# Galilean, Carrollian, and Flat Space Holography from AdS/CFT

**Andrea Fontanella**

Trinity College Dublin

CORFU 2025 Workshop on Quantum Gravity and Strings

Based on:

- NR holography: Phys.Rev.Lett. 133 (2024) 15, 151601
  Phys.Rev.D 111 (2025) 2, 026003
  JHEP 06 (2025) 058
  (with J.M. Nieto García)
- Carroll holography: arXiv:2508.10085 (with O. Payne)

# Motivations

In recent years, good progress in Non-Lorentzian (NL) limits

NL limits: $c \to \infty$ (non-relativistic), or $c \to 0$ (ultra-relativistic, Carroll)

Spacetime geometry changes: becomes 'Newton-Cartan'

The Lorentz group is broken. The metric splits into degenerate tensors

Holography here is completely different than usual AdS/CFT

Spacetime is i) non-AdS, but also ii) non-Lorentzian

Main question:

*New Gauge/Gravity dualities by taking limits of AdS/CFT?*

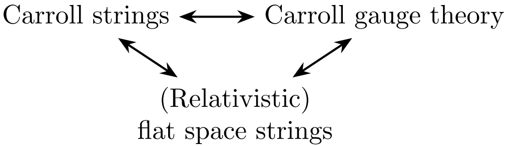
In this talk we address this question for NL + flat space limits

# Plan of the talk

1. Background 1: Maldacena's construction of $AdS_5/CFT_4$
2. Background 2: NL limits
3. Non-relativistic limit of AdS/CFT
4. Carroll limit of AdS/CFT
5. Flat Space limit of AdS/CFT

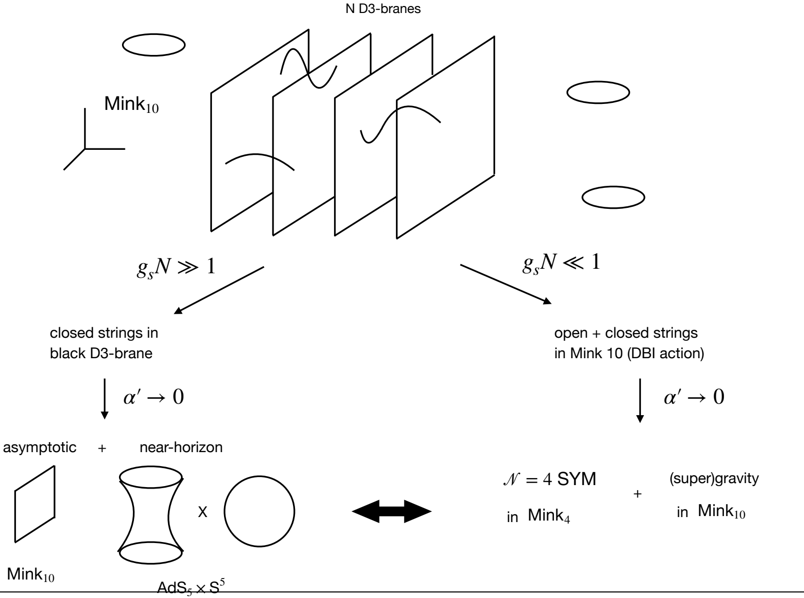
Main results:

- New NR + Carroll Gauge/Gravity dualities
- Suggest Holographic Triality:



# Maldacena's construction of $AdS_5/CFT_4$ (review)

Type IIB string theory in $\mathrm{Mink}_{10}$ + stack $N$ D3-branes [Maldacena (1997)]



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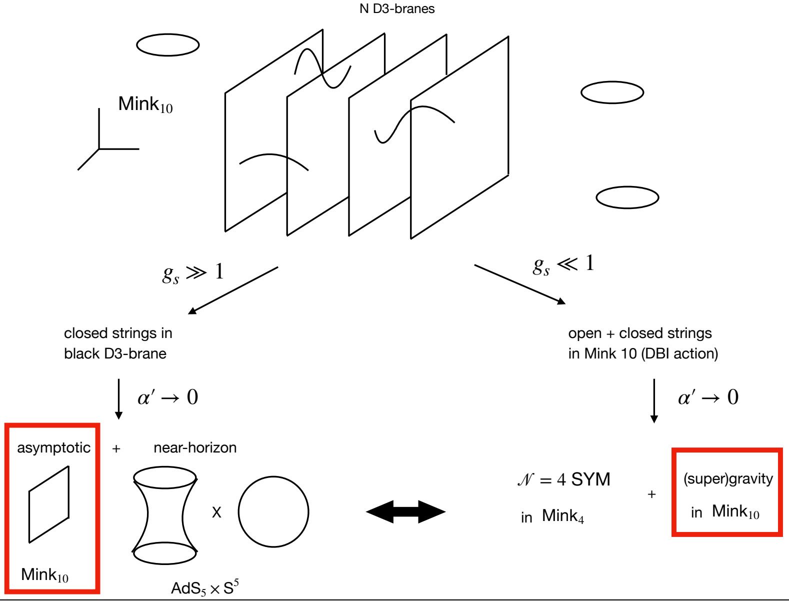

