **KLT relations** [Kawai et al. 1988]

closed tree amplitudes ⇔ combination of open tree amplitudes

In the limit $\alpha' \to 0$, this implies

SUGRA amplitudes $\mathscr{A}_{SUGRA}$ ⇔ "square" of SYM amplitudes $\mathscr{A}_{SYM}$

$$\mathscr{A}_{SUGRA} = \mathscr{A}_{SYM} \cdot S_{KLT} \cdot \widetilde{\mathscr{A}}_{SYM}$$

We obtain $\mathscr{A}_{SYM}$ from database of open string amplitudes: https://tools.aei.mpg.de/purespinor/pss.html

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Let us look at amplitudes with **two axion insertions** χ^2

	Tree
$\chi^2 h^3$	$\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) 4R^3\left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\chi)(\partial^\lambda\chi) - \frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\chi)^2\right]$

	Tree	Loop
$\phi^2 h^3$	$\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) 4R^3\left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\phi)(\partial^\lambda\phi) - \frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\phi)^2\right]$	

MATCH EXACTLY!

The contribution in the Lagrangian is given by

$$\mathscr{L}^{(3)} \supset f_0(\tau,\bar{\tau})\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) R^3\left[3\,\delta^\nu_\mu\,\mathscr{P}_\sigma\overline{\mathscr{P}}^\lambda - \frac{1}{6}\,\delta^\nu_\mu\,\delta^\lambda_\sigma\,\left|\mathscr{P}\right|^2\right]$$

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Consistent with $\mathcal{N}=1$ SUSY in **four dimensions!**

[BBHL hep-th/0204254, Bonetti et al. 1608.01300]

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Let us look at amplitudes with **two axion insertions** χ^2

	Tree
$\chi^2 h^3$	$\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) 4R^3 \left[\frac{3}{4}\delta^\nu_\mu(\partial_\sigma\chi)(\partial^\lambda\chi) - \frac{1}{24}\delta^\nu_\mu\delta^\lambda_\sigma(\partial\chi)^2\right]$
$\chi^2\phi h^2$	$-2\phi\epsilon_8\epsilon_8\, R^2(\nabla\nabla\chi)^2 + \frac{3}{20}\epsilon_8\epsilon_8 R^2((\delta\delta\partial\chi\cdot\partial\chi)(\nabla\nabla\phi) - (\delta\delta\partial\chi\cdot\partial\phi)(\nabla\nabla\chi))$

	Tree
$\phi^2 h^3$	$\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right) 4R^3$
$\phi^3 h^2$	$-2\phi\epsilon_8\epsilon_8\, R^2(\nabla\nabla\phi)^2$

The $\mathrm{SL}(2,\mathbb{Z})$-completed expression is

$$\mathscr{L}^{(3)} \supset f_1(\tau,\bar{\tau})\,\epsilon_8\epsilon_8\, R^2 \left[-\delta\,\left|\mathscr{P}\right|^2(\nabla\mathscr{P}) - (\delta\mathscr{P}^2)(\nabla\overline{\mathscr{P}}) + \frac{3}{40}\left\{(\delta\delta\mathscr{P}_\alpha\overline{\mathscr{P}}^\alpha)(\nabla\mathscr{P}) - (\delta\delta\mathscr{P}_\alpha\mathscr{P}^\alpha)(\nabla\overline{\mathscr{P}})\right\}\right] + \text{c.c.}$$

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Let us look at amplitudes with **two axion insertions** χ^2

	Tree
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$\chi^2\phi h^2$	$-2\phi\epsilon_8\epsilon_8\, R^2(\nabla\nabla\chi)^2 + \frac{3}{20}\epsilon_8\epsilon_8 R^2((\delta\delta\partial\chi\cdot\partial\chi)(\nabla\nabla\phi) - (\delta\delta\partial\chi\cdot\partial\phi)(\nabla\nabla\chi))$

	Tree
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“Funny kinematics!”

