The Dark Side of the Universe - DSU2024 SEP 8 - 14, 2024

Domain walls beyond Z_2

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- Brief introduction of Z_2 domain walls
- Domain walls from Z_N breaking ($N > 2$)
- Non-Abelian domain walls, tetrahedral/octahedral cases (A_4/S_4)
- Gravitational waves from domain walls beyond Z_2
- Application in the testability of discrete flavour symmetries

Talk based on

[1] *Gravitational wave signatures from discrete flavor symmetries,* G. Gelmini, S. Pascoli, E. Vitagliano, YLZ, 2009.01903

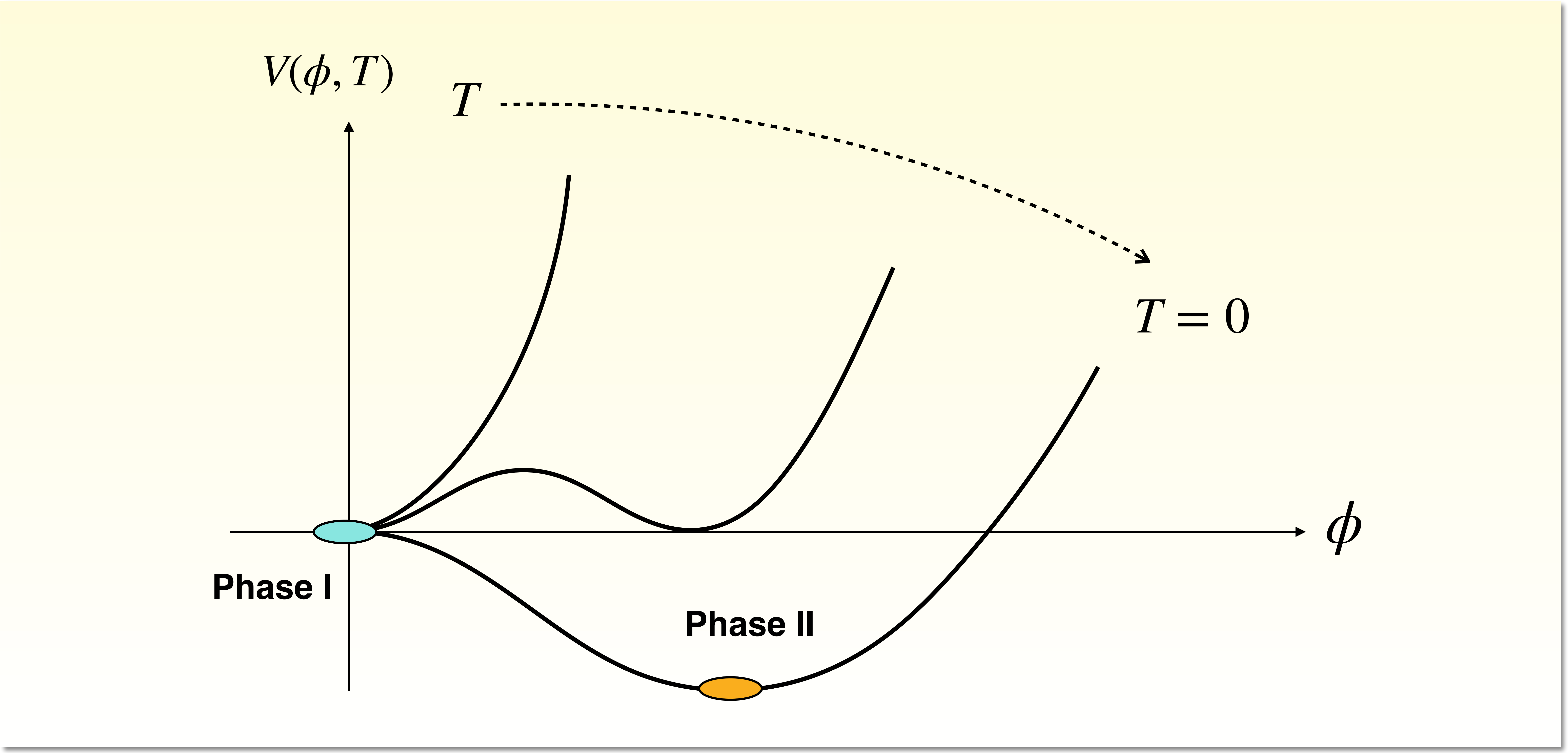
[2] *Collapsing domain walls beyond Z_2,* Y. Wu, K.P. Xie, YLZ, 2204.04374

[3] *Classification of Abelian domain walls,* Y. Wu, K.P. Xie, YLZ, 2205.11529

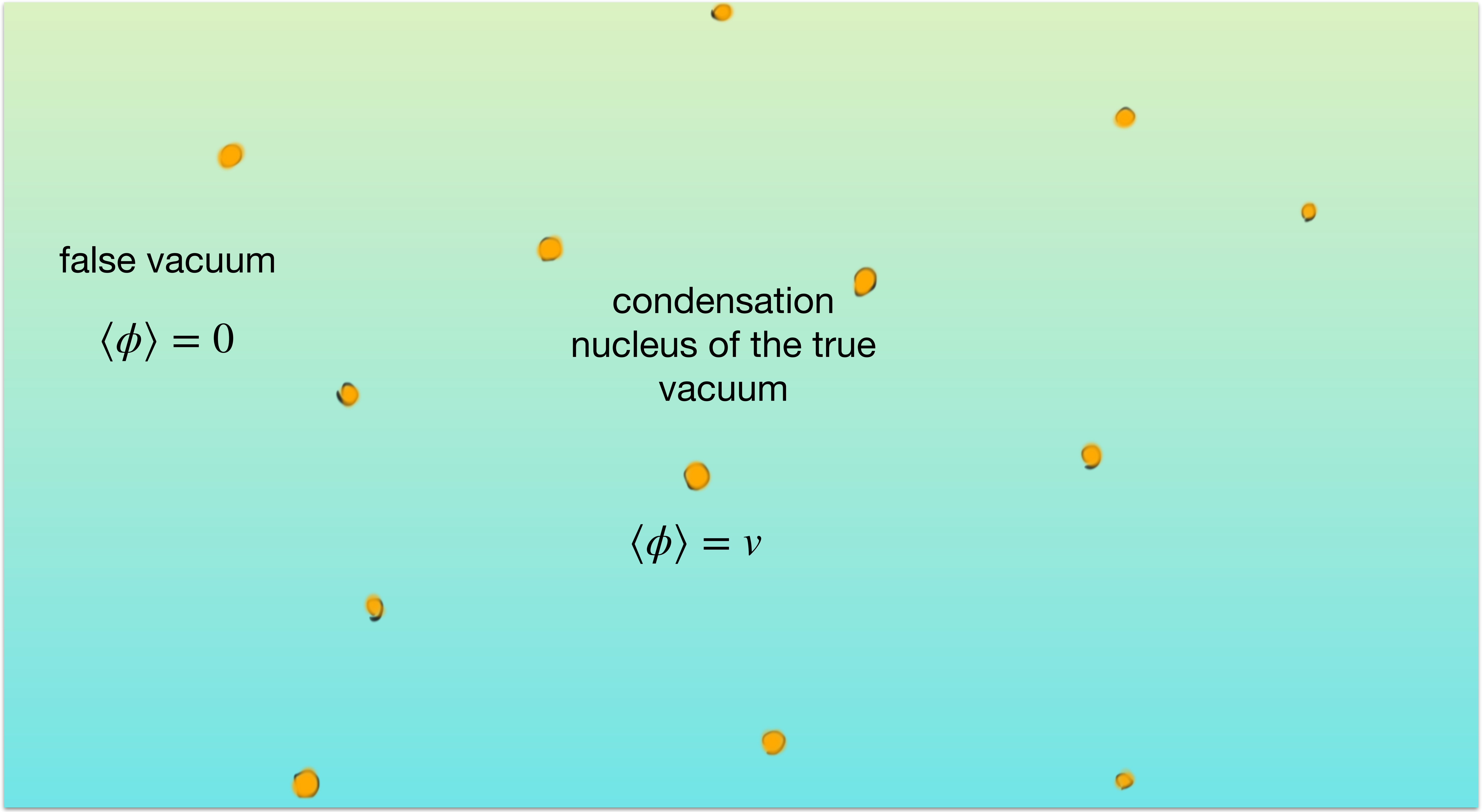
[4] *Non-Abelian domain walls,* B. Fu, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409.xxxxx

A general picture of phase transition (1st-order)

Effective potential including finite-T corrections $V(\phi, T) \approx D(T^2 - T_0^2)\phi^2 - \tilde{\mu}_T\,\phi^3 + \frac{\lambda_T}{4}\phi^4$



A general picture of phase transition (1st-order)



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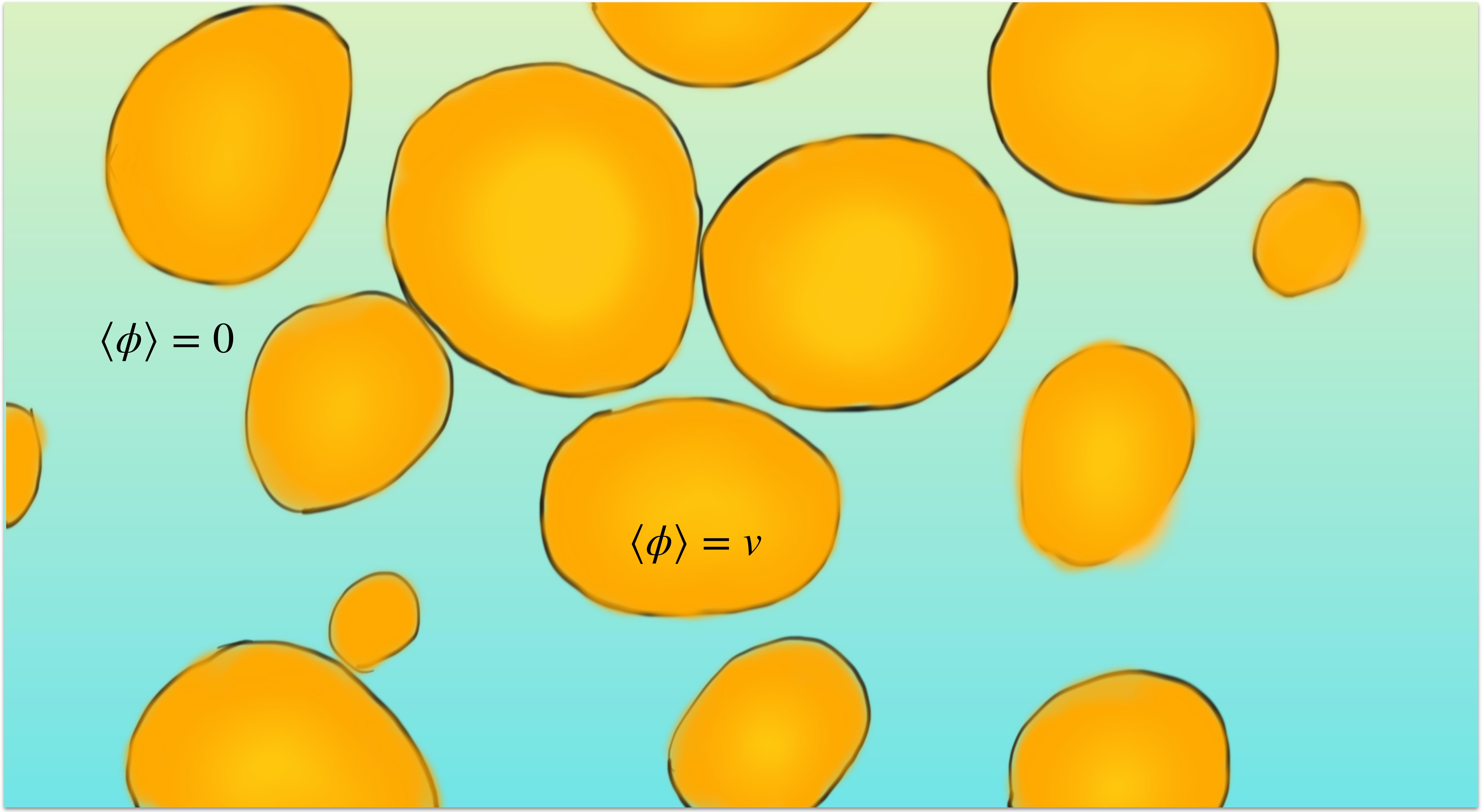

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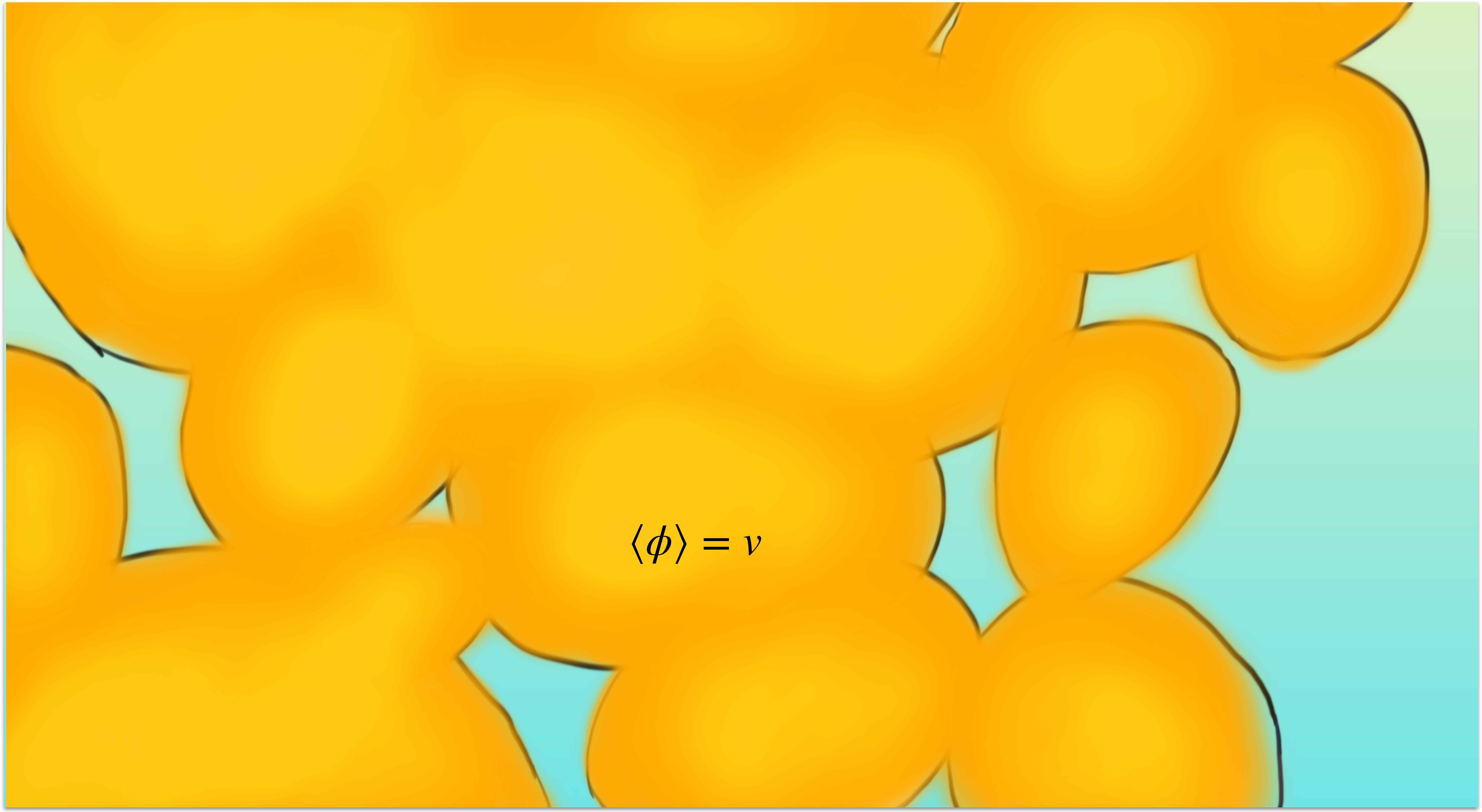
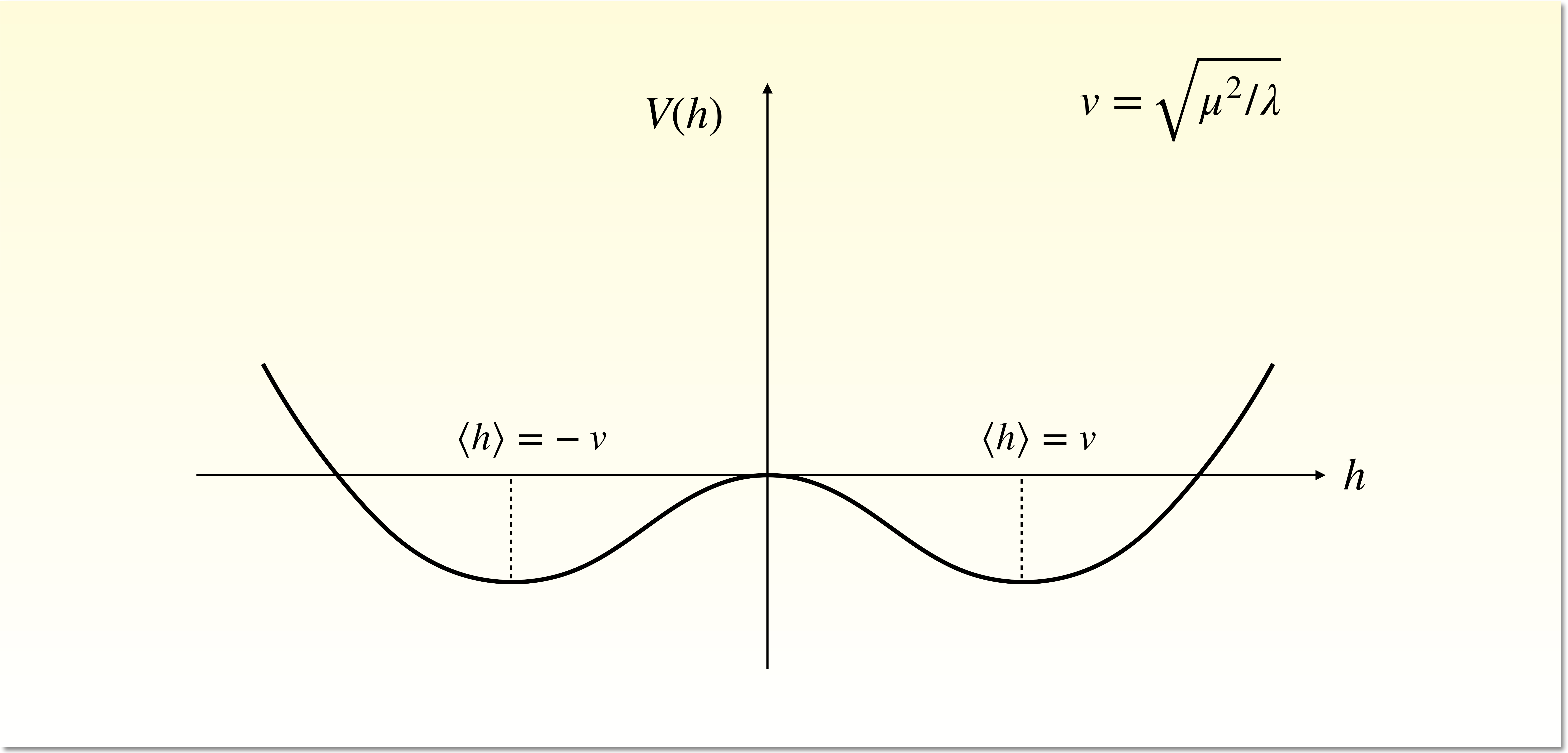
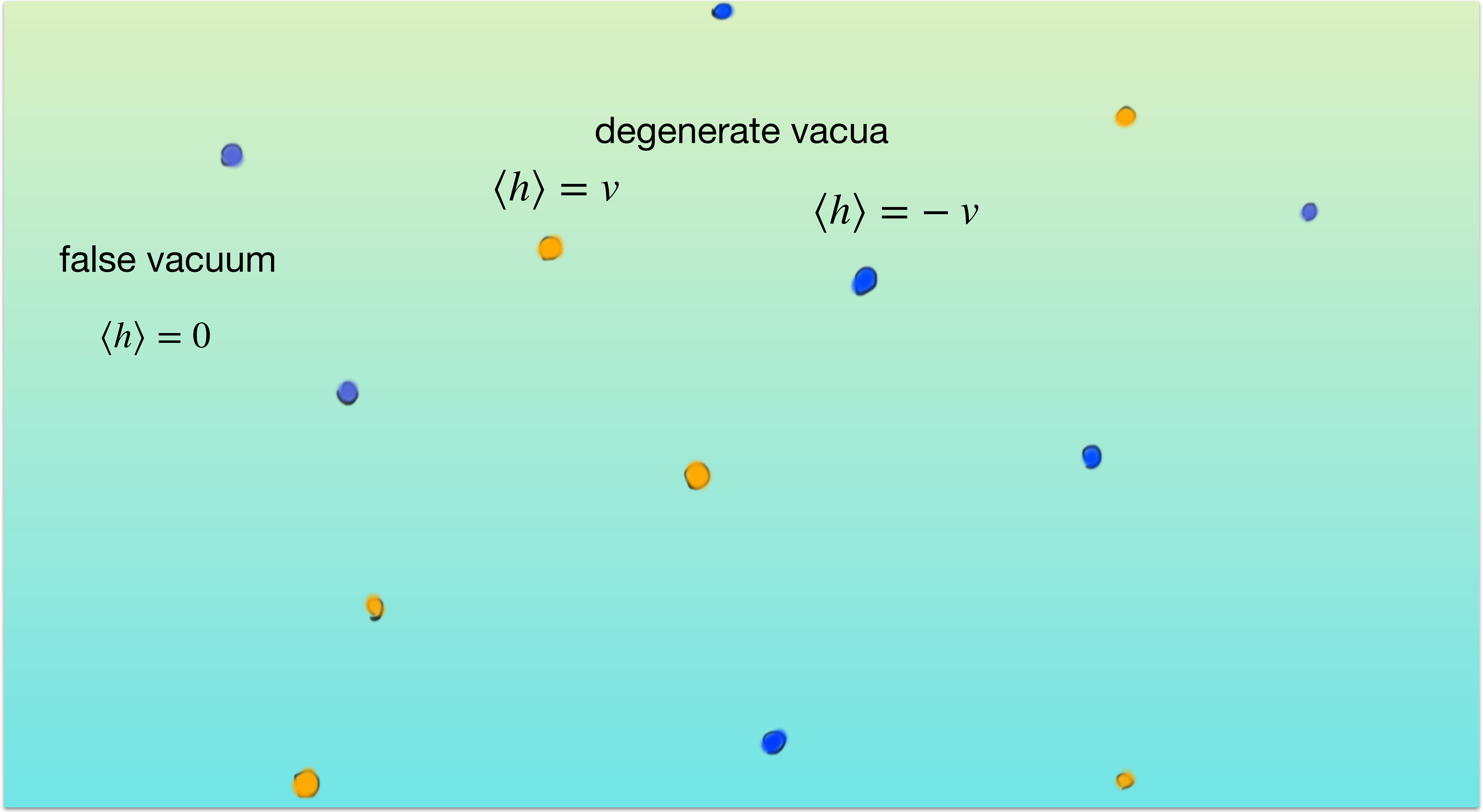


