Quantum M2-branes and Holography

Jesse van Muiden

Abdus Salam Centre for Theoretical Physics
Imperial College London

Workshop on Quantum Gravity and Strings

Corfu, September 2025

Based on
[2304.12340] with Fridrik Gautason and Valentina Puletti
[2503.16597], [2505.21633] & [w.i.p.] with Fridrik Gautason

Quantum Membranes

Zero Modes

Localisation

Membrane Ensembles

Holography

Emergence

In the 1980s Fradkin and Tseytlin constructed a **background field method** to compute the string generating functional

$$\mathcal{Z}_{\text{string}}[G, \Phi, B_2, \ldots]$$

which depends on target space fields as background sources.

Fradkin, Tseytlin '84-'85

Evaluating the functional in all generality is out of reach, instead an expansion in g_s and ℓ_s is applied.

The pointlike genus-0 contribution at long wavelengths reproduces the ten-dimensional supergravity action.

Note: many subtleties regarding (log) divergences.

Tesytlin '88-'89

At long wavelengths we expand the functional in saddles

$$\mathcal{Z}_{\text{string}} \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots = -S_{10\text{d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots$$

-) Degenerate string → perturbative contributions

$$\partial^2 + \partial^8 \ldots \sim \text{loops + \textbf{0-modes}} \sim \int_{\text{0-modes}} R + t_8 t_8 R^4 + \ldots$$

-) Non-degenerate strings → non-perturbative contributions

At long wavelengths we expand the functional in saddles

$$\mathcal{Z}_{\text{string}} \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots = -S_{10\text{d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots$$

-) Degenerate string → perturbative contributions

$$\partial^2 + \partial^8 \ldots \sim \text{loops + } \textbf{0-modes} \sim \int_{\text{0-modes}} R + t_8 t_8 R^4 + \ldots$$

-) Non-degenerate strings → non-perturbative contributions

0-modes: $\int$ free position of string

Equivariant localisation acts on zero-mode sector!

At long wavelengths we expand the functional in saddles

$$\mathcal{Z}_{\text{string}} \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots = -S_{\text{10d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots$$

-) Degenerate string → perturbative contributions

$$\partial^2 + \partial^8 \ldots \sim \text{loops} + \textbf{0-modes} \sim \int_{\text{0-modes}} R + t_8 t_8 R^4 + \ldots$$

-) Non-degenerate strings → non-perturbative contributions

0-modes: $\int$ free position of string

Equivariant localisation acts on zero-mode sector!

Sensible? → Holography!

$$\log Z_{\text{QFT}} \approx -S_{\text{10d}} + \ldots + \sum \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} \approx \mathcal{Z}_{\text{string}}$$

Gautason, Puletti, JvM '23

The <u>best understood</u> examples of holography include

$$\begin{array}{ccc} \mathrm{AdS}_4 \times S^7/\mathbf{Z}_k & \mathrm{AdS}_5 \times S^5 & \mathrm{AdS}_7 \times S^4 \\ \text{ABJM} & \mathcal{N}=4\,\text{SYM} & \text{6d }(2,0). \end{array}$$

Observables computable through SUSY localisation

- Partition functions ($S^d,\ M^{d-1}\times S^1,\ \ldots$).
- VEVs of Wilson line and defect operators.

Focus: the ABJM and (2,0) theories.

Watch out! Backgrounds of interest are in M-theory!

Proposal: promote background field method to M2-branes

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx -S_{\mathrm{11d}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

Proposal: promote background field method to M2-branes

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx -S_{\mathrm{11d}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

-) Degenerate M2 $\rightarrow$ perturbative contributions

$\partial^2 + \partial^8 \ldots \sim$ loops + **0-modes** $\sim \int_{\text{0-modes}} R + R^4 + A_3 \wedge I_8 + \ldots$

-) Non-degenerate M2 $\rightarrow$ non-perturbative contributions

Proposal: promote background field method to M2-branes

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx -S_{\mathrm{11d}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

-) Degenerate M2 → perturbative contributions

$\partial^2 + \partial^8 \ldots \sim$ loops + **0-modes** $\sim \int_{\text{0-modes}} R + R^4 + A_3 \wedge I_8 + \ldots$

-) Non-degenerate M2 → non-perturbative contributions

0-modes: $\int$ free position of membrane

Equivariant localisation acts on zero-mode sector!

Proposal: promote background field method to M2-branes

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx -S_{11\mathrm{d}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

-) Degenerate M2 $\rightarrow$ perturbative contributions

$$\partial^2 + \partial^8 \ldots \sim \text{loops} + \textbf{0-modes} \sim \int_{\text{0-modes}} R + R^4 + A_3 \wedge I_8 + \ldots$$

-) Non-degenerate M2 $\rightarrow$ non-perturbative contributions

0-modes: $\int$ free position of membrane

Equivariant localisation acts on zero-mode sector!

Sensible? $\rightarrow$ *Holography!*

$$\log Z_{\mathrm{QFT}} \approx -S_{11\mathrm{d}} + \ldots + \sum \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}} \approx \mathcal{Z}_{\mathrm{M2}}$$

Gautason, JvM '25

$\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ computed at **fixed source** $A_3 \sim \mu$;

$\mathcal{Z}_{\rm M2}[G, A_3]$ computed at **fixed source** $A_3 \sim \mu$;

The **conjugate quantity** $\star G_4$ is an expectation value

$$\text{e.g. in AdS}_4 \times S^7: \qquad \frac{2\pi}{(2\pi\ell_p)^9}\int G_4 \wedge \star G_4 = \mu N \,.$$

$\mathcal{Z}_{\rm M2}[G, A_3]$ computed at **fixed source** $A_3 \sim \mu$;

The **conjugate quantity** $\star G_4$ is an expectation value

$$\text{e.g. in AdS}_4 \times S^7: \qquad \frac{2\pi}{(2\pi\ell_p)^9} \int G_4 \wedge \star G_4 = \mu N\,.$$

$\mathcal{Z}_{\rm M5}(G, A_6)$ computed at fixed A_6.

$\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ computed at **fixed source** $A_3 \sim \mu$;

The **conjugate quantity** $\star G_4$ is an expectation value

$$\text{e.g. in } \mathrm{AdS}_4 \times S^7: \qquad \frac{2\pi}{(2\pi\ell_p)^9} \int G_4 \wedge \star G_4 = \mu N \,.$$

$\mathcal{Z}_{\mathrm{M5}}(G, A_6)$ computed at fixed A_6.

Transform between ensembles is given by

$$\mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}(N)} = \frac{1}{2\pi\mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu \, \mathrm{e}^{\mathcal{Z}_{\mathrm{M2}}(\mu) - \mu N} \,, \quad \mathrm{e}^{\mathcal{Z}_{\mathrm{M2}}(\mu)} = \sum_{N=0}^{\infty} \mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}(N) + \mu N} \,.$$

$\mathcal{Z}_{\rm M2}[G, A_3]$ computed at **fixed source** $A_3 \sim \mu$;

The **conjugate quantity** $\star G_4$ is an expectation value

e.g. in $\mathrm{AdS}_4 \times S^7$: $$\frac{2\pi}{(2\pi\ell_p)^9} \int G_4 \wedge \star G_4 = \mu N\,.$$

$\mathcal{Z}_{\rm M5}(G, A_6)$ computed at fixed A_6.