The $\mathrm{SL}(2,\mathbb{Z})$-completed expression is

$$\mathscr{L}^{(3)} \supset f_1(\tau,\bar{\tau})\,\epsilon_8\epsilon_8\,R^2\left[-\delta\left|\mathscr{P}\right|^2(\nabla\mathscr{P}) - (\delta\mathscr{P}^2)(\nabla\overline{\mathscr{P}}) + \frac{3}{40}\left\{(\delta\delta\mathscr{P}_\alpha\overline{\mathscr{P}}^\alpha)(\nabla\mathscr{P}) - (\delta\delta\mathscr{P}_\alpha\mathscr{P}^\alpha)(\nabla\overline{\mathscr{P}})\right\}\right] + \mathrm{c.c.}$$

In particular, there are terms that **vanish in the pure NSNS-sector**:

$$(\delta\delta\mathscr{P}_\alpha\overline{\mathscr{P}}^\alpha)(\nabla\mathscr{P}) - (\delta\delta\mathscr{P}_\alpha\mathscr{P}^\alpha)(\nabla\overline{\mathscr{P}}) \sim (\delta\delta\partial\chi\cdot\partial\chi)(\nabla\nabla\phi) - (\delta\delta\partial\chi\cdot\partial\phi)(\nabla\nabla\chi)$$

The scalar-graviton sector at five points

RR-sector couplings at five points

Liu, Minasian, Savelli, **AS**: 2507.07934

Remaining 2-axion couplings…

$\chi^2\phi^2 h$

$$\begin{aligned}\mathcal{L}^{\text{tree}}_{\chi^2\phi^2 h} = 96\,&\left[4(\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\chi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2(\nabla_\sigma\nabla_\rho\chi)^2\right]_{5\,\text{pt}}\\ &+24R^{\mu\nu\rho\sigma}\Big[-4(\nabla_\mu\nabla^\alpha\phi)(\nabla_\sigma\nabla_\alpha\phi)(\nabla_\nu\chi)(\nabla_\rho\chi) - 4(\nabla_\mu\nabla^\alpha\chi)(\nabla_\sigma\nabla_\alpha\chi)(\nabla_\nu\phi)(\nabla_\rho\phi)\\ &\quad+8(\nabla_\mu\nabla^\alpha\phi)(\nabla_\sigma\nabla_\alpha\chi)(\nabla_\nu\phi)(\nabla_\rho\chi) - 8(\nabla_\mu\nabla^\alpha\phi)(\nabla_\sigma\nabla_\alpha\chi)(\nabla_\nu\chi)(\nabla_\rho\phi)\\ &\quad+24(\nabla_\mu\nabla^\alpha\phi)(\nabla_\nu\nabla_\alpha\chi)(\partial_\sigma\phi)(\partial_\rho\chi) - 24(\nabla_\mu\nabla^\alpha\phi)(\nabla_\nu\nabla_\alpha\chi)(\partial_\sigma\chi)(\partial_\rho\phi)\\ &\quad-19(\nabla_\mu\nabla_\sigma\phi)(\nabla_\nu\nabla_\rho\phi)\,(\partial\chi)^2 - 19(\nabla_\mu\nabla_\sigma\chi)(\nabla_\nu\nabla_\rho\chi)\,(\partial\phi)^2\\ &\quad+40(\nabla_\mu\nabla_\sigma\phi)(\nabla_\nu\nabla_\rho\chi)(\partial^\alpha\phi)(\partial^\alpha\chi)\Big]\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{tree}}_{\chi^2\phi^3} = 96\,&\Big[\frac{\phi}{2}(4(\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\chi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2(\nabla_\sigma\nabla_\rho\chi)^2)\\ &+8(\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\chi)(\nabla_\sigma\nabla_\rho\phi\,\partial^\sigma\phi\partial^\rho\chi) - 4(\nabla_\mu\nabla_\nu\phi)^2(\nabla_\sigma\nabla_\rho\chi\,\partial^\sigma\phi\partial^\rho\chi)\Big]\\ &+\frac{8}{75}\phi\epsilon_8\epsilon_8(\nabla\nabla\phi)^2(\nabla\nabla\chi) + \frac{4}{175}\Big(3\epsilon_8\epsilon_8\nabla\nabla\phi(\nabla\nabla\chi)^2(\delta\delta(\partial\phi)^2)\\ &\quad-4\epsilon_8\epsilon_8(\nabla\nabla\phi)^2\nabla\nabla\chi(\delta\delta(\partial\phi)(\partial\chi)) + \epsilon_8\epsilon_8(\nabla\nabla\phi)^3(\delta\delta(\partial\chi)^2)\Big)\end{aligned}$$