Phase transition for a Z_2 breaking

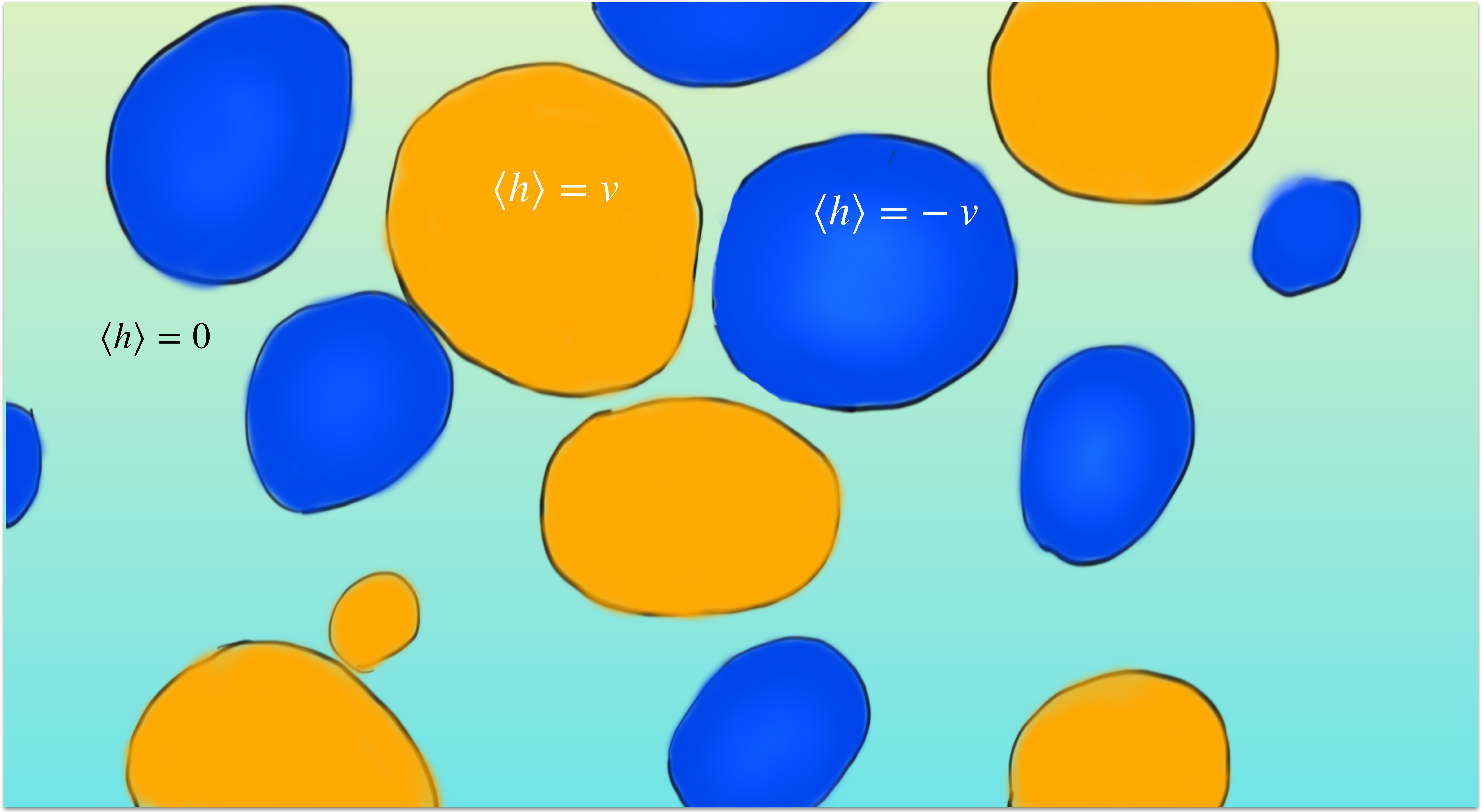
Tree-level potential for a real scalar h in Z_2 $\qquad V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$



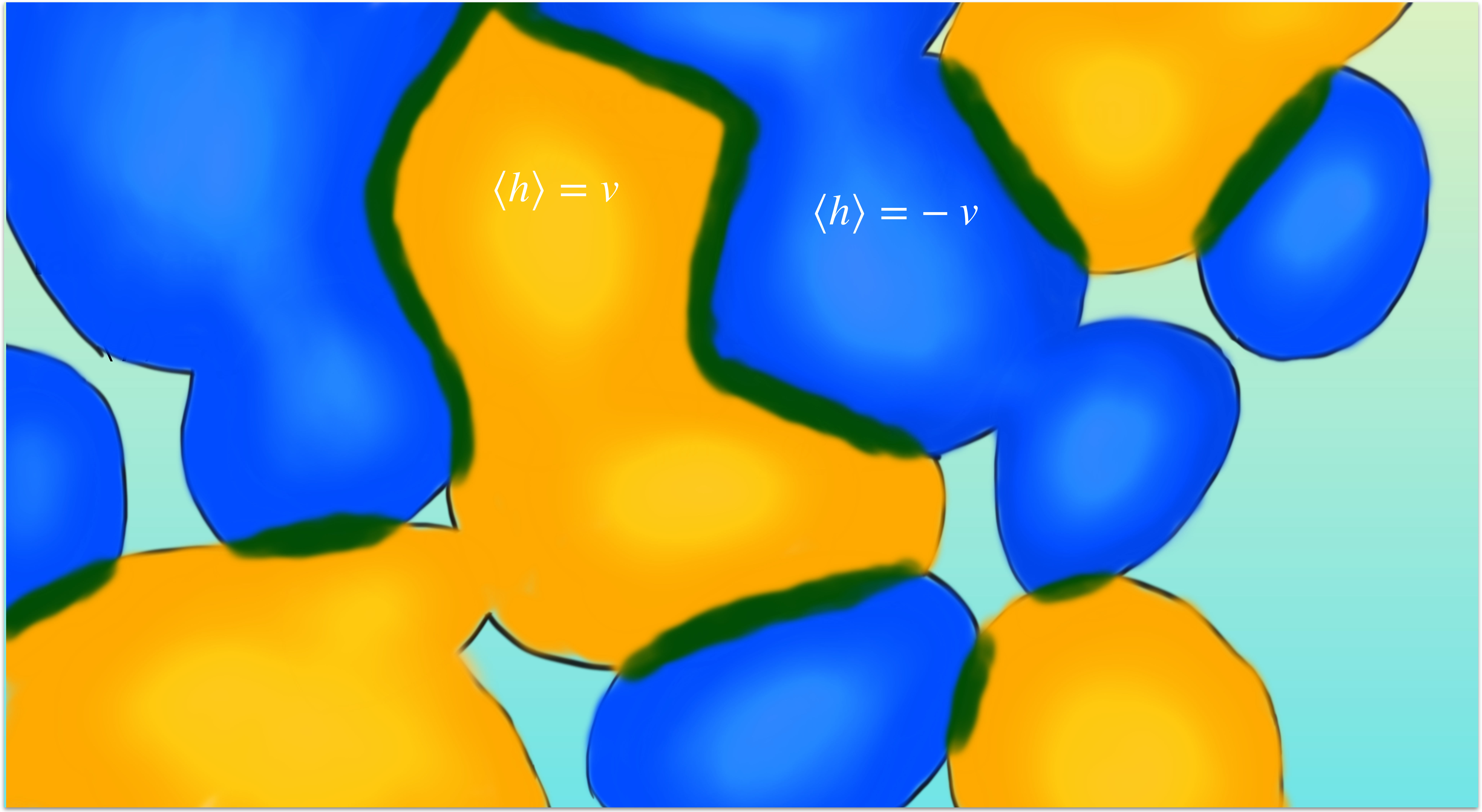
Phase transition for a Z_2 breaking



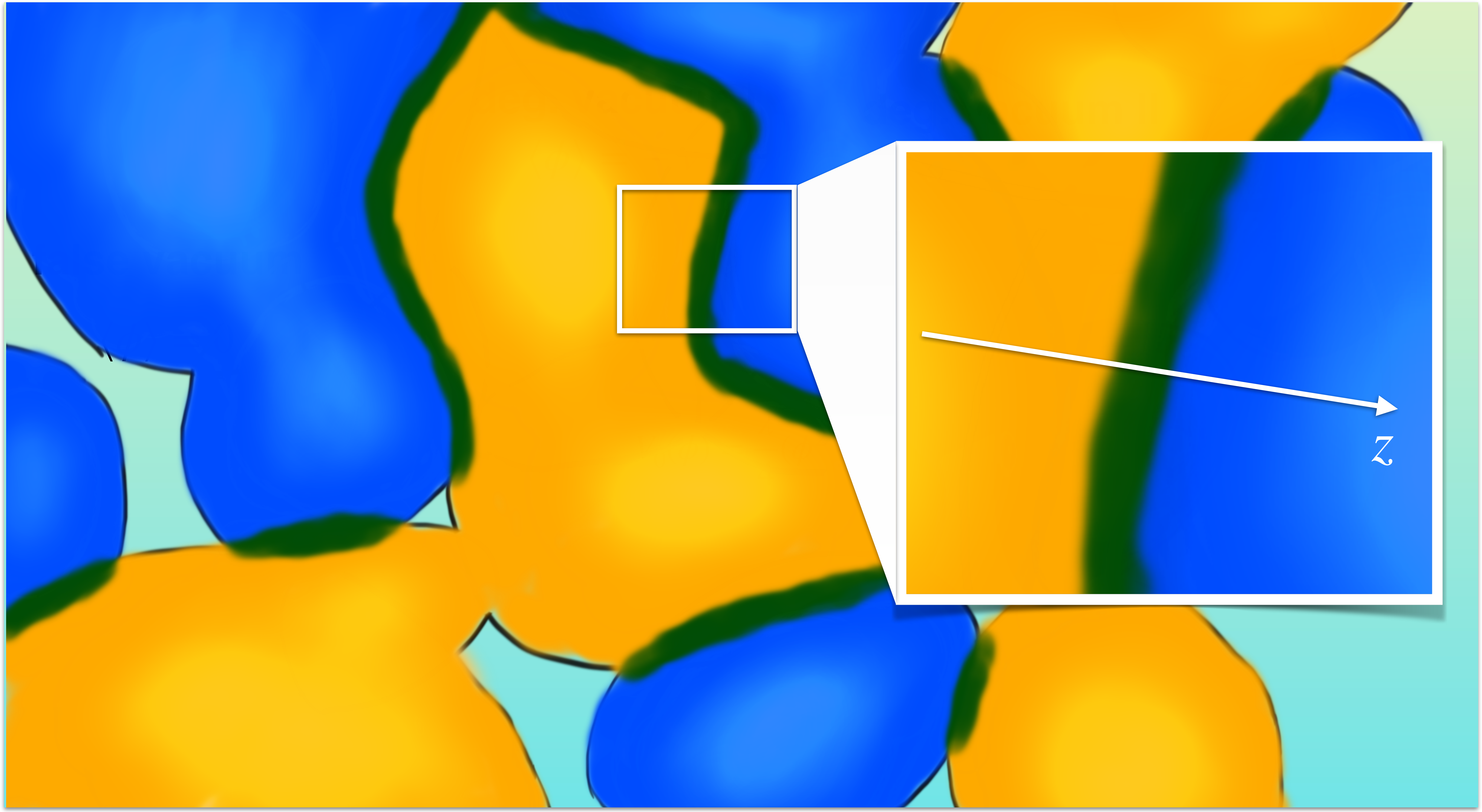
Phase transition for a Z_2 breaking



Phase transition for a Z_2 breaking



Domain walls: static solution of classic field in 1D

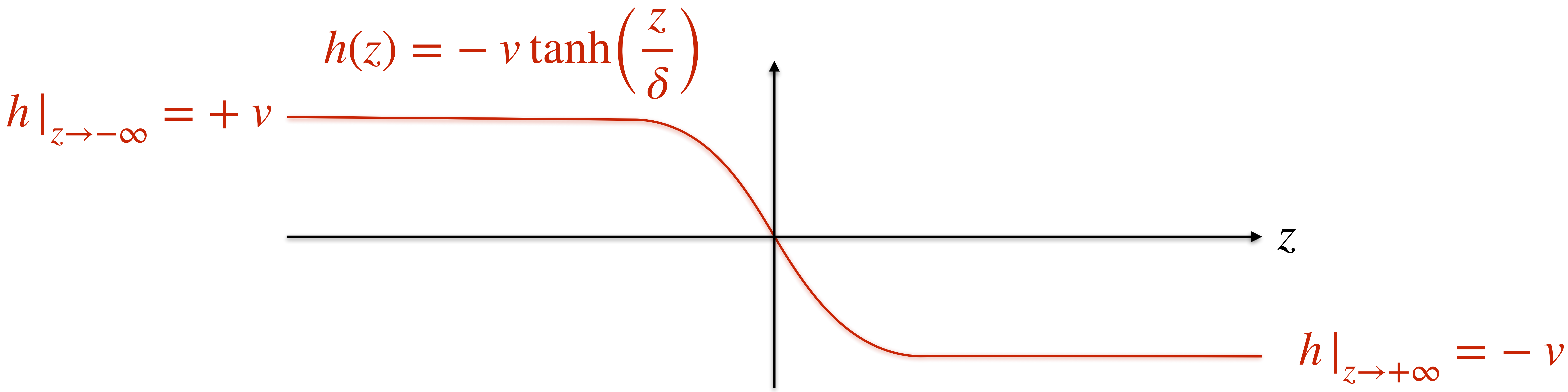


Z_2 domain wall —— the simplest domain wall

Given a toy potential for a real scalar in Z_2 $\qquad V = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$

EOM of field $\quad \partial^2 h + \frac{\partial V(h)}{\partial h} = 0 \;\Rightarrow\; \frac{d^2}{dz^2}h(z) = \lambda h(h^2 - v^2) \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$

