Journey Towards Asymptotically Safe Grand Unified Theory

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Sep. 7th, 2024

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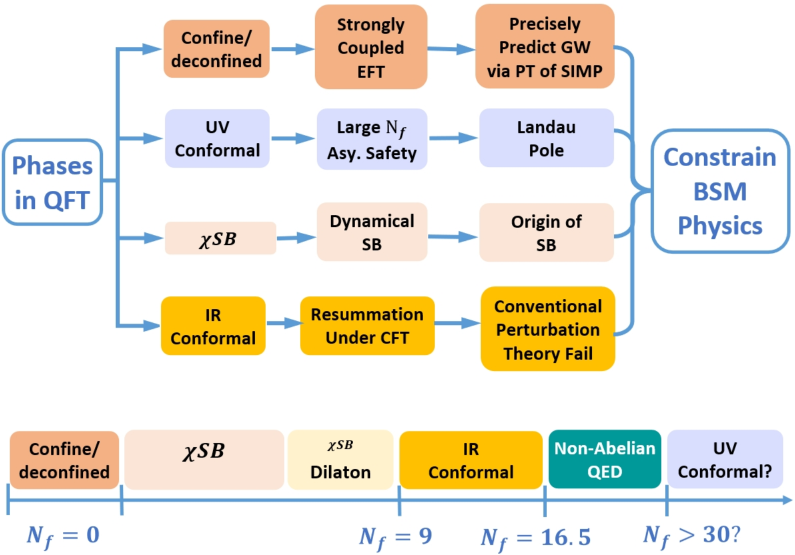
Quantum Gravity, Strings and the Swampland
Sep. 3rd - Sep. 9th 2024, Corfu, Greece

A Landscape of Phases in QFT and its Relation to BSM Physics



Part 1: Towards Asymptotically Safe Standard Model

- The Standard Model is not a fundamental theory since it runs into Landau Pole at UV due to the abelian $U(1)$ gauge group.
- Question 1: how to make the Standard Model UV complete without gravity?
- Many GUTs (in particular supersymmetric ones) are even worse.
- Due to the presence of large representations, the RGE of the unified coupling will hit Landau pole right above the unification scale.
- Question 2: can we make the Standard Model UV complete via a GUT embedding?
- Question 3: can UV completion provides an alternative guiding principle to BSM like naturalness/fine-tuning?

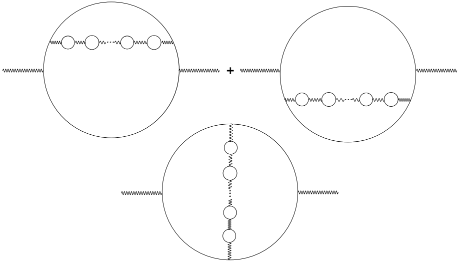
Fundamental Theory

- A fundamental theory has an UV fixed point (K. G. Wilson, Phys. Rev. B **4** (1971) 3174.)
- Couplings stop running with the energy scale at the fixed point
- The Standard Model is not a fundamental theory since it runs into Landau Pole at UV due to the abelian $U(1)$ gauge group
- Asymptotically Free: non-interacting (Gaussian) fixed point (D. J. Gross and F. Wilczek, Phys. Rev. D **8** (1973) 3633; D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.)
    - non-interacting in the UV
    - coupling runs with logarithmic scale dependence
    - Perturbation theory in UV
- Asymptotically Safe: Interacting fixed point (S. Weinberg(1979). "Ultraviolet divergences in quantum theories of gravitation".)
    - interacting in the UV
    - coupling runs with power law scale dependence
    - Perturbative/Non perturbative theory in UV
    - Smaller critical surface dimension $\Rightarrow$ more IR predictiveness

Large N_f Expansion

- $1/N_f$ expansion in Abelian/non-Abelian gauge theory was firstly developed respectively by Pascual and Gracey and later on summarized by Bob Holdom with initial analysis of the pole structure

  A. Palanques-Mestre and P. Pascual, Commun. Math. Phys. **95** (1984) 277; J. A. Gracey, Phys. Lett. B **373** (1996) 178; B. Holdom, Phys. Lett. B **694**, 74 (2011).
- Pascual noticed that it is possible to sum up a subset of the diagrams and the resulting power series is so well behaved to provide a closed-form expression at $1/N_f$ order



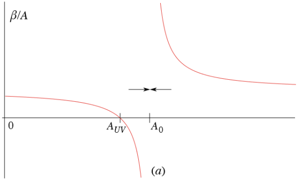
The Summation Function and its Pole Structure

- The resummed U(1) beta function reads (with summation function $F_1(A)$):

$$\beta_A = \frac{2A^2}{3}\left[1 + \frac{1}{N_f}F_1(A)\right], \qquad A \equiv 4N_f\alpha = 4N_f\frac{g_1^2}{(4\pi)^2}$$

$$F_1(A) = \frac{3}{4}\int_0^A \mathrm{d}x\,\tilde{F}\left(0, \tfrac{2}{3}x\right), \quad \tilde{F}(0,x) = \frac{(1-x)(1-\frac{x}{3})(1+\frac{x}{2})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}$$

- $1/N_f$ expansion encodes all order loop contributions.
- $F_1(A)$ has a pole structure at $A = 15/2$ (Non-abelian at $A = 3$).
- $\beta - A$ diagram(B. Holdom, Phys. Lett. B **694**, 74 (2011)).



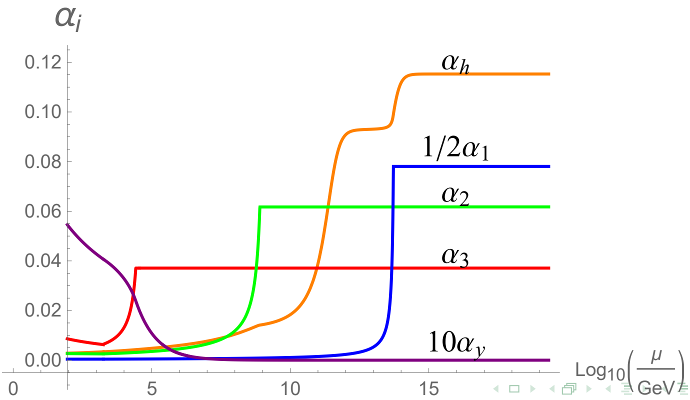
- The pole structure guarantees the **UV fixed point** of the gauge coupling.

Mann, Meffe, Sannino, Steele, Z. W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802

Safe Standard Model: $SU(3) \times SU(2) \times U(1)$

Mann, Meffe, Sannino, Steele, Z.W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802

- We introduce vector-like fermions under the SM group $SU(3) \times SU_L(2) \times U(1)$: $N_{F3}\ (3,1,0) \oplus N_{F2}\ (1,3,0) \oplus N_{F1}\ (1,1,1)$
- The gauge couplings $\alpha_1,\ \alpha_2,\ \alpha_3$ and Higgs quartic coupling α_h are safe while the top Yukawa coupling α_y is free
- The transition scale of the interacting fixed point is dependent on N_f



Conformal Window 2.0 O. Antipin and F. Sannino, Phys. Rev. D **97** (2018) 116007 .

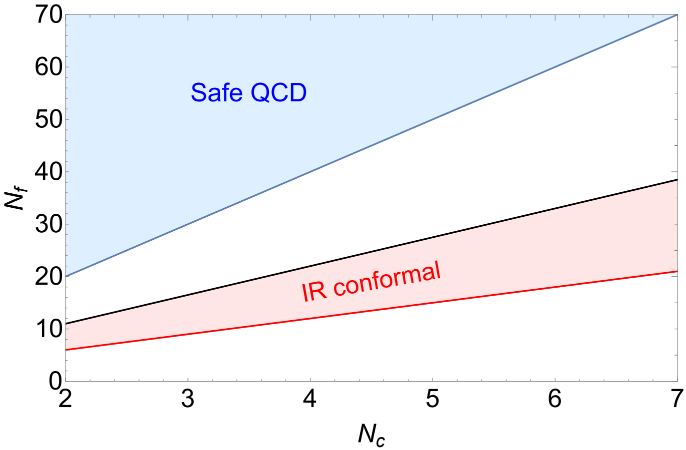


图: Phase diagram of $SU(N_c)$ gauge theories with fermionic matter in the fundamental representation. The shaded areas depict the corresponding conformal windows where the theories develop an IRFP (light red region) or an UVFP (light blue region).

Further Testing: Lattice and Beyond $1/N_f$ Calculations

- R. Shrock has shown the summation function is scheme independent at large N_f and thus natural to expect also the pole structure is scheme independent PRD 89 (2014) 045019
- The existence of UV fixed point is based on the truncation of diagram resummation at $1/N_f$ order.
- It is important to check whether higher order results say $1/N_f^2$ order will destablize the fixed point or not. Dondi, Dunne, Reichert and Sannino, "Towards the QED beta function and renormalons at $1/N_f^2$ and $1/N_f^3$," PRD **102** (2020) 035005.
- Helsinki Lattice group has studied $SU(2)$ case with 24 and 48 Dirac fermions by using the gradient flow method. Leino, Rindlisbacher, Rummukainen, Sannino and Tuominen, "Safety versus triviality on the lattice," PRD **101** (2020) 074508.
- However, they found the current lattice actions is unable to explore the deep ultraviolet region where safety might emerge.
- Their work constitutes an **essential** step towards determining the UV conformal phase of non asymptotically free gauge theories (lattice group lead by Oliver Witzel at Siegen University is also studying this).

Generalize the Large N_f beta Functions

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- We generalize the Large N_f gauge beta functions to more general semi-simple gauge groups.
- We insert the bubble chain to quartic and Yukawa interactions and have obtained large N_f beta functions for the first time for quartic and Yukawa couplings.

Large N_f beta Functions: Semi-Simple Gauge

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Vector-like fermions charged
  under only simple gauge group (PRL **119** (2017) 261802)
  $\Rightarrow$ semi-simple gauge group (Note: two different gauge lines below)

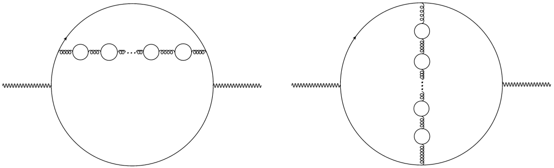


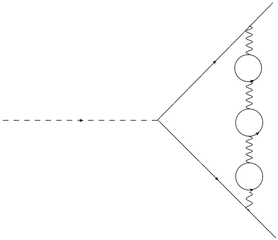
图: Two extra Feynman diagrams for the 2-point functions giving mixed terms to the beta functions.

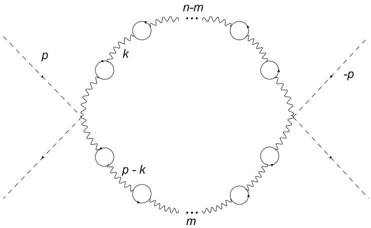
- $$\beta_i^{\rm ho} = \frac{2A_i\alpha_i}{3}\left(\frac{d(G_i)H_{1_i}(A_i)}{N_f\,\prod_k d\left(R_\psi^k\right)} + \frac{\sum_j\, d(G_j)\,F_{1_j}(A_j)}{N_f\prod_k d\left(R_\psi^k\right)}\right)$$

Large N_f beta Functions: Yukawa and Quartic

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Bubble chain insertion only for gauge couplings (PRL **119** (2017) 261802)
  $\Rightarrow$ all gauge, Yukawa and Quartic couplings (PRD 98 (2018) 016003)





Recipe of Bubble Improved RG Function: Yukawa

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

See also (Kowalska, Sessolo, JHEP 1804 (2018) 027; Phys.Rev. D97 (2018) 095013)

- Recipe for Yukawa

$$\beta_y = c_1 y^3 + y \sum_\alpha c_\alpha A_\alpha I_y\left(A_\alpha\right), \text{ where}$$

$$I_y\left(A_\alpha\right) = H_\phi\left(0, \tfrac{2}{3}A_\alpha\right)\left(1 + A_\alpha \frac{C_2\left(R_\phi^\alpha\right)}{6\left(C_2\left(R_\chi^\alpha\right) + C_2\left(R_\xi^\alpha\right)\right)}\right),$$

$$H_\phi(0,x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}.$$

- The summation function H_ϕ has a pole at $x = 5$ corresponding to $A = 15/2$.
- For models where c_1 and c_α are known, we can immediately obtain the bubble diagram contributions following this recipe.

Recipe of Bubble Improved RG Function: Quartic

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.