$\chi^2\phi^3$

The scalar-graviton sector at five points

RR-sector completion — Summary

Liu, Minasian, Savelli, **AS**: 2507.07934

To summarise, the full action is given by

$$\mathscr{L}^{(5)} = \alpha f_0(\tau,\bar{\tau})\left[\mathscr{L}_{2\,\text{scalars}} + \mathscr{L}_{4\,\text{scalars}}\right] + \alpha f_1(\tau,\bar{\tau})\left[\mathscr{L}_{3\,\text{scalars}} + \mathscr{L}_{5\,\text{scalars}}\right] + \text{h.c.}$$

where the **even-scalar** contributions are

$$\mathscr{L}_{2\,\text{scalars}} = (t_8t_8 - \tfrac{1}{4}\epsilon_8\epsilon_8)R^3\left(12\delta\mathscr{P}\overline{\mathscr{P}} - \tfrac{2}{3}\delta\delta\,|\mathscr{P}|^2\right)$$

$$\mathscr{L}_{4\,\text{scalars}} = 384R^{\mu\nu\sigma\rho}\left(5(\nabla_\mu\overline{\mathscr{P}}_\sigma)(\nabla_\nu\overline{\mathscr{P}}_\rho)\,\mathscr{P}^2 + (\nabla_\mu\mathscr{P}_\sigma)\left[5(\nabla_\nu\mathscr{P}_\rho)\,\overline{\mathscr{P}}\,2 - 9(\nabla_\nu\overline{\mathscr{P}}_\rho)\,|\mathscr{P}|^2\right] + \ldots\right)$$

and the **odd-scalar** interactions are

$$\mathscr{L}_{3\,\text{scalars}} = \epsilon_8\epsilon_8R^2\left[8(\nabla\mathscr{P})(\delta\mathscr{P}\,\overline{\mathscr{P}}) + \frac{3}{10}\left((\nabla\mathscr{P})(\delta\delta\mathscr{P}\cdot\overline{\mathscr{P}}) - (\nabla\overline{\mathscr{P}})(\delta\delta\mathscr{P}\cdot\mathscr{P})\right)\right]$$

$$\mathscr{L}_{5\,\text{scalars}} = \epsilon_8\epsilon_8(\nabla\mathscr{P})\left[-\frac{64}{75}(\nabla\mathscr{P})^2(\delta\overline{\mathscr{P}}\,2) + \frac{16}{175}\left\{-3(\nabla\mathscr{P})^2(\delta\delta\overline{\mathscr{P}}\,2) + 2\ \nabla\mathscr{P}\ ^2\delta\delta\,|\mathscr{P}|^2 + (\nabla\overline{\mathscr{P}})^2(\delta\delta\mathscr{P}^2)\right\}\right]$$

M-theory vs. Type IIA vs. Type IIB

Dilaton? What Dilaton?

Liu, Minasian, Savelli, **AS**: 2507.07934

In the pure NSNS sector, the tree-level $(\alpha')^3$ action of Type II can be written without explicit dilaton couplings in string frame.

σ-model [Tseytlin hep-th/0612296] T-duality [Garousi 2012.15091]

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This is **not true** anymore in the mixed NSNS/RR-sector.

This has led to some confusion and potential inconsistencies in the duality web (IIB $\leftrightarrow$ IIA $\leftrightarrow$ M-theory) already at the 4-point function level, see in particular [Wulff et al. 2506.16391]

M-theory vs. Type IIA vs. Type IIB

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We show that there are no such inconsistencies

- by computing missing RR-sector 4-point couplings in Type IIA, and
- after carefully applying field redefinitions.