Soliton solution: scalar solution along z direction



Vilenkin, Phys. Rept.121 (1985) 263

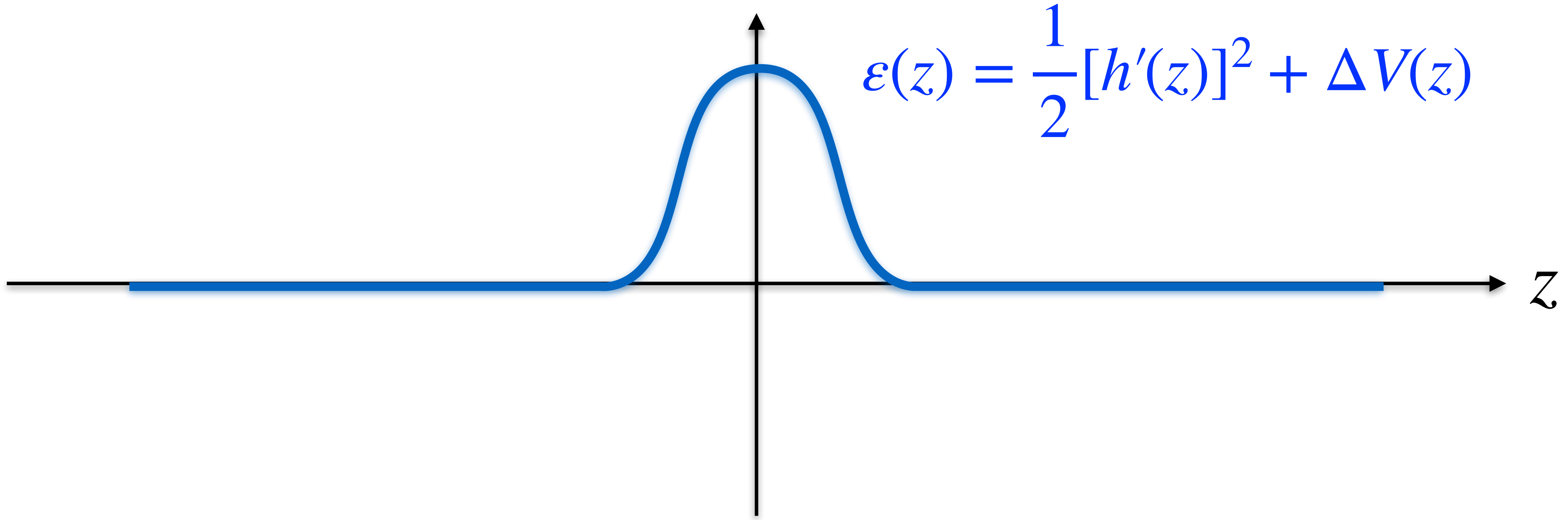
Z_2 domain wall —— the simplest domain wall

Given a toy potential for a real scalar in Z_2 $\quad V = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$

EOM of field $\quad \partial^2 h + \frac{\partial V(h)}{\partial h} = 0 \Rightarrow \frac{d^2}{dz^2}h(z) = \lambda h(h^2 - v^2) \quad v = \sqrt{\frac{\mu^2}{\lambda}}$

Tension / surface energy $\quad \sigma = \frac{4}{3}\sqrt{\frac{\lambda}{2}}v_0^3 \quad \sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz$

Thickness $\quad \delta = \sqrt{\frac{2}{\lambda v_0^2}}$



$$\varepsilon(z) = \frac{1}{2}[h'(z)]^2 + \Delta V(z) \qquad \Delta V = V - V_{min}$$

Vilenkin, Phys. Rept.121 (1985) 263

Necessity to include a bias term

- Stable domain wall leads to cosmological problem

$$\rho_{\rm DW} \sim \sigma H \quad \text{(scaling solution)} \quad \Longrightarrow \quad \frac{\rho_{\rm DW}}{\rho_c} \sim \frac{\sigma G}{H} \sim \frac{\lambda^{1/2} v^3}{M_{\rm pl} T^2} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

- No fundamental rules to force discrete symmetry to be an exact symmetry

- Gravity and chiral anomaly may break discrete symmetries explicitly at quantum level

- Bias term: $\delta V = \epsilon v h \left(\frac{1}{3} h^2 - v^2 \right)$ Hiramatsu, Kawasaki, Saikawa, 1002.1555

$\Longrightarrow$ Vacua splitting $(V_{\rm bias})_{10} = V|_{+v} - V|_{-v} = -\frac{4}{3}\epsilon v^4$

Sufficient small to stabilise vacuum configuration and to survive the domain walls for a certain period

Not to small to provide enough vacuum pressure to push the wall outside the horizon at a certain stage before BBN

- Gravitational waves spectrum is peaked during the collapsing domain walls

—> see Alexander Vikman's talk

Go beyond Z_2 —— motivations

- Z_N from $U(1)_{\mathrm{PQ}}$ breaking

  P. Sikivie, 1982

- Discrete symmetries in SUSY

  e.g. Z_3 in NMSSM

  Review in Chung, Everett, Kane, King, Lykken, Wang, 0312378

- Z_N as flavour symmetry
- Non-Abelian discrete flavour symmetries A_4, S_4, …

  Reviews e.g.,
  Altarelli, Feruglio, 1002.0211
  King, Luhn, 1301.1340;
  Xing, 1909.09610;
  Feruglio, Romanino, 1912.06028
  ……

- ……

Z_N $(N > 2)$ and its vacuum configuration

Z_N-invariant potential for a complex scalar $\phi = (h + ia)/\sqrt{2}$

$$V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu^{4-N}(\phi^N + \phi^{*N})$$

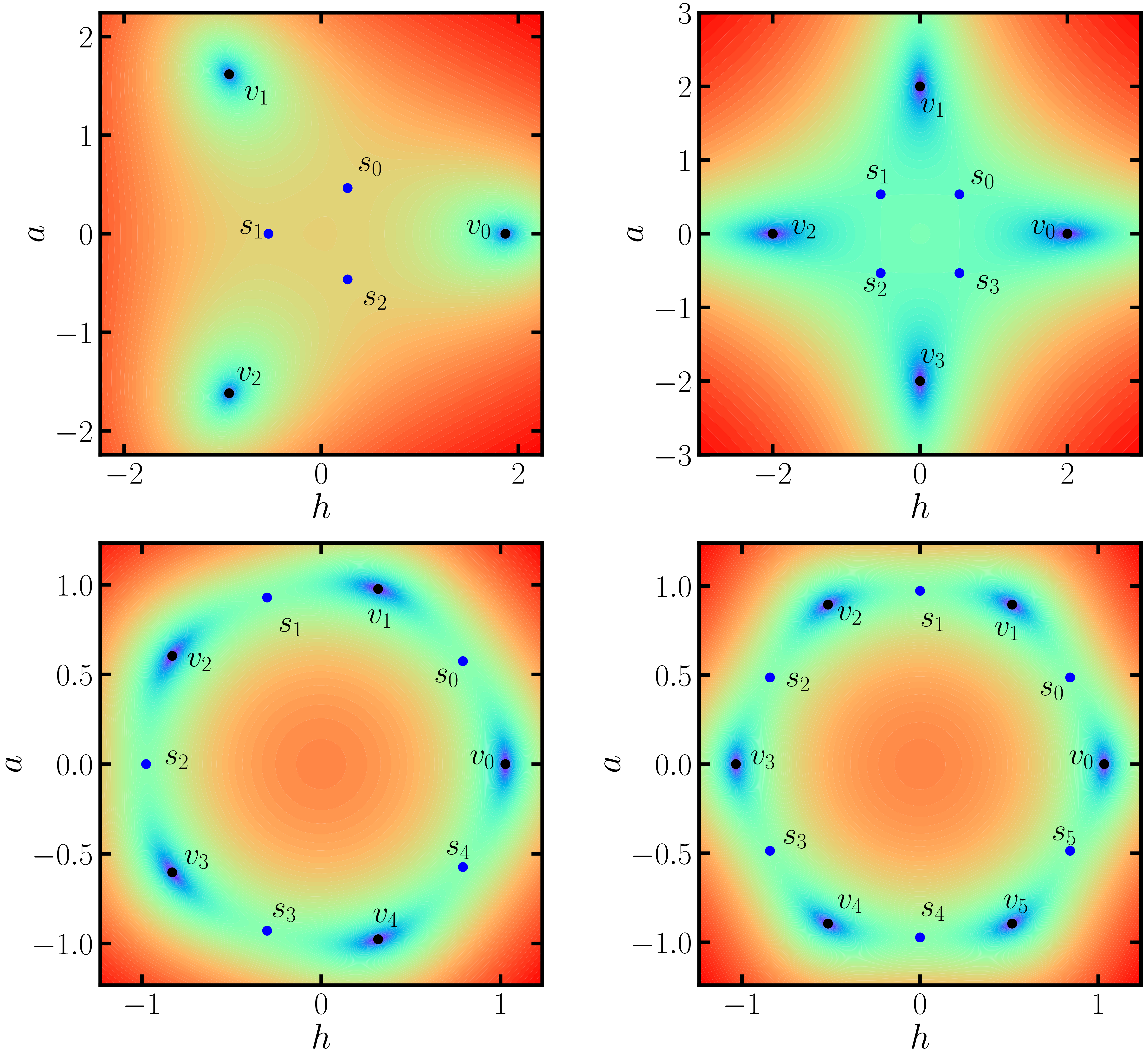
(assuming CP conservation, simplest form)

N degenerate vacua:

$$v_k = v_0 e^{i2\pi\frac{k}{N}}$$

$$k = 0, 1, \ldots, N-1$$

Y.C. Wu, K.P. Xie, **YLZ**, 2205.11529

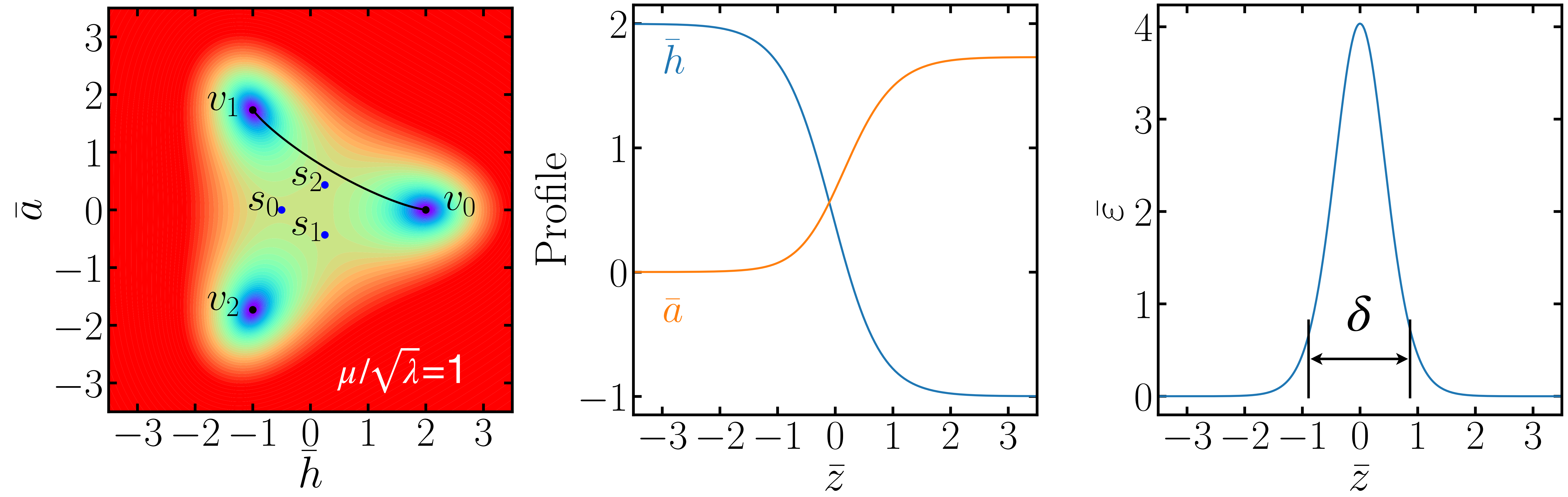


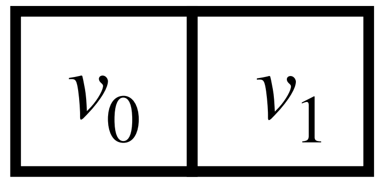
Z_3 domain walls

Z_3-invariant potential $\quad V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu(\phi^3 + \phi^{*3})$ $\qquad \beta = 3\lambda_2/\sqrt{8\lambda_1} > 0$

3 vacua $\quad v_k = v_0\, e^{i2\pi k/3} \qquad k = 0,1,2 \qquad v_0 = \frac{\mu}{\sqrt{2\lambda_1}}(\beta + \sqrt{1+\beta^2})$

$\beta = 3/4$





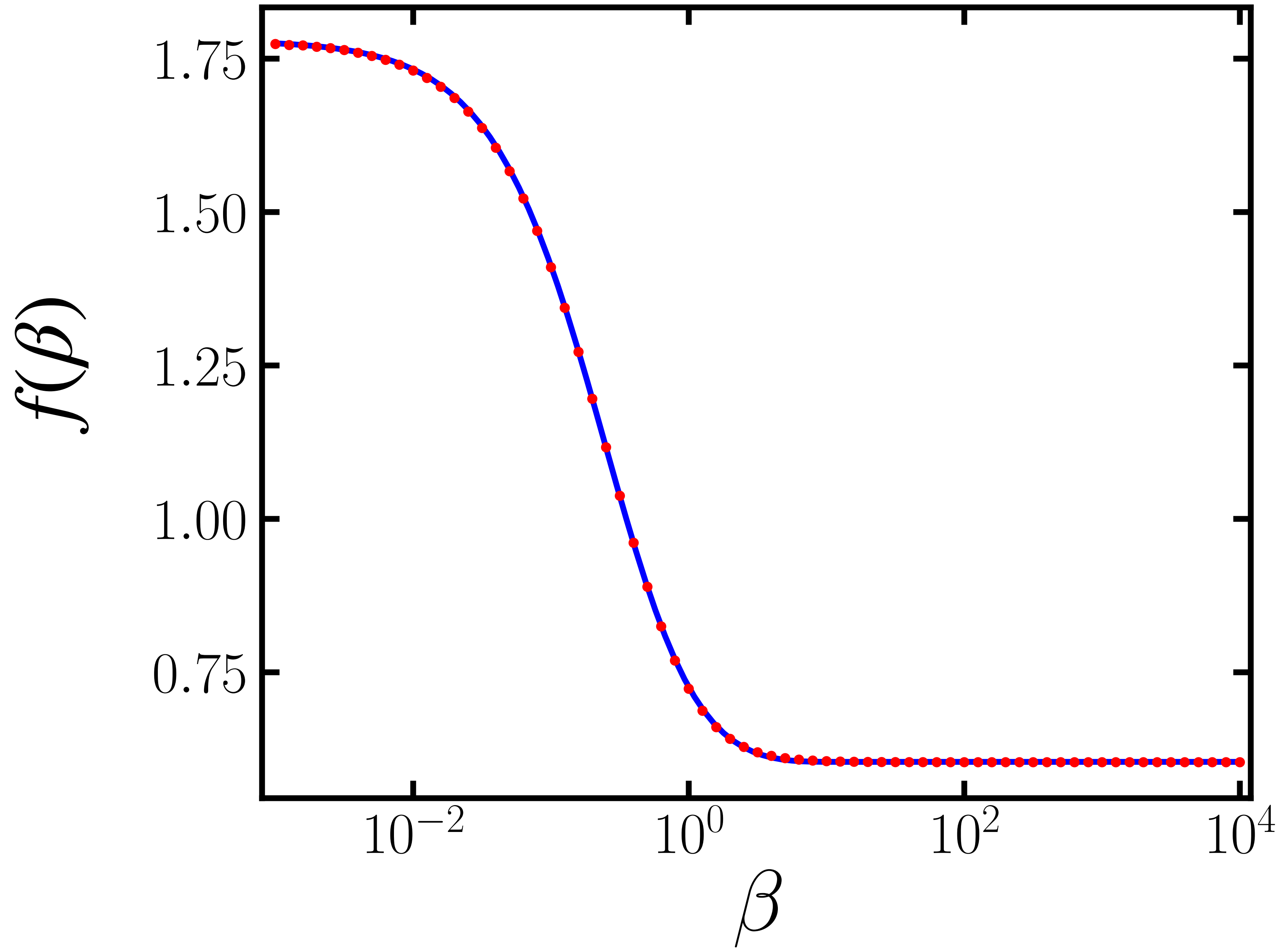
$$\varepsilon(z) = \frac{1}{2}\{[h'(z)]^2 + [a'(z)]^2\} + \Delta V(z)$$

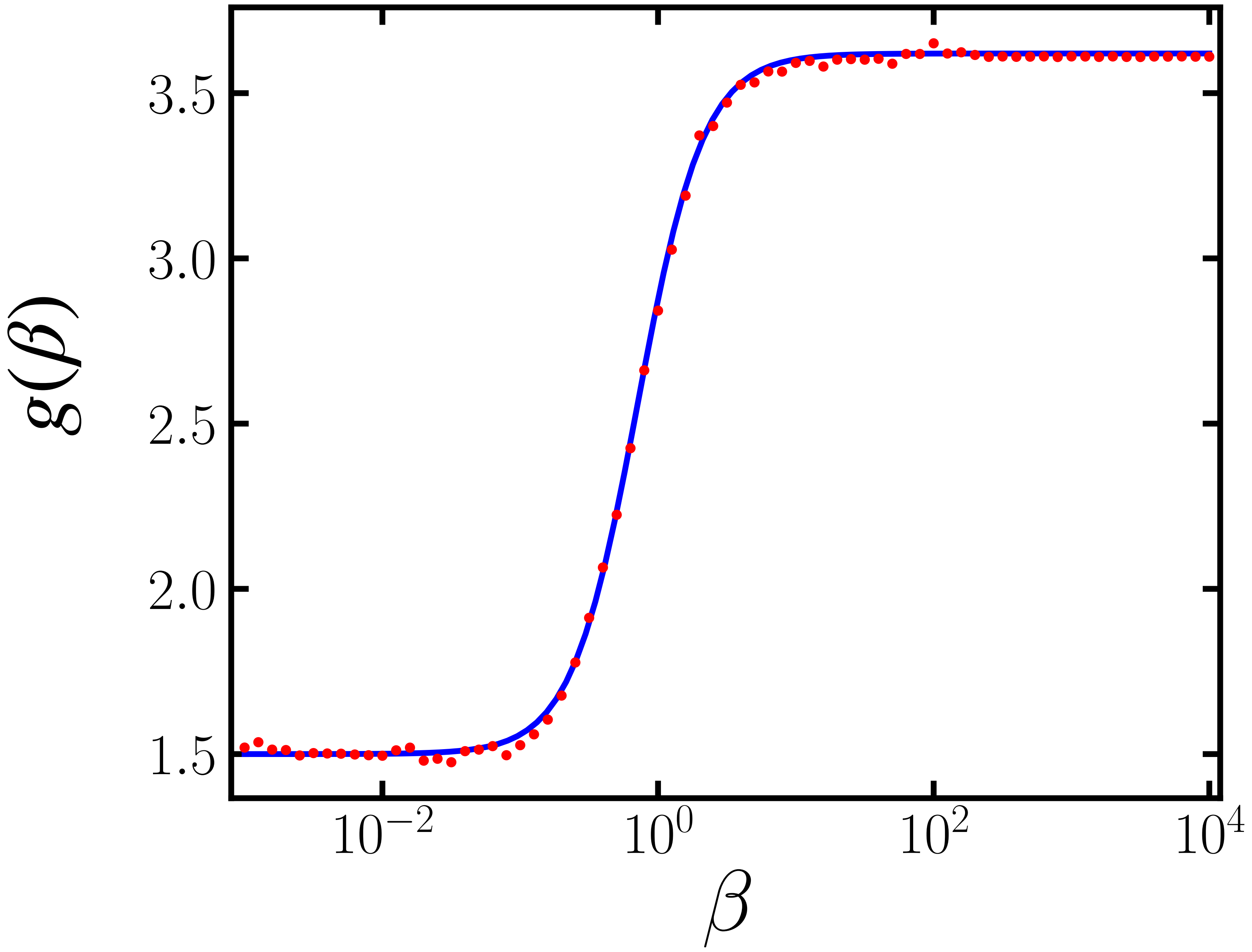
Tenson $\Rightarrow$ $\sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz$ $\qquad$ Thickness $\Rightarrow$ $\int_{-\delta/2}^{\delta/2} dz\varepsilon(z) = 64\% \times \sigma\,.$