G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Recipe for Quartic Coupling

$$\beta_\lambda = c_1\lambda^2 + \lambda\sum_\alpha c_\alpha A_\alpha I_{\lambda g^2}\left(A_\alpha\right) + \sum_\alpha c'_\alpha A_\alpha^2 I_{g^4}\left(K_\alpha\right) + \sum_{\alpha<\beta} c_{\alpha\beta} A_\alpha A_\beta I_{g_1^2 g_2^2}\left(A_\alpha,\, A_\beta\right)\,,$$

$$I_{\lambda g^2}\left(A_\alpha\right) = H_\phi\left(0, \tfrac{2}{3}A_\alpha\right)$$

$$I_{g^4}\left(A_\alpha\right) = H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) + A_\alpha \frac{dH_\lambda\left(1, \tfrac{2}{3}A_\alpha\right)}{dA_\alpha}$$

$$I_{g_1^2 g_2^2}\left(A_\alpha,\, A_\beta\right) = \frac{1}{A_\alpha - A_\beta}\left[A_\alpha H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) - A_\beta H_\lambda\left(1, \tfrac{2}{3}A_\beta\right)\right]$$

$$H_\lambda(1, x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{6\Gamma^3(2-\frac{x}{2})\Gamma(1+\frac{x}{2})}\,.$$

Pole Structure Crisis

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770;
G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Pole in the summation functions:

$$H_\phi\left(0, \tfrac{2}{3}A_\alpha\right) \sim \frac{1}{\frac{15}{2} - A_\alpha},\; H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) \sim \frac{1}{A_\alpha - \frac{15}{2}},$$

$$\frac{\partial}{\partial A_\alpha} H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) \sim -\frac{1}{\left(A_\alpha - \frac{15}{2}\right)^2}$$

- Pole structure of Yukawa coupling (multiplicative proportional to y): approaching asymptotically free quickly

$$\beta_y \sim c_1 y^3 + y A_\alpha \left(\frac{1}{A_\alpha - \frac{15}{2}}\right)(c_2 + c_3 A_\alpha)$$

- Pole structure of Quartic Coupling (**Not** multiplicative proportional to λ): blow up to very negative value!

$$\beta_\lambda \sim c_1 \lambda^2 + c_2 \lambda A_\alpha \left(\frac{1}{A_\alpha - \frac{15}{2}}\right) + c_3 A_\alpha^2 \left(\frac{1}{A_\alpha - \frac{15}{2}} - \frac{1}{\left(A_\alpha - \frac{15}{2}\right)^2}\right)$$

$U(1)$ Landau Pole Problem Recap and Alternative Motivation to Study Safe GUT Embedding

- Two ways to address the $U(1)$ problem
  - Embedding in a non-abelian group
  - $U(1)$ safety with large N_f
- $U(1)$ problem is not successfully addressed in the large N_f framework
  - the mass anomalous dimension blows up at the Abelian pole place (Antipin and Sannino, Phys. Rev. D **97** (2018) 116007, arXiv:1709.02354.)
  - the quartic coupling will also blow up at the abelian pole (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Semi-simple gauge does not help (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Yukawa summation does not help (T. Alanne and S. Blasi, Phys. Rev. D **98** (2018) 116004, arXiv:1808.03252.)
- The **incompatibility** between the $U(1)$ and Higgs self coupling motivates the study of a safe GUT theory where $U(1)$ is embedded in a non-abelian group.
- Or alternatively there is no Higgs self coupling.

Question 2：Can we make the Standard Model UV safe through GUT embedding?

- Safe Pati-Salam: $G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R$
  (Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.)
- Safe Trinification: $G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$
  (Z.W. Wang, Balushi, Mann and Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.)

Safety of Grand Unified Theory: Pati-Salam Model

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Pati-Salam model is under gauge symmetry group G_{PS}

  J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974) Erratum: [Phys. Rev. D **11**, 703 (1975)].

$$G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \, .$$

- The SM quark and lepton fields are unified into the $G_{\rm PS}$ irreducible representations

$$\psi_{Li} = \begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \end{pmatrix}_i \sim (4,2,1)_i \, ,$$

$$\psi_{Ri} = \begin{pmatrix} u_R & u_R & u_R & \nu_R \\ d_R & d_R & d_R & e_R \end{pmatrix}_i \sim (4,1,2)_i \, ,$$

- Symmetry breaking pattern: $G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

Pati-Salam Model: Gauge Field Content

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The gauge fields of G_{PS} can be written as follows:

$$\hat{W}_{L\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{L\mu} & \sqrt{2}W^+_{L\mu} \\ \sqrt{2}W^-_{L\mu} & -W^0_{L\mu} \end{pmatrix},$$

$$\hat{W}_{R\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{R\mu} & \sqrt{2}W^+_{R\mu} \\ \sqrt{2}W^-_{R\mu} & -W^0_{R\mu} \end{pmatrix},$$

$$\hat{G}_{\mu} \equiv \frac{1}{2}\begin{pmatrix} G_{3\mu}+\frac{G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{12\mu} & \sqrt{2}G^+_{13\mu} & \sqrt{2}X^+_{1\mu} \\ \sqrt{2}G^-_{12\mu} & -G_{3\mu}+\frac{G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{23\mu} & \sqrt{2}X^+_{2\mu} \\ \sqrt{2}G^-_{13\mu} & \sqrt{2}G^-_{23\mu} & -\frac{2G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}X^+_{3\mu} \\ \sqrt{2}X^-_{1\mu} & \sqrt{2}X^-_{2\mu} & \sqrt{2}X^-_{3\mu} & -\frac{3B_\mu}{\sqrt{6}} \end{pmatrix}$$

- $W^0_{L\mu}$ and $W^\pm_{L\mu}$ correspond to the electroweak (EW) gauge bosons, $G_{3\mu}$, $G_{8\mu}$, $G^\pm_{12\mu}$, $G^\pm_{13\mu}$ and $G^\pm_{23\mu}$ are the $SU(3)_C$ gluons, B_μ is the $B-L$ gauge field, and $X^\pm_{1\mu}$, $X^\pm_{2\mu}$ and $X^\pm_{3\mu}$ are leptoquarks.

Pati-Salam Model: Scalar Fields and Couplings

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Scalar field $\phi_R \sim (4,1,2)$ triggers Pati-Salam symmetry breaking:

$$\phi_R = \begin{pmatrix} \phi_R^{u_1} & \phi_R^{u_2} & \phi_R^{u_3} & \phi_R^0 \\ \phi_R^{d_1} & \phi_R^{d_2} & \phi_R^{d_3} & \phi_R^- \end{pmatrix}, \quad G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- Scalar bi-doublet $\Phi \sim (1,2,2)$ triggers electroweak symmetry breaking:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi_1 & \Phi_2 \end{pmatrix}$$

- The scalar potential can be written as:

$$\begin{aligned} V(\Phi,\phi_R) = & -\mu_1^2 \,\mathrm{Tr}\left(\Phi^\dagger\Phi\right) + \mathrm{Re}\left[\mu_{12}^2 \mathrm{Tr}\left(\Phi^\dagger\Phi^c\right)\right] - \mu_R^2 \mathrm{Tr}\left(\phi_R^\dagger\phi_R\right) \\ & + \lambda_1 \,\mathrm{Tr}^2\left(\Phi^\dagger\Phi\right) + \mathrm{Re}\left[\lambda_2 \mathrm{Tr}^2\left(\Phi^\dagger\Phi^c\right)\right] + \mathrm{Re}\left[\lambda_3 \mathrm{Tr}\left(\Phi^\dagger\Phi\right)\mathrm{Tr}\left(\Phi^\dagger\Phi^c\right)\right] \\ & + (\lambda_4 - 2\mathrm{Re}\lambda_2)\left|\mathrm{Tr}\left(\Phi^\dagger\Phi^c\right)\right|^2 + \lambda_{R1}\mathrm{Tr}^2\left(\phi_R^\dagger\phi_R\right) + \lambda_{R2}\,\mathrm{Tr}\left(\phi_R^\dagger\phi_R\phi_R^\dagger\phi_R\right) \\ & + \lambda_{R\Phi1}\mathrm{Tr}\left(\phi_R^\dagger\phi_R\right)\mathrm{Tr}\left(\Phi^\dagger\Phi\right) + \mathrm{Re}\left[\lambda_{\mathrm{R}\Phi_2}\mathrm{Tr}\left(\phi_\mathrm{R}\phi_\mathrm{R}^\dagger\right)\mathrm{Tr}\left(\Phi^\dagger\Phi^c\right)\right] \\ & + \lambda_{R\Phi3}\mathrm{Tr}\left(\phi_R^\dagger\phi_R\Phi^\dagger\Phi\right). \end{aligned}$$

Pati-Salam Model: Yukawa Sector

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The most general Yukawa Lagrangian for the matter fields $\psi_{L/R}$ is:

$$\mathcal{L}_{\text{Yuk}}^{\psi} \quad = \quad y\, \text{Tr}\left[\overline{\psi_L}\,\Phi\,\psi_R\right] + y_c\, \text{Tr}\left[\overline{\psi_L}\,\Phi^c\,\psi_R\right] + \text{h.c.}$$

- Electroweak symmetry breaking is induced by a nonzero vev of Φ:

$$\langle\Phi\rangle \;=\; \begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix}, \qquad \text{generically } u_1 \neq u_2$$

- The fermion mass spectrum:

$$m_t = m_{\nu_\tau} = (y\sin\beta + y_c\cos\beta)v\,,$$
$$m_b = m_\tau = (y\cos\beta + y_c\sin\beta)v\,,$$

  where $v \equiv \sqrt{u_1^2 + u_2^2} = 174$ GeV and $\tan\beta \equiv u_1/u_2$.
- To separate the neutrino and top masses, a new chiral fermion singlet $N_L \sim (1, 1, 1)$ added:

$$\mathcal{L}_{\text{Yuk}}^{N} \;=\; -y_\nu\,\overline{N_L}\,\text{Tr}\left[\phi_R^\dagger\,\psi_R\right] + \text{h.c.}$$

Pati-Salam Model: Yukawa Sector (backup)

- To separate the neutrino and top masses, we implement the seesaw mechanism by adding a new chiral fermion singlet $N_L \sim (1,1,1)$

$$\mathcal{L}_{\rm Yuk}^{N} = -y_\nu \overline{N_L}\mathrm{Tr}\left[\phi_R^\dagger \psi_R\right] + \mathrm{h.c.}$$

  The latter generates a Dirac mass term $M_R\overline{N_L}\nu_R$, with $M_R \equiv y_\nu v_R$.

- The resulting Majorana mass term for the neutral fermion fields reads:

$$\mathcal{L}_{\rm mass}^{\nu} = -\frac{1}{2}\begin{pmatrix} \overline{\nu_R^c} & \overline{\nu_R} & \overline{N_R^c} \end{pmatrix}\begin{pmatrix} 0 & m_t & 0 \\ m_t & 0 & M_R \\ 0 & M_R & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_L^c \\ N_L \end{pmatrix} + \mathrm{h.c.}$$

- The mass spectrum consists of one massless neutrino and one Dirac neutrino N_D with mass $m_D = \sqrt{m_t^2 + M_R^2}$
- Add a Majorana mass term for the N_L singlet $-\frac{1}{2}M_N\overline{N_R^c}N_L$, the spectrum consists of three massive neutrinos.
- Taking $M_N \ll m_t, M_R$, give one light active Majorana neutrino $m_{\nu_\tau} = M_N\frac{m_t^2}{m_D^2}$ and two quasi-degenerate heavy Majorana neutrinos $N_{1,2}$ with opposite CP parities and masses $M_{1,2} = m_D \pm \frac{M_N}{2}\frac{M_R^2}{m_D^2}$

Pati-Salam Model: Simplest Scenario $\Phi = \Phi^c$

R. R. Volkas, Phys. Rev. D **53** (1996) 2681

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- For a self-conjugate bi-doublet field $\Phi \equiv \Phi^c$, one has $u_1 = u_2$ and equality between fermion masses is enforced at tree-level, namely

$$m_t = m_b = m_\tau = m_{\nu_\tau}$$

- We further extend the matter content of the theory with a new vector-like fermion $F \sim (10, 1, 1)$ with mass M_F and Yukawa interactions:

$$\mathcal{L}^F_{\text{Yuk}} = y_F \, \text{Tr} \left(\overline{F_L} \, \phi_R^T \, i\tau_2 \, \psi_R \right) + \text{h.c.}$$

The field F can be decomposed as

$$F = \begin{pmatrix} S & B\sqrt{2} \\ B^T\sqrt{2} & E \end{pmatrix} ,$$

where S, B and E denote a 3×3 symmetric matrix representing the colour sextet, 3×1 triplet and singlet, respectively.

Pati-Salam Model: Simplest Scenario $\Phi = \Phi^c$

R. R. Volkas, Phys. Rev. D **53** (1996) 2681

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The fields B and E mix with the right-handed components of ψ_R, b_R and τ_R, respectively, giving the overall Dirac mass terms with $m_B \equiv y_F\, v_R/\sqrt{2}$:

$$\mathcal{L}^b_{\rm mass} = \begin{pmatrix} \overline{b_L} & \overline{B_L} \end{pmatrix} \begin{pmatrix} m_t & 0 \\ m_B & M_F \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} + \text{h.c.}\,,$$

$$\mathcal{L}^\tau_{\rm mass} = \begin{pmatrix} \overline{\tau_L} & \overline{E_L} \end{pmatrix} \begin{pmatrix} m_t & 0 \\ \sqrt{2}\, m_B & M_F \end{pmatrix} \begin{pmatrix} \tau_R \\ E_R \end{pmatrix} + \text{h.c.}\,,$$

- Via mixing, the top quark becomes naturally heavier than the other SM fermions. In the limit $m_B \gg m_t,\, M_F$, the b quark and τ charged lepton masses satisfy the tree-level relation:

$$m_b = \sqrt{2}\, m_\tau \approx \frac{M_F\, m_t}{\sqrt{2}\, m_B}\,.$$

- We have a new vector-like quark, $\hat{B}$, and a new vector-like lepton, $\hat{E}$, with corresponding masses $M_B = M_E/\sqrt{2} \approx m_B$,

Pati-Salam Model: A Summary of the Couplings

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- All the couplings:

Gauge Couplings	Yukawa Couplings	Scalar Couplings
$SU(4) : g_4$	$\psi_{L/R} : y, y_c$	$\phi_R : \lambda_{R1}, \lambda_{R2}$
$SU(2)_L : g_L$	$N_L : y_\nu$	portal: $\lambda_{R\Phi_1}, \lambda_{R\Phi_2}, \lambda_{R\Phi_3}$
$SU(2)_R : g_R$	$F : y_F$	$\Phi : \lambda_1, \lambda_2, \lambda_3, \lambda_4$

表: Gauge, Yukawa and scalar quartic couplings of the Pati-Salam model.