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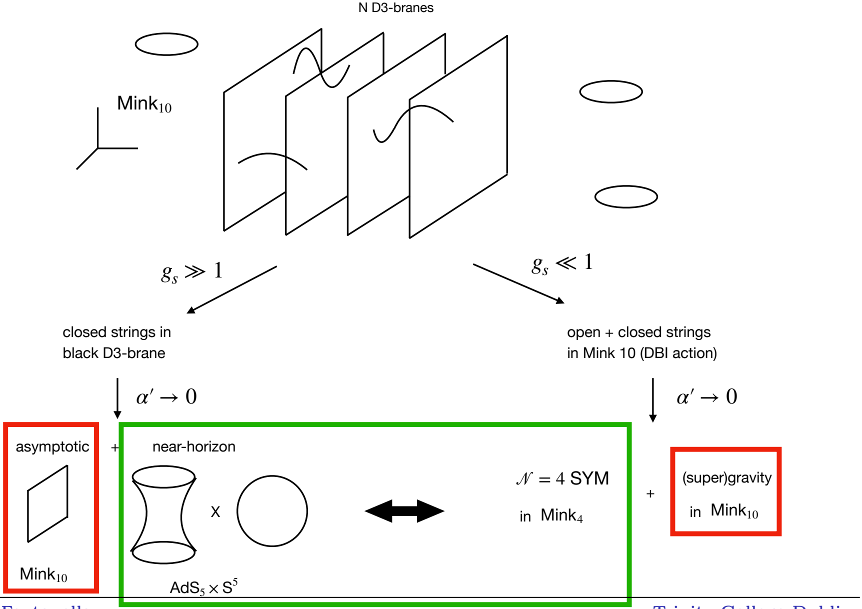


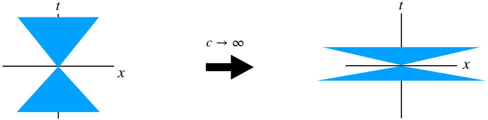
# Non-Lorentzian limits

① **Non-relativistic**: slow velocity dynamics

$$v \ll c \qquad \Longleftrightarrow \qquad c \to \infty$$

($c \to \omega c$, then $\omega \to \infty$)

e.g. Minkowski $\quad \mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^i\mathrm{d}x_i$




Newton-Cartan geometry

$$g_{\mu\nu} = c^2\tau_{\mu\nu} + h_{\mu\nu}\ , \qquad \tau_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}\ , \qquad h_{\mu\nu} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

$\tau_{\mu\nu}$: longitudinal metric (leading)

$h_{\mu\nu}$: transverse tensor (next-to-leading)

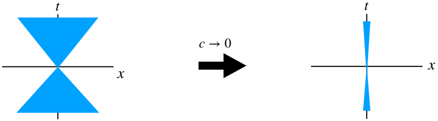
Galilei boost: $\quad x^i \quad \to \quad t \quad \to \quad 0$

② **Ultra-relativistic (Carroll)**: Lévy-Leblond (1965) and Gupta (1966), as a pure exercise, considered the singular limit

$$c \to 0$$

nowadays many applications : flat space holography,
fractons, null hypersurfaces,
black hole horizons, hydrodynamics, ...

e.g. Minkowski $\quad \mathrm{d}s^2 = -c^2 \mathrm{d}t^2 + \mathrm{d}x^i \mathrm{d}x_i$

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$h_{\mu\nu}$: transverse metric (leading)

$\tau_{\mu\nu}$: longitudinal tensor (next-to-leading)