Z_3 domain walls

Tension $\sigma = m_a v_0^2 f(\beta)$

Thickness $\delta = m_a^{-1} g(\beta)$





$$f(\beta) = 0.604 + \frac{0.234}{e^{0.826\beta} + 0.435\beta^2 - 0.801}$$

$$g(\beta) = 3.62 - \frac{2.12}{1 + 1.85\beta^{1.81}}$$

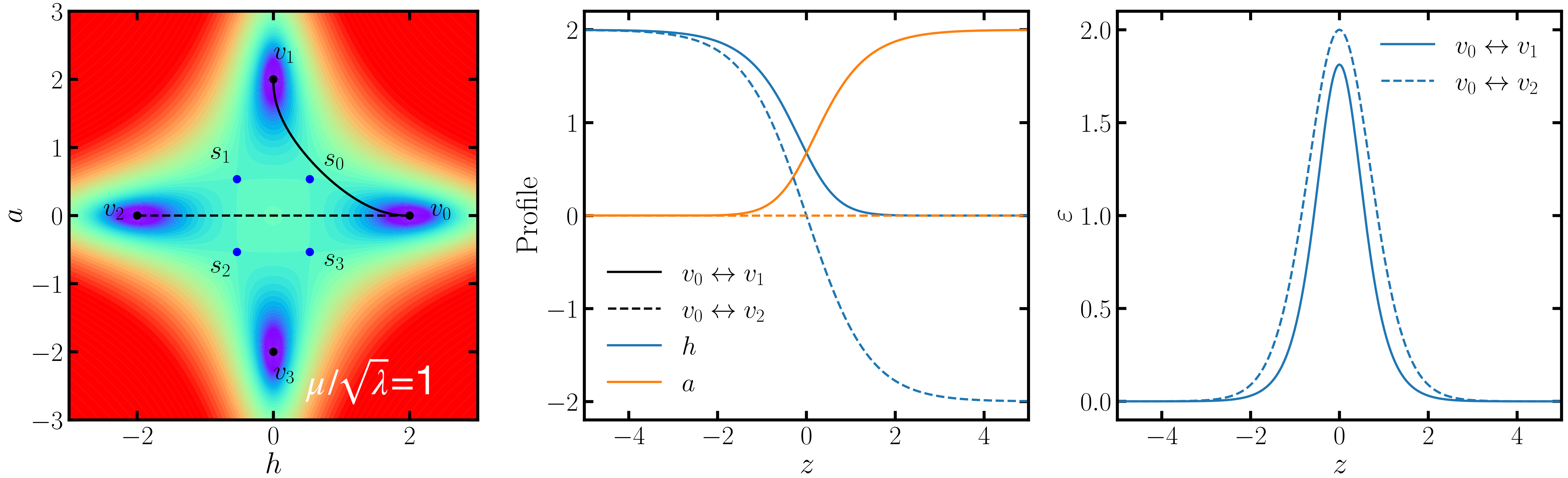
m_a mass of pseudo Nambu-Goldstone boson

Z_4 domain walls

Z4-invariant potential $V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2(\phi^4 + \phi^{*4})$ $\beta \equiv 2\lambda_2/\lambda_1$

$v_k = v_0 e^{i\frac{2\pi}{4}k}$ $k = 0,1,2,3$ $v_0 = \dfrac{\mu}{\sqrt{2\lambda_1(1-\beta)}}$

$\beta = 3/4$



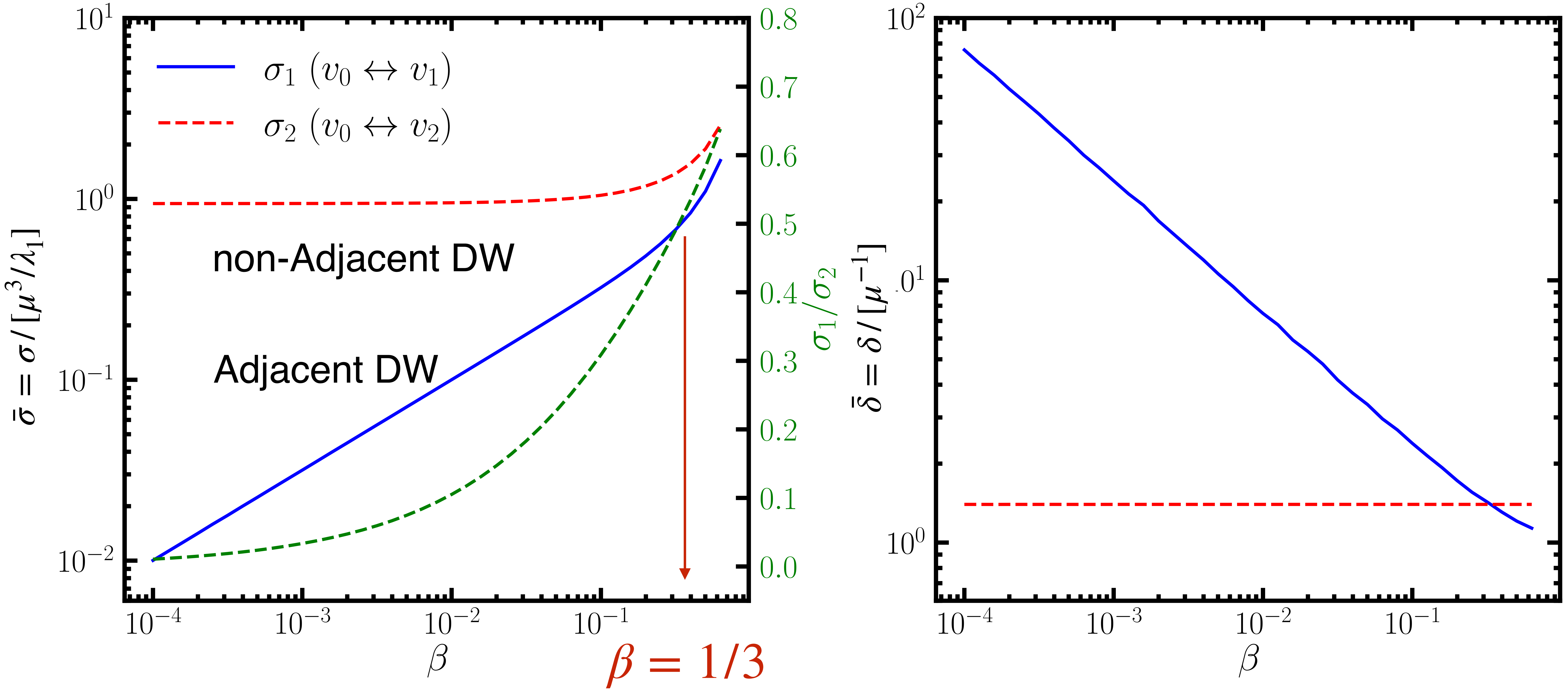
Adjacent walls: separating adjacent walls in the field space

e.g., that separating v_0 and v_1 | v_0 | v_1 |

Non-adjacent walls： separating non-adjacent walls

e.g., that separating v_0 and v_2 | v_0 | v_2 |

Z_4 domain walls

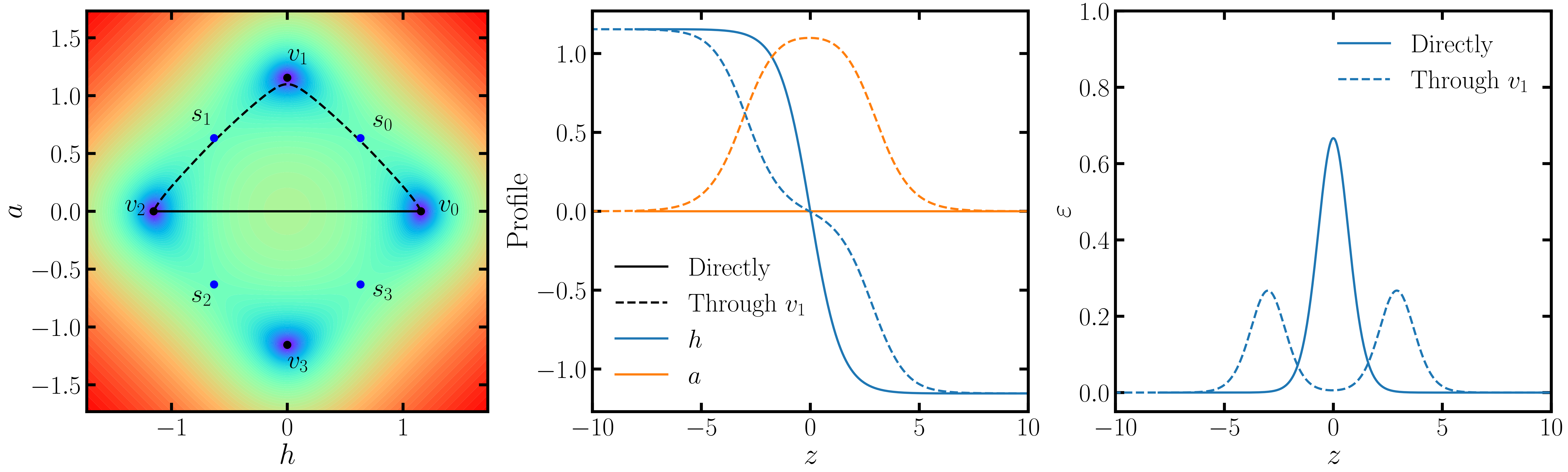


$\beta = 1/3$

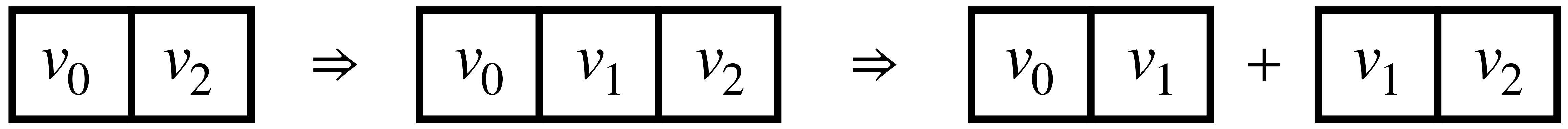
$$\bar{\sigma}_{\text{non-adj}} = 2\,\bar{\sigma}_{\text{adj}} = \sqrt{2}$$

Z_4 domain walls

$\beta = 1/4$

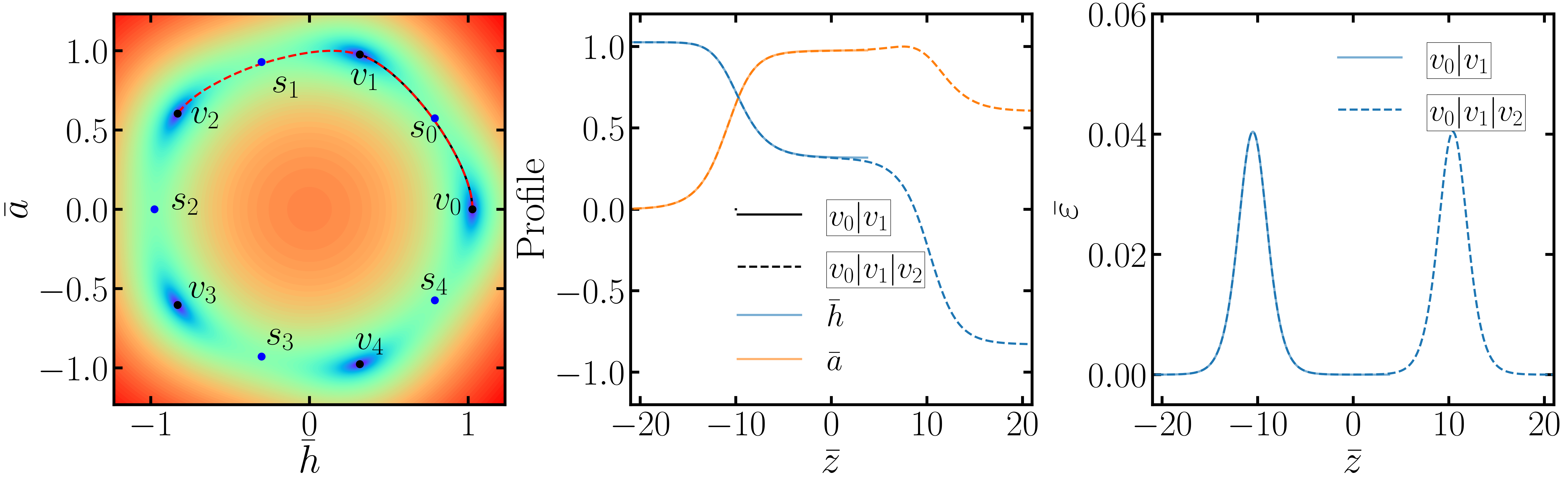


For $\beta < 1/3$, $\sigma_2 > 2\sigma_1$, non-Adjacent DWs are unstable, decaying to two adjacent DWs



Z_N domain walls with small Z_N effects

$\phi^N (N > 4)$ unrenormalisable



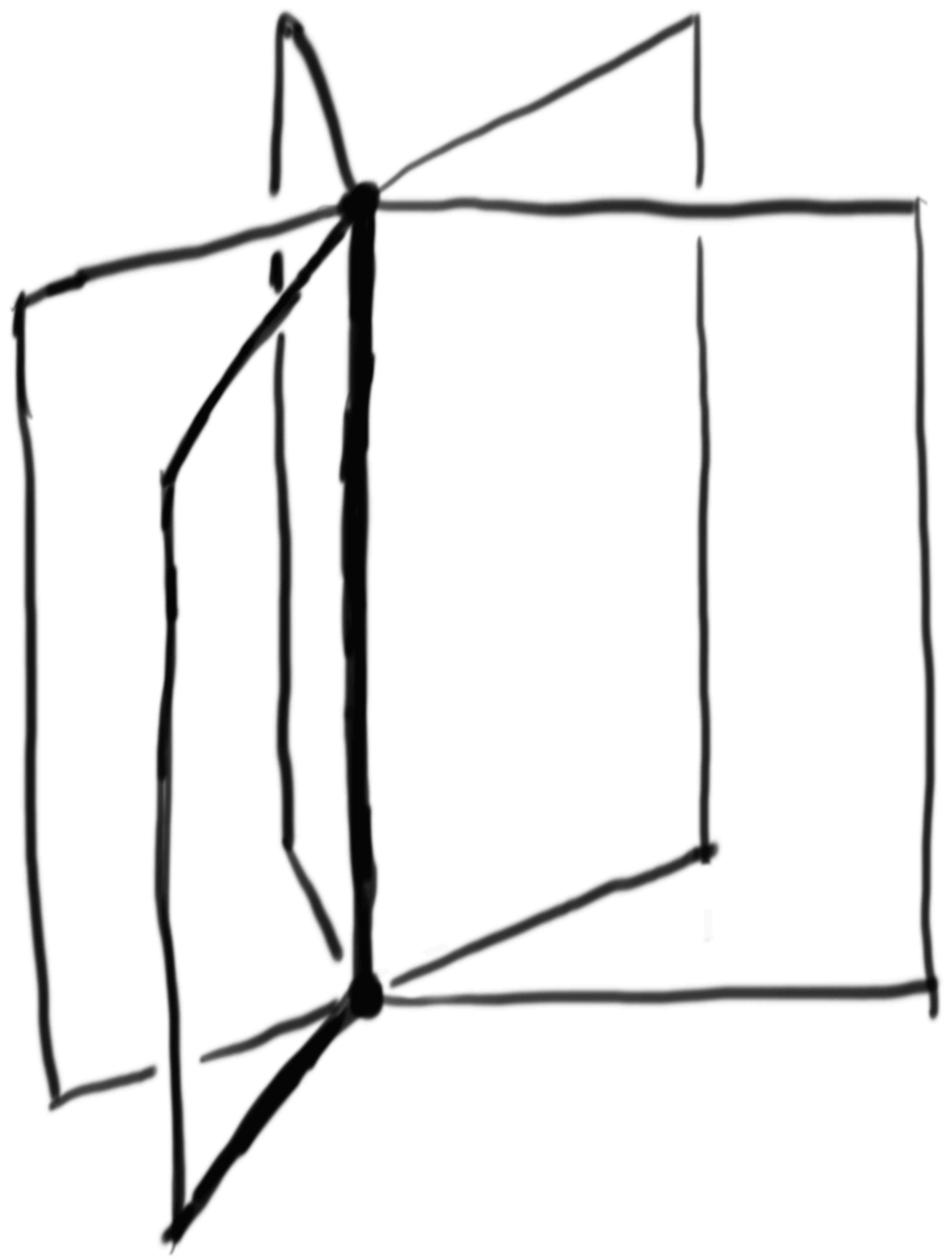
Approx U(1) $\phi = |\phi| e^{i\theta}$

$\downarrow \langle |\phi| \rangle \approx v$

Z_N

$\downarrow \langle \theta \rangle = 2\pi k/N$

1



String-bounded walls

Similar to axion domain walls, e.g., Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 1207.3166; 1412.0789

Z_N walls with multi-scalars

e.g., Z_6-invariant potential with two scalars

Scalar	ϕ	ξ
Z_6 charge	1	3

$$V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4$$
$$-\frac{1}{2}\mu_\xi^2\xi^2 + \frac{1}{4}\lambda_\xi\xi^4$$
$$-\lambda_{\phi\xi}(\phi^3 + \phi^{*3})\xi$$