Vector-like Fermions Charges & Large N_f Gauge Beta

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- We consider three sets of vector-like fermions charged under $G_{\rm PS}$, with the charge assignment:

$$N_{f_4}\,(4,1,1)\oplus N_{f_{2L}}\,(1,3,1)\oplus N_{f_{2R}}\,(1,1,2)$$

  where the $N_{f_{2L}}$ vector-like fermions are chosen in the adjoint representation of $SU(2)_L$ to avoid fractional electrical charges.
- The charge assignments are chosen to avoid the extra contributions in the summation of semi-simple group
- The large N_f gauge beta functions are given by:

$$\beta^{tot}_{\alpha_{2L}} = \beta^{1loop}_{\alpha_{2L}} + \beta^{\rm ho}_{\alpha_{2L}} = -6\alpha^2_{2L} + \frac{2A_{2L}\alpha_{2L}}{3}\,\frac{H_{1_{2L}}\,(A_{2L})}{N_{f_{2L}}}$$

$$\beta^{tot}_{\alpha_{2R}} = \beta^{1loop}_{\alpha_{2R}} + \beta^{\rm ho}_{\alpha_{2R}} = -\frac{14}{3}\alpha^2_{2R} + \frac{2A_{2R}\alpha_{2R}}{3}\,\frac{H_{1_{2R}}\,(A_{2R})}{N_{f_{2R}}}$$

$$\beta^{tot}_{\alpha_4} = \beta^{1loop}_{\alpha_4} + \beta^{\rm ho}_{\alpha_4} = -18\alpha^2_4 + \frac{2A_4\alpha_4}{3}\,\frac{H_{1_4}\,(A_4)}{N_{f_4}}\,.$$

Alternative Picture of Gauge Coupling Unification

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- A sample case of gauge unification with $N_{f_{2L}} = 35$, $N_{f_{2R}} = N_{f_4} = 140$:

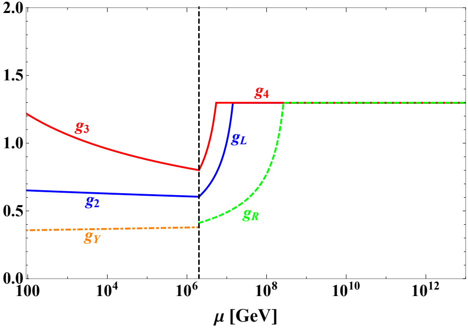


图: The dashed line represents the Pati-Salam symmetry breaking scale at $2000\,\mathrm{TeV}$ where all the vector-like fermions are introduced. The three couplings g_Y, g_2, g_3 at the left hand side of the dashed line are determined by the running of the SM gauge couplings.

Pole Structure PreCheck: Advantage of Pati-Salam

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The beta functions of Yukawa and Quartic couplings have the poles only at the Abelian pole where $A = \frac{15}{2}$
- When Abelian gauge coupling reaches a fixed point, the Yukawa coupling will be asymptotically free while the quartic coupling will blow up (very negative)
- In certain GUTs (only Non-abelian Gauge group involved), the UV fixed point at $A = 3$ in gauge sector is away from the pole in the quartic and Yukawa couplings allowing the existence of UV fixed points in all couplings.
- **Pati-Salam model has the potential to be asymptotically safe**

Classification of UV Fixed Point: Relevant & Irrelevant

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Vacuum stability condition

$$\lambda_{R1} + \lambda_{R2} > 0 \qquad \lambda_1 - \lambda_2 + \lambda_4 > 0, \qquad \lambda_1 > 0$$

λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.12	0.05	0	0.13	0.02	0	0.13	-0.01	0.78	0.78	0.84	0
Irev	Rev	0	Irev	Irev	0	Irev	Rev	Irev	Irev	Irev	0

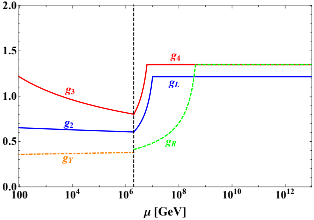
λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.05	0.02	0	0.01	0.04	0	0.02	0.08	0.24	0.24	0.57	0.74
Irev	Rev	0	Irev	Irev	0	Irev	Irev	Irev	Irev	Irev	Irev

表: These tables summarize the sample UV fixed point solution with two sample values ($N_{f_{2L}} = 40,\ N_{f_{2R}} = 150,\ N_{f_4} = 200;\ N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. The UV fixed point solutions of the couplings are classified with relevant (Rev) and irrelevant (Irev) characteristics. “0” denotes Gaussian Fixed points.

RG Flow: Gauge and Yukawa

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the gauge and Yukawa couplings by using the UV to IR approach.



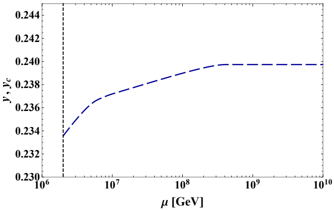


图: We have chosen $N_{f_{2L}} = 40$, $N_{f_{2R}} = 130$, $N_{f_4} = 130$. We have used the matching conditions at IR to set the initial conditions of g_L, g_R, g_4 at IR. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

RG Flow: Quartic Coupling of Bi-doublet Φ

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the Quartic Coupling by using the UV to IR approach.

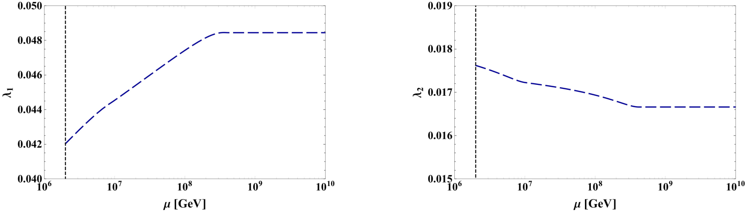
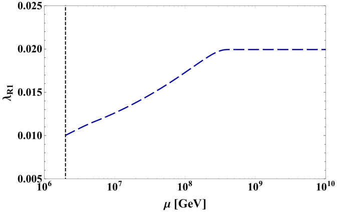


图: We have chosen $N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

RG Flow: Quartic Coupling of ϕ_R

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the Quartic Coupling by using the UV to IR approach.



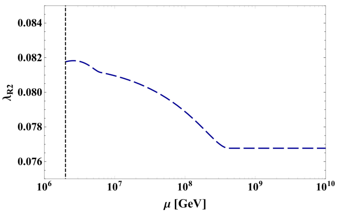


图: We have chosen $N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

Matching the Standard Model: Top Yukawa Coupling

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The top Yukawa mass term is given by (with CP symmetry $y = y_c$ and $\tan\beta = 1$):

$$\mathcal{L}^{\psi}_{\text{Yuk}} = y\,\text{Tr}\left[\overline{\psi_L}\,\Phi\,\psi_R\right] + y_c\,\text{Tr}\left[\overline{\psi_L}\,\Phi^c\,\psi_R\right] + \text{h.c.}$$
$$m_{\text{top}} = (y\sin\beta + y_c\cos\beta)v \to \sqrt{2}yv = m_{\text{top}}$$

- Thus at electroweak scale, y is smaller than the conventional SM top Yukawa coupling value $\sim \frac{0.93}{\sqrt{2}} \sim 0.66$
- It can be shown that by choosing $N_{f_{2L}} = 32,\ N_{f_{2R}} = 108,\ N_{f_4} = 56$, we obtain $y \sim 0.614$ as required.

Matching the Standard Model: Higgs Mass

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The low energy scalar sector of the Pati-Salam model is the two Higgs doublet model

$$
\begin{aligned}
V_H = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \\
& + \frac{1}{2}\bar{\lambda}_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\bar{\lambda}_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \bar{\lambda}_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) \\
& + \bar{\lambda}_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left[\frac{1}{2}\bar{\lambda}_5 \left(\Phi_1^\dagger \Phi_2\right)^2 \right. \\
& \left. + \bar{\lambda}_6 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_1^\dagger \Phi_2\right) + \bar{\lambda}_7 \left(\Phi_2^\dagger \Phi_2\right)\left(\Phi_1^\dagger \Phi_2\right) + \text{H.c.} \right]
\end{aligned}
$$

- The matching conditions (by comparing with the UV bi-doublet scalar potential) are (at the Pati-Salam symmetry breaking scale):

$$
\begin{aligned}
& \bar{\lambda}_1 = \lambda_1, \quad \bar{\lambda}_2 = \lambda_1, \quad \bar{\lambda}_3 = 2\lambda_1, \quad \bar{\lambda}_4 = 4\left(-2\lambda_2 + \lambda_4\right) \\
& \bar{\lambda}_5 = 4\lambda_2, \quad \bar{\lambda}_6 = -\lambda_3, \quad \bar{\lambda}_7 = \lambda_3 \,.
\end{aligned}
$$

Matching the Standard Model: Higgs Mass

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The mass matrix (neutral scalar fields) of the two Higgs doublet model is given by:

$$M^2_{\text{neutral}} = \begin{bmatrix} \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_1 v_1^2 & -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 \\ -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 & \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_2 v_2^2 \end{bmatrix}$$

- This matrix is defined at the electroweak scale. By using the two Higgs doublet beta functions and the matching conditions, we obtain the quartic couplings $\lambda_i \ (i = 1, \cdots 5)$ at the electroweak scale.
- The phenomenological constraint are: both of the eigenvalues of the mass matrix should be positive and the lighter one should close to the $125\,\text{GeV}$ Higgs mass.

Matching the Standard Model

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- It can be shown that by choosing $N_{F2} = 32,\ N_{F3} = 108,\ N_{F4} = 56$, we obtain:

$$\bar{\lambda}_1 = 0.135,\ \bar{\lambda}_2 = 0.360,\ \bar{\lambda}_3 = 0.25,\ \bar{\lambda}_4 = -0.379,\ \bar{\lambda}_5 = 0.259,\ y = 0.614\,.$$

- Matching of the scalar quartic coupling: two neutral scalar mass with $M_{H1} \sim 125\,\mathrm{GeV}$ (lighter Higgs) and the heavier one $M_{H2} > 238\,\mathrm{GeV}$ with $m_{12} > 150\,\mathrm{GeV}$
- Matching of the top Yukawa coupling: the IR value of y is around 0.66 as required
- Asymptotic Safe Pati-Salam model can roughly match the SM at IR.
- In this minimal model, most of the RG flows lead to much lighter Higgs mass and Pati-Salam symmetry breaking scale above $10000\,\mathrm{TeV}$ is required.

Safety of Grand Unified Theory: Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- Trinification model is under gauge symmetry group G_{TR} (note: without Z_3)
  K. S. Babu, X. G. He and S. Pakvasa, Phys. Rev. D **33** (1986) 763.

$$G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$$

- The coloured fermions are given by $\psi_{Q_L} \sim (3, \bar{3}, 1)$ & $\psi_{Q_R} \sim (3, 1, \bar{3})$:

$$\psi_{Q_L} = \begin{pmatrix} u_L^1 & u_L^2 & u_L^3 \\ \mathscr{D}_L^1 & \mathscr{D}_L^2 & \mathscr{D}_L^3 \\ \mathscr{D}_L'^1 & \mathscr{D}_L'^2 & \mathscr{D}_L'^3 \end{pmatrix}, \; \psi_{Q_R} = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ \mathscr{D}_R^1 & \mathscr{D}_R^2 & \mathscr{D}_R^3 \\ \mathscr{D}_R'^1 & \mathscr{D}_R'^2 & \mathscr{D}_R'^3 \end{pmatrix},$$

  (note: instead of $\psi_{Q_L}^c \sim (\bar{3}, 1, 3)$ we use $\psi_{Q_R} \sim (3, 1, \bar{3})$ since no attempt to unify three gauge group)

- The lepton content in this minimal Trinification model is given by:

$$\psi_E = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ \bar{e}_R & \bar{\nu}_R & \nu' \end{pmatrix} \sim (1, 3, \bar{3}),$$

Vector-like Fermions Charges & Large N_F Gauge Beta

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- We consider three sets of vector-like fermions charged under $G_{\rm TR}$, with the charge assignment:

$$N_{F_C}\,(3,1,1)\oplus N_{F_L}\,(1,3,1)\oplus N_{F_R}\,(1,1,3)\ ,$$

  where the charge assignments are chosen to avoid the extra contributions in the summation of semi-simple group
- For simplicity, all new vector-like fermions are introduced at the Trinification symmetry breaking scale assumed at few TeV scale.
- The large N_F gauge beta functions are given by:

$$\beta^{tot}_{\alpha_L}=\beta^{1loop}_{\alpha_L}+\beta^{\rm ho}_{\alpha_L}=-\left(10+n_H\right)\alpha_L^2+\frac{2A_L\alpha_L}{3}\frac{H_{1_L}\left(A_L\right)}{N_{F_L}}$$

$$\beta^{tot}_{\alpha_R}=\beta^{1loop}_{\alpha_R}+\beta^{\rm ho}_{\alpha_R}=-\left(10+n_H\right)\alpha_R^2+\frac{2A_R\alpha_R}{3}\frac{H_{1_R}\left(A_R\right)}{N_{F_R}}$$

$$\beta^{tot}_{\alpha_c}=\beta^{1loop}_{\alpha_c}+\beta^{\rm ho}_{\alpha_c}=-10\alpha_c^2+\frac{2A_c\alpha_c}{3}\frac{H_{1_c}\left(A_c\right)}{N_{F_C}}\ ,$$

  n_H denotes the number of scalar triplets ($n_H=2$ in our case)