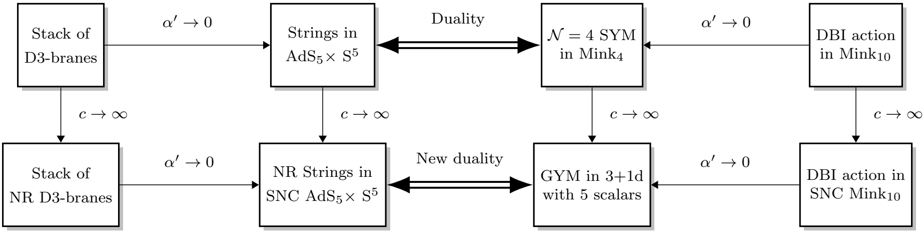
Carroll boost: $\quad t \ \to \ x^i \ \to \ 0$

# Holography from Limits of AdS/CFT: Consistency

We demand consistency conditions:

1. **Horizon**. Limit of D3-brane retains a horizon + asymptotic regions
2. **Commuting**. Limit commutes with $\alpha' \to 0$
3. **Holographic**. gauge theory is formulated on conformal boundary of gravity theory
4. **Symmetry matching**.
   symmetries of gauge theory = symmetries of bulk theory @ boundary
5. **Quantitative test**. $\exists$ dictionary of observables, expectation values match

# 1. Non-relativistic




Consistency conditions:

1. Horizon ✓

   SNC D3-brane metric: $\tau \sim H^{-1}\mathrm{d}t^2 + H\mathrm{d}z^2\,, \qquad h \sim H^{-1}\mathrm{d}\vec{x}^2 + H\mathrm{d}\vec{y}^2$

2. Commuting ✓

3. Holographic ✓

   $$\text{Penrose boundary}\ \left[\text{SNC AdS}_5\times\text{S}^5\right] \quad = \quad \text{NC Mink}_4\ (3{+}1\text{d})$$

4. Symmetry matching ✓

   Conformal Milne Algebra + $\infty$-dim internal symm.

5. Quantitative test: *in progress*

# Quantitative test

$E = \Delta$

$E$ = NR string spectrum in light-cone gauge
$\Delta$ = scaling dimensions

$E$ complicated, due to "winding" modes along non-isometry direction

Perturbatively ($T \gg 1$)

$$S = S_2 + \frac{1}{\sqrt{T}} S_3 + \frac{1}{T} S_4 + \frac{1}{\sqrt{T^3}} S_5 + \frac{1}{T^2} S_6 + \dots$$

After complicated field redefinitions

to scare you a bit... →

[de Leeuw, AF, Nieto García (2024)]

$$\begin{aligned} S^{(2)} &= \text{free fields in } \mathrm{AdS}_2 \\ S^{(i)} &= 0 \qquad \forall i \geq 3 \qquad \text{(checked up to } i = 6\text{)} \end{aligned}$$

Now, match this with $\Delta$ from the GYM + 5 scalars! (*in progress*)

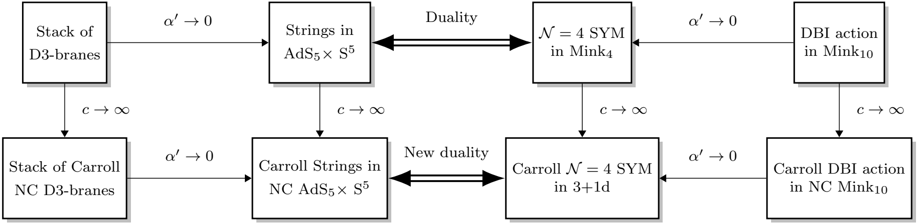
<u>Integrability</u>: can we derive the exact $E$ via integrability principles?

Hard problem, since the spectral curve trivialise [AF, Nieto García, Ohlsson-Sax (2022)]

We need a new spectral curve that works for NL theories.