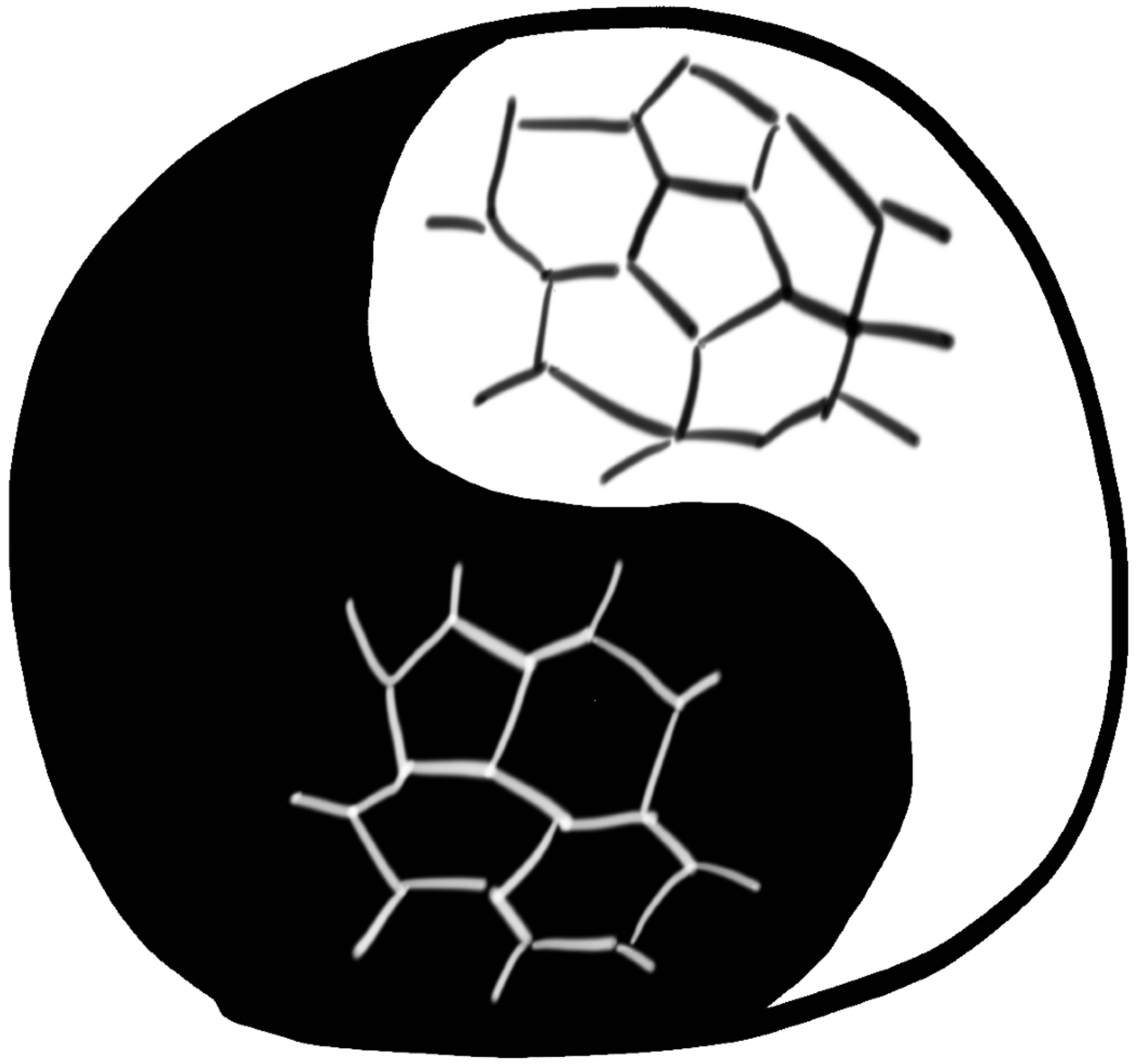
Z_6

↓ $\langle\xi\rangle = \pm\sqrt{\mu_\xi^2/2\lambda_\xi}$

Z_3

↓ $\langle\phi\rangle = \frac{\mu(\beta + \sqrt{1+\beta^2})}{\sqrt{2\lambda_1}} e^{\pm i2\pi k/3}$

1



Walls wrapped by walls

Classification of Abelian domain walls

A incomplete list

Y. Wu, K.P. Xie, YLZ, 2205.11529

Potential forms		breaking chains	textures of domain walls
single scalar	large ϕ^N	$Z_N \to 1$	adj. walls | non-adj. walls ($N \geqslant 4$)
	small ϕ^N	appr. $U(1) \to Z_N \to 1$	string-bounded adj. walls
multiscalar (ϕ, ξ with charges q_ϕ, q_ξ)	C1	appr. $U(1) \to Z_N \to 1$	string-bounded adj. walls
	C2	$Z_N \to Z_{\gcd(q_\xi, N)} \to 1$	walls wrapped by walls
	C3	$Z_N \to \begin{cases} Z_{\gcd(q_\xi, N)} \\ Z_{\gcd(q_\phi, N)} \end{cases}$	walls blind among diff. types

C1) Charges of ϕ and ξ are coprime with N, i.e., gcd(q_ξ, N) = gcd(q_ϕ, N) = 1.

C2) q_ξ has a non-trivial common divisor of N, but q_ϕ is still coprime with N, i.e., gcd(q_ξ, N) > 1 and gcd(q_ϕ, N) = 1.

(gcd: greatest common divisor)

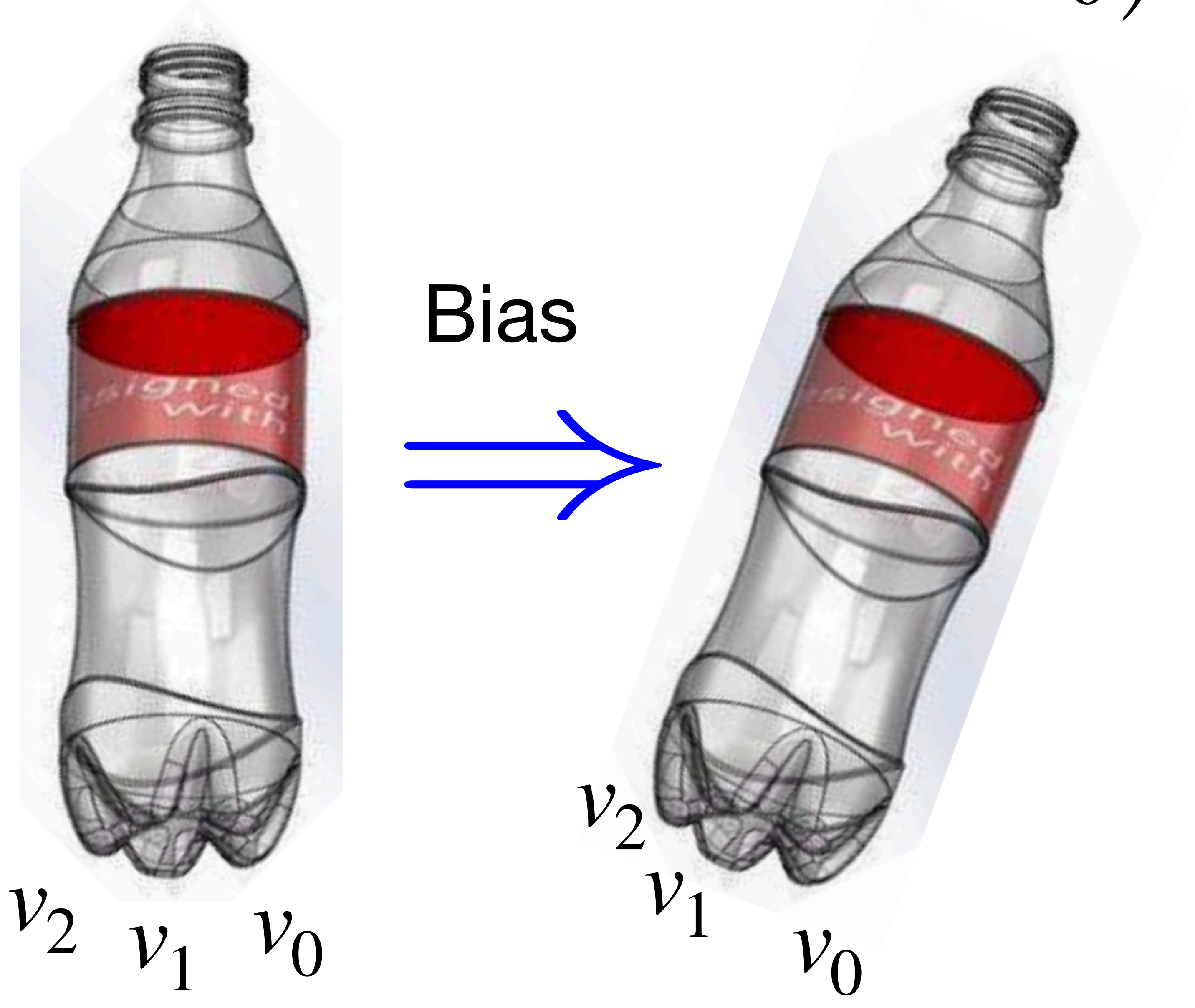
C3) Both q_ϕ and q_ξ have non-trivial common divisors with N, i.e., gcd(q_ϕ, N), gcd(q_ξ, N) > 1. We further require these two gcds are coprime with each other without loss of generality, otherwise, the essential symmetry is not Z_N but Z_N / gcd(gcd(q_ϕ, N),gcd(q_ξ, N)).

Bias term and GWs from collapsing domain walls

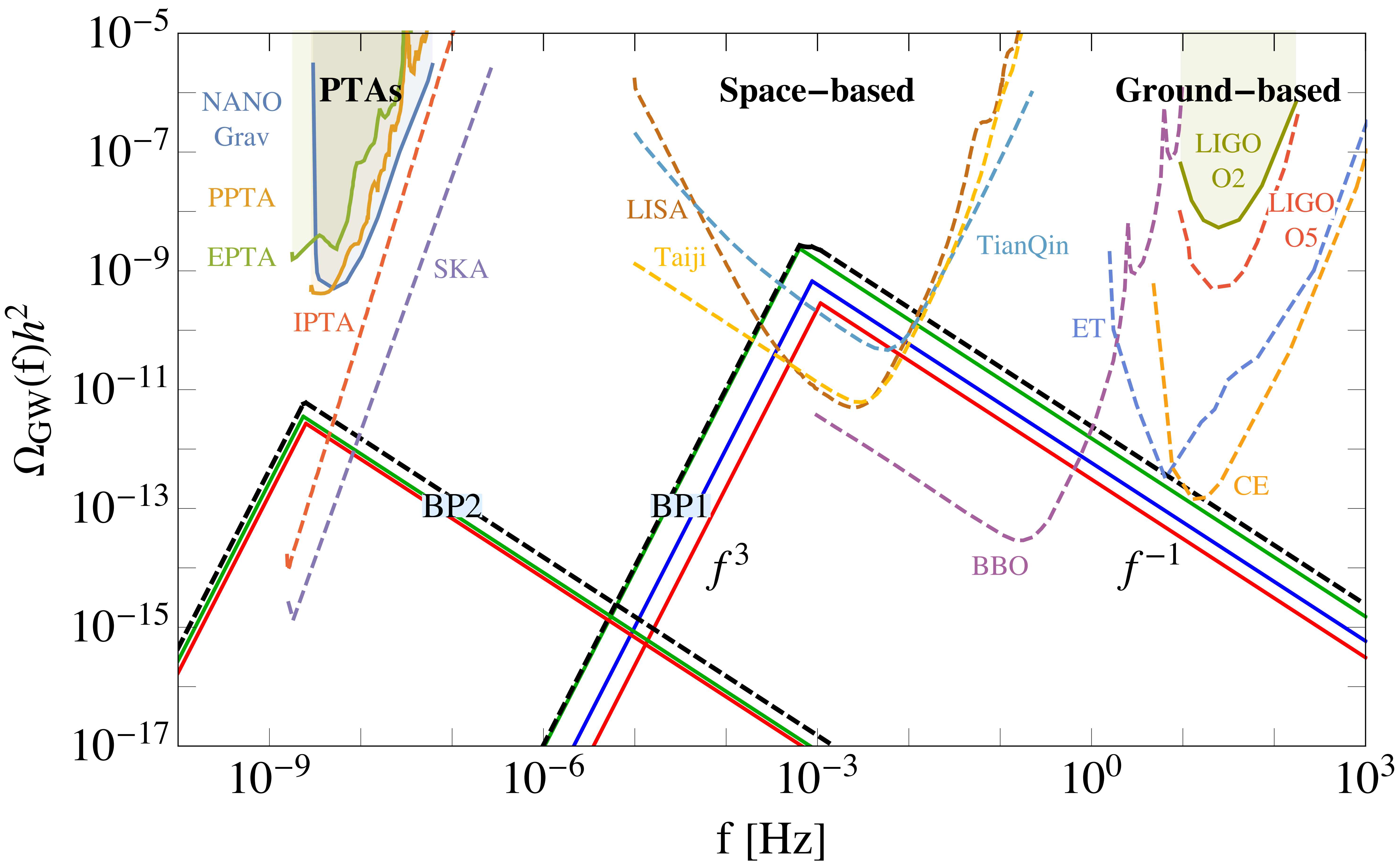
$$\delta V = \frac{2e^{i\alpha}}{3\sqrt{3}}\epsilon\phi\left(\frac{1}{4}\phi^3 - v_0^3\right) + \mathrm{h.c.}$$

$$(V_{\rm bias})_{10} = V|_{v_1} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha + \frac{\pi}{6}\right)$$

$$(V_{\rm bias})_{20} = V|_{v_2} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha - \frac{\pi}{6}\right)$$



GW spectrum, broken power laws based on Saikawa [1703.02576]

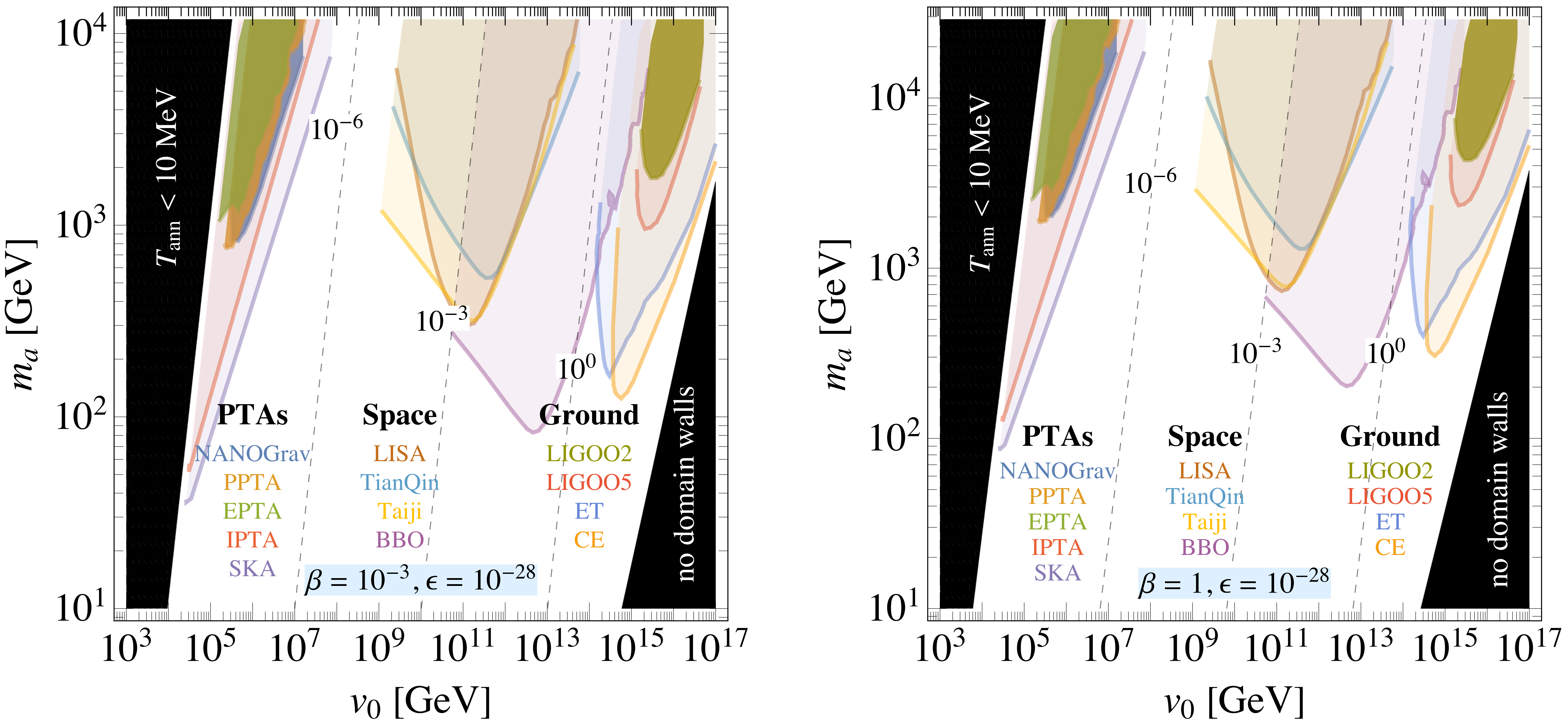


$$\mathrm{BP1}: v_0 = 10^{11}\ \mathrm{GeV}, \quad m_a = 2\ \mathrm{TeV}, \quad \alpha = \frac{2\pi}{9};$$

$$\mathrm{BP2}: v_0 = 10^{5}\ \mathrm{GeV}, \quad m_a = 500\ \mathrm{GeV}, \quad \alpha = \frac{\pi}{27}.$$

Y. Wu, K.P. Xie, YLZ, 2204.04374

Testability of Z_N walls via GWs: taking Z_3 as a case study



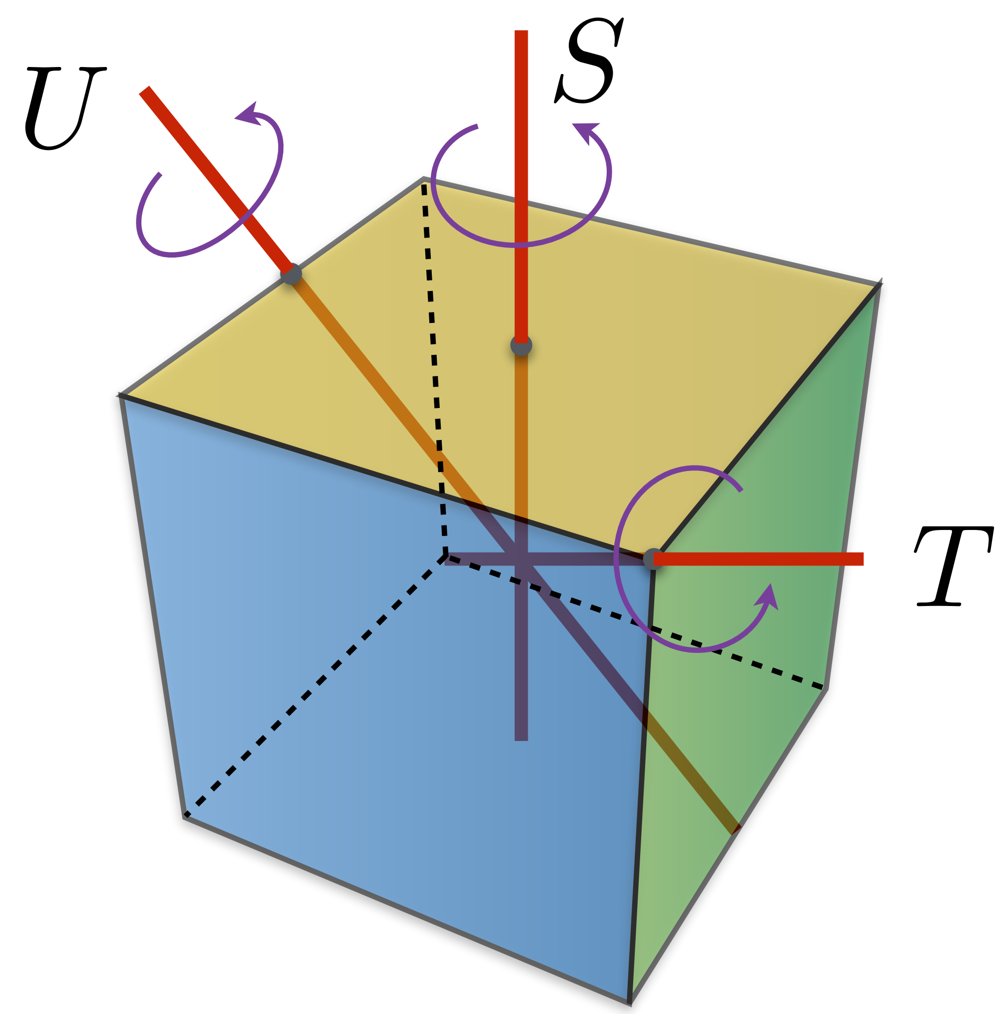
- Due to the different dynamics of Z_3 DW from Z_2 DW, we expect a different GW spectrum.
- However, a quantitative study requires a detailed simulation of domain walls.