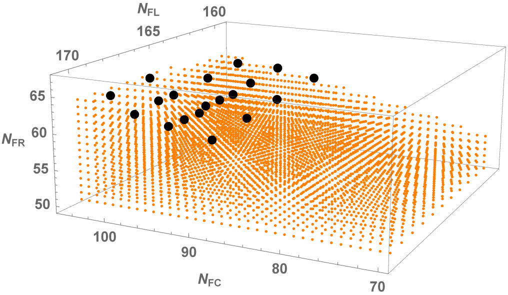
The Matching of the Standard Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- By choosing $N_{FC} = 95,\ N_{FL} = 165,\ N_{FR} = 62$, we obtain:

$$M^{\rm Pre}_{\rm Higgs} = 125\,{\rm GeV} \quad {\rm y}^{\rm Pre}_{\rm top} = 0.806, \quad {\rm y}^{\rm Pre}_{\rm bottom} = 0.019, \quad {\rm y}^{\rm Pre}_{\rm tau} = 0.011$$
$$M^{\rm SM}_{\rm Higgs} = 126\,{\rm GeV} \quad {\rm y}^{\rm SM}_{\rm top} = 0.780, \quad {\rm y}^{\rm SM}_{\rm bottom} = 0.019, \quad {\rm y}^{\rm SM}_{\rm tau} = 0.008\,.$$

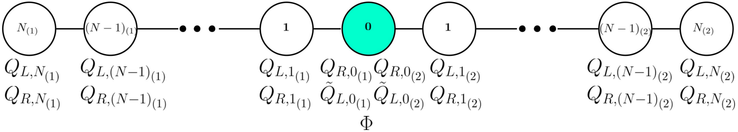
- 3D scan of the parameter space



Safe Clockwork: Clockwork Gears Setting

(Sannino, Smirnov and ZW. Wang, Phys. Rev. D **100** (2019) 075009, arXiv:1902.05958.

- We combine the clockwork mechanism with the Safe Pati-Salam model



- **Basic Idea**: A fundamental interaction (third generation Yukawa) is confined only at the zero node. This interaction will be diluted by each chain providing effective Yukawa couplings for 1st and 2nd generations.
- The large number of vector-like fermions play the role of clockwork gears
- We introduce $N_{(i)}$, $(i = 1, 2)$ pair of vector like fermions $\left(Q_{L,1_{(i)}}, Q_{R,1_{(i)}}\right), \cdots, \left(Q_{L,N_{(i)}}, Q_{R,N_{(i)}}\right)$ with one extra chiral fermion $Q_{R,0_{(i)}}$ (i.e. each generation (i) of the PS fermions is associated with a clockwork chain with $N_{(i)}$ nodes).
- For any number of $N_{(i)}$, the chiral fermions $Q_{L,N_{(i)}}$ and $Q_{R,N_{(i)}}$ are charged respectively under the fundamental representation $(4, 2, 1)$ and $(4, 1, 2)$ of PS gauge group $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$.

Safe Clockwork: Clockwork Chain Interaction

(Sannino, Smirnov and ZW. Wang, Phys. Rev. D **100** (2019) 075009, arXiv:1902.05958.

- We also introduce $\tilde{Q}_{L,0_{(1)}}$ and $\tilde{Q}_{L,0_{(2)}}$ which will only interact respectively with the zero node fields $Q_{R,0_{(1)}}$ and $Q_{R,0_{(2)}}$ through the Yukawa contributions i.e.

$$\mathcal{L}^{Q}_{\text{Yuk}} = y_1 \bar{\tilde{Q}}_{L,0_{(1)}} \Phi\, Q_{R,0_{(1)}} + y_2 \bar{\tilde{Q}}_{L,0_{(2)}} \Phi\, Q_{R,0_{(2)}} \,,$$

- The clockwork mechanism is realized by the following clockwork chain interaction:

$$\begin{aligned}
\mathcal{L}^{Q_R}_{\text{clock}} &= -m_{(1)} \sum_{j=1}^{N_{(1)}} \left(\bar{Q}_{L,j_{(1)}} Q_{R,j_{(1)}} - q_{(1)} \bar{Q}_{L,j_{(1)}} Q_{R,j-1_{(1)}} \right) \\
&\quad - m_{(2)} \sum_{j=1}^{N_{(2)}} \left(\bar{Q}_{L,j_{(2)}} Q_{R,j_{(2)}} - q_{(2)} \bar{Q}_{L,j_{(2)}} Q_{R,j-1_{(2)}} \right) \,,
\end{aligned}$$

- For simplicity, we set $q_{(1)} = q_{(2)} = q$ and $m_{(1)} = m_{(2)} = m$ (i.e. all the clockwork vectorlike fermions are introduced at one scale). Actually, when turning on the difference between q and m, we will have more freedom and bigger parameter space to explore.

Safe Clockwork: Massless Mode

(Sannino, Smirnov and ZW. Wang, Phys. Rev. D **100** (2019) 075009, arXiv:1902.05958.

- After diagonalizing the mass matrix, we obtain $M_{Q_{(i)}} = diag\left(0, M_{1_{(i)}}, \cdots, M_{N_{(i)}}\right), \; (i=1,2)$ where there is always one massless mode $\psi_{R,0_{(i)}}, \; (i=1,2)$.
- The massless modes (eigenstate) $\psi_{R,0_{(i)}}, \; (i=1,2)$ overlaps with the fields at the zero node of the chain (non-eigenstate) with a suppression factor i.e. $\psi_{R,0_{(i)}} = 1/q^{N_{(i)}} Q_{R,0_{(i)}}$.
- The Yukawa coupling of the $i-$th generations of the PS fermions which originates from the Yukawa interaction terms between $\tilde{Q}_{L,0_{(i)}}$ and the massless mode $\psi_{R,0_{(i)}}$ will also be suppressed by $1/q^{N_{(i)}}$ leading to:

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}}^{\text{eff}} &= y_1^{\text{eff}} \bar{\tilde{Q}}_{L,0_{(1)}} \Phi \, \psi_{R,0_{(1)}} + y_2^{\text{eff}} \bar{\tilde{Q}}_{L,0_{(2)}} \Phi \, \psi_{R,0_{(2)}} \\
&= \frac{y_1}{q^{N_{(1)}}} \bar{\tilde{Q}}_{L,0_{(1)}} \Phi \, \psi_{R,0_{(1)}} + \frac{y_2}{q^{N_{(2)}}} \bar{\tilde{Q}}_{L,0_{(2)}} \Phi \, \psi_{R,0_{(2)}}
\end{aligned}$$

Safe Clockwork

(Sannino, Smirnov and ZW. Wang, Phys. Rev. D **100** (2019) 075009, arXiv:1902.05958.

- The clockwork vector-like fermions are charged under G_{PS} with the following charge assignment:

$$N_F\,(4,1,2) \oplus N_F\,(4,2,1)\ , \quad N_F = N_{(1)} + N_{(2)}\,.$$

- We searched the full parameter space in the range of $N_F \in (10,\,200)$ and find for $N_F = 13$ we can match both the Higgs mass and the top Yukawa coupling at the electroweak scale.
- The relations among $q^{N_{(1)}}$, $q^{N_{(2)}}$ and the light quark masses are

$$q^{N_{(1)}} = \frac{m_{top}}{m_u},\ q^{N_{(2)}} = \frac{m_{top}}{m_c},\ N_{(1)} + N_{(2)} = 13\,,$$

where $m_{top} = 173\,\mathrm{GeV}$, $m_c = 1.29\,\mathrm{GeV}$ and $m_u = 2.3\,\mathrm{MeV}$.

- By solving above Eq., we find

$$N_{(1)} = 9, \quad N_{(2)} = 4, \quad q = 3.46\,.$$

Conclusions So far

- Large N_f resummation offers a possibility to realize UV completion (asymptotic safety) of the Standard Model via a non-trivial UV fixed point and GUT embedding
- Large N_f procedure provides a framework which faciliates the UV completion of various kinds of BSM model building such as GUTs and composite Higgs
- Higher order calculations and in particular, lattice calculations are required to solidfy/confirm the UV fixed point (UV conformal phase)
- Renormalization group flow as a bridge connects UV (boundary condition) and IR physics
- UV completion (asymptotic safety) as a guiding principle provides strong constraints on the RG flows and thus also IR BSM physics

Further Study

- Testing asymptotic safety using gravitational Wave
- From large N_f to large charge Q using CFT
- Alternative way to realize asymptotic safety via extra dimension

Gravitational Waves from Pati-Salam Dynamics

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The detection of stochastic gravitational wave generated through strong 1st order phase transition can help to explore the high energy physics beyond Collider.
- Pati-Salam model is particularly interesting because strong first order phase transition at few $1000\,\mathrm{TeV}$ scale will typically generate gravitational wave with peak frequency at $10-100\,\mathrm{Hz}$ in the detection region of aLIGO.
- We study the stochastic gravitational wave signatures from both safe and non-safe Pati-Salam model. For safe scenario, we introduce $N_f : (4,1,2) \oplus (4,2,1)$ and use large N_f resummation to realize safety.
- Safe scenario has strong predictive power providing a much smaller parameter space which we use as seed values to explore the full parameter space beyond safety.

Relevant Scalar Sector of Pati-Salam Model

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- In order to induce the breaking of $G_{\rm PS}$ to the SM gauge group, we introduce a scalar field ϕ_R which transforms as the fermion multiplet ψ_R, that is, $\phi_R \sim (4,1,2)$:

$$\phi_R \; = \left(\begin{array}{cccc} \phi_R^{u_1} & \phi_R^{u_2} & \phi_R^{u_3} & \phi_R^{0} \\ \phi_R^{d_1} & \phi_R^{d_2} & \phi_R^{d_3} & \phi_R^{-} \end{array} \right) ,$$

  where the neutral component ϕ_R^0 takes a non-zero vev, $\langle \phi_R^0 \rangle \equiv v_R$, such that $G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

- The relevant terms in the tree level effective potential can be written as:

$$V_{\rm tree}\left(\phi_R\right) = \lambda_{R1} {\rm Tr}^2 \left(\phi_R^\dagger \phi_R\right) + \lambda_{R2}\, {\rm Tr} \left(\phi_R^\dagger \phi_R \phi_R^\dagger \phi_R\right) .$$

- Note that we do not include any explicit mass terms in the tree level potential. The symmetry breaking in this work is induced by Coleman-Weinberg mechanism.

Strong First Oder Phase Transition

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The total finite temperature effective potential of the Pati-Salam model:

$$V_{\rm eff}\left[\phi, T\right] = V_{\rm tree} + V_{\rm 1loop} + V_T^{\rm tot} + V_{\rm ring}^{\rm scalar,tot} + V_{\rm ring}^{\rm gauge,tot} \, .$$

- A positive non-trivial (away from the origin) minimum occurs for $\phi_{Rc} \sim 8400\,\mathrm{TeV}$ and thus $\phi_{Rc}/T_c \sim 3.13 > 1$. This shows that the associated phase transition is a **strong first order** one.

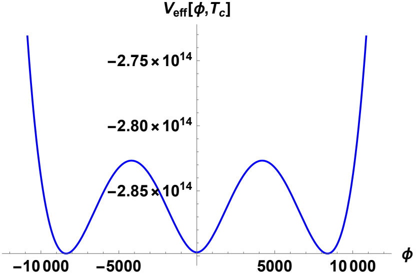


图: We plot the finite temperature effective potential. The renormalization scale μ is set at $5000\,\mathrm{TeV}$ while the temperature is chosen at $T = T_c = 2680\,\mathrm{TeV}$ which is the critical temperature.

Using Stream Plot to Locate the Parameter Space

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The parameter space to have strong first order phase transition is located at the right lower corner!