Transform between ensembles is given by

$$\mathrm{e}^{\mathcal{Z}_{\rm M5}(N)} = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu\, \mathrm{e}^{\mathcal{Z}_{\rm M2}(\mu) - \mu N}\,, \quad \mathrm{e}^{\mathcal{Z}_{\rm M2}(\mu)} = \sum_{N=0}^{\infty} \mathrm{e}^{\mathcal{Z}_{\rm M5}(N) + \mu N}\,.$$

Attention

AdS_4: $\mathcal{Z}_{\rm M2}(\mu)$ does not fix N; **grand canonical CFT ensemble**.
AdS_7: $\mathcal{Z}_{\rm M2}(N)$ does fix N; **canonical CFT ensemble**.

- **M2-brane saddles**

  M2-brane partition function is expanded in saddles of degenerate and non-degenerate branes.

- **Equivariant Localisation**

  Zero-modes need to be integrated over. Equivariant localisation simplifies this integral to a sum.

- **Membrane Ensembles**

  *M2-brane couples to a background source A_3. The conjugate variable $\star G_4$ is **not** fixed. Change of ensembles achieved through Laplace transform.*

We choose a background in M-theory

$$\text{AdS}_4 \times S^7/\mathbf{Z}_k\,, \qquad \text{AdS}_4|_\partial = S^3$$

Note: generalisable to other asymptotic AdS_4 geometries.

We choose a background in M-theory

$$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k\,, \qquad \mathrm{AdS}_4|_\partial = S^3$$

Note: generalisable to other asymptotic AdS_4 geometries.

Check 1: on-shell action

$$\mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}} \approx Z^{(0)}_{\text{1-loop}} + \ldots = -S_{\mathrm{11d}} + \mathcal{O}(1/\mu)$$

Check 2: membrane instantons

$$\mathcal{Z}^{\mathrm{np}}_{\mathrm{M2}} = \mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} + \ldots + \mathcal{O}(\mathrm{e}^{-2S_{\mathrm{cl}}}) \quad \leftarrow \quad S^3/\mathbf{Z}_k \subset S^7/\mathbf{Z}_k$$

Check 3: Wilson lines and defects

We choose a background in M-theory

$$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k\,, \qquad \mathrm{AdS}_4|_\partial = S^3$$

Note: generalisable to other asymptotic AdS_4 geometries.

Check 1: on-shell action

$$\mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}} \approx Z^{(0)}_{\text{1-loop}} + \ldots = -S_{\mathrm{11d}} + \mathcal{O}(1/\mu)$$

Check 2: membrane instantons

$$\mathcal{Z}^{\mathrm{np}}_{\mathrm{M2}} = \mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} + \ldots + \mathcal{O}(\mathrm{e}^{-2S_{\mathrm{cl}}}) \quad \leftarrow \quad S^3/\mathbf{Z}_k \subset S^7/\mathbf{Z}_k$$

Check 3: Wilson lines and defects

Note: QFT historically studied in grand canonical ensemble!

$$\mathrm{e}^{J(\mu)} = \sum Z^{S^3}_{\mathrm{ABJM}}(N)\,\mathrm{e}^{\mu N}\,, \quad J(\mu) = C_k\,\mu^3/3 + B_k\mu + A_k + \mathcal{O}(\mathrm{e}^{-\mu})$$

Drukker, Marino, Putrov '10-'11; Hatsuda, Moriyama, Okuyama '12, …

Check 1: on-shell action.

Match: $\mathcal{Z}_{\text{M2}}(\mu) \approx Z^{(0)}_{\text{1-loop}} = -S_{\text{11d}}(\mu) = \dfrac{2\mu^3}{3\pi^2 k} = C_k\, \mu^3/3 \approx J(\mu)\,.$

Mismatch: $-\,S_{\text{11d}}(N) \neq F_{\text{ABJM}}(N)\,.$

Check 1: on-shell action.

Match: $$\mathcal{Z}_{\rm M2}(\mu) \approx Z^{(0)}_{\text{1-loop}} = -S_{\rm 11d}(\mu) = \frac{2\mu^3}{3\pi^2 k} = C_k\,\mu^3/3 \approx J(\mu)\,.$$

Mismatch: $$-\,S_{\rm 11d}(N) \neq F_{\rm ABJM}(N)\,.$$

Instead ensemble change is needed $\rightarrow$ Legendre trafo:

$$F_{\rm ABJM}(N) \approx \mathcal{Z}_{\rm M2}(\mu_*) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3}\,, \quad \mu_* \approx \frac{Nk\pi^2}{2}\,.$$

Check 1: on-shell action.

Match: $\mathcal{Z}_{\mathrm{M2}}(\mu) \approx Z^{(0)}_{\text{1-loop}} = -S_{11\mathrm{d}}(\mu) = \dfrac{2\mu^3}{3\pi^2 k} = C_k\,\mu^3/3 \approx J(\mu)\,.$

Mismatch: $-\,S_{11\mathrm{d}}(N) \neq F_{\mathrm{ABJM}}(N)\,.$

Instead ensemble change is needed → Legendre trafo:

$$F_{\mathrm{ABJM}}(N) \approx \mathcal{Z}_{\mathrm{M2}}(\mu_*) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3}\,, \quad \mu_* \approx \frac{Nk\pi^2}{2}\,.$$

Holography dictates thus that

$$J^{\mathrm{p}}(\mu) = C_k\,\mu^3/3 + B_k\mu + A_k \sim R + R^4 + \text{non-local} = \mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}}(\mu)$$

and **no further corrections!**

Check 1: on-shell action.

Match: $\mathcal{Z}_{\text{M2}}(\mu) \approx Z^{(0)}_{\text{1-loop}} = -S_{\text{11d}}(\mu) = \frac{2\mu^3}{3\pi^2 k} = C_k\,\mu^3/3 \approx J(\mu)\,.$

Mismatch: $-S_{\text{11d}}(N) \neq F_{\text{ABJM}}(N)\,.$

Instead ensemble change is needed → Legendre trafo:

$$F_{\text{ABJM}}(N) \approx \mathcal{Z}_{\text{M2}}(\mu_*) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3}\,, \quad \mu_* \approx \frac{Nk\pi^2}{2}\,.$$

Holography dictates thus that

$$J^{\text{p}}(\mu) = C_k\,\mu^3/3 + B_k\mu + A_k \sim R + R^4 + \text{non-local} = \mathcal{Z}^{\text{p}}_{\text{M2}}(\mu)$$

and **no further corrections!** **Infinitely** many $1/N$ in canonical:

$$Z^{\text{p}}(N,k) = \mathrm{e}^{A_k} C_k^{-1/3} \mathrm{Ai}[C_k^{-1/3}(N - B_k)]\,.$$

LOCALISATION IN M-THEORY

4D supergravity **localises equivariantly**

$$S^{(4\mathrm{D})}_{\mathrm{SUGRA}} = \int \Phi = \sum_{\xi^*} \Phi \qquad \text{for} \qquad \begin{cases} (d - i_\xi)\Phi = 0 \\ \xi^* \sim \epsilon = \pm\gamma_{(4)}\epsilon \end{cases}$$

Benetti Genolini, Gauntlett, Ipina, Jiao, Lüscher, Sparks, . . . ’19-’25

Localisation in M-theory

4D supergravity **localises equivariantly**

$$S^{(4D)}_{\text{SUGRA}} = \int \Phi = \sum_{\xi^*} \Phi \qquad \text{for} \qquad \begin{cases} (d - i_\xi)\Phi = 0 \\ \xi^* \sim \epsilon = \pm\gamma_{(4)}\epsilon \end{cases}$$

Benetti Genolini, Gauntlett, Ipina, Jiao, Lüscher, Sparks, … '19-'25

<u>In M-theory equivariant localisation acts on 0-modes!</u>

$$\mathcal{Z}_{\text{M2}}[G, A_3] \approx Z^{(0)}_{\text{1-loop}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-nS_{\text{cl}}} Z^{(n)}_{\text{1-loop}} + \ldots$$

where

$$Z_{\text{1-loop}} = Z_{\text{0-modes}}\, \text{Sdet}'[\mathbb{K}]^{-1/2} = \int \mathrm{d}\varphi\, \text{Jac}_\varphi\, \mathrm{e}^{-S[\Phi_{\text{cl}}]} \text{Sdet}'[\mathbb{K}]^{-1/2}$$

$$Z_{\text{0-modes}} \sim 0 \times \infty ?!$$

Integral localises to ξ-fixed points!