Non-Abelian domain walls

- Symmetry: the octahedral group S_4

Representation matrices in the triplet $\mathbf{3}'$

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad U = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$



$$T^3 = S^2 = (ST)^3 = \mathbf{1}$$

$$U^2 = (SU)^2 = (TU)^2 = (STU)^4 = \mathbf{1}$$

Irreps: $\mathbf{1}, \mathbf{1}', \mathbf{2}, \mathbf{3}, \mathbf{3}'$

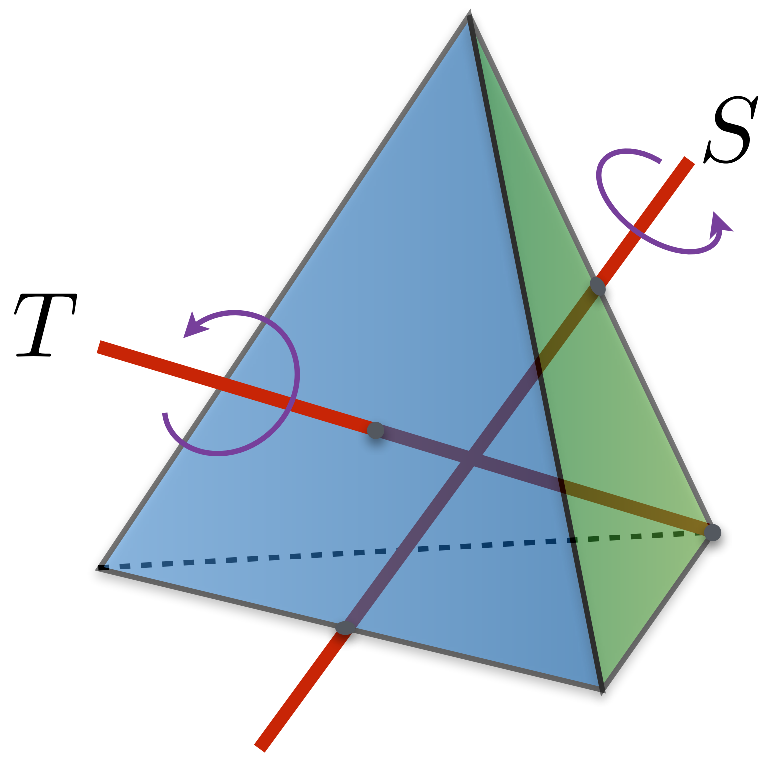
- Renormalisable potential

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 \qquad g_1 > 0 \quad g_2 > -4g_1$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

$$I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

also applies to $A_4 \times Z_2^P$ $(\phi \leftrightarrow -\phi)$



Non-Abelian domain walls

- Vacuum configuration

$g_2 > 0$ $\quad$ Z_2-preserving vacua

$$v_m \in \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix} \right\} v$$

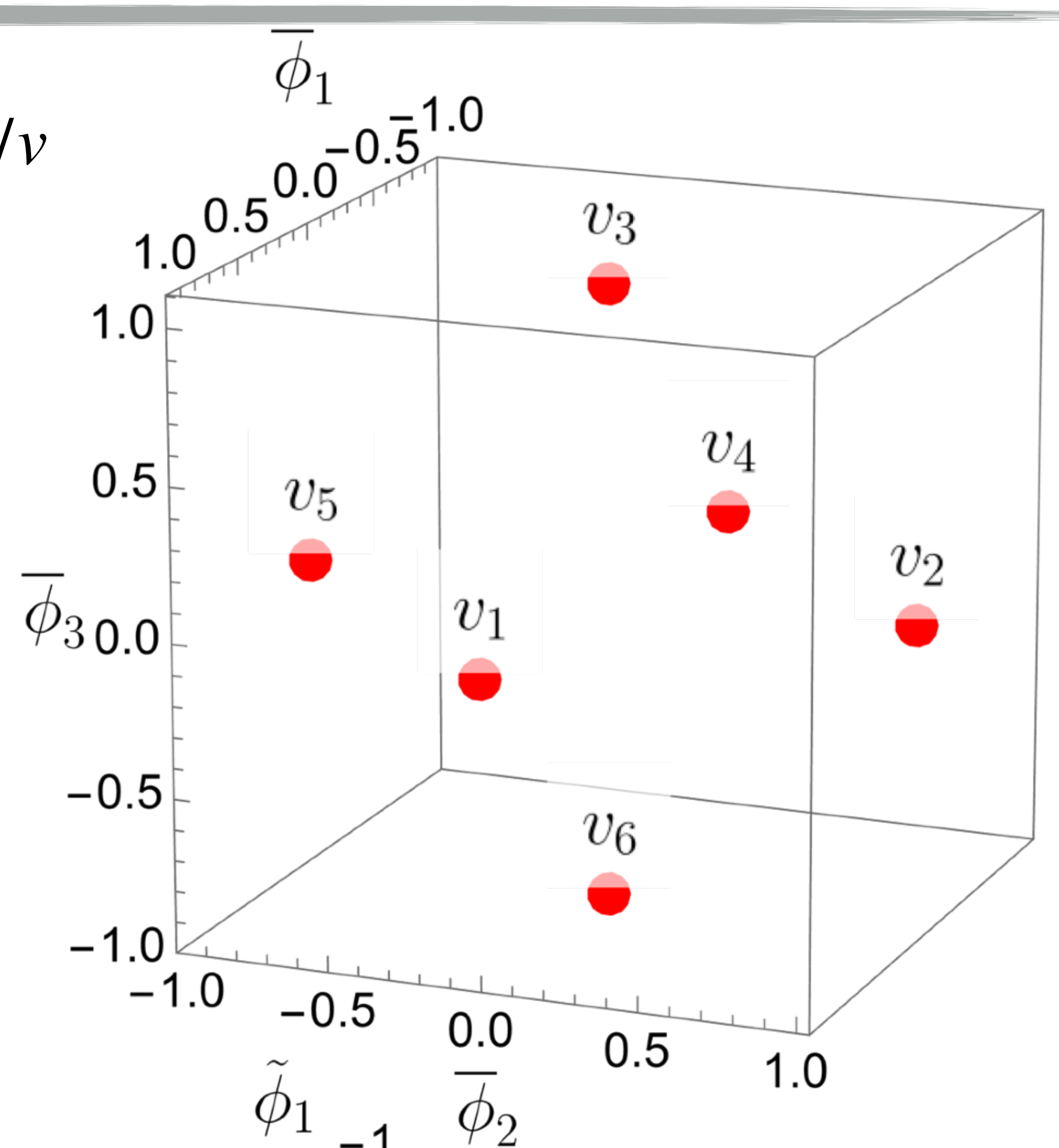
$$m = 1,2,3,4,5,6$$

$g_2 < 0$ $\quad$ Z_3-preserving vacua

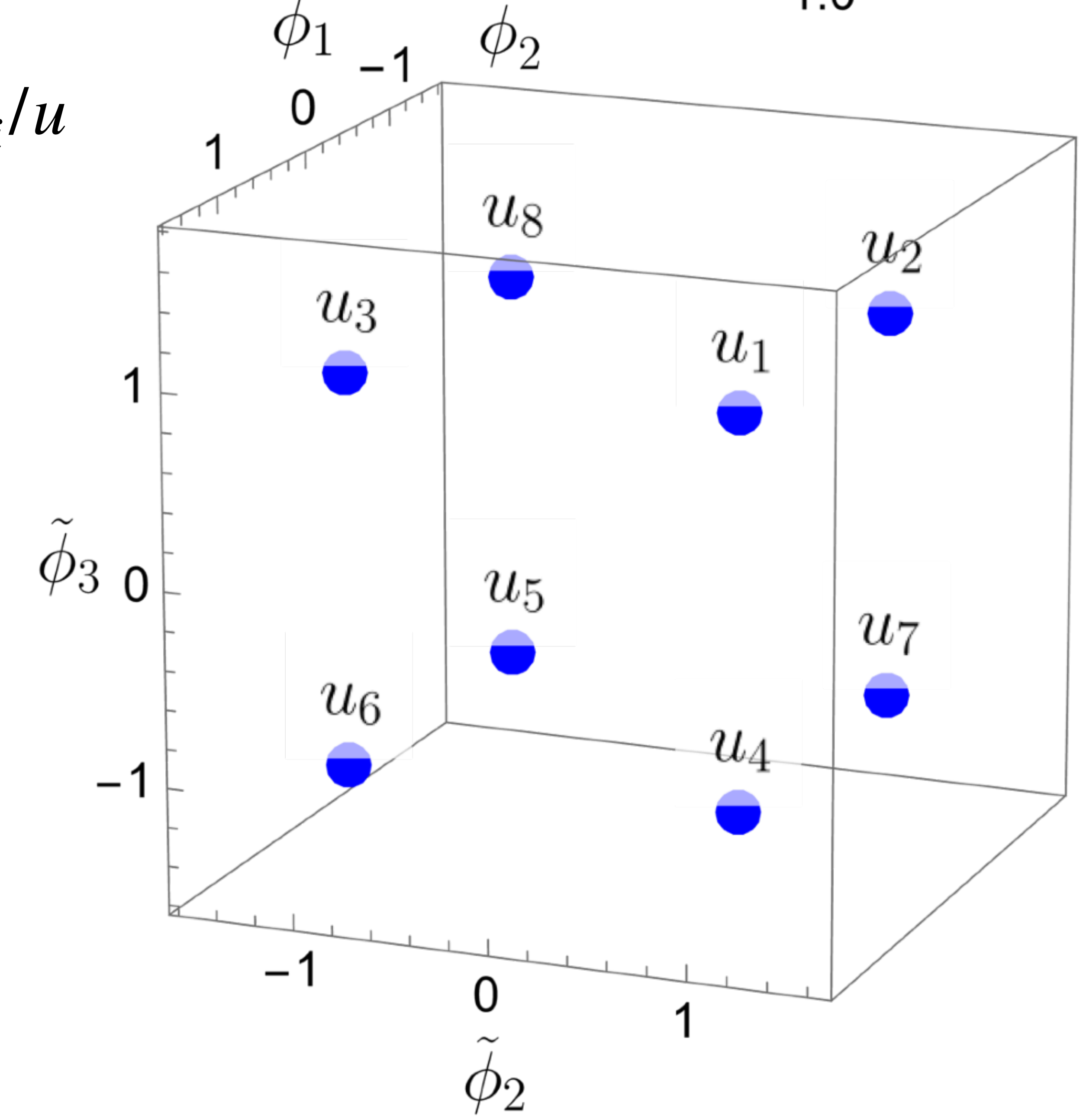
$$u_n = \left\{ \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\end{pmatrix}, \begin{pmatrix}-1\\-1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\end{pmatrix}, \begin{pmatrix}-1\\1\\-1\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\} u$$