M-theory vs. Type IIA vs. Type IIB

Duality web for scalar couplings

Liu, Minasian, Savelli, **AS**: 2507.07934

M-theory

$t_8 t_8 \hat{R}^4$

$t_8 t_8 \hat{R}^2 (\nabla G_4)^2$

$\ldots$

→ reduction

M-theory on S^1

$R^2 (\nabla F_2)^2$

$R(\nabla\nabla\phi)(\nabla F_2)^2$

$(\nabla\nabla\phi)^2 (\nabla F_2)^2$

$\ldots$

M-theory vs. Type IIA vs. Type IIB

Duality web for scalar couplings

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M-theory		M-theory on S^1		Type IIA		Type IIB
$t_8 t_8 \hat{R}^4$		$R^2(\nabla F_2)^2$	$\longleftrightarrow$	$R^2(\nabla F_2)^2$		$R(\nabla\nabla\phi)(\nabla F_3)^2$
$t_8 t_8 \hat{R}^2(\nabla G_4)^2$	$\longrightarrow$ reduction	$R(\nabla\nabla\phi)(\nabla F_2)^2$	$\longleftrightarrow$	$R(\nabla\nabla\phi)(\nabla F_2)^2$	$\longleftrightarrow$ T-duality	$R(\nabla\nabla\phi)(\nabla\nabla\chi)(\nabla F_3)$
$\dots$		$(\nabla\nabla\phi)^2(\nabla F_2)^2$	$\longleftrightarrow$	$(\nabla\nabla\phi)^2(\nabla F_2)^2$		$\dots$
		$\dots$	matching	$\dots$		

M-theory vs. Type IIA vs. Type IIB

Duality web for scalar couplings

Liu, Minasian, Savelli, **AS**: 2507.07934

M-theory		M-theory on S^1		Type IIA		Type IIB
$t_8 t_8 \hat{R}^4$		$R^2(\nabla F_2)^2$	⟷	$R^2(\nabla F_2)^2$		$R(\nabla\nabla\phi)(\nabla F_3)^2$
$t_8 t_8 \hat{R}^2(\nabla G_4)^2$	⟶ reduction	$R(\nabla\nabla\phi)(\nabla F_2)^2$	⟷	$R(\nabla\nabla\phi)(\nabla F_2)^2$	⟷ T-duality	$R(\nabla\nabla\phi)(\nabla\nabla\chi)(\nabla F_3)$
$\dots$		$(\nabla\nabla\phi)^2(\nabla F_2)^2$	⟷ matching	$(\nabla\nabla\phi)^2(\nabla F_2)^2$		$\dots$
		$\dots$		$\dots$		

↓

We computed the tree/one-loop 4-point amplitudes $h^2F_2^2$, $h\phi F_2^2$, $\phi^2 F_2^2$ directly!

4-point effective actions in Type II

[Policastro, Tsimpis hep-th/0603165]

[Liu, Minasian 1304.3137, 1912.10974]

M-theory vs. Type IIA vs. Type IIB

Duality web for scalar couplings

Liu, Minasian, Savelli, **AS**: 2507.07934

These terms were missing in [Wulff et al. 2506.16391]!

M-theory

$t_8 t_8 \hat{R}^4$

$t_8 t_8 \hat{R}^2 (\nabla G_4)^2$

$\dots$

reduction

M-theory on S^1

$R^2 (\nabla F_2)^2$

$R(\nabla\nabla\phi)(\nabla F_2)^2$

$(\nabla\nabla\phi)^2 (\nabla F_2)^2$

$\dots$

matching

Type IIA

$R^2 (\nabla F_2)^2$

$R(\nabla\nabla\phi)(\nabla F_2)^2$

$(\nabla\nabla\phi)^2 (\nabla F_2)^2$

$\dots$

T-duality

Type IIB

$R(\nabla\nabla\phi)(\nabla F_3)^2$

~~$R(\nabla\nabla\phi)(\nabla\nabla\chi)(\nabla F_3)$~~

$\dots$

Vanishes kinematically!