Puletti, Gautason, JvM '23, Gautason, JvM '25

Check 2: membrane instantons.

$$\mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} : \quad S_{\mathrm{cl}} = \frac{2\pi}{(2\pi \ell_s)^3} \left(\int \mathrm{d}^3 x \sqrt{g} + \mathrm{i} q \int A_3 \right) = \frac{4L^3}{k \ell_p^3} \equiv \frac{4\mu}{k}$$

Check 2: membrane instantons.

$$\mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} : \quad S_{\mathrm{cl}} = \frac{2\pi}{(2\pi \ell_s)^3} \left(\int \mathrm{d}^3 x \sqrt{g} + \mathrm{i} q \int A_3 \right) = \frac{4L^3}{k \ell_p^3} \equiv \frac{4\mu}{k}$$

SUSY localises membrane to the ξ-fixed points!

$$(1 - \mathrm{i} q \Gamma_{\mathrm{M2}})\epsilon = 0 \quad \Rightarrow \quad \gamma_{(4)} \epsilon = q\epsilon \, ,$$

Check 2: membrane instantons.

$$\mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} : \quad S_{\mathrm{cl}} = \frac{2\pi}{(2\pi\ell_s)^3}\left(\int \mathrm{d}^3x\sqrt{g} + \mathrm{i}q\int A_3\right) = \frac{4L^3}{k\ell_p^3} \equiv \frac{4\mu}{k}$$

SUSY localises membrane to the ξ-fixed points!

$$(1 - \mathrm{i}q\Gamma_{\mathrm{M2}})\epsilon = 0 \quad \Rightarrow \quad \gamma_{(4)}\epsilon = q\epsilon\,,$$

Including the 1-loop determinant:

$$\mathcal{Z}^{(1)}_{\mathrm{M2}} = 2\sum_{\xi^*} \frac{s\,(2/k)^{2k} s(x_+)^{-k} s(x_-)^{-k}}{t(x_+)t(x_-)}\mathrm{e}^{-4\mu/k}$$

-) s and t are standard functions for 3d partition functions

$$s(z) = e^{\frac{i\,\mathrm{Li}_2(e^{2\pi i z})}{2\pi} - \frac{i\pi}{12} - z\log\left[1-e^{2\pi i z}\right] + \frac{i\pi z^2}{2}}\,, \quad t(z) = 4\sin\pi z/(kz).$$

-) $x_\pm$ fixes boundary; S^3, S^3_b, $S^1\times\Sigma_{\mathfrak{g}}, \ldots$.

Gautson, JvM '25

Check 2: membrane instantons.

$$\mathrm{e}^{-S_{\mathrm{cl}}} Z^{(1)}_{\text{1-loop}} : \quad S_{\mathrm{cl}} = \frac{2\pi}{(2\pi \ell_s)^3} \left(\int \mathrm{d}^3 x \sqrt{g} + \mathrm{i} q \int A_3 \right) = \frac{4L^3}{k \ell_p^3} \equiv \frac{4\mu}{k}$$

SUSY localises membrane to the ξ-fixed points!

$$(1 - \mathrm{i} q \Gamma_{\mathrm{M2}})\epsilon = 0 \quad \Rightarrow \quad \gamma_{(4)}\epsilon = q\epsilon \,,$$

Including the 1-loop determinant:

$$\mathcal{Z}^{(1)}_{\mathrm{M2}} = 2 \sum_{\xi^*} \frac{s\,(2/k)^{2k} s(x_+)^{-k} s(x_-)^{-k}}{t(x_+) t(x_-)} \mathrm{e}^{-4\mu/k}$$

-) s and t are standard functions for 3d partition functions

$$s(z) = e^{\frac{i\,\mathrm{Li}_2(e^{2\pi i z})}{2\pi} - \frac{i\pi}{12} - z \log\left[1 - e^{2\pi i z}\right] + \frac{i\pi z^2}{2}} \,, \quad t(z) = 4 \sin \pi z/(kz).$$

-) $x_\pm$ fixes boundary; S^3, S^3_b, $S^1 \times \Sigma_{\mathfrak{g}}, \ldots$.

Gautson, JvM '25

-) S^3: $x_\pm = 2/k$ **matches field theory in μ-ensemble!**

Gautason, Puletti, JvM '23; Beccaria, Giombi, Tseytlin '23

Structure of M2-brane partition functions.

	$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$	$\mathrm{AdS}_{7,\beta} \times S^4$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{p}}$	$C\mu^3 + B\mu + A$	$cN^3 + bN + a$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{np}(\ell)}$	$\frac{\exp(-4\ell\mu/k)}{4\ell\sin^2 2\pi\ell/k}$	$\frac{\exp(-\ell N\beta)}{4\ell\sinh^2 \ell\beta/2}$	$\leftarrow$ **1-loop**
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{WL}(\ell)}$	$\frac{\exp(2\ell\mu/k)}{2\sin 2\pi\ell/k}$	$\frac{\exp(\ell\beta N)}{2\sinh \ell\beta/2}$	$\leftarrow$ **1-loop**
QFT ensemble	grand-canonical	canonical	

Gautason, JvM '25 + w.i.p.

Structure of M2-brane partition functions.

	$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$	$\mathrm{AdS}_{7,\beta} \times S^4$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{p}}$	$C\mu^3 + B\mu + A$	$cN^3 + bN + a$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{np}(\ell)}$	$\frac{\exp(-4\ell\mu/k)}{4\ell \sin^2 2\pi\ell/k}$	$\frac{\exp(-\ell N\beta)}{4\ell \sinh^2 \ell\beta/2}$	$\leftarrow$ **1-loop**
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{WL}(\ell)}$	$\frac{\exp(2\ell\mu/k)}{2\sin 2\pi\ell/k}$	$\frac{\exp(\ell\beta N)}{2\sinh \ell\beta/2}$	$\leftarrow$ **1-loop**
QFT ensemble	grand-canonical	canonical	

*Cubic structure holds for class-$\mathcal{S}$ anomalies; asymptotic AdS_5!

Gautason, JvM '25 + w.i.p.

Structure of M2-brane partition functions.

	$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$	$\mathrm{AdS}_{7,\beta} \times S^4$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{p}}$	$C\mu^3 + B\mu + A$	$cN^3 + bN + a$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{np}(\ell)}$	$\frac{\exp(-4\ell\mu/k)}{4\ell\sin^2 2\pi\ell/k}$	$\frac{\exp(-\ell N\beta)}{4\ell\sinh^2 \ell\beta/2}$	$\leftarrow$ **1-loop**
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{WL}(\ell)}$	$\frac{\exp(2\ell\mu/k)}{2\sin 2\pi\ell/k}$	$\frac{\exp(\ell\beta N)}{2\sinh \ell\beta/2}$	$\leftarrow$ **1-loop**
QFT ensemble	grand-canonical	canonical	

*Cubic structure holds for class-$\mathcal{S}$ anomalies; asymptotic AdS_5!

-) SUSY M2-brane partition functions truncate!

$$\mathcal{Z}_{\mathrm{M2}}^{(0)} \sim a\mu^3 + b\mu + c \sim R + R^4 + \text{non-local}$$

Gautason, JvM '25 + w.i.p.