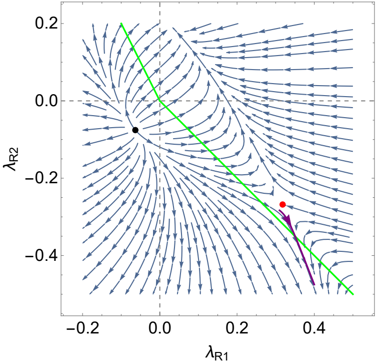
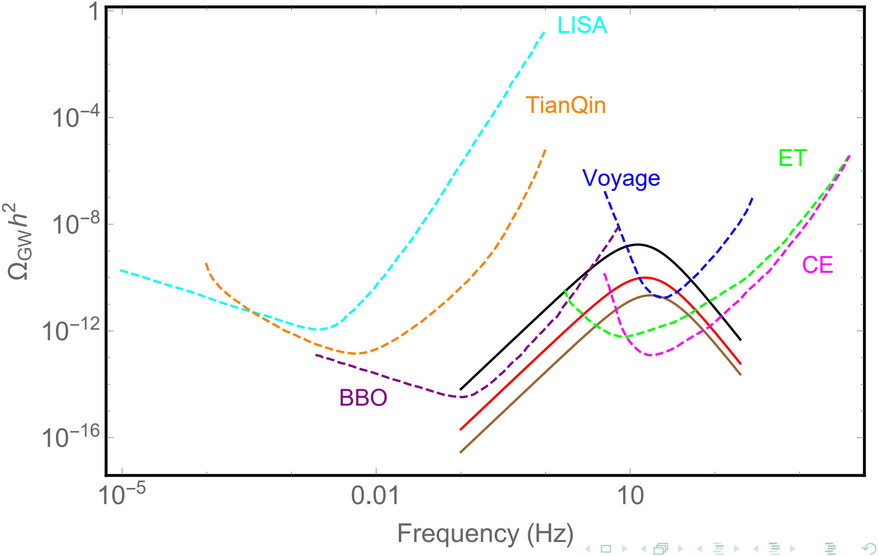


图: The flow direction is defined from UV to IR. The red and black plots are both the fixed point. The two green lines are the symmetry breaking lines which are defined as $\lambda_{R1} + \lambda_{R2} = 0$ for $\lambda_{R2} < 0$ and $\lambda_{R2}/2 + \lambda_{R1} = 0$ for $\lambda_{R2} > 0$. The purple line is the particular RG flow corresponding to the safe solution.

Gravitational Wave Signals and Bounds

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)



Question

From large N_f to large Q?

Comparing Different Large Parameter Expansion

- Large N_c: Planar limit: t'Hooft coupling $A_c \equiv g^2 N_c$ is fixed
- Large N_f: Bubble diagrams: t'Hooft coupling $A_f \equiv g^2 N_f$ is fixed
- Large Q: t'Hooft coupling $A_Q \equiv \lambda Q$ is fixed
- We have:

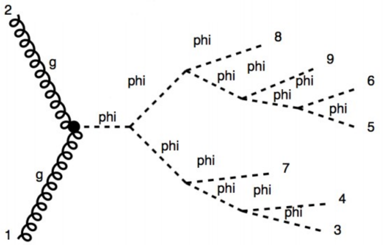
$$\text{Observable} \sim \sum_{l=\text{loops}} g^l P_l\left(N\right) = \sum_k \frac{1}{N_i^k} F_k\left(\mathcal{A}_i\right)$$

where $N = \{N_c,\, N_f,\, Q\}$ and $\mathcal{A} = \{A_c,\, A_f,\, A_Q\}$.

Multiparticle Production Problem: Higgs Explosion

(Khoze, “Higgs Explosion", indico.cern.ch/event/677640/contributions/2938636)

- It was proposed that multiparitlce production processes are problematic: higgs explosion and instanton-like processes in baryogenesis.
  (V. A. Rubakov, “Nonperturbative aspects of multiparticle production,” hep-ph/9511236.)
- In the process of $A_{gg\to n\times h} = \sum_{\text{polygons}} A^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\cdots+n_k=n} \Pi^k_{i=1} A_{h^*_i\to n_i\times h}$, perturbation theory fails when around 130 Higgses are produced at $O(100\,\text{TeV})$.



Multiparticle Production Problem: Higgs Explosion

(V. A. Rubakov, “Nonperturbative aspects of multiparticle production,” hep-ph/9511236.)

- The “exact" result for the tree amplitude at threshold $(E = nm)$ is

$$A_{1\to n}^{\text{tree}} = n! \left(\frac{\lambda}{8m^2} \right)^{\frac{n-1}{2}}$$

- For multiparticle production of $\lambda\phi^4$ theory (mimic Higgs explosion), the amplitude at one loop level:

$$A_{1\to n}^{\text{tree}} + A_{1\to n}^{\text{one loop}} = A_{1\to n}^{\text{tree}} \left(1 + B\lambda n^2\right) \ , \quad A_{1\to n}^{\text{tree}} = n! \left(\frac{\lambda}{8m^2} \right)^{\frac{n-1}{2}}$$

- Two folds problems:
  - The factorial behavior of the tree amplitudes indicates that the cross section also increase with n and at $n \sim 1/\lambda$, the cross section will exceed the unitarity limit at sufficent large n

$$\sigma_{1\to n}^{\text{tree}} \sim \frac{1}{n!} \left|A_{1\to n}^{\text{tree}}\right|^2 \times (\text{phase space}) \sim n! \lambda^n \epsilon^n \qquad \epsilon = \frac{E - nm}{n}$$

  - Loop corrections fails and conventional perturbation theory fails!

Towards the Holy Grail Summation Function $F(\lambda n, \epsilon)$

(V. A. Rubakov, “Nonperturbative aspects of multiparticle production,” hep-ph/9511236.)

- Rubakov’s insight: $\sigma_{1\to n}(E) \propto \exp\left[nF(\lambda n, \epsilon)\right]$
- However, only a few terms in the expansion of $F(\lambda n, \epsilon)$ at small λn and ϵ are known:

$$F(\lambda n, \epsilon) = \ln\frac{\lambda n}{16} + \frac{1}{2} + \frac{3}{2}\ln\frac{\epsilon}{3\pi} - \frac{17}{12}\epsilon + B\lambda n + \cdots$$

- Using Large charge method, we can instead calculate LO and NLO scaling dimensions of fixed charge operator $[\phi^n]$ in $U(1)$ symmetric $\lambda\left(\bar{\phi}\phi\right)^2$ theory. (G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

$$Z^2_{\phi^n}\lambda_0^n\langle\left[\bar{\phi}^n\right](x_f)\left[\phi^n\right](x_i)\rangle$$
$$=\lambda_0^n n!\exp\left[\frac{1}{\lambda_0}\Gamma_{-1}(\lambda_0 n, x_{fi}) + \Gamma_0(\lambda_0 n, x_{fi}) + \Gamma_1(\lambda_0 n, x_{fi}) + \cdots\right]$$

Weyl Map and State-Operator Correspondence

- At the (Wilson-Fisher) fixed point, we can exploit the power of conformal invariance.
- Perform a Weyl map from the plane to the cylinder $\mathbb{R}^d \to \mathbb{R} \times S^{d-1}$ where dilatons (scaling dimensions $\Delta_{\mathcal{O}}$)on the plane are mapped to time translations (energy spectrum) on the cylinder.
- In coordinates i.e. $(r, \Omega_{d-1}) \to (\tau, \Omega_{d-1})$ using $r = Re^{\tau/R}$. The cylinder metric is related to the flat one by a Weyl re-scaling:

$$ds^2_{\text{cyl}} = d\tau^2 + R^2 d\Omega^2_{d-1} = \frac{R^2}{r^2} ds^2_{\text{flat}}$$

- The two-point function of a scalar primary operator $\mathcal{O}$ on the cylinder is related to the flat space one by:

$$\langle \mathcal{O}^\dagger (x_f) \mathcal{O}(x_i) \rangle_{\text{cyl}} = |x_f|^{\Delta_{\mathcal{O}}} |x_i|^{\Delta_{\mathcal{O}}} \langle \mathcal{O}^\dagger (x_f) \mathcal{O}(x_i) \rangle_{\text{flat}} \equiv \frac{|x_f|^{\Delta_{\mathcal{O}}} |x_i|^{\Delta_{\mathcal{O}}}}{|x_f - x_i|^{2\Delta_{\mathcal{O}}}}$$

- In the limit $x_i \to 0$ on the plane translates to $\tau_i \to -\infty$ on the cylinder and obtain:

$$\langle \mathcal{O}^\dagger (x_f) \mathcal{O}(x_i) \rangle_{\text{cyl}} \overset{\tau_i \to -\infty}{=} e^{-E_{\mathcal{O}}(\tau_f - \tau_i)}, \qquad E_{\mathcal{O}} = \Delta_{\mathcal{O}}/R$$

Leading Order Scaling Dimension Δ_{-1}: $U(1)$ Example

(G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

- We compute the expectation of the evolution operator e^{-HT} in an arbitrary state $|\psi_n\rangle$ with fixed charge n. In the limit $T \to \infty$, the expectation gets saturated by the **lowest energy state**.

$$\langle \psi_n | e^{-HT} | \psi_n \rangle \overset{T \to \infty}{=} \tilde{\mathcal{N}} e^{-E_{\phi^n} T}$$

- It corresponds to the lowest lying operator i.e. operator with minimal classical scaling dimension with the give fixed charge (ϕ^n in $U(1)$).
- Introduce polar coordinates for the field $\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}$, we have:

$$\langle \psi_n | e^{-HT} | \psi_n \rangle = \mathcal{Z}^{-1} \int_{\rho=f}^{\rho=f} \mathcal{D}\rho \mathcal{D}\chi e^{-S_{eff}}$$

$$S_{eff} = \int d\tau \int d\Omega \left[\frac{1}{2} (\partial\rho)^2 + \frac{1}{2} \rho^2 (\partial\chi)^2 + \frac{m^2}{2} \rho^2 + \frac{\lambda_0}{16} \rho^4 + i \frac{n}{R^{d-1}\Omega} \dot{\chi} \right]$$

- Here, $i \frac{n}{R^{d-1}\Omega} \dot{\chi}$ is the boundary term which fixes the value of the total charge and can not be dropped while $m^2 = \left(\frac{d-2}{2R}\right)^2$ arises from the $\mathcal{R}(g)\bar{\phi}\phi$ coupling to the Ricci scalar enforced by conformal invariance.

Leading Order Scaling Dimension Δ_{-1}: $U(1)$ Example

(G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

- The variation of the S_{eff} provides the E.O.M. and total charge constraint:

$$-\partial^2\rho+\left[(\partial\chi)^2+m^2\right]\rho+\frac{\lambda_0}{4}\rho^3=0,\ i\partial_\mu\left(\rho^2 g^{\mu\nu}\partial_\nu\chi\right)=0;\ i\rho^2\dot{\chi}=\frac{n}{R^{d-1}\Omega_{d-1}}$$

- The stationary configuration (ground state) is spatially homogeneous given by:

$$\rho=f,\qquad \chi=-i\mu\tau+\text{const}.$$

- Using E.O.M. and the total charge constraint, we obtain the constraints of the VEV f and chemical potential μ as

$$\left(\mu^2-m^2\right)=\frac{\lambda_0}{4}f^2,\qquad \mu f^2 R^{d-1}\Omega_{d-1}=n$$

- The effective action S_{eff} evaluated on the stationary configuration provides the leading order value for the energy and thus Δ_{-1}. We set $\lambda_0=\lambda_*$ at the fixed point and $d=4$ to obtain:

$$\frac{1}{\lambda_*}\Delta_{-1}=\frac{S_{eff}}{T}\bigg|_{\lambda_0=\lambda_*}=\frac{n}{2}\left(\frac{3}{2}\mu+\frac{1}{2}\frac{m^2}{\mu}\right)\bigg|_{\lambda_0=\lambda_*}$$

The Results of Δ_{-1}: $U(1)$ Example

(G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

- We obtain the leading order result with a closed form

$$\frac{\Delta_{-1}}{\mathcal{A}^*} = \frac{1}{4} F(x)\,, \qquad F(x) \equiv \frac{3^{\frac{2}{3}} x^{\frac{1}{3}}}{3^{\frac{1}{3}} + x^{\frac{2}{3}}} + \frac{3^{\frac{1}{3}}\left(3^{\frac{1}{3}} + x^{\frac{2}{3}}\right)}{x^{\frac{1}{3}}},$$

$$x = 9\frac{\mathcal{A}^*}{(4\pi)^2} + \sqrt{-3 + 81\frac{\mathcal{A}^{*2}}{(4\pi)^4}}\,, \qquad \mathcal{A}^* = \lambda^* n$$

- $F(x)$ is a real and positive function for $x > 0$.
- The closed form results can be expanded in two extreme regimes, $\lambda_* n \ll (4\pi)^2$ and $\lambda_* n \gg (4\pi)^2$

$$\frac{\Delta_{-1}}{\lambda^*} = \begin{cases} n\left[1 + \frac{1}{2}\left(\frac{\lambda^* n}{16\pi^2}\right) - \frac{1}{2}\left(\frac{\lambda^* n}{16\pi^2}\right)^2 + \mathcal{O}\left(\frac{(\lambda^* n)^3}{(4\pi)^6}\right)\right], & \lambda^* n \ll (4\pi)^2, \\ \frac{8\pi^2}{\lambda^*}\left[\frac{3}{4}\left(\frac{\lambda^* n}{8\pi^2}\right)^{4/3} + \frac{1}{2}\left(\frac{\lambda^* n}{8\pi^2}\right)^{2/3} + \mathcal{O}(1)\right], & \lambda^* n \gg (4\pi)^2. \end{cases}$$

The Next-to-Leading-Order Δ_0: $U(1)$ Example

(G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

- The NLO scaling dimension Δ_0 arises from the fluctuation determinant of the Gaussian integrals after we expand the fields around the saddle point configurations $\rho(x) = f + r(x)$, $\chi(x) = -i\mu\tau + \frac{1}{f\sqrt{2}}\pi(x)$.

$$\Delta_0(\lambda n) = \frac{R}{T}\log\frac{\sqrt{\det S^{(2)}}}{\det\left(-\partial_\tau^2 - \Delta_{s^{d-1}} + m^2\right)} = \frac{R}{2}\sum_{\ell=0}^{\infty} n_\ell \left[\sum_i g_i \omega_i(\ell)\right]$$

- Here, n_ℓ is the multiplicity of the Laplacian on the $(d-1)-$dim. sphere where $n_\ell = (1+\ell)^2$ for $d=4$ and ω_i's are the dispersion relations of the fluctuations and g_i is the multiplicity for each ω_i.
- In the small and large $\lambda_* n$ limit, we expand Δ_0 to obtain respectively:

$$\Delta_0 = -\frac{3\lambda_* n}{(4\pi)^2} + \frac{\lambda_*^2 n^2}{2(4\pi)^4} + \mathcal{O}\left(\frac{\lambda_*^3 n^3}{(4\pi)^6}\right), \quad \text{for } (\lambda_* n << 1)$$

$$\Delta_0 = \left[\alpha + \frac{5}{24}\log\left(\frac{\lambda_* n}{8\pi^2}\right)\right]\left(\frac{\lambda_* n}{8\pi^2}\right)^{4/3} + \left[\beta - \frac{5}{36}\log\left(\frac{\lambda_* n}{8\pi^2}\right)\right]\left(\frac{\lambda_* n}{8\pi^2}\right)^{2/3},$$

$$\text{for } (\lambda_* n >> 1) \quad \text{where } \alpha = -0.5753315(3), \quad \beta = -0.93715(9).$$

From Abelian to Non-abelian: the $O(N)$ Model

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, "Charging the $O(N)$ model," PRD 102 (2020) 045011.)