$$n = 1,2,3,4,5,6,7,8$$

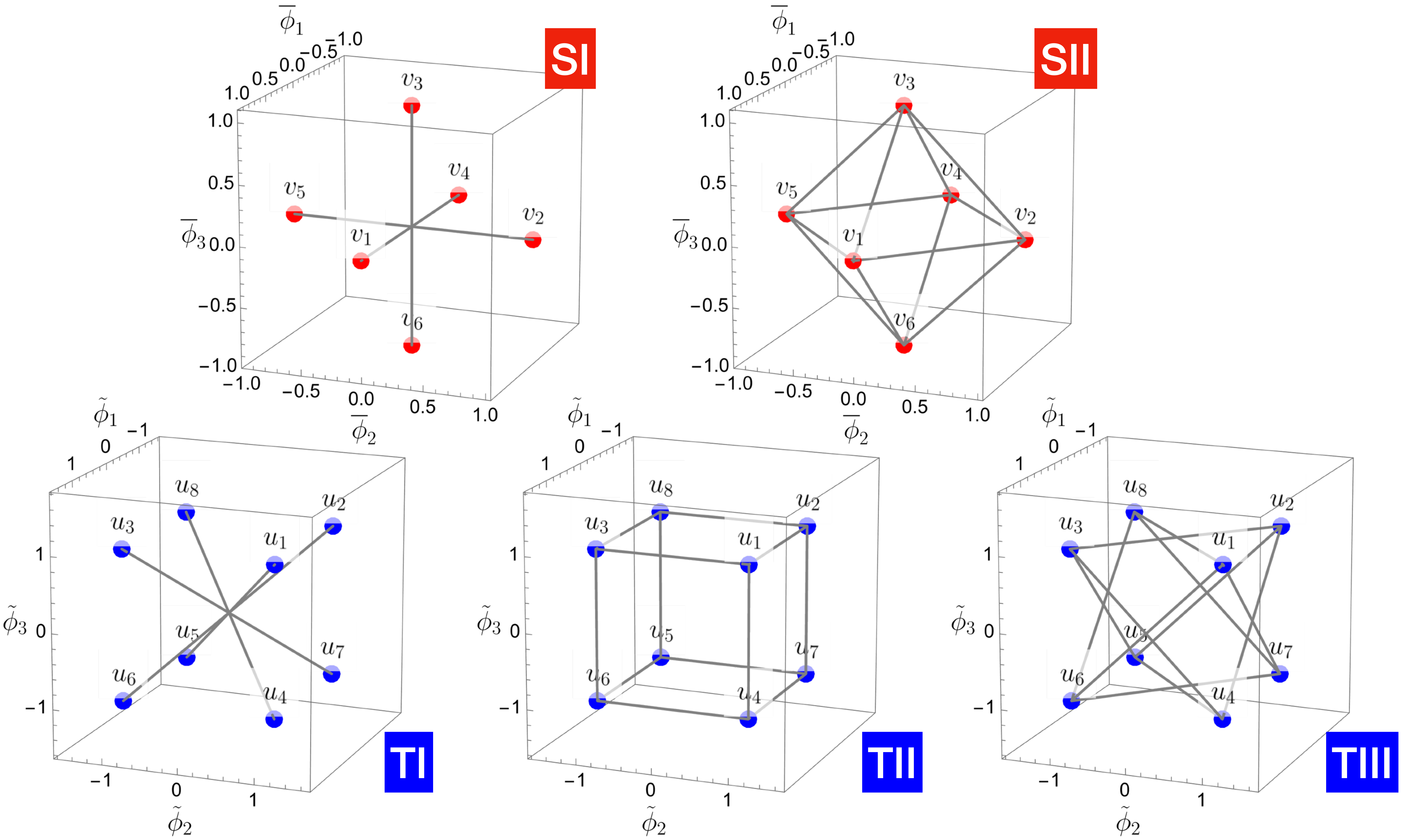
$\bar{\phi}_i = \phi_i / v$



$\tilde{\phi}_i = \phi_i / u$

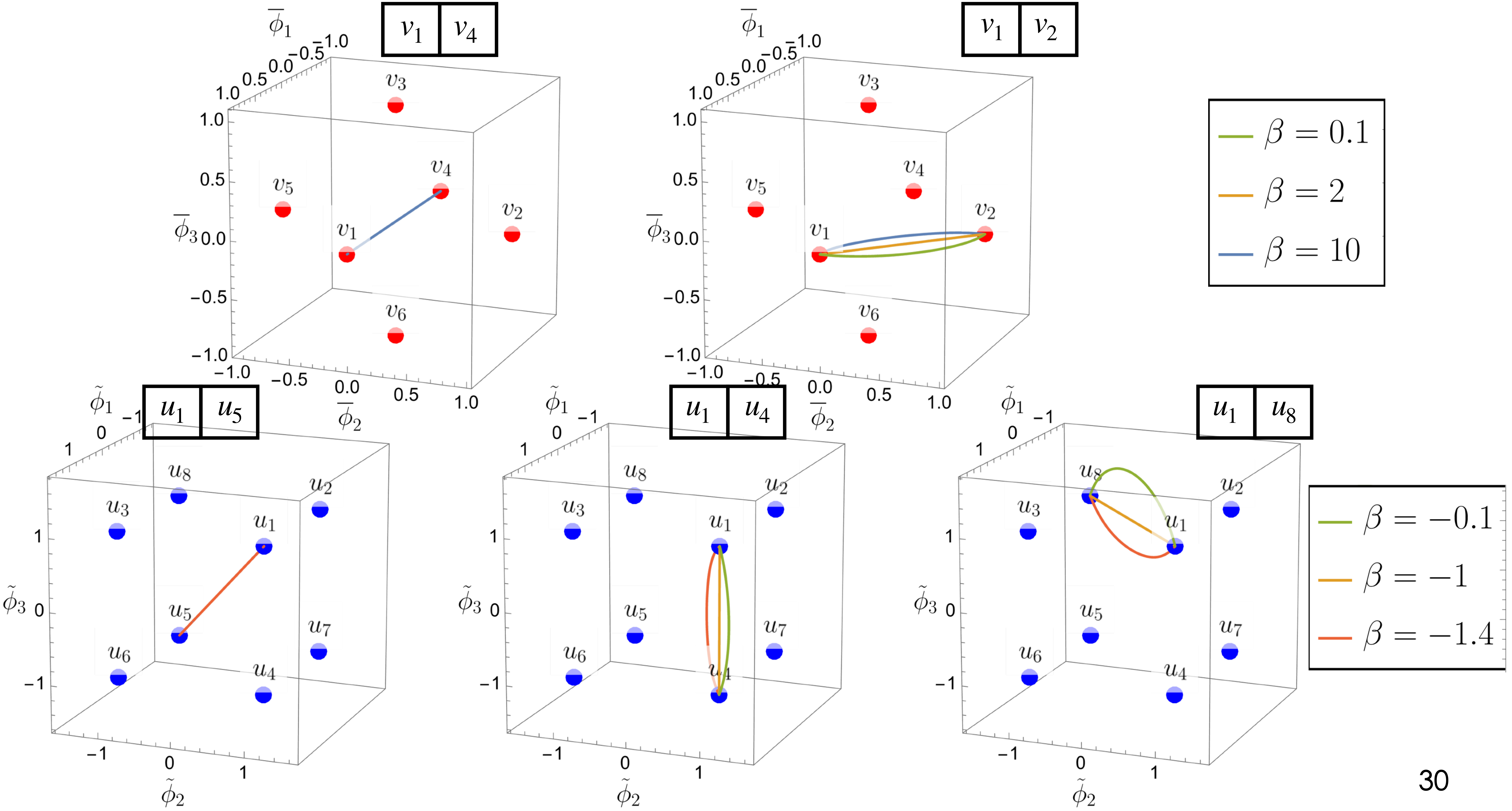


All possible domain walls from S_4 breaking $\Rightarrow$ 5 types



Domain wall solutions

B. Fu, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409.xxxxx



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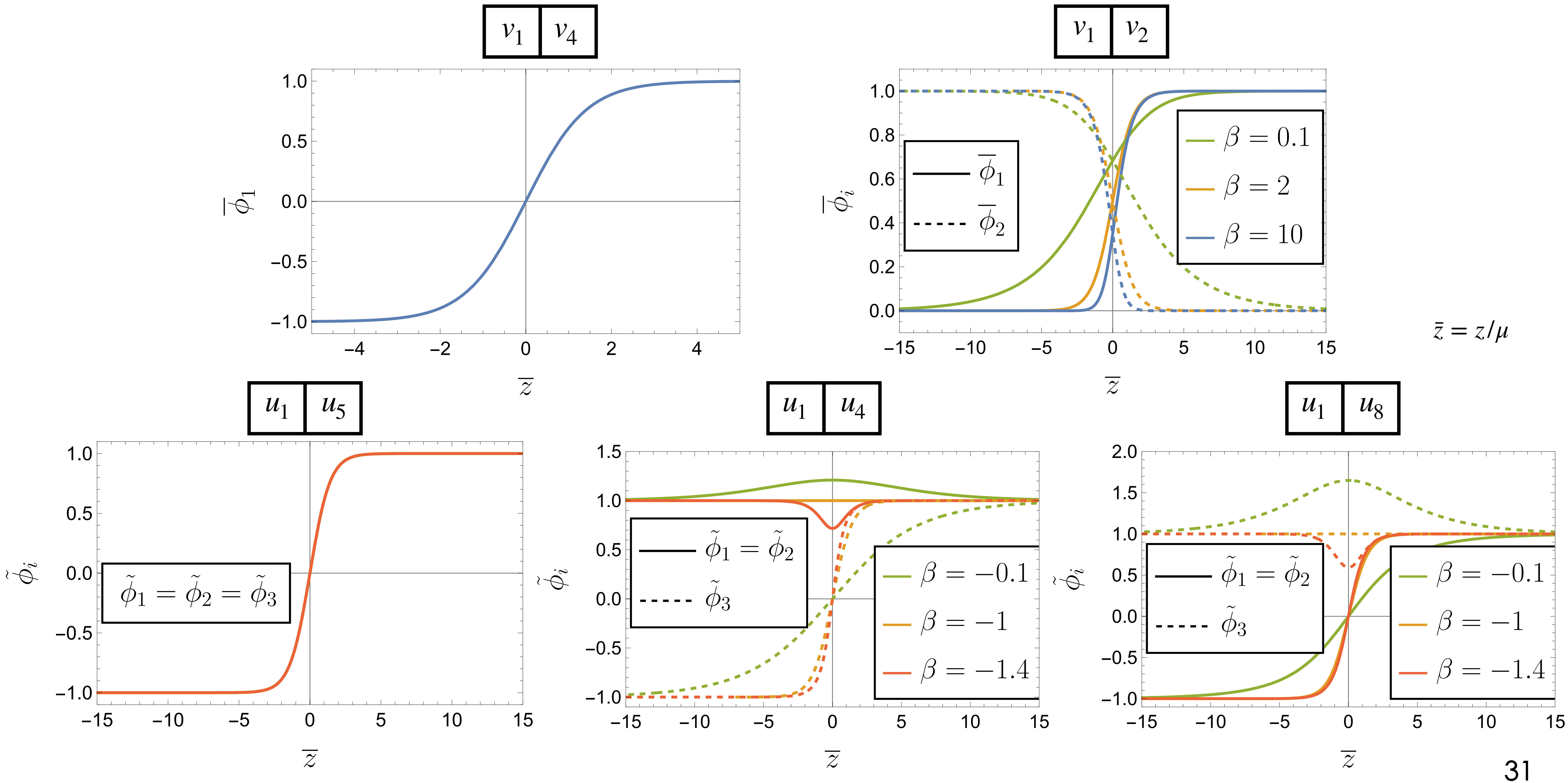
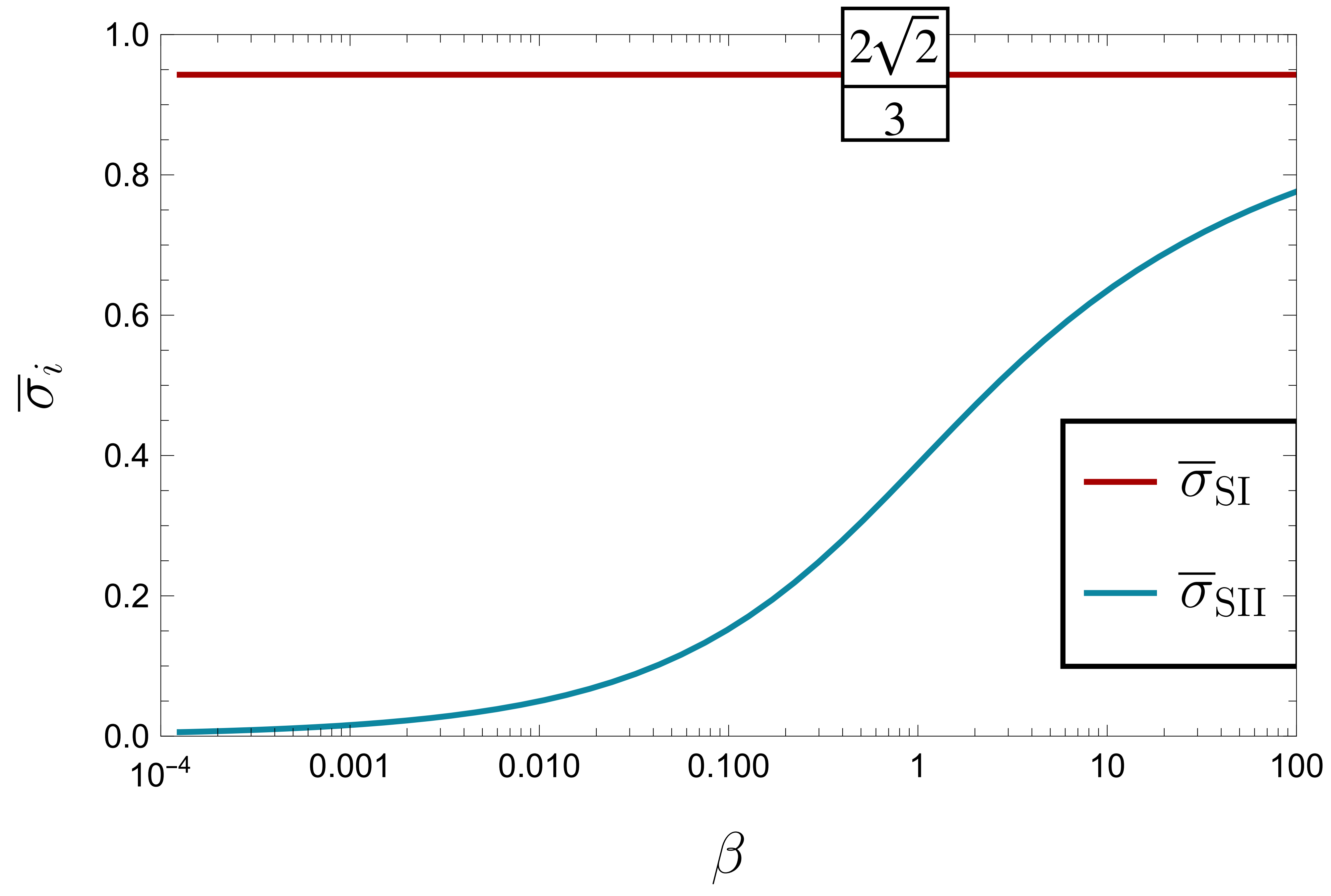


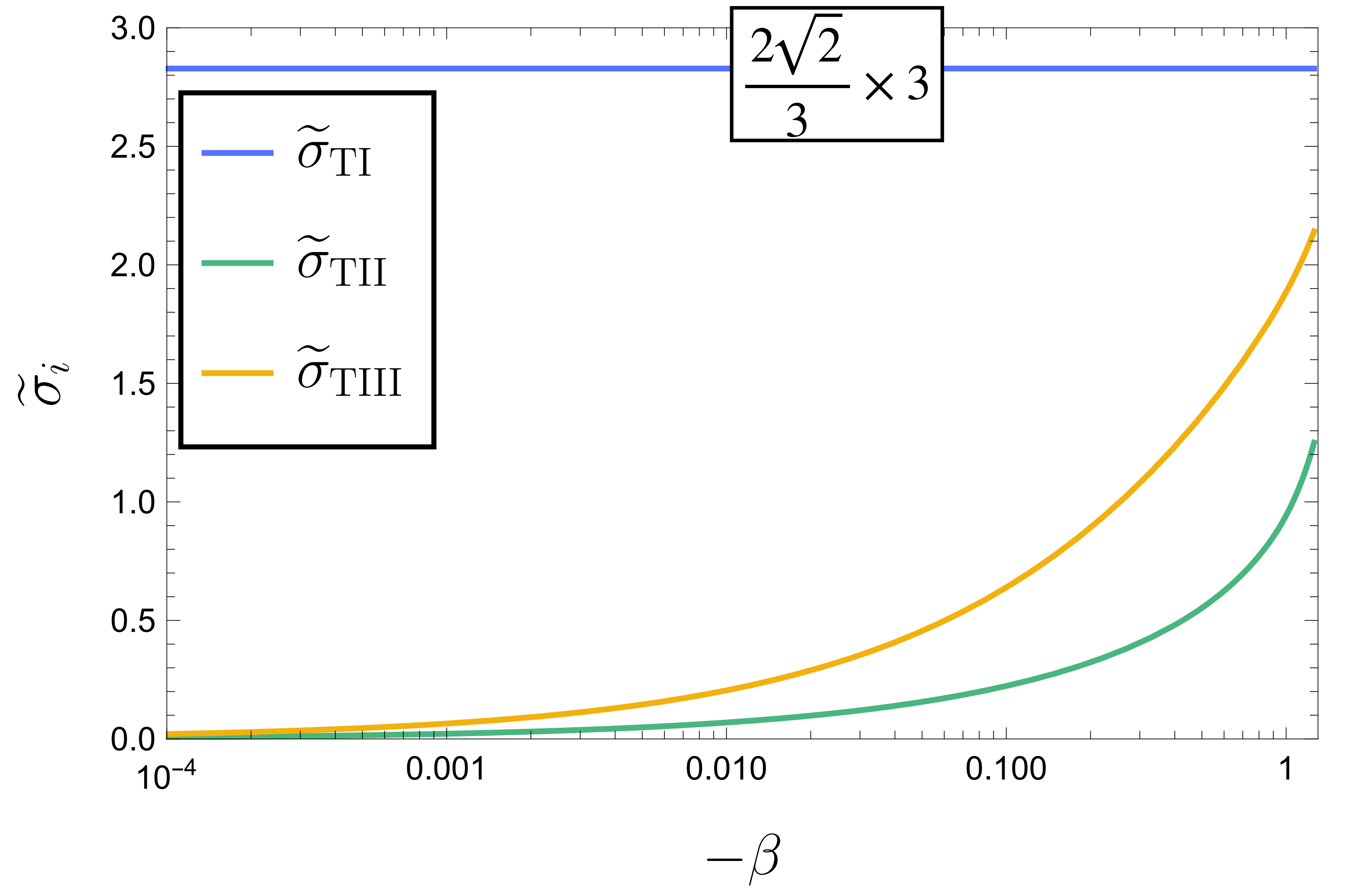
Domain wall solutions

B. Fu, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409.xxxxx

$$\bar{\sigma}_i = \frac{\sigma_i}{\mu v^2}$$

$$\tilde{\sigma}_i = \frac{\sigma_i}{\mu u^2}$$





$$\overline{\sigma}_{\rm SII}(\beta) \approx \frac{2\sqrt{2}}{3} \frac{1}{1+1.875\beta^{-1/2}} \left[1 + 0.5 \frac{\beta^{1/2}}{1+2\beta}\right]$$

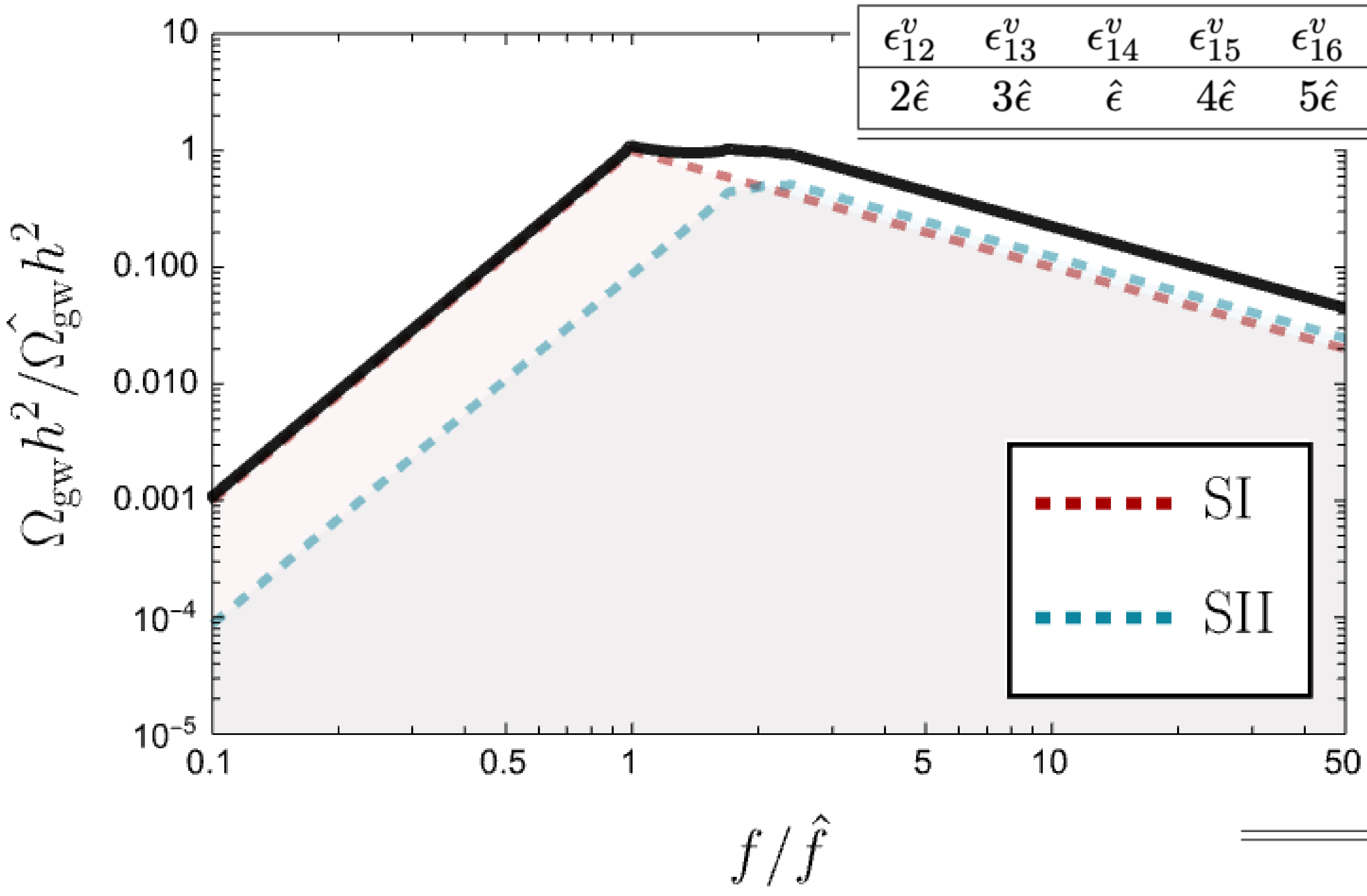
$$\tilde{\sigma}_{\rm TII}(\beta) = \frac{0.77(-\beta)^{0.5}}{(1.5+\beta)^{0.25}} \qquad \tilde{\sigma}_{\rm TIII}(\beta) = \frac{2.06(-\beta)^{0.5}}{1+0.09(-\beta)^{0.6}}$$