Structure of M2-brane partition functions.

	$\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$	$\mathrm{AdS}_{7,\beta} \times S^4$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{p}}$	$C\mu^3 + B\mu + A$	$cN^3 + bN + a$	
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{np}(\ell)}$	$\frac{\exp(-4\ell\mu/k)}{4\ell \sin^2 2\pi\ell/k}$	$\frac{\exp(-\ell N\beta)}{4\ell \sinh^2 \ell\beta/2}$	$\leftarrow$ **1-loop**
$\mathcal{Z}_{\mathrm{M2}}^{\mathrm{WL}(\ell)}$	$\frac{\exp(2\ell\mu/k)}{2\sin 2\pi\ell/k}$	$\frac{\exp(\ell\beta N)}{2\sinh \ell\beta/2}$	$\leftarrow$ **1-loop**
QFT ensemble	grand-canonical	canonical	

*Cubic structure holds for class-$\mathcal{S}$ anomalies; asymptotic AdS_5!

-) SUSY M2-brane partition functions truncate!

$$\mathcal{Z}_{\mathrm{M2}}^{(0)} \sim a\mu^3 + b\mu + c \sim R + R^4 + \text{non-local}$$

-) Non-perturbative saddles are 1-loop exact!

$$\mathcal{Z}_{\mathrm{M2}}^{(\ell)}(\mu, \beta) = \ell^{1-\chi} \mathcal{Z}_{\mathrm{M2}}^{(1)}(\mu, \ell\beta)$$

Gautason, JvM '25 + w.i.p.

Emergence

Proposed a background field formalism for M2-branes

$$\mathcal{Z}_{\text{M2}}[G, A_3] \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots = -S_{\text{11d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} \,.$$

Emergence

Proposed a background field formalism for M2-branes

$$\mathcal{Z}_{\text{M2}}[G, A_3] \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} + \ldots = -S_{\text{11d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\text{cl}}} Z_{\text{1-loop}} \,.$$

Quantising the point-like saddle is conceptually challenging!

Emergence

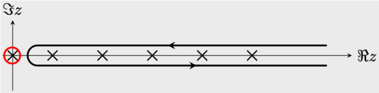
Proposed a background field formalism for M2-branes

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx \sum_{\text{saddles}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}} + \ldots = -S_{\mathrm{11d}} + \ldots + \sum_{\text{inst.}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}} \,.$$

Quantising the point-like saddle is conceptually challenging!

Circumvent issue by integrating out instantons instead

Gopakumar Vafa '; Dedushenko, Witten '14; Hattab, Palti '24



$$\mathcal{Z}^{\mathrm{AdS}_7}_{\mathrm{M2}} = \sum_n \int \frac{\mathrm{d}z}{z} \frac{\mathrm{e}^{2\pi \mathrm{i} n z} \mathrm{e}^{-z\beta N}}{4 \sinh^2 z\beta/2} = \frac{\beta N^3}{6} - \frac{\beta N}{8} + E_c + \sum_{\text{inst.}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

w.i.p.

- **Background field formalism** for M2-branes.
- **Membrane ensembles** in holography.
- Partition functions **uniform (cubic!) form across dimensions**.
- **Equivariant localisation** acts on M2-brane zero-modes.
- Semi-classical M2-branes **1-loop exact**.
- Predictions for **new non-perturbative effects** in holographic QFTs.

Future work:

- Quantisation of **point-like** M2-branes.
- Explain **cubic structure** of perturbative saddle.
- **Two-loop quantisation** of M2-branes.
- Show existence of **worldvolume localisation**.
- Ensembles in **different type II and M-theory setups**.

Thank you!

Extra

Localization of the M2-brane

[2503.16597]

Background

M-theory on SE_7 backgrounds can be consistently truncated to 4d minimal $\mathcal{N} = 2$ gauged supergravity

$$\frac{-2\pi}{(2\pi\ell_p)^9}\int \star(R - \frac{1}{2}|G_4|^2) - \frac{1}{6}A_3 \wedge G_4 \wedge G_4 \,, \quad \delta_\epsilon \Psi_M = 0 \,,$$

$$\updownarrow$$

$$\frac{-1}{16\pi G_N^{(4)}}\int \star_4(R_4 + \Lambda - \frac{1}{2}|F|^2) \,, \quad \delta_\eta \psi_\mu = 0 \,.$$

Background

M-theory on SE_7 backgrounds can be consistently truncated to 4d minimal $\mathcal{N}=2$ gauged supergravity

$$\frac{-2\pi}{(2\pi\ell_p)^9}\int \star\Big(R-\frac{1}{2}|G_4|^2\Big)-\frac{1}{6}A_3\wedge G_4\wedge G_4\,,\qquad \delta_\epsilon \Psi_M = 0\,,$$

$$\updownarrow$$

$$\frac{-1}{16\pi G_N^{(4)}}\int \star_4\Big(R_4+\Lambda-\frac{1}{2}|F|^2\Big)\,,\qquad \delta_\eta\psi_\mu = 0\,.$$

These simple backgrounds 'factorise':

$$\mathrm{d}s_{11}^2 = L^2\left(\mathrm{d}s_4^2 + 4\,\mathrm{d}s^2_{\mathrm{KE}_6} + \big(\mathrm{d}y + 2\sigma + \tfrac{1}{2}A\big)^2\right),$$

$$G_4 = iL^3\,(3\,\mathrm{vol}_4 + 2J\wedge\star_4 F)\,,$$

$$\epsilon = \eta\otimes\chi^+ + \eta^c\otimes\chi^-\,,\qquad \not{J}\chi^\pm \propto \pm\chi^\pm\,.$$

For simplicity we take $S^7/\mathbf{Z}_k$ as internal space.

Gauntlett, Varela '07

Note: 4d solutions determined by (anti-)self-dual flux $F^{(\pm)}$.

Localization in Supergravity

4d minimal $\mathcal{N} = 2$ gauged supergravity can be localized using Berline-Vergne-Atiyah-Bott fixed point formula:

- Any SUSY solution has a Killing spinor η
- The Killing spinor defines a Killing vector ξ, and an associated equivariant derivative $\mathrm{d}_\xi = \mathrm{d} - i_\xi$
- Equivariantly closed polyforms localise to fixed points (nuts or bolts) of the Killing vector ξ (BVAB)
- Fixed points are in one-to-one correspondence with chiral points of the Killing spinor: $\eta = \pm\gamma_{(4)}\eta$

Localization in Supergravity

4d minimal $\mathcal{N} = 2$ gauged supergravity can be localized using Berline-Vergne-Atiyah-Bott fixed point formula:

- Any SUSY solution has a Killing spinor η
- The Killing spinor defines a Killing vector ξ, and an associated equivariant derivative $\mathrm{d}_\xi = \mathrm{d} - i_\xi$
- Equivariantly closed polyforms localise to fixed points (nuts or bolts) of the Killing vector ξ (BVAB)
- Fixed points are in one-to-one correspondence with chiral points of the Killing spinor: $\eta = \pm\gamma_{(4)}\eta$

Application: on-shell actions in gauged supergravity.

Benetti Genolini, Gauntlett, Ipina, Jiao, Lüscher, Sparks, …'19-'25

Localization in Supergravity

4d minimal $\mathcal{N} = 2$ gauged supergravity can be localized using Berline-Vergne-Atiyah-Bott fixed point formula:

- Any SUSY solution has a Killing spinor η
- The Killing spinor defines a Killing vector ξ, and an associated equivariant derivative $\mathrm{d}_\xi = \mathrm{d} - i_\xi$
- Equivariantly closed polyforms localise to fixed points (nuts or bolts) of the Killing vector ξ (BVAB)
- Fixed points are in one-to-one correspondence with chiral points of the Killing spinor: $\eta = \pm\gamma_{(4)}\eta$

Application: on-shell actions in gauged supergravity.