- In euclidean spacetime, the $O(N)$ theory is defined by the action

$$\mathcal{S} = \int d^d x \left(\frac{1}{2} \partial^\mu \phi_a \partial^\mu \phi_a + \frac{(4\pi)^2 g_0}{4!} (\phi_a \phi_a)^2 \right) \qquad a = 1, \ldots, N \ .$$

- The $O(N)$ model with even or odd N has rank $\frac{N}{2}$ or $\frac{N-1}{2}$ corresponding to the number of charges we can fix. Only total charge and VEV matter.

$$\mu^2 - m^2 = \frac{(4\pi)^2}{6} g_0 v_{tot}^2 \quad \text{(EOM)}; \qquad \frac{\bar{Q}}{\text{Vol.}} = \mu v_{tot}^2 \quad \text{(Noether Charge)}$$

$$v_{tot}^2 \equiv \sum_i v_i^2 \quad \text{(Sum of VEVs)}; \qquad \bar{Q} \equiv \sum_i Q_i \quad \text{(Sum of charges)}$$

- Charge configurations with the same total charge are equivalent
  - Consider a generic charge configuration for $O(N)$

$$\mathcal{Q} = (m_1, m_2, \cdots, m_{N/2})$$

  - The corresponding fixed-charge operator is

$$\mathcal{O} = \Pi_{i=1}^{i=N/2} (\varphi_i)^{m_i} \qquad \varphi_k \equiv \frac{1}{\sqrt{2}} (\phi_{2k-1} + i\phi_{2k})$$

The Results of $O(N)$ Model

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, "Charging the $O(N)$ model," PRD 102 (2020) 045011.)

- We calculate the LO and NLO in charge expansion (with small 't Hooft coupling) but to all order in couplings scaling dimensions of $\bar{Q}$-index traceless symmetric tensor operator $T_{\bar{Q}} \equiv T^{(\bar{Q})}_{i_1 \dots i_{\bar{Q}}}$ in the $O(N)$ model

$$\Delta_{T_{\bar{Q}}} = \bar{Q} + \left(-\frac{\bar{Q}}{2} + \frac{\bar{Q}(\bar{Q}-1)}{8+N}\right)\epsilon - \left[\frac{184+N(14-3N)}{4(8+N)^3}\bar{Q} + \frac{(N-22)(N+6)}{2(8+N)^3}\bar{Q}^2 + \frac{2}{(8+N)^2}\bar{Q}^3\right]\epsilon^2 + \left[\frac{8}{(8+N)^3}\bar{Q}^4 + \frac{-456-64N+N^2+2(8+N)(14+N)\zeta(3)}{(8+N)^4}\bar{Q}^3 - \frac{-31136-8272N-276N^2+56N^3+N^4+24(N+6)(N+8)(N+26)\zeta(3)}{4(N+8)^5}\bar{Q}^2 + \frac{-65664-8064N+4912N^2+1116N^3+48N^4-N^5+64(N+8)(178+N(37+N))\zeta(3)}{16(N+8)^5}\bar{Q}\right]\epsilon^3 + \mathcal{O}\left(\epsilon^4\right) .$$

- At each ϵ order, the semi-classical computation provides term with leading Q and next leading Q shown in red.

Towards the Standard Model

Antipin, Bersini, Panopoulos, Sannino, Wang, “Infinite order results for charged sectors of the SM,” JHEP 2024.

- We calculate the LO and NLO in charge expansion (with small ’t Hooft coupling) but to all order in couplings scaling dimensions of the Higgs operator in the Standard Model written as $H^{I_1}...H^{I_Q}$

$$\Delta_Q = Q + \left\{\frac{1}{3}\lambda Q^2 + \left[N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l - \frac{3}{4}g'^2 - \frac{\lambda}{3}\right]Q\right\} - \left\{\frac{2}{9}\lambda^2 Q^3 - \left[2N\mathcal{Y}_{uu} + 2N\mathcal{Y}_{dd} + 2\mathcal{Y}_{ll} - \frac{2}{3}\lambda(N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l) - \frac{1}{3}\lambda g'^2 + \frac{g'^4}{16} + \frac{\lambda^2}{9}\right]Q^2 + C_{22}Q\right\} + \left\{\frac{8}{27}\lambda^3 Q^4 + \left[\frac{1}{16}g'^6(9\zeta(3)-1) - \frac{1}{6}g'^4\lambda(1+3\zeta(3)) + \frac{1}{3}g'^2\lambda^2(3-2\zeta(3)) + \frac{4}{27}\lambda^3(9\zeta(3)-8) + \frac{4}{27}\left(3N\left(\lambda^2\mathcal{Y}_u - 3\lambda\mathcal{Y}_{uu} + 9\zeta(3)(\lambda\mathcal{Y}_{uu} - 2\mathcal{Y}_{uuu})\right) + 3N\left(\lambda^2\mathcal{Y}_d - 3\lambda\mathcal{Y}_{dd} + 9\zeta(3)(\lambda\mathcal{Y}_{dd} - 2\mathcal{Y}_{ddd})\right) + 3\left(\lambda^2\mathcal{Y}_l - 3\lambda\mathcal{Y}_{ll} + 9\zeta(3)(\lambda\mathcal{Y}_{ll} - 2\mathcal{Y}_{lll})\right)\right)\right]Q^3 + C_{23}Q^2 + C_{33}Q\right\} + \mathcal{O}\left(\kappa_I^4 Q^5\right). \tag{7.1}$$

where red, blue, and orange colors highlight the terms stemming from the small $\kappa_I Q$ expansion of Δ_{-1}, Δ_0^{fm} and Δ_0^{bos}.

- $Q = 1$ denotes the Higgs field scaling dimension and can be matched up to three loop order

Asymptotic Safety via Extra Dimension?

Asymptotic GUT (aGUT)

(Cacciapaglia, Deandrea, Pasechnik, Z.W.Wang, PLB **852** (2024) 138629)

- Asymptotic unification is NOT unification in the usual sense.
- Gauge couplings are never equal, but tend to the same UV fixed point.
- The RGE of the gauge couplings with contributions from the extra dimension can be written as:

$$2\pi \frac{d\alpha_i}{dt} = b_i^{\mathrm{SM}} \alpha_i^2 + (S(t) - 1)\, b_5 \alpha_i^2$$

- Here $S(t)$ encodes the sum of the KK contributions (the number of Kaluza-Klein levels that have been crossed) to the running:

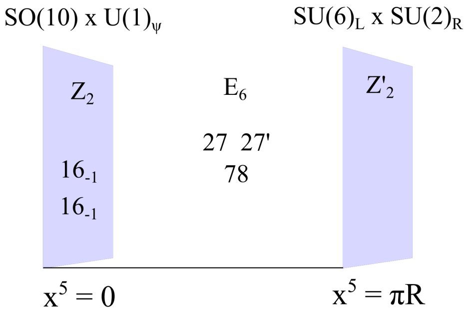
$$S(t) = \begin{cases} \mu R = m_Z R e^t & \text{for } \mu > 1/R\,, \\ 1 & \text{for } m_Z < \mu < 1/R\,. \end{cases}$$

- We define the 't Hooft coupling in 5D as $\tilde{\alpha} = \alpha S(t)$ and obtain (Gies, PRD 68 (2008)):

$$2\pi \frac{d\tilde{\alpha}_i}{dt} = 2\pi\tilde{\alpha}_i - b_5 \tilde{\alpha}_i^2, \quad \tilde{\alpha}_{UV} = \frac{2\pi}{b_5}, \quad b_5 = \frac{\pi}{2}\left(C(G) - \sum_i T_i(R_i) \right).$$

The E_6 Model

(Cacciapaglia, Deandrea, Pasechnik, Z.W.Wang, PLB **852** (2024) 138629)



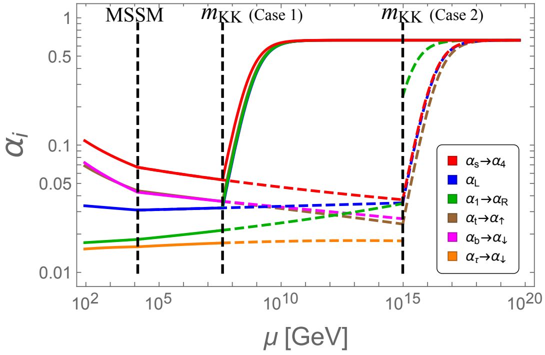
- We consider a supersymmetric E_6 gauge theory in 5D. The fifth dimension is compactified on the orbifold $S^1/Z_2 \times Z_2'$
- Orbifold projection breaks both $N = 2$ SUSY and the gauge symmetry

$$E_6 \to SO(10) \times U(1), \qquad E_6 \to SU(6)_L \times SU(6)_L \times SU(2)_R$$

- Left-handed SM fermions and Higgs from the 27 ; right handed SM fermions from the adjoint; $27'$ give mass to unwanted states

Asymptotic Safety via Extra Dimension

(Cacciapaglia, Deandrea, Pasechnik, Z.W.Wang, PLB **852** (2024) 138629)



- The couplings are the usual 4D ones up to the scale m_{KK} (assumed PS scale $\sim m_{KK}$), above which they are replaced by the corresponding 5D 't Hooft couplings
- The gauge and Yukawa couplings do unify to a single value thanks to the UV fixed point

Acknowledgements

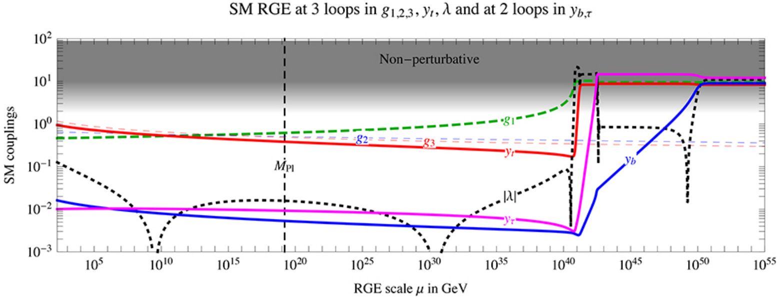
Thank You

Important Concepts

- $U(1)$ Landau Pole Problem
- Asymptotically free vs asymptotically safe
- Large N_f Expansion
- from large N_f to large Q

The Standard Model Running Couplings

- Field: Gauge fields + Fermions + Scalars
- Interactions: Gauge ($SU(3) \times SU(2)_L \times U(1)$) + Yukawa (Fermions Mass) + Scalar self-interaction
- Not UV Complete: the theory is not well defined at very high energy scale
- $U(1)$ gauge coupling runs into Landau Pole



G. M.

Pelaggi, F. Sannino, A. Strumia and E. Vigiani, Front. in Phys. **5** (2017) 49

Fundamental Theory

- A fundamental theory has an UV fixed point (K. G. Wilson, Phys. Rev. B **4** (1971) 3174.)
- Couplings stop running with the energy scale at the fixed point
- The Standard Model is not a fundamental theory since it runs into Landau Pole at UV due to the abelian $U(1)$ gauge group
- Asymptotically Free: non-interacting (Gaussian) fixed point (D. J. Gross and F. Wilczek, Phys. Rev. D **8** (1973) 3633; D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.)
  - non-interacting in the UV
  - coupling runs with logarithmic scale dependence
  - Perturbation theory in UV
- Asymptotically Safe: Interacting fixed point (S. Weinberg(1979). "Ultraviolet divergences in quantum theories of gravitation".)
  - interacting in the UV
  - coupling runs with power law scale dependence
  - Perturbative/Non perturbative theory in UV
  - Smaller critical surface dimension $\Rightarrow$ more IR predictiveness