$$
\begin{aligned}
f^{(4)}_{z_m} = & -\frac{\kappa^6}{48}\left[33 - 26\cos(\kappa\sigma) + \cos(2\kappa\sigma)\right]\sec^4\left(\frac{\kappa\sigma}{2}\right)\sec(\kappa\sigma)\delta z_1^4 \delta z_m \\
& + \frac{\kappa^5}{6}\sec^3\left(\frac{\kappa\sigma}{2}\right)\sec(\kappa\sigma)\left[-11\sin\left(\frac{\kappa\sigma}{2}\right) + \sin\left(\frac{3\kappa\sigma}{2}\right)\right]\delta z_1^4 \delta z_m' \\
& + \frac{\kappa^5}{2}\sec^3\left(\frac{\kappa\sigma}{2}\right)\sec(\kappa\sigma)\left[-11\sin\left(\frac{\kappa\sigma}{2}\right) + \sin\left(\frac{3\kappa\sigma}{2}\right)\right]\delta z_1^3 \delta z_1' \delta z_m \\
& + \kappa^4 \frac{1 - 2\sec(\kappa\sigma)}{1 + \cos(\kappa\sigma)}\delta z_1^4 \delta z_m'' + 3\kappa^4 \sec^2\left(\frac{\kappa\sigma}{2}\right)\left[1 - 2\sec(\kappa\sigma)\right]\delta z_1^3 \delta z_1' \delta z_m' \\
& + \kappa^4 \sec^2\left(\frac{\kappa\sigma}{2}\right)\left[1 - 2\sec(\kappa\sigma)\right]\delta z_1^3 \delta z_1'' \delta z_m \\
& + 3\kappa^4 \sec^2\left(\frac{\kappa\sigma}{2}\right)\left[1 - 2\sec(\kappa\sigma)\right]\delta z_1^2 (\delta z_1')^2 \delta z_m \\
& - \frac{2\kappa^3}{3}\sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^4 \delta z_m''' + 6\kappa^3 \sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^3 \delta z_1' \delta z_m'' \\
& + 4\kappa^3 \sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^3 \delta z_1'' \delta z_m' + 12\kappa^3 \sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^2 (\delta z_1')^2 \delta z_m' \\
& - \frac{2\kappa^3}{3}\sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^3 \delta z_1''' \delta z_m - 6\kappa^3 \sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1^2 \delta z_1' z_1'' \delta z_m \\
& - 4\kappa^3 \sec(\kappa\sigma)\tan\left(\frac{\kappa\sigma}{2}\right)\delta z_1 (\delta z_1')^3 \delta z_m + \frac{\kappa^2}{6}\delta z_1^4 \delta z_m'''' + 2\kappa^2 \delta z_1^3 \delta z_1' \delta z_m''' \\
& + 2\kappa^2 \delta z_1^3 \delta z_1'' \delta z_m'' + 6\kappa^2 \delta z_1^2 (\delta z_1')^2 \delta z_m'' + 4\kappa^2 \delta z_1 (\delta z_1')^3 \delta z_m' \\
& + 6\kappa^2 \delta z_1^2 \delta z_1'' \delta z_1' \delta z_m' + \frac{2\kappa^2}{3}\delta z_1^3 \delta z_1''' \delta z_m' \,, \qquad \text{(B.11)}
\end{aligned}
$$

# 2. Carroll




Consistency conditions:

1. Horizon ✓

   NC D3-brane metric: $h \sim H^{-1}\mathrm{d}\vec{x}^2 + H(\mathrm{d}\Omega_5^2 + \mathrm{d}z^2)\,, \qquad \tau \sim H^{-1}\mathrm{d}t^2$

2. Commuting ✓

3. Holographic ✓

   $$\text{Penrose boundary}\left[\text{Carroll NC } AdS_5\times S^5\right] \quad = \quad \text{Carroll NC } Mink_4$$

4. Symmetry matching: $\to$ *need Carroll type IIB supergravity*

5. Quantitative test $\to$ *need light-cone spectrum of Carroll strings*

# Symmetries of magnetic Carroll $\mathcal{N}=4$ SYM

Magnetic Carroll $\mathcal{N}=4$ SYM bosonic action:

$$S=\int \mathrm{d}t\,\mathrm{d}^3x\left[-\frac{1}{4}(F^{ij\,a})^2-\frac{1}{2}(\mathcal{D}_i S^{I\,a})^2-\frac{1}{4}\kappa^{i\,a}F^a_{ti}+\frac{1}{2}\lambda^a_I\mathcal{D}_t S^{I\,a}+\frac{1}{4}\left(\mathfrak{f}_{bc}{}^a S^{I\,b}S^{J\,c}\right)^2\right].$$

$\kappa^{i\,a},\lambda^a_I$: Lagrange multipliers transforming non-linearly under spacetime symm.

Symmetries:

Conformal Carroll Algebra $\mathrm{CCA}_4$ + $\infty$-dim internal symm.