Benetti Genolini, Gauntlett, Ipina, Jiao, Lüscher, Sparks, . . . '19-'25

Application in M-theory?

$$\mathcal{Z}_{\mathrm{M2}}[G, A_3] \approx -S_{11\mathrm{d}} + \ldots + \sum_{\text{instantons}} \mathrm{e}^{-S_{\mathrm{cl}}} Z_{\text{1-loop}}$$

Gautason, JvM '25

Localization of the M2-brane

M2-branes of charge q have action and susy constraint

$$S_{\text{M2}} = \frac{2\pi}{(2\pi\ell_s)^3}\left(\int \mathrm{d}^3x\sqrt{g} + \mathrm{i}q\int A_3\right)\,, \qquad (1 - \mathrm{i}q\Gamma_{\text{M2}})\epsilon = 0$$

with $\Gamma_{\text{M2}} = \frac{1}{3!}\varepsilon^{abc}\Gamma_{abc}$.

Localization of the M2-brane

M2-branes of charge q have action and susy constraint

$$S_{\rm M2} = \frac{2\pi}{(2\pi\ell_s)^3}\left(\int \mathrm{d}^3x\sqrt{g} + \mathrm{i}q\int A_3\right)\,, \qquad (1-\mathrm{i}q\Gamma_{\rm M2})\epsilon = 0$$

with $\Gamma_{\rm M2} = \frac{1}{3!}\varepsilon^{abc}\Gamma_{abc}$.

On-shell action of an instanton evaluates to

$$S^3/\mathbf{Z}_k \subset S^7/\mathbf{Z}_k: \qquad S^{\rm cl}_{\rm M2} = 4L^3/k\ell_p^3 \equiv 4\mu/k\,.$$

Localization of the M2-brane

M2-branes of charge q have action and susy constraint

$$S_{\mathrm{M2}} = \frac{2\pi}{(2\pi\ell_s)^3}\left(\int \mathrm{d}^3x\sqrt{g} + \mathrm{i}q\int A_3\right), \qquad (1-\mathrm{i}q\Gamma_{\mathrm{M2}})\epsilon = 0$$

with $\Gamma_{\mathrm{M2}} = \frac{1}{3!}\varepsilon^{abc}\Gamma_{abc}$.

On-shell action of an instanton evaluates to

$$S^3/\mathbf{Z}_k \subset S^7/\mathbf{Z}_k: \qquad S^{\mathrm{cl}}_{\mathrm{M2}} = 4L^3/k\ell_p^3 \equiv 4\mu/k\,.$$

The SUSY constraint imposes

$$\Gamma_{\mathrm{M2}} = \gamma_{(4)}\gamma_{\mathrm{M2}}: \quad \mathrm{i}\gamma_{\mathrm{M2}}\chi^{\pm} = \chi^{\pm}, \quad \gamma_{(4)}\eta = q\eta\,,$$

localising the instantons to the ξ-fixed points.

Quantum M2-branes

Quantisation: compute 1-loop determinant of $8+8$ d.o.f.

$$Z_{\text{1-loop}} = Z_{\text{0-modes}}\, \mathrm{Sdet}'[\mathbb{K}]^{-\frac{1}{2}} = \int \mathrm{d}\Phi\, \mathrm{Jac}_{\Phi}\, \mathrm{e}^{-S[\Phi]} \mathrm{Sdet}'[\mathbb{K}]^{-\frac{1}{2}}$$

Integral over zero-modes is difficult.

Quantum M2-branes

Quantisation: compute 1-loop determinant of $8+8$ d.o.f.

$$Z_{\text{1-loop}} = Z_{\text{0-modes}}\,\text{Sdet}'[\mathbb{K}]^{-\frac{1}{2}} = \int \mathrm{d}\Phi\,\text{Jac}_\Phi\,\mathrm{e}^{-S[\Phi]}\text{Sdet}'[\mathbb{K}]^{-\frac{1}{2}}$$

Integral over zero-modes is difficult.

SUSY localises to ξ-fixed points

Quantum M2-branes

Quantisation: compute 1-loop determinant of $8+8$ d.o.f.

$$Z_{\text{1-loop}} = Z_{\text{0-modes}}\,\text{Sdet}'[\mathbb{K}]^{-\frac{1}{2}} = \int \mathrm{d}\Phi\,\text{Jac}_\Phi\,\mathrm{e}^{-S[\Phi]}\text{Sdet}'[\mathbb{K}]^{-\frac{1}{2}}$$

Integral over zero-modes is difficult.

SUSY localises to ξ-fixed points

Result: $$\mathcal{Z}^{(1)}_{\text{M2}}[G, A_3] = 2\sum_{\text{fxd pts}} \frac{s\left(\frac{2}{k}\right)^{2k} s(x_+)^{-k} s(x_-)^{-k}}{t(x_+)t(x_-)}\mathrm{e}^{-4\mu/k}\,,$$

with $x_\pm = 2(1\pm F^{(q)})/k$ determining the boundary, and:

$$s(z) = e^{\frac{i\,\text{Li}_2(e^{2\pi i z})}{2\pi} - \frac{i\pi}{12} - z\log\left[1-e^{2\pi i z}\right] + \frac{i\pi z^2}{2}}\,, \quad t(z) = \frac{4}{k}\frac{\sin\pi z}{z}.$$

Gautason, JvM '25

QUANTUM M2-BRANES

Quantisation: compute 1-loop determinant of $8+8$ d.o.f.

$$Z_{\text{1-loop}} = Z_{\text{0-modes}}\, \mathrm{Sdet}'[\mathbb{K}]^{-\frac{1}{2}} = \int \mathrm{d}\Phi\, \mathrm{Jac}_\Phi\, \mathrm{e}^{-S[\Phi]} \mathrm{Sdet}'[\mathbb{K}]^{-\frac{1}{2}}$$

Integral over zero-modes is difficult.

SUSY localises to ξ-fixed points

Result: $$\mathcal{Z}^{(1)}_{\mathrm{M2}}[G, A_3] = 2 \sum_{\text{fxd pts}} \frac{s\left(\frac{2}{k}\right)^{2k} s(x_+)^{-k} s(x_-)^{-k}}{t(x_+)t(x_-)} \mathrm{e}^{-4\mu/k}\,,$$

with $x_\pm = 2(1 \pm F^{(q)})/k$ determining the boundary, and:

$$s(z) = e^{\frac{i\,\mathrm{Li}_2(e^{2\pi i z})}{2\pi} - \frac{i\pi}{12} - z\log\left[1 - e^{2\pi i z}\right] + \frac{i\pi z^2}{2}}\,, \quad t(z) = \frac{4}{k}\frac{\sin \pi z}{z}.$$

Gautason, JvM ’25

S^3 bdry; $F^{(q)} = 0$: $$\mathcal{Z}^{(1)}_{\mathrm{M2}}[G, A_3] = \sin^{-2}\frac{2\pi}{k}\exp(-4\mu/k)\,.$$

Gautason, Puletti, JvM ’23; Beccaria, Giombi, Tseytlin ’23

Ensembles in M-theory and Holography

[2505.21633]

Ensembles in AdS_4 Holography

Computed $\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

ENSEMBLES IN AdS_4 HOLOGRAPHY

Computed $\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

Field theory instead studied at **fixed** $\star G_4 \sim N$.

ENSEMBLES IN AdS_4 HOLOGRAPHY

Computed $\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

Field theory instead studied at **fixed** $\star G_4 \sim N$.