Two Ways to Address $U(1)$ Landau Pole Problem

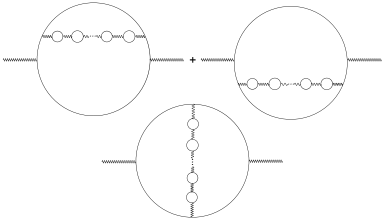
- Assuming $U(1)$ group is not a fundamental group and should be embedded in a non-abelian group at high energy scale (asymptotically free/safe). Typical example is GUT embedding.
- $U(1)$ gauge coupling will run into an interacting fixed point. (asymptotically safe)$\Rightarrow$ Highly non-perturbative (without large N_f strategy requiring an extremely large Yukawa coupling)

Large N_f Expansion

- Pascual noticed that it is possible to sum up a subset of the diagrams and the resulting power series is so well behaved to provide a closed-form expression at $1/N_f$ order
- The resummed U(1) beta function reads:

$$\beta_A = \frac{2A^2}{3}\left[1 + \frac{1}{N_f}F_1(A)\right], \qquad A \equiv 4N_f\alpha = 4N_f\frac{g_1^2}{(4\pi)^2}$$

$$F_1(A) = \frac{3}{4}\int_0^A \mathrm{d}x\,\tilde{F}\left(0, \tfrac{2}{3}x\right), \quad \tilde{F}(0,x) = \frac{(1-x)(1-\frac{x}{3})(1+\frac{x}{2})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}$$



The Attractive Features of Pati-Salam Model

J. C. Pati, Int. J. Mod. Phys. A **32** (2017) 1741013

- Unification of all 16 members of a family (SM matter content + Right Handed neutrino) within one left-right self-conjugate multiplet
- Quantization of electric charge i.e. $Q_{e^-} + Q_P = 0$
- Quark-Lepton unification through $SU(4)$ colour
- The right-handed neutrino as a compelling member of each family
- Universality for the weak interactions with respect to quarks and leptons
- conservation of parity at a fundamental level
- B-L as a local symmetry
- No proton decay issue
- Possible to be embedded in a simple group $SO(10)$ to address gauge coupling unification

Game of Asymptotic Safety (Non-Abelian): One Example

- Consider a special gauge-Yukawa system (Gauge: $SU(3)$ and $SU(2)$). It more or less mimic standard model gauge Yukawa sector.
- Positive and Negative (see also Bond, Hiller, Kowalska and Litim, JHEP **1708** (2017) 004)

$$
\begin{aligned}
SU(3): \quad & \beta_3 = -B_3\alpha_3^2 + \left(C_3\alpha_3 + G_3\alpha_2 - D_3\alpha_y\right)\alpha_3^2 \\
SU(2): \quad & \beta_2 = -B_2\alpha_2^2 + \left(C_2\alpha_2 + G_2\alpha_3 - D_2\alpha_y\right)\alpha_2^2 \\
Yukawa: \quad & \beta_y = \left(E\alpha_y - F_2\alpha_2 - F_3\alpha_3\right)\alpha_y
\end{aligned}
$$

- At two loop order gauge and one loop order Yukawa coupling, the Yukawa terms are the only negative terms in the gauge RG functions.
- The Yukawa terms occur at next leading order rather than leading order $\Rightarrow$ Non-perturbative Yukawa coupling is required to blance the positive contributions (with lowest dimension of representation) and highly non-perturbative to involve $U(1)$

Non-Perturbative Issue: Veneziaono Limit and Litim-Sannino Model

- In Litim-Sannino Model, the Veneziano limit is implemented to make the leading order terms as small as possible

  (D. F. Litim and F. Sannino, “Asymptotic safety guaranteed,” JHEP **1412** (2014) 178.)

- Define $\varepsilon = \frac{N_F}{N_c} - \frac{11}{2}$, the general leading order term of the gauge RG function is:

$$\beta_\alpha = -\frac{4}{3}\varepsilon\alpha^2 + O(\alpha^3)$$

- In the limit $N_F \to \infty$ and $N_c \to \infty$, ε could be as small as possible and the perturbative analysis is under control.
- Large N_c will make it difficult to connect to phenomenologies.
- $U(1)$ Landau pole problem is not addressed

Generalize to the Standard Model: $SU(3) \times SU(2) \times U(1)$

- Holdom's system only involves two kinds fields (gauge field+fermions) and one coupling (gauge coupling g)
- Safety can be realized in a more general gauge-Yukawa system: the Standard Model

  Mann, Meffe, Sannino, Steele, Z. W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802
- The standard Model can be safe at UV when including reasonably number of vector-like fermions.
- For $U(1)$, it requires $N_F > 16$ while for $SU(3)$, it requires $N_F > 32$ to suppress $1/N_f^2$ contributions.
- Use vector-like fermions for simplification without involving extra scalars to generate their mass terms

Bottom-Top Mass Splitting in Pati-Salam Model

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669

- To obtain the bottom-top mass splitting, we introduce new vector-like fermion $F \sim (10,1,1)$ with mass M_F and Yukawa interactions:

$$\mathcal{L}_{\text{Yuk}}^{F} = y_F \, \text{Tr}\left(\overline{F_L}\, \phi_R^T \, i\tau_2 \, \psi_R\right) + \text{h.c.} \quad F = \begin{pmatrix} S & B\sqrt{2} \\ B^T\sqrt{2} & E \end{pmatrix}$$

- The bottom quark Dirac mass term:

$$\mathcal{L}_{\text{mass}}^{b} = \begin{pmatrix} \overline{b_L} & \overline{B_L} \end{pmatrix} \begin{pmatrix} m_t & 0 \\ m_B & M_F \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} + \text{h.c.}\,,$$

- We obtain:

$$m_b = \sqrt{2}\, m_\tau \approx \frac{M_F\, m_t}{\sqrt{2}\, m_B} \quad m_B \equiv y_F\, v_R/\sqrt{2}$$

Top and Right-handed Neutrino Mass Spliting in Pati-Salam

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669

- In order to split the top and neutrino mass, we implement the inverse seesaw mechanism by adding a new chiral fermion singlet $N_L \sim (1,1,1)$, which has Yukawa interaction

$$\mathcal{L}_{\text{Yuk}}^{N} = -y_\nu \overline{N_L} \text{Tr}\left[\phi_R^\dagger \psi_R\right] + \text{h.c.}$$

- It generates a Dirac mass term $M_R \overline{N_L} \nu_R$, with $M_R \equiv y_\nu v_R \sim 10000\,\text{GeV}$.
- The Majorana mass term for the neutral fermion fields reads:

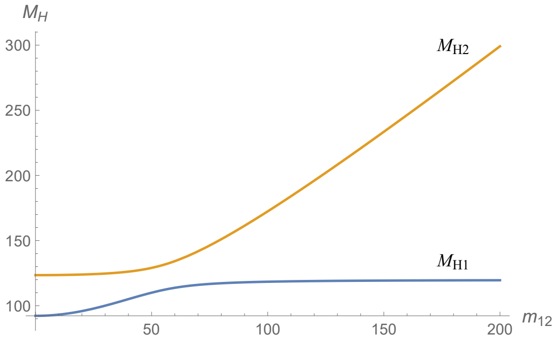
$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_R^c} & \overline{\nu_R} & \overline{N_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_t & 0 \\ m_t & 0 & M_R \\ 0 & M_R & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \\ N_L \end{pmatrix} + \text{h.c.}$$

- By introducing very light Majorana mass term for $-\frac{1}{2} M_N \overline{N_R^c} N_L$, we obtain one light active Majorana neutrino ν_τ with mass

$$m_{\nu_\tau} = M_N \frac{m_t^2}{m_D^2}; \quad m_D = \sqrt{m_t^2 + M_R^2}$$

Matching Higgs Mass

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669



Pole Structure PreCheck: Advantage of Pati-Salam

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The beta functions of Yukawa and Quartic couplings have the poles only at the Abelian pole where $K = \frac{15}{2}$
- When Abelian gauge coupling reaches a fixed point, the Yukawa coupling will be asymptotically free (negative pole contributions and multiplicative proportional to the Yukawa coupling itself).
- The quartic coupling will blow up (very negative) due to the negative pole contributions and not multiplicative proportional to the quartic coupling itself for the g^4 term.
- Higher order singular structure is involved in the quartic beta. When the gauge coupling logarithmically approaching the pole, the pole contribution in the quartic beta will blow up.
- In certain GUT (only Non-abelian Gauge group involved), the UV fixed point at $K = 3$ in gauge sector is away from the pole in the quartic and Yukawa couplings allowing the existence of UV fixed points in all couplings.
- **Pati-Salam model has the potential to be asymptotically safe**

The Symmetry Breaking of Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- To induce the breaking of $G_{\rm TR}$ to the SM gauge group, we introduce two scalar triplet fields Φ_1, Φ_2 which transform under the $G_{\rm TR}$ as $(1,3,3)$:

$$\Phi_a = \begin{pmatrix} \phi_1^a & \phi_2^a & \phi_3^a \\ S_1^a & S_2^a & S_3^a \end{pmatrix} , \qquad (a=1,2) ,$$

  where ϕ_i^a, $(i=1,2,3)$ denotes the Higgs doublets while S_i^a, $(i=1,2,3)$ denotes the singlets.
- The vacuum configuration of the scalar triplet is given as:

$$\langle\Phi_1\rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & v_1 \end{pmatrix} , \qquad \langle\Phi_2\rangle = \begin{pmatrix} n_1 & 0 & n_3 \\ 0 & n_2 & 0 \\ v_2 & 0 & v_3 \end{pmatrix} .$$

- G_{TR} to left-right model (through v_3, v_1 at few TeV):
  $SU(3)_C \times SU(3)_L \times SU(3)_R \to SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$
  left-right model breaking to the SM (through v_2 also at few TeV)
  u_1, u_2, n_1, n_2, n_3 further breaks the Standard Model

The Yukawa Sector of Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- The Yukawa terms for the quark sector is given by:

$$\mathcal{L}^{Q}_{\rm Yuk} = m_{d'}\overline{d}'_L d'_R + \sum_{a=1}^{2} y_{\psi_{Qa}}\Big[\left(-s_\alpha \overline{d}_R + c_\alpha \overline{d}'_R\right) Q\phi_1^a + \overline{u}_R Q\phi_2^a + \left(c_\alpha \overline{d}_R + s_\alpha \overline{d}'_R\right) Q\phi_3^a\Big] + \text{h.c.}$$

- The Yukawa terms $\mathcal{L}^{E}_{\rm Yuk}$ for the lepton sector is given by:

$$m_E \overline{E}E + \sum_{a=1}^{2} y_{\psi_{Ea}}\Big\{ -\left[\overline{E}\left(-c_\beta \overline{\nu}_R - s_\beta \nu'\right) - \left(c_\beta E + s_\beta L\right)\overline{e}_R\right]\phi_1^a + \left(E\nu' - L\overline{\nu}_R\right)\phi_2^a + \left[\overline{E}\left(s_\beta \overline{\nu}_R - c_\beta \nu'\right) - \left(-s_\beta E + c_\beta L\right)\overline{e}_R\right]\phi_3^a \Big\} + \text{h.c.}$$

- The quark and lepton masses are given by:

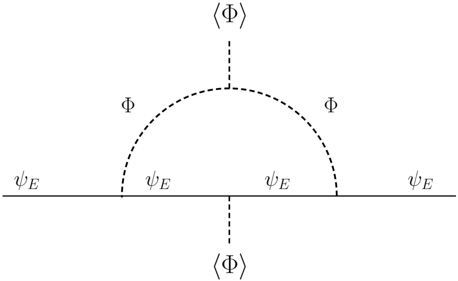
$$m_t = y_{\psi_{Q1}} u_2, \quad m_b = y_{\psi_{Q1}} u_1 s_\alpha\,, \quad m_e = y_{\psi_{E1}} u_1 s_\beta\,, \quad m_{\nu_L,\nu_R} = y_{\psi_{E1}} u_2\,.$$

- Here the right handed neutrino mass is the same order as electron and will be significantly changed by radiative loop corrections.

Safe Trinification: Radiative Neutrino Mass I

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- We show the one loop radiative correction to the neutrino mass below:



- The total contribution to the two point function is proportional to:

$$F_E = \frac{m_E}{(4\pi)^2}\frac{1}{2}\left(\frac{m_{H1}^2}{m_E^2 - m_{H1}^2}\log\frac{m_{H1}^2}{m_E^2} - \frac{m_{H2}^2}{m_E^2 - m_{H2}^2}\log\frac{m_{H2}^2}{m_E^2}\right)$$

Safe Trinification: Radiative Neutrino Mass II

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- The one-loop neutrino mass matrix is thus written as:

$$M_\nu^{\rm 1loop} = \left[\begin{array}{ccc} 0 & -y_{\psi_{E1}} u_2 & 0 \\ -y_{\psi_{E1}} u_2 & s_{\alpha-\beta} c_\beta y^2_{\psi_{E1}} F_E & (s_{2\beta} s_\alpha - c_\alpha)\, y^2_{\psi_{E1}} F_E \\ 0 & (s_{2\beta} s_\alpha - c_\alpha)\, y^2_{\psi_{E1}} F_E & c_{\alpha-\beta} s_\beta y^2_{\psi_{E1}} F_E \end{array} \right] .$$

- To obtain a phenomenological viable neutrino mass, the elements of the above matrix should have the following form:

$$M_\nu^{\rm 1loop} = \left[\begin{array}{ccc} 0 & 10\,{\rm GeV} & 0 \\ 10\,{\rm GeV} & 0 - 1\,{\rm TeV} & 0.33 - 1\,{\rm TeV} \\ 0 & 0.33 - 1\,{\rm TeV} & 1\,{\rm KeV} \end{array} \right] ,$$

  where the three mass eigenvalues will correspond to the physical mass of the two sterile neutrinos $(\nu_R,\ \nu')$ and the SM neutrino ν_L.