($\mathrm{CCA}_d\cong\mathrm{BMS}_{d+1}$)

Find <u>unusual</u> behaviour:

$$\begin{aligned}[\mathrm{CCA},\mathrm{CCA}] &\sim \mathrm{CCA}\\ [\mathrm{Internal},\mathrm{Internal}] &\sim \mathrm{Internal}\\ [\mathrm{CCA},\mathrm{Internal}] &\sim \mathrm{Internal}+\textbf{non-symmetry}\end{aligned}$$

E.g. dilatation $D=t\partial_t+x^i\partial_i$; $Q^{(l,m,n,p)}_{(ij)}=x^l y^m z^n\left(pt^{p-1}\,F_{tj}+2t^p\,\partial_t F_{tj}\right)\frac{\partial}{\partial\kappa^i}$

$$\left[D,Q^{(l,m,n,p)}_{ij}\right]=(l+m+n+p)Q^{(l,m,n,p)}_{ij}-\underbrace{\frac{1}{4}pt^{p-1}x^l y^m z^n\left(F_{tj}\frac{\partial}{\partial\kappa^i}+F_{ti}\frac{\partial}{\partial\kappa^j}\right)}_{\text{not symmetry}}$$

$p\notin\{0,1\}$

# 3. Flat Space

The method fails immediately:

1. Horizon ✗

   Flat D3-brane metric: $g \sim -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2$

$\rightarrow$ Give up D3-brane. Here the interpretation is a bit different:

$$
\begin{array}{ccc}
\mathrm{AdS}_{d+1} & \xrightarrow{\text{boundary}} & \mathrm{Mink}_d \\
\Big| & & \Big| \\
\text{large radius} & & \text{large radius} \\
\downarrow & & \downarrow \\
\mathrm{Mink}_{d+1} & \xrightarrow{\text{boundary}} & \text{Carroll Mink}_d
\end{array}
$$

radius $R$ is <u>same</u> for bulk and boundary

**Issue when applied to AdS/CFT**

Bulk:

$$\mathrm{AdS}_5 \times \mathrm{S}^5 \xrightarrow{R\to\infty} \mathrm{Mink}_{10}$$

Boundary:

$$\mathcal{N} = 4 \text{ SYM in 4d} \xrightarrow{R\to\infty} \text{Carroll } \mathcal{N} = 4 \text{ SYM in 3+1d}$$

Holographic boundary miss-match: $\partial \mathrm{Mink}_{10} \neq \mathrm{Carroll}_4$

# Possible resolutions: A Triality?

Two possible resolutions:

1. Compactification $\quad \text{Mink}_{10} \to \text{Mink}_5 \times X^5$
2. Bulk is Carroll as well $\rightarrow$ Carroll holography seen before

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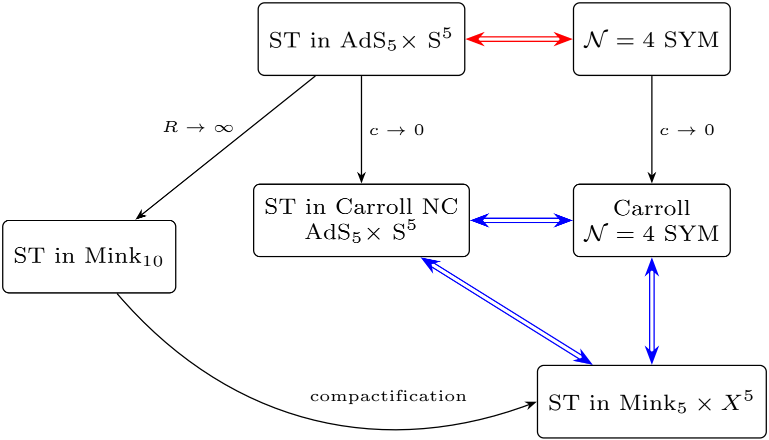

# Conclusions

Systematic approach to obtain gauge/gravity duality from limits of AdS/CFT

Novel Galilean + Carrollian holographies

Proposed a triality with flat space holography

**Open problems**

- show that $\Delta$ in Galilean $\mathcal{N} = 4$ SYM is <u>exact</u>
- non-relativistic spectral curve?
- Carroll type IIB supergravity?
- non-closure of symm. Connection with problem of magnetic Carroll susy?
- light-cone gauge spectrum of Carroll $\mathrm{AdS}_5 \times \mathrm{S}^5$?
- prove the triality.

  <u>Hint</u>: $\mathrm{AdSC}_d = \mathrm{Ti}_d$, with '$\mathrm{Ti}_d$' blow-up of timelike infinity of $\mathrm{Mink}_d$ (à la Ashtekar-Hansen) [Figueroa-O'Farrill, Have, Prohazka, Salzer (2021)]

  <u>Application</u>: AdSC has better IR properties than flat spacetime


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