The quantaties are **conjugates** to each other. At leading order

$$\frac{2\pi}{(2\pi\ell_p)^9}\int G_4 \wedge \star G_4 = \mu N\,.$$

Ensembles in AdS_4 Holography

Computed $\mathcal{Z}_{\rm M2}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

Field theory instead studied at **fixed** $\star G_4 \sim N$.

The quantaties are **conjugates** to each other. At leading order

$$\frac{2\pi}{(2\pi\ell_p)^9} \int G_4 \wedge \star G_4 = \mu N \,.$$

Natural candidate coupling to background A_6: $\mathcal{Z}_{\rm M5}(G, A_6)$.

Ensembles in AdS_4 Holography

Computed $\mathcal{Z}_{\mathrm{M2}}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

Field theory instead studied at **fixed** $\star G_4 \sim N$.

The quantaties are **conjugates** to each other. At leading order

$$\frac{2\pi}{(2\pi \ell_p)^9} \int G_4 \wedge \star G_4 = \mu N \,.$$

Natural candidate coupling to background A_6: $\mathcal{Z}_{\mathrm{M5}}(G, A_6)$.

Transform between ensembles is given by

$$\mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}(N)} = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu \, \mathrm{e}^{\mathcal{Z}_{\mathrm{M2}}(\mu) - \mu N} \,, \quad \mathrm{e}^{\mathcal{Z}_{\mathrm{M2}}(\mu)} = \sum_{N=0}^{\infty} \mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}(N) + \mu N} \,.$$

Ensembles in AdS_4 Holography

Computed $\mathcal{Z}_{\rm M2}[G, A_3]$ for **fixed source** $A_3 \sim \mu$; (M2-ensemble).

Field theory instead studied at **fixed** $\star G_4 \sim N$.

The quantaties are **conjugates** to each other. At leading order

$$\frac{2\pi}{(2\pi\ell_p)^9}\int G_4 \wedge \star G_4 = \mu N\,.$$

Natural candidate coupling to background A_6: $\mathcal{Z}_{\rm M5}(G, A_6)$.

Transform between ensembles is given by

$$\mathrm{e}^{\mathcal{Z}_{\rm M5}(N)} = \frac{1}{2\pi \mathrm{i}}\int_{\mathcal{C}} \mathrm{d}\mu\, \mathrm{e}^{\mathcal{Z}_{\rm M2}(\mu)-\mu N}\,, \quad \mathrm{e}^{\mathcal{Z}_{\rm M2}(\mu)} = \sum_{N=0}^{\infty} \mathrm{e}^{\mathcal{Z}_{\rm M5}(N)+\mu N}\,.$$

This **defines** $\mathcal{Z}_{\rm M5}(N)$ and identifies

$$\mathcal{Z}_{\rm M5}(N) = \log Z_{\rm ABJM}(N)\,.$$

Not clear how to compute $\mathcal{Z}_{\rm M5}(N)$ or its relation to eleven-dimensional supergravity.

ENSEMBLES IN AdS_4 HOLOGRAPHY

In ABJM, historically, computations are performed in the **grand-canonical ensemble**, i.e. with fixed μ

$$e^{J(\mu,k)} = \sum_{N=0}^{\infty} Z_{S^3}(N,k)\, e^{\mu N}\,, \quad J(\mu,k) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k + \mathcal{O}(\mathrm{e}^{-\mu})$$

Drukker, Marino, Putrov ’11; Hatsuda, Honda, Fuji, Moriyama, Okuyama, Okazaki, Nosaka, Yokoyama, Grassi, Klemm, Schiereck, Soroush, Bobev, Hristov, Hong, Reys, de Smet, Zhang, Chester, Kalloor, Sharon, Armanini, Griguolo, Guerrini, Gaiotto, Abajian, Kubo, Pang, ...

Ensembles in AdS_4 Holography

In ABJM, historically, computations are performed in the **grand-canonical ensemble**, i.e. with fixed μ

$$e^{J(\mu,k)} = \sum_{N=0}^{\infty} Z_{S^3}(N,k)\, e^{\mu N}\,, \quad J(\mu,k) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k + \mathcal{O}(\mathrm{e}^{-\mu})$$

Drukker, Marino, Putrov '11; Hatsuda, Honda, Fuji, Moriyama, Okuyama, Okazaki, Nosaka, Yokoyama, Grassi, Klemm, Schiereck, Soroush, Bobev, Hristov, Hong, Reys, de Smet, Zhang, Chester, Kalloor, Sharon, Armanini, Griguolo, Guerrini, Gaiotto, Abajian, Kubo, Pang, ...

Identify grand canonical potential J directly with the M2-brane partition function

$$J(\mu,k) = \mathcal{Z}_{\mathrm{M2}}(\mu,k)\,.$$

M2-ensemble = grand canonical M5-ensemble = canonical

ENSEMBLES IN AdS_4 HOLOGRAPHY

In ABJM, historically, computations are performed in the **grand-canonical ensemble**, i.e. with fixed μ

$$e^{J(\mu,k)} = \sum_{N=0}^{\infty} Z_{S^3}(N,k)\, e^{\mu N}\,, \quad J(\mu,k) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k + \mathcal{O}(\mathrm{e}^{-\mu})$$

Drukker, Marino, Putrov ’11; Hatsuda, Honda, Fuji, Moriyama, Okuyama, Okazaki, Nosaka, Yokoyama, Grassi, Klemm, Schiereck, Soroush, Bobev, Hristov, Hong, Reys, de Smet, Zhang, Chester, Kalloor, Sharon, Armanini, Griguolo, Guerrini, Gaiotto, Abajian, Kubo, Pang, ...

Identify grand canonical potential J directly with the M2-brane partition function

$$J(\mu,k) = \mathcal{Z}_{\mathrm{M2}}(\mu,k)\,.$$

M2-ensemble = grand canonical M5-ensemble = canonical

Now we run through a list of holographic observables, supporting this identification.

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}_{\mathrm{M2}}^{\mathrm{P}}$

1) The two-derivative supergravity action:

$$\mathcal{Z}_{\mathrm{M2}}(\mu, k) \approx -S_{11\mathrm{d}}(\mu, k) = \frac{2\mu^3}{3\pi^2 k} = \frac{C_k}{3}\mu^3\,,$$

does not equal the leading contribution in canonical ensemble!

Ensembles in Holography; $\mathcal{Z}^{\mathrm{P}}_{\mathrm{M2}}$

1) The two-derivative supergravity action:

$$\mathcal{Z}_{\mathrm{M2}}(\mu, k) \approx -S_{11\mathrm{d}}(\mu, k) = \frac{2\mu^3}{3\pi^2 k} = \frac{C_k}{3}\mu^3 \, ,$$

does not equal the leading contribution in canonical ensemble!

Legendre transform at large N solves the issue

$$F_{\mathrm{ABJM}}(N, k) \approx \mathcal{Z}_{\mathrm{M2}}(\mu_*, k) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3} \, , \quad \mu_* \approx \frac{Nk\pi^2}{2} \, .$$

where $N = \frac{\partial \mathcal{Z}}{\partial \mu}\Big|_{\mu=\mu^*}$.

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}}$

1) The two-derivative supergravity action:

$$\mathcal{Z}_{\mathrm{M2}}(\mu, k) \approx -S_{11\mathrm{d}}(\mu, k) = \frac{2\mu^3}{3\pi^2 k} = \frac{C_k}{3}\mu^3 \,,$$

does not equal the leading contribution in canonical ensemble!