We computed the tree/one-loop 4-point amplitudes $h^2 F_2^2$, $h\phi F_2^2$, $\phi^2 F_2^2$ directly!

4-point effective actions in Type II

[Policastro, Tsimpis hep-th/0603165]

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Conclusions

We provided an incomplete list of $\mathrm{SL}(2,\mathbb{Z})$-invariant contact terms from 5-point scattering such as $G_3^2 R^3$ or $\left|\mathscr{P}\right|^2 (\nabla \mathscr{P})\, R^2 \ldots$

- Computed RR-couplings that **cannot be unambiguously** inferred from just the NSNS-sector
- Found expected matching of tree/one-loop kinematics for scalars (**special at 5-points**)
- Proved **consistency with T-duality** and circle reductions of **M-theory**

… but a more systematic approach is required to determine the structure of couplings from amplitudes with $P > 5$ points


Thank you!

CY compactifications

K3 reductions to 6 dimensions

[Liu, Minasian, Savelli, **AS**: 2205.11530]

$$\mathscr{L} = f_0\, t_{16}\, R^4 + f_0 \left[T(t_8,\epsilon_{10}) - t_{18}\right] \left|G_3\right|^2 R^3 + \frac{3}{2}\left(f_1\, t_{18} G_3^2 R^3 + f_{-1}\, t_{18} \overline{G}_3^2 R^3\right) \qquad T(t_8,\epsilon_{10}) = 2\, t_8\, t_8 - \frac{1}{2}\epsilon_8\,\epsilon_8 - \frac{1}{3}\epsilon_9\,\epsilon_9$$

We extend results of [Liu, Minasian 1304.3137, 1912.10974] to RR-sector by focussing on factorised pieces:

$$\int_{K3} G_3^2 R^3 \supset G_3^2 R \int_{K3} R^2$$

As required by SUSY [Lin et al. 1508.07305], we verify that the 3-point functions $\left|G_3\right|^2 R, G_3^2\, R$ vanish:

- There is no factorised piece coming from t_{18} :

$$t_{18} \; G_3 \;{}^2 R^3 = t_{18}\, G_3^2 R^3 = t_{18}\, \overline{G}_3^2 R^3 = 0$$

- The kinematics from generalised geometry cancels

$$\left(2 t_8 t_8 - \frac{1}{2}\epsilon_8\epsilon_8\right)\left|G_3\right|^2 R^3 = 0$$

- After integrating by parts and using the Bianchi identity (ignoring the dilaton), one arrives at

$$-\frac{1}{3}\epsilon_9\epsilon_9 \left|G_3\right|^2 R^3 + 6\left(t_8 t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right)\left|\nabla G_3\right|^2 R^2 = 0$$

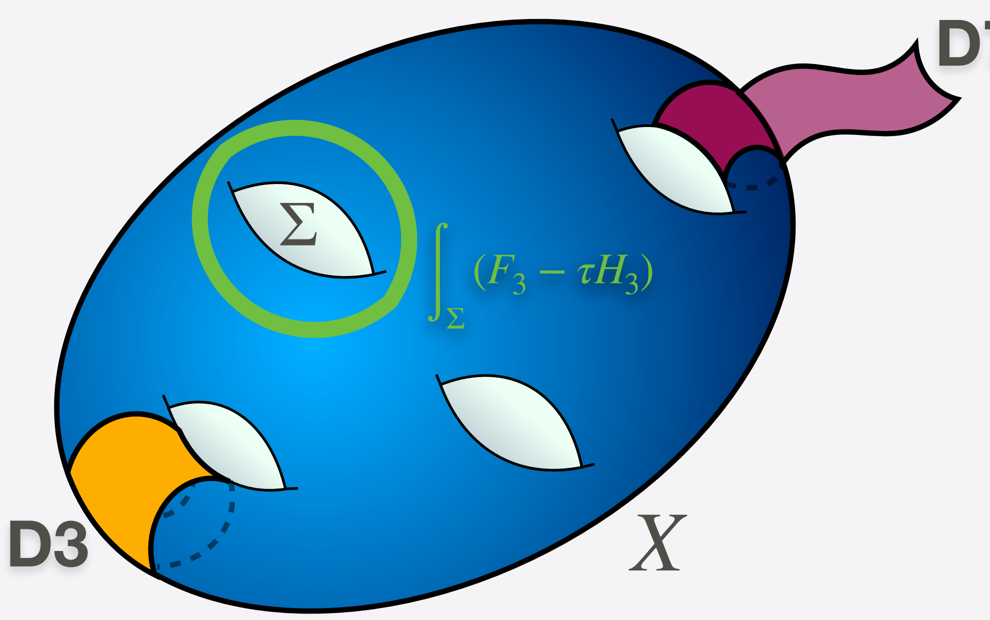
TESTS ONLY ODD-ODD SECTOR COUPLINGS LIKE $\epsilon_9\epsilon_9$ AND $\epsilon_8\epsilon_8$ TO WHICH CY THREEFOLD REDUCTIONS ARE INSENSITIVE TO!