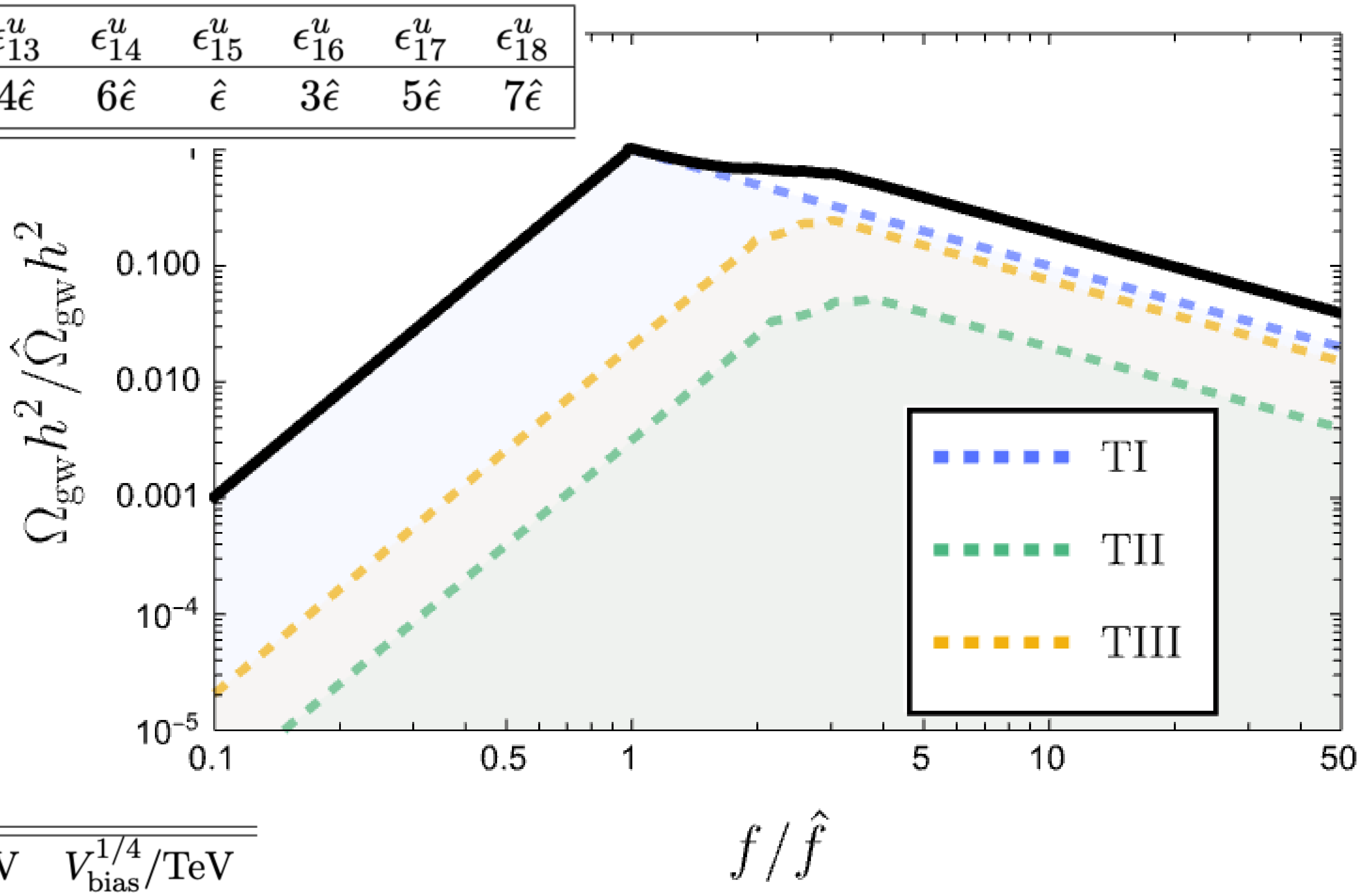
GW spectrums, for illustration

With bias $V_{\text{bias}}^{ij} = \epsilon_{ij} v^4$

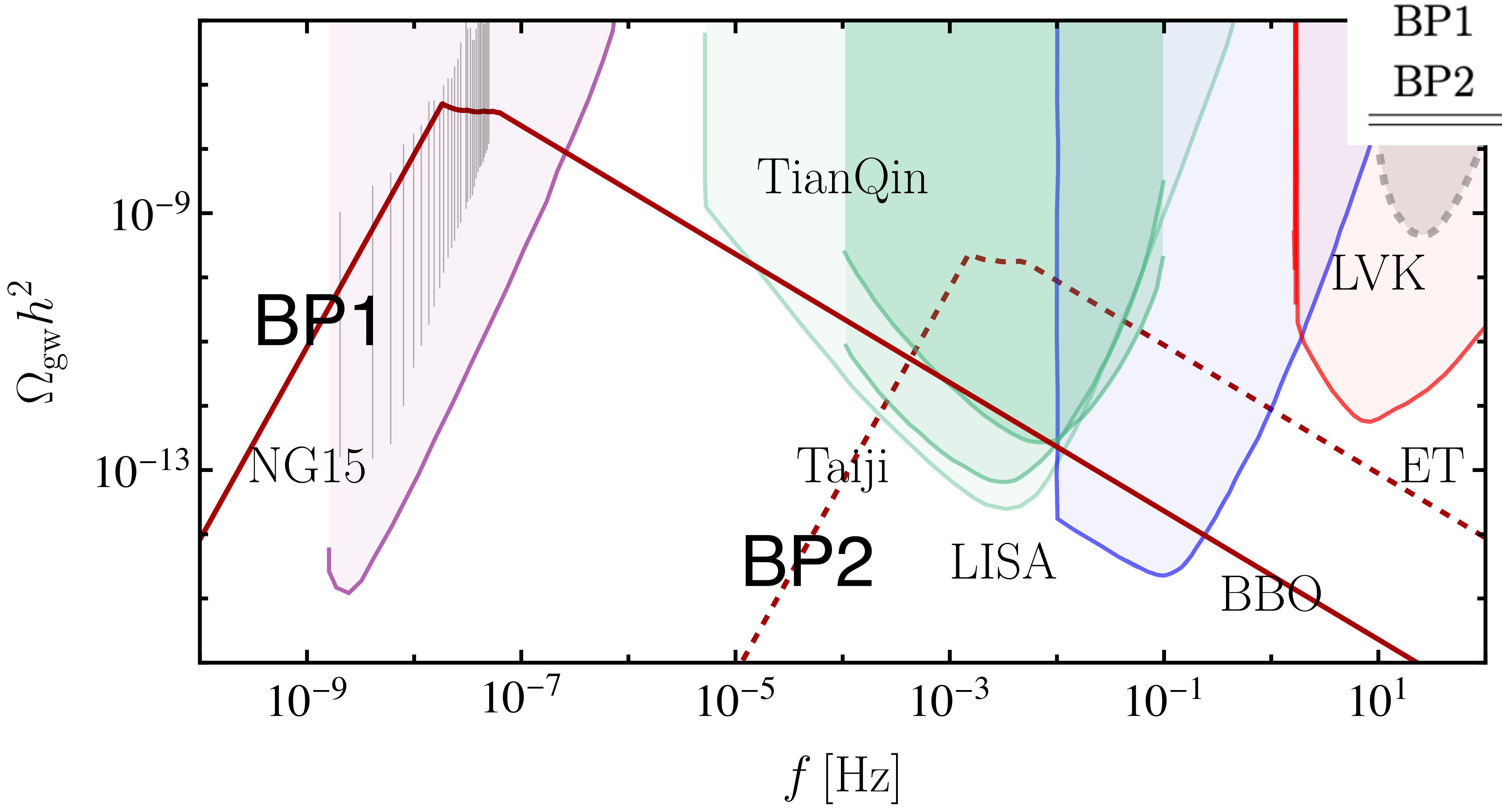
ϵ^v_{12}	ϵ^v_{13}	ϵ^v_{14}	ϵ^v_{15}	ϵ^v_{16}
$2\hat{\epsilon}$	$3\hat{\epsilon}$	$\hat{\epsilon}$	$4\hat{\epsilon}$	$5\hat{\epsilon}$

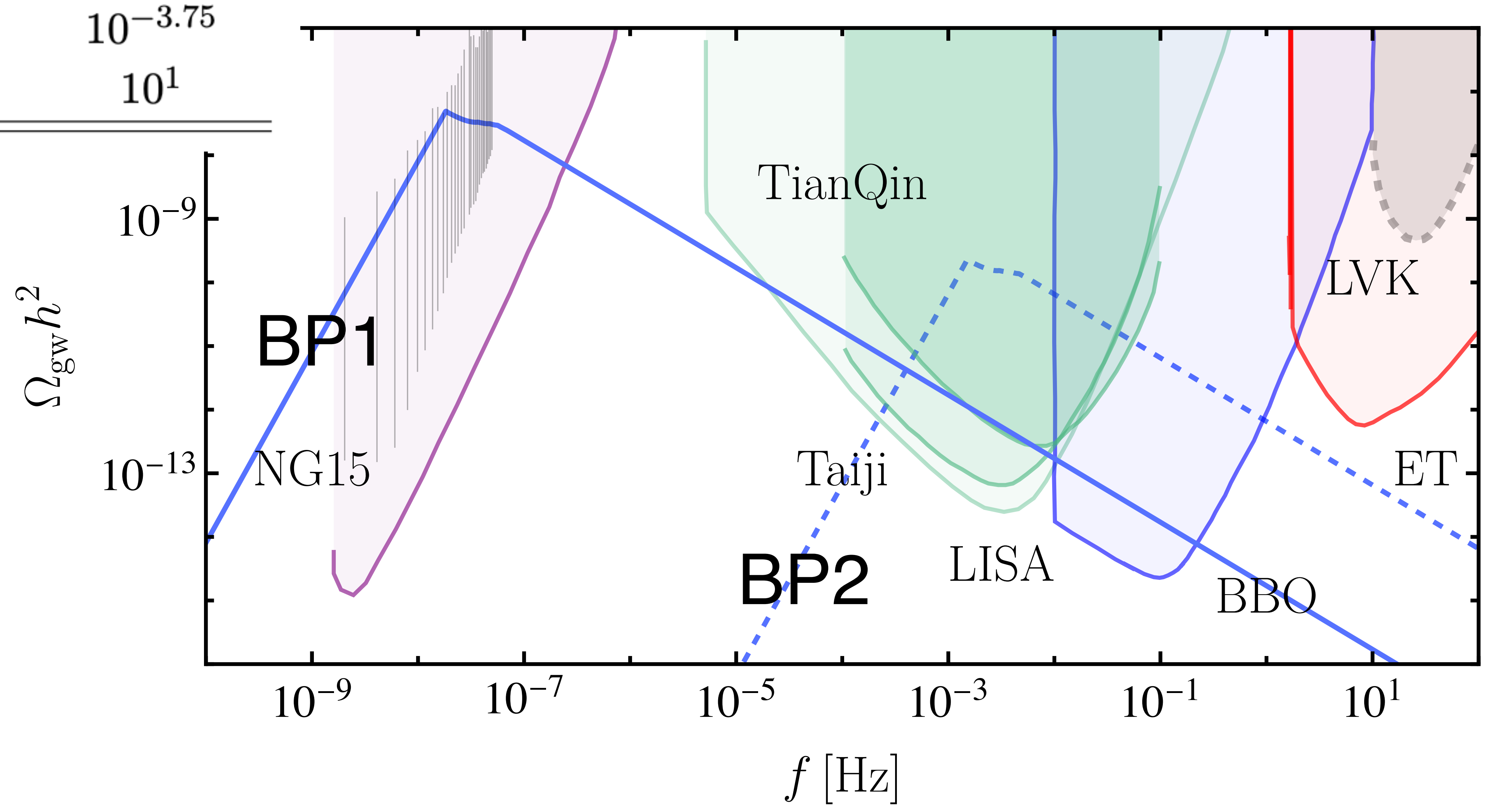
ϵ^u_{12}	ϵ^u_{13}	ϵ^u_{14}	ϵ^u_{15}	ϵ^u_{16}	ϵ^u_{17}	ϵ^u_{18}
$2\hat{\epsilon}$	$4\hat{\epsilon}$	$6\hat{\epsilon}$	$\hat{\epsilon}$	$3\hat{\epsilon}$	$5\hat{\epsilon}$	$7\hat{\epsilon}$





	$\sigma^{1/3}$/TeV	$V_{\text{bias}}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1





Testing discrete flavour symmetries

Gelmini, Pascoli, Vitagliano, YLZ, 2009.01903

A lepton flavour model in A_4

$$-\mathcal{L}_{l,\nu} \supset y_D \bar{L}_i \tilde{H} N_i + y_N \bar{N}_i N_j^c \chi_k + \frac{1}{2} u \bar{N}_i^c N_i + \frac{\varphi_i}{\Lambda} \bar{L}_i H (y_e e_R + \omega^{1-i} y_\mu \mu_R + \omega^{i-1} y_\tau \tau_R) + \text{h.c.}$$

$i \neq j \neq k \neq i,\ \omega = e^{i2\pi/3}$

with explicit breaking

$$-\mathcal{L}_{\not{A}_4} = \frac{1}{2} \epsilon_{ij} v_\chi \bar{N}_i^c N_j + \text{h.c.}$$

$$\epsilon_{12} = \epsilon_{13} = 0,\ \epsilon_{22} = -\epsilon_{33}$$

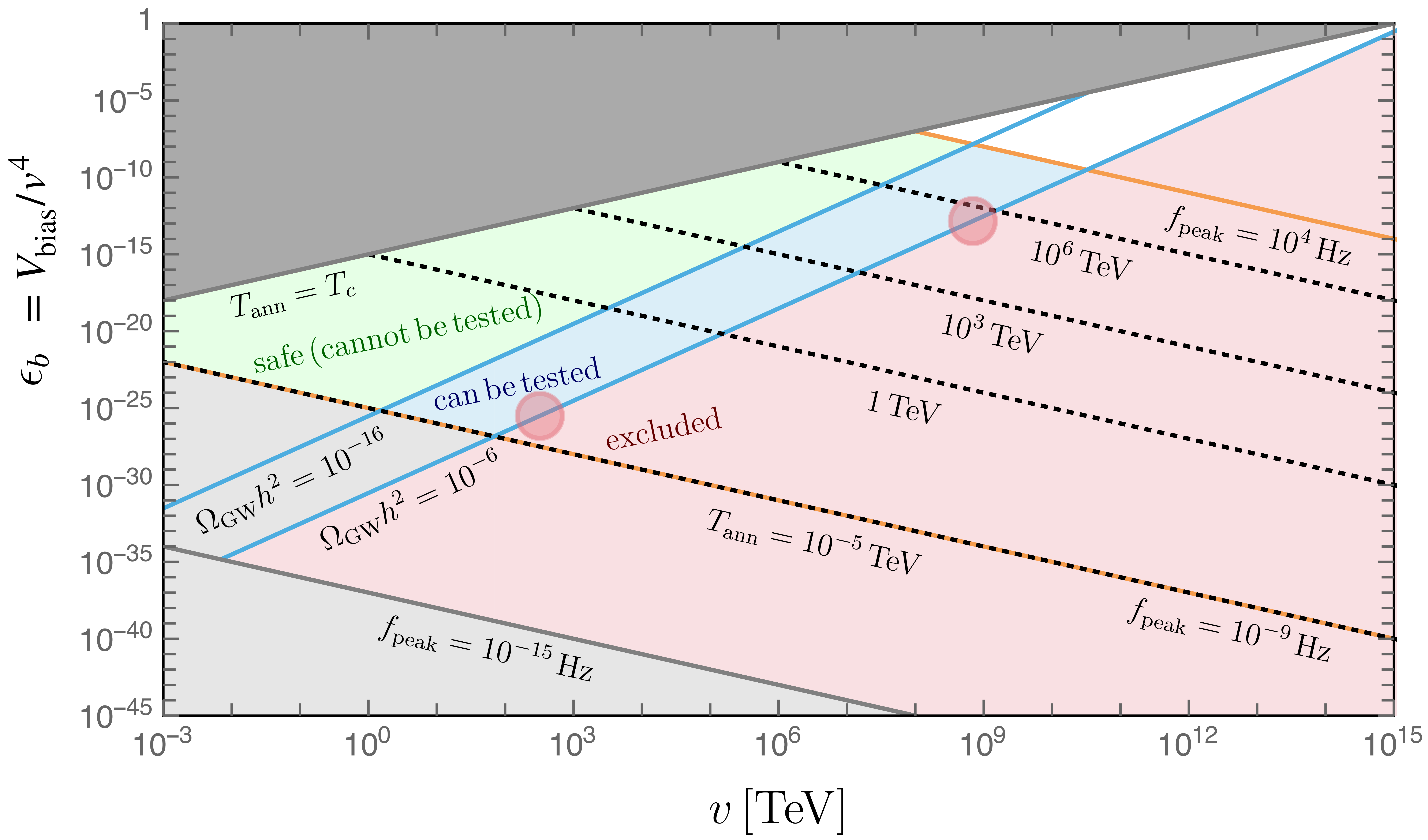
$\mu - \tau$ reflection symm. & TM2 mixing

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} \cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} |\sin\theta| \\ \left|\frac{1}{\sqrt{6}} \cos\theta - \frac{i}{\sqrt{2}} \sin\theta\right| & \frac{1}{\sqrt{3}} & \left|\frac{1}{\sqrt{6}} \sin\theta + \frac{i}{\sqrt{2}} \cos\theta\right| \\ \left|\frac{1}{\sqrt{6}} \cos\theta - \frac{i}{\sqrt{2}} \sin\theta\right| & \frac{1}{\sqrt{3}} & \left|\frac{1}{\sqrt{6}} \sin\theta - \frac{i}{\sqrt{2}} \cos\theta\right| \end{pmatrix}$$

$\theta_{23} = 45^\circ$ $\delta = \pm 90^\circ$ $\alpha_{21}, \alpha_{31} = 0, 180^\circ$ $\sin\theta_{12} = \frac{1}{\sqrt{3}\cos\theta_{13}}$ $\sin\theta_{13} \simeq 2\sqrt{\frac{2}{3}} \frac{|\epsilon_{22}| v_\chi M_2}{|\Delta M_{31}^2|}$

Testability of domain walls in flavour symmetries

Testing non-Abelian discrete flavour symmetry



Summary & Outlook

- Abelian DWs, in general, can have properties very different from the Z_2 DW.
  - Two types of Z_4 DWs: adjacent and non-adjacent DWs. The latter may be unstable.
  - There might be more complicated DWs in Z_N, e.g., string-bounded DWs, DW-wrapped DWs. We give an incomplete classification of Z_N DWs.
- Non-Abelian DW, taking S_4 as an example, is studied for the first time.
  - Five types are classified, SI, SII, TI, TII, TIII. The former two separate Z_2 vacua and the latter three separate Z_3 vacua.
  - SI, TI, and TIII are unstable in some parameter space.
- Bias terms are required for domain walls to collapse before they dominates the Universe. Due to the existence of different DWs and different biases among the vacua, we expect a different dynamics of DW collapsing and the consequent GW spectrum should be different from Z_2 DW.

Thank you very much!