Legendre transform at large N solves the issue

$$F_{\mathrm{ABJM}}(N, k) \approx \mathcal{Z}_{\mathrm{M2}}(\mu_*, k) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3} \,, \quad \mu_* \approx \frac{Nk\pi^2}{2} \,.$$

where $N = \frac{\partial \mathcal{Z}}{\partial \mu}\Big|_{\mu=\mu^*}$. Holography dictates thus that

$$J^{\mathrm{p}}(\mu, k) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k \sim R + R^4 + \text{non-local} = \mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}}(\mu, k)$$

and **no further corrections!**

Ensembles in Holography; $\mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}}$

1) The two-derivative supergravity action:

$$\mathcal{Z}_{\mathrm{M2}}(\mu,k) \approx -S_{\mathrm{11d}}(\mu,k) = \frac{2\mu^3}{3\pi^2 k} = \frac{C_k}{3}\mu^3\,,$$

does not equal the leading contribution in canonical ensemble!

Legendre transform at large N solves the issue

$$F_{\mathrm{ABJM}}(N,k) \approx \mathcal{Z}_{\mathrm{M2}}(\mu_*,k) - \mu_* N = \frac{-\sqrt{2kN^3}\pi}{3}\,, \quad \mu_* \approx \frac{Nk\pi^2}{2}\,.$$

where $N = \frac{\partial \mathcal{Z}}{\partial \mu}\Big|_{\mu=\mu^*}$. Holography dictates thus that

$$J^{\mathrm{p}}(\mu,k) = \frac{C_k}{3}\mu^3 + B_k\mu + A_k \sim R + R^4 + \text{non-local} = \mathcal{Z}^{\mathrm{p}}_{\mathrm{M2}}(\mu,k)$$

and **no further corrections!** Infinitely many $1/N$ in canonical:

$$Z^{\mathrm{p}}(N,k) = \mathrm{e}^{A_k} C_k^{-1/3}\mathrm{Ai}[C_k^{-1/3}(N-B_k)]\,.$$

Ensembles in Holography; $\mathcal{Z}_{\text{M2}}^{\text{NP}}$

2) M2-instantons match $J^{(\text{np})}$ in M2-ensemble

$$\mathcal{Z}_{\text{M2}}^{(1)}(\mu) = C_{\text{GV}} \frac{\mathrm{e}^{-4\mu/k}}{4\sin^2 2\pi/k} = J^{(1)}(\mu, k) \quad \leftarrow \quad \textbf{1-loop exact}$$

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}_{\mathrm{M2}}^{\mathrm{NP}}$

2) M2-instantons match $J^{(\mathrm{np})}$ in M2-ensemble

$$\mathcal{Z}_{\mathrm{M2}}^{(1)}(\mu) = C_{\mathrm{GV}} \frac{\mathrm{e}^{-4\mu/k}}{4\sin^2 2\pi/k} = J^{(1)}(\mu, k) \quad \leftarrow \quad \textbf{1-loop exact}$$

Not true in canonical ensemble:

$$\mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}^{(1)}(N,k)} = C_{\mathrm{GV}} \frac{1}{4\sin^2 2\pi/k} Z^{\mathrm{p}}(N + 4/k, k) \sim \mathrm{Airy}\,.$$

Shift in Z^{p} explained by on-shell action + ensemble change.

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}_{\mathrm{M2}}^{\mathrm{NP}}$

2) M2-instantons match $J^{(\mathrm{np})}$ in M2-ensemble

$$\mathcal{Z}_{\mathrm{M2}}^{(1)}(\mu) = C_{\mathrm{GV}} \frac{\mathrm{e}^{-4\mu/k}}{4\sin^2 2\pi/k} = J^{(1)}(\mu, k) \quad \leftarrow \quad \textbf{1-loop exact}$$

Not true in canonical ensemble:

$$\mathrm{e}^{\mathcal{Z}_{\mathrm{M5}}^{(1)}(N,k)} = C_{\mathrm{GV}} \frac{1}{4\sin^2 2\pi/k} Z^{\mathrm{p}}(N + 4/k, k) \sim \mathrm{Airy}\,.$$

Shift in Z^{p} explained by on-shell action + ensemble change.

1-loop exactness for multi-wrapped M2-branes:

$$J^{(m)}(\mu, k) = C_{\mathrm{GV}} \frac{\mathrm{e}^{-4m\mu/k}}{4m\sin^2 \frac{2\pi m}{k}} = \frac{1}{m} J^{(1)}(\mu, k/m)\,.$$

Ensembles in Holography; $\mathcal{Z}_{\rm M2}^{\rm NP}$

2) M2-instantons match $J^{(\rm np)}$ in M2-ensemble

$$\mathcal{Z}_{\rm M2}^{(1)}(\mu) = C_{\rm GV}\frac{\mathrm{e}^{-4\mu/k}}{4\sin^2 2\pi/k} = J^{(1)}(\mu,k) \quad \leftarrow \quad \textbf{1-loop exact}$$

Not true in canonical ensemble:

$$\mathrm{e}^{\mathcal{Z}_{\rm M5}^{(1)}(N,k)} = C_{\rm GV}\frac{1}{4\sin^2 2\pi/k} Z^{\rm p}(N+4/k,k) \sim \text{Airy}\,.$$

Shift in $Z^{\rm p}$ explained by on-shell action + ensemble change.

1-loop exactness for multi-wrapped M2-branes:

$$J^{(m)}(\mu,k) = C_{\rm GV}\frac{\mathrm{e}^{-4m\mu/k}}{4m\sin^2\frac{2\pi m}{k}} = \frac{1}{m}J^{(1)}(\mu,k/m)\,.$$

Note: Some instanton partition functions are simple quadratic polynomials in μ.

Ensembles in Holography; $\mathcal{Z}_{\mathrm{M2}}^{\mathrm{WL}}$

3) Insertions are transformed with the same integral

$$\langle \mathcal{O}(N,k) \rangle = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu \mathrm{e}^{J(N,k)-\mu N} \langle \mathcal{O}(\mu,k) \rangle$$

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}^{\mathrm{WL}}_{\mathrm{M2}}$

3) Insertions are transformed with the same integral

$$\langle \mathcal{O}(N,k) \rangle = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu \mathrm{e}^{J(N,k)-\mu N} \langle \mathcal{O}(\mu,k) \rangle$$

For Wilson loops we find an exact match in M2-ensemble

$$\mathcal{Z}^{\mathrm{WL}}_{\mathrm{M2}}(\mu,k) = \frac{\mathrm{e}^{2\mu/k}}{2\sin 2\pi/k} = \langle W(\mu,k) \rangle \quad \leftarrow \quad \textbf{1-loop exact}$$

Ensembles in Holography; $\mathcal{Z}^{\rm WL}_{\rm M2}$

3) Insertions are transformed with the same integral

$$\langle \mathcal{O}(N,k)\rangle = \frac{1}{2\pi \mathrm{i}}\int_{\mathcal{C}} \mathrm{d}\mu \mathrm{e}^{J(N,k)-\mu N}\langle \mathcal{O}(\mu,k)\rangle$$

For Wilson loops we find an exact match in M2-ensemble

$$\mathcal{Z}^{\rm WL}_{\rm M2}(\mu,k) = \frac{\mathrm{e}^{2\mu/k}}{2\sin 2\pi/k} = \langle W(\mu,k)\rangle \quad \leftarrow \quad \textbf{1-loop exact}$$

Not true in canonical ensemble:

$$\mathrm{e}^{\mathcal{Z}^{\rm WL}_{\rm M5}(N,k)} = \frac{1}{2\sin 2\pi/k} Z^{\rm p}(N-2/k,k) \sim \text{Airy}\,,$$

Shift explained again by on-shell action + ensemble change.