Kinetic terms for B_2/C_2-axions
We also derived 4D kinetic terms, but issues remain! At NSNS tree level, we trivially agree with [Grimm et al.: 1702.08404].

CY compactifications

CY3 reductions to 4 dimensions

[Liu, Minasian, Savelli, **AS**: 2205.11530]



From reducing $f_0\, t_{16}\, R^4$, we can identify the correction to the Kähler potential [Antoniadis et al. hep-th/9707013]

$$K = K^{(0)} - 2\log\left[\mathcal{V} + \frac{\zeta}{4} f_0(\tau,\bar\tau)\right],\quad \zeta = -\frac{\chi(X_3)}{2(2\pi)^3},\quad K^{(0)} = -\log[-i(\tau-\bar\tau)] - \log\left[-i\int_{X_3}\Omega\wedge\overline{\Omega}\right],\quad W = \int_{X_3} G_3\wedge\Omega$$

The 4D F-term scalar potential to order $(\alpha')^3$ originating from the reduction of 10D 8-derivative terms reads

$$V_\zeta = \frac{3\zeta e^{K^{(0)}}}{8\mathcal{V}^3}\left[f_0\left(\,\left|W\right|^2 - (\tau-\bar\tau)^2\left|D_\tau W\right|^2\,\right) + (\tau-\bar\tau)\left(f_{-1}\,\overline{W}D_\tau W - f_1\,W\overline{D_\tau W}\right)\right]$$

$$f_w(\tau,\bar\tau) = a_T + \frac{a_L}{1-4w^2} + \mathcal{O}(e^{-\mathrm{Im}(\tau)})$$

$$V_\zeta = \frac{\zeta e^{K^{(0)}}}{\mathcal{V}^3}\left(-\frac{1}{4}\right)\left[(-6a_T - 2a_L)e^{-2\phi}\int_{X_3} H_3\wedge\Omega\int_{X_3} H_3\wedge\overline{\Omega} + (-4a_L)\int_{X_3} F_3\wedge\Omega\int_{X_3} F_3\wedge\overline{\Omega} + \ldots\right]$$

4D SUSY implies that F_3 flux does not contribute to V_ζ at tree level [Becker et al. hep-th/0204254].

We find that **our results are consistent** with the 4D perspective **provided** (equivalently for $H_3^2R^3$ as well as $\left|G_3\right|^2 R^3$, $G_3^2R^3$)

$$\alpha\int_{X_3}\left(t_{18} + \frac{2}{3}\delta_1\right)F_3^2R^3 = -\frac{\zeta e^{K^{(0)}}}{4\mathcal{V}^3}\int_{X_3} F_3\wedge\Omega\int_{X_3} F_3\wedge\overline{\Omega}$$

Here, δ_1 is a potential backreaction effect entering in the MUV sector. Apart from that, **only the MUV kinematics t_{18} is relevant to determine V_ζ !**