ENSEMBLES IN HOLOGRAPHY; $\mathcal{Z}^{\text{WL}}_{\text{M2}}$

3) Insertions are transformed with the same integral

$$\langle \mathcal{O}(N,k) \rangle = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{C}} \mathrm{d}\mu \mathrm{e}^{J(N,k)-\mu N} \langle \mathcal{O}(\mu,k) \rangle$$

For Wilson loops we find an exact match in M2-ensemble

$$\mathcal{Z}^{\text{WL}}_{\text{M2}}(\mu,k) = \frac{\mathrm{e}^{2\mu/k}}{2\sin 2\pi/k} = \langle W(\mu,k) \rangle \quad \leftarrow \quad \textbf{1-loop exact}$$

Not true in canonical ensemble:

$$\mathrm{e}^{\mathcal{Z}^{\text{WL}}_{\text{M5}}(N,k)} = \frac{1}{2\sin 2\pi/k} Z^{\text{p}}(N-2/k,k) \sim \text{Airy}\,,$$

Shift explained again by on-shell action + ensemble change.

Same structure for multi-wound Wilson Loops:

$$\langle W^{(m)}(\mu,k) \rangle = \frac{\mathrm{e}^{2m\mu/k}}{2\sin 2\pi m/k} = \langle W^{(1)}(\mu,k/m) \rangle\,.$$

Ensembles in AdS_7 holography

[2505.21633]

Ensembles in AdS_7 holography

Similar structures arise in AdS_7 holography.

Ensembles in AdS_7 holography

Similar structures arise in AdS_7 holography.

M2-ensemble now does correspond to canonical ensemble!

The four-form flux is quantised to N on the four-sphere.

ENSEMBLES IN AdS_7 HOLOGRAPHY

Similar structures arise in AdS_7 holography.

M2-ensemble now does correspond to canonical ensemble!
The four-form flux is quantised to N on the four-sphere.

-) Anomalies (holds for A_{N-1}, D_N, and class-$\mathcal{S}$):

$$a = C_a N^3 - B_a\, N - A_a\,, \quad c = C_c N^3 - B_c N - A_c$$

Harvey, Minasian, Moore '98; Intrilligator '00; Yi '01; Gaiotto, Maldacena '12; …

ENSEMBLES IN AdS_7 HOLOGRAPHY

Similar structures arise in AdS_7 holography.

M2-ensemble now does correspond to canonical ensemble!

The four-form flux is quantised to N on the four-sphere.

-) Anomalies (holds for A_{N-1}, D_N, and class-$\mathcal{S}$):

$$a = C_a N^3 - B_a\, N - A_a\,, \quad c = C_c N^3 - B_c N - A_c$$

Harvey, Minasian, Moore '98; Intrilligator '00; Yi '01; Gaiotto, Maldacena '12; …

-) Superconformal index from 5d SYM:

Kim, Kim, Kim, Lee '12-'13

$$\log Z_{(2,0)}^{S^5\times S^1_\beta}(N,\beta) = \beta/6\, N^3 - \beta/8\, N + \text{const} + \sum_{m=1}^{\infty} \frac{\exp(-m\beta N)}{4m\sinh^2(m\beta/2)}$$

$$\sim \int R + R^4 + \text{non-local} + \sum_{m=1}^{\infty} \mathcal{Z}^{(m)}_{\text{M2}}(N,\beta)\,.$$

The leading instanton has been confirmed.

Gautason, Puletti, JvM '23; Beccaria, Giombi, Tseytlin '23; Gautason, JvM '25

COORDS

On $S^7/\mathbf{Z}_k$ we choose the following coordinates.

$$
\begin{aligned}
e^5 &= 2L\,\mathrm{d}\theta, \\
e^6 &= L\sin\theta\,\mathrm{d}\theta_1, \quad e^7 = L\sin\theta\sin\theta_1\,\mathrm{d}\phi_1, \\
e^8 &= L\cos\theta\,\mathrm{d}\theta_2, \quad e^9 = L\cos\theta\sin\theta_2\,\mathrm{d}\phi_2, \\
e^{10} &= L\sin\theta\cos\theta\left(2\,\mathrm{d}\varphi + \cos\theta_1\,\mathrm{d}\phi_1 - \cos\theta_2\,\mathrm{d}\phi_2\right), \\
e^{11} &= L\left(\mathrm{d}y + 2\sigma + \tfrac{1}{2}A\right).
\end{aligned}
$$

FRADKIN TSEYTLIN REFERENCE I

EFFECTIVE ACTION APPROACH TO SUPERSTRING THEORY

E.S. FRADKIN and A.A. TSEYTLIN
Department of Theoretical Physics, P N. Lebedev Physical Institute,
Leninsky pr. 53, Moscow 117924, USSR

Received 15 April 1985

We define the quantum effective action for local fields corresponding to superstring excitation modes It is given by a superstring theory path integral with a generalized superstring action containing couplings to background fields of $D = 10$ super-Yang–Mills and supergravity multiplets Dilaton couplings in a low-energy approximation for the effective action are shown to be the same as in corresponding $D = 10$ supergravity actions

. . .

FRADKIN TSEYTLIN REFERENCE II

...

The approach of ref. [2] is based on an off-shell effective action Γ for the infinite set of local fields corresponding to the modes of a free string spectrum. Γ accumulates the full quantum dynamics of the "second-quantized" string theory: it is a generating functional of all possible off-shell scattering amplitudes on arbitrary backgrounds (i.e. is a direct analog of the ordinary QFT effective action). In the case of the theory of interacting open and closed oriented Bose strings Γ is defined as follows [2]:

$$\Gamma\left[\Phi, G_{\mu\nu}, A_{\mu\nu}, \varphi, \ldots \mid \phi, A_\mu, \ldots\right] = \sum_{\chi=1,0,-1,} e^{\sigma\chi} \int_\Sigma \mathrm{d}e(t)\, \mathrm{d}x^\mu(t) \int_{M^2} \mathrm{d}g_{ab}(z)\, \mathrm{d}x^\mu(z) \exp(-I_2 - I_1), \tag{1}$$

$$I_2 = \int_{M^2} \mathrm{d}^2 z \left\{ \sqrt{g}\,\Phi(x(z)) + (1/4\pi)\sqrt{g}\,R\varphi(x(z)) + (1/4\pi\alpha')\sqrt{g}\,g^{ab}\partial_a x^\mu \partial_b x^\nu G_{\mu\nu}(x(z)) + \mathrm{i}\varepsilon^{ab}\partial_a x^\mu \partial_b x^\nu A_{\mu\nu}(x(z)) + \ldots \right\}, \tag{2}$$

$$I_1 = \int_\Sigma \mathrm{d}t \left\{ e\phi(x(t)) + \mathrm{i}\dot{x}^\mu A_\mu(x(t)) + \ldots \right\}. \tag{3}$$

SPECTRUM ON M2-BRANE

Field	d.o.f.	Q_1	Q_2	ML
Scalars	4	f	0	$if/2$
	4	1	1	$i\sqrt{3}/2$
Fermions	4	0	0	$3q/4$
	2	$1+f$	1	$-q(3/4-f/2)$
	2	$1-f$	1	$-q(3/4+f/2)$

$$f = \tfrac{1}{2}(F + q \star_4 F)$$

The kinetic operators equal

$$\mathcal{K} = -D^2 + M^2 \,, \quad \mathcal{D} = \mathrm{i}\not{D} + M \,,$$

with

$$D = \nabla - \mathrm{i}Q_1\mathcal{A}_1 - \mathrm{i}Q_2\mathcal{A}_2$$

$$\mathcal{A}_1 = \frac{1}{2}(\mathrm{d}\varphi - \cos\theta_2 \mathrm{d}\phi_2) \,, \quad \mathcal{A}_2 = -\frac{1}{2}\cos\theta_2 \mathrm{d}\phi_2 \,.$$