- There are only two solutions to satisfy the above constraint either $\beta << 1$ or $\alpha - \beta - \frac{\pi}{2} \sim \pm 10^{-9}$. For $\beta << 1$, it will lead to extremely large Yukawa $y_{\psi_{E1}}$ which is not acceptable. Thus, we choose:

$$\alpha - \beta - \frac{\pi}{2} \sim \pm 10^{-9} .$$

Question 3

Can we address the Yukawa Hierarchies in the Safe GUT?

Answer 3: Safe Clockwork

(Sannino, Smirnov and ZW. Wang, Phys. Rev. D **100** (2019) 075009, arXiv:1902.05958.

- We kill two birds with one stone!
- Ordinary clockwork theory requires large number of gauge charged extra fermions resulting in Landau pole at low energy scale.
- In safe scenario, the fixed point can not tell the difference between generations.
- We combine these two and reinterpret the extra vector-like fermions introduced in the safe scenario as clockwork gears.

Pati-Salam: Finite Temperature Effective Potential

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- One loop contribution without temperature:
  - $$V_{\rm 1loop} = \sum_i \pm n_i \frac{m_i^4}{64\pi^2}\left(\log\left[\frac{m_i^2}{\mu^2}\right] - C_i\right)$$
  - The total one-loop effective potential is:

$$V_{\rm 1loop} = V_{\rm Higgs} + V_{\rm Gold} + V_{\rm lepto} + V_{W_R^\pm} + V_{Z'} + V_\nu + V_F\,.$$

- Finite temperature effective potential:
  - The one loop finite temperature effective potential:

$$V_T = \sum_i \pm n_i \frac{T^4}{2\pi^2}\int_0^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+m_i^2/T^2}}\right]\,,$$

  - The total finite temperature effective potential (without ring contributions) as:

$$V_T^{\rm tot} = \frac{T^4}{2\pi^2}\left(I_B\left[\frac{M^2_{\rm Higgs1}}{T^2}\right] + 6I_B\left[\frac{M^2_{\rm Higgs2}}{T^2}\right] + 9I_B\left[\frac{M^2_{\rm Gold}}{T^2}\right] + 6I_B\left[\frac{M^2_{W_R^\pm}}{T^2}\right]\right.$$
$$\left. + 3I_B\left[\frac{M^2_{Z'}}{T^2}\right] + 18I_B\left[\frac{M^2_{\rm lepto}}{T^2}\right] + 4I_F\left[\frac{M^2_\nu}{T^2}\right] + 16I_F\left[\frac{M^2_F}{T^2}\right]\right)\,.$$

Effective Potential: Ring Contributions to the Scalar

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The general formula for the ring contributions (scalar in the big ring):

$$V^{i}_{\rm ring} = -\frac{T}{12\pi}\left(\left[m_i^2\left(\rho\right) + \sum_{\text{bosons } j} \pi_i^j\left(0\right)\right]^{3/2} - m_i^3\left(\rho\right)\right) ,$$

- The contributions of different species j in the outside ring of the daisy diagram i.e.

$$\pi^{j}_{\rm scalar}\left(0\right) = \frac{1}{12}\frac{m_j^2\left(v\right)}{v^2}T^2 .$$

- The total thermal mass contributions to the Higgs field $\sum_j \pi^j_{\rm scalar}\left(0\right)$:

$$= \pi^{\rm Higgs1}_{\rm scalar}\left(0\right)+6\pi^{\rm Higgs2}_{\rm scalar}\left(0\right)+9\pi^{\rm Gold}_{\rm scalar}\left(0\right)+18\pi^{\rm lepto}_{\rm scalar}\left(0\right)+6\pi^{W_R^{\pm}}_{\rm scalar}\left(0\right)+3\pi^{Z'}_{\rm scalar}\left(0\right)$$

- The total ring contributions to the scalar fields:

$$V^{\rm scalar,tot}_{\rm ring} = V^{\rm Higgs1}_{\rm ring} + 6V^{\rm Higgs2}_{\rm ring} + 9V^{\rm Gold}_{\rm ring} .$$

Effective Potential: Ring Contributions to the Gauge

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The ring contributions (gauge field in the big ring):

$$V_{\rm ring}^{\text{gauge,tot}} = -\frac{T}{12\pi}\,\text{Tr}\left(\left[\mathbf{M^2}(\rho) + \mathbf{\Pi}(\mathbf{0})\right]^{3/2} - \mathbf{M^3}(\rho)\right),$$

  where both $m_i^2(\rho)$ and $\sum_i \pi_i^j(0)$ are rewritten as matrices $\mathbf{M^2}(\rho)$ and $\mathbf{\Pi}(\mathbf{0})$ respectively since $\mathbf{M^2}(\rho)$ in the gauge field basis is not diagonalized.

- $\mathbf{\Pi}(\mathbf{0})$ is a diagonal matrix and it's eigenvalues are calculated through:

$$SU(N): \quad \pi_{\rm gauge}^{L,S} = \frac{g^2T^2}{3}\sum_S t_2(R_S),$$

$$\pi_{\rm gauge}^{L,F} = \frac{g^2T^2}{6}\sum_F t_2(R_F),$$

$$\pi_{\rm gauge}^{L,V} = \frac{N}{3}g^2T^2.$$

Bubble nucleation

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The tunnelling rate per unit volume $\Gamma(T)$ from the metastable (false) vacuum to the stable one is suppressed by the three dimensional Euclidean action $S_3(T)$:

$$\Gamma(T) = \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} T^4 e^{-S_3(T)/T},$$

where the Euclidean action has the form:

$$S_3(\rho, T) = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2}\left(\frac{d\rho}{dr}\right)^2 + V(\rho, T) - V(0, T)\right].$$

- The bubble configuration (instanton solution) is give by solving the equation of motion of the action (over shooting under shooting method):

$$\frac{d^2\rho}{dr^2} + \frac{2}{r}\frac{d\rho}{dr} - \frac{\partial F}{\partial \rho}(\rho, T) = 0,$$

with the boundary conditions:

$$\frac{d\rho}{dr}(0, T) = 0, \qquad \lim_{r\to\infty} \rho(r, T) = 0.$$

First Oder Phase Transition and Coleman Weinberg

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- Question: Is asymptotic safety compatible with the Coleman-Weinberg symmetry breaking? (Remember: Gildener's PhD thesis with Coleman showed the compatibility between Coleman-Weinberg symmetry breaking and asymptotically free.)
- Answer: yes! We find Pati-Salam symmetry can be broken by Coleman-Weinberg mechanism in the **Safe** Pati-Salam scenario.
- We find a strong first order phase transition occurs **only** when spontaneous symmetry breaking happens via the Coleman-Weinberg mechanism (in our model).

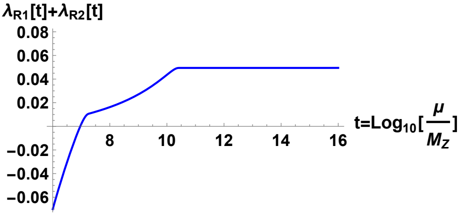


图: We plot the RG running of $\lambda_{R1}(t) + \lambda_{R2}(t)$ from UV to IR. The transition point (the scale $\lambda_{R1}(t) + \lambda_{R2}(t) = 0$) defines the Coleman-Weinberg symmetry breaking scale of the Pati-Salam model.

Inverse duration of the Phase Transition β and the ratio of the latent heat α

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The nucleation temperature which is defined as the temperature at which the rate of bubble nucleation per Hubble volume and time is approximately one:

$$\Gamma(T) \sim H^4\,, \quad T \ln\frac{T}{m_{pl}} \simeq -\frac{S_3(T)}{4} \Rightarrow T_n \sim 1260\,\mathrm{TeV}.$$

- The inverse duration of the PT β relative to the Hubble rate H_* at T_n is:

$$\frac{\beta}{H_*} = \left[T\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\right]\Bigg|_{T=T_n} \Rightarrow \beta/H_* \simeq 183\,.$$

- The ratio of the latent heat released by the phase transition α:

$$\alpha = \frac{\epsilon}{\rho_{\rm rad}} = \frac{1}{\frac{\pi^2}{30}g_* T_n^4}\left(-\Delta V + T_n \Delta s\right), \quad \Delta s = \frac{\partial V}{\partial T}\left(v_{T_n}, T_n\right) - \frac{\partial V}{\partial T}\left(0, T_n\right)$$

  where $\Delta V = V\left(v_{T_n}, T_n\right) - V\left(0, T_n\right)$ and we find $\alpha_{T_n} \equiv \alpha(T = T_n) = 0.217.$

Gravitational Wave Spectrum

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The power spectrum of the acoustic gravitational wave is given by (sound wave only; Collision of scalar field shells and turbulence sub-leading):

$$h^2\Omega_{sw}(f) = 8.5 * 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Gamma_{AI}^2\overline{U}_f^4\left(\frac{H_*}{\beta}\right)v_w S_{sw}(f)\ ,$$

  where the adiabatic index $\Gamma_{AI} = \overline{\omega}/\overline{\epsilon} \simeq 4/3$. $\overline{U}_f$ is a measure of the root-mean-square (rms) fluid velocity and is:

$$\overline{U}_f^2 \simeq \frac{3}{4}\kappa_f\alpha_{T_n}\ , \qquad \kappa_f \sim \frac{\alpha}{0.73 + 0.083\sqrt{\alpha} + \alpha}$$

  where κ_f is the efficiency parameter and v_ω (wall speed) $\to 1$.
- The spectral shape $S_{sw}(f)$ and peak frequency f_{sw} are:

$$S_{sw}(f) = \left(\frac{f}{f_{sw}}\right)^3\left(\frac{7}{4 + 3(f/f_{sw})^2}\right)^{\frac{7}{2}}$$

$$f_{sw} = 8.9\mu\text{Hz}\,\frac{1}{v_\omega}\left(\frac{\beta}{H_*}\right)\left(\frac{z_p}{10}\right)\left(\frac{T_n}{100\,\text{GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\ .$$

From Large N_f to Large Q Expansion

- Scaling dimension of the fixed charge operator $\phi^{\bar{Q}}$ in the $U(1)$ model.

$$\frac{\Delta_{\phi^{\bar{Q}}}}{\bar{Q}} = \begin{pmatrix} a_{00} & 0 & 0 & 0 & 0 & \dots \\ a_{11}\hat{\lambda}\bar{Q} & a_{10}\hat{\lambda} & 0 & 0 & 0 & \dots \\ a_{22}\hat{\lambda}^2\bar{Q}^2 & a_{21}\hat{\lambda}^2\bar{Q} & a_{20}\hat{\lambda}^2 & 0 & 0 & \dots \\ a_{33}\hat{\lambda}^3\bar{Q}^3 & a_{32}\hat{\lambda}^3\bar{Q}^2 & a_{31}\hat{\lambda}^3\bar{Q} & a_{30}\hat{\lambda}^3 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{ll}\hat{\lambda}^l\bar{Q}^l & a_{l,l-1}\hat{\lambda}^l\bar{Q}^{l-1} & a_{l,l-2}\hat{\lambda}^l\bar{Q}^{l-2} & a_{l,l-3}\hat{\lambda}^l\bar{Q}^{l-3} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

- Sum the column:

$$\frac{\Delta_{\phi^{\bar{Q}}}}{\bar{Q}} = \Delta_{-1}\left(\mathcal{A}^*\right) + \frac{1}{\bar{Q}}\Delta_0\left(\mathcal{A}^*\right) + + \frac{1}{\bar{Q}^2}\Delta_1\left(\mathcal{A}^*\right) + \cdots \quad , \qquad \mathcal{A} \equiv \lambda\bar{Q}$$

What is the lowest lying operator?

How to choose the charge configuration?

Charge Config. Model Building Recipe: $U(N) \times U(N)$

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, Phys. Rev. D **102** (2020) 125033, arXiv:2006.10078 [hep-th].
Antipin, Bersini, Sannino, Z.W. Wang and Zhang, JHEP **12** 204 (2021), arXiv:2102.04390 [hep-th].)

- The charge configuration matters in the $U(N) \times U(N)$ model which in Euclidean spacetime, is written as (H is a $N \times N$ complex matrix)

$$\mathcal{L} = \text{Tr}(\partial_\mu H^\dagger \partial^\mu H) + u_0 \, \text{Tr}(H^\dagger H)^2 + v_0 (\, \text{Tr} H^\dagger H)^2 \, .$$

1. Give a total classical scaling dimension $\bar{Q}$ of the operator $\mathcal{O}$ we are interested
2. Construct all the possible charge configurations satisfying:
   - diagonal elements can only be integer or half-integer;
   - the sum of absolute value of the diagonal element is $\bar{Q}/2$;
   - $\text{Tr}\mathcal{Q} = 0$ in $U(N) \times U(N)$ model
3. Determine the “Matrix Form" chemical potential and vacuum expectation value through the E.O.M. and total charge constraints.
4. Following the semi-classical computation recipe to calculate the scaling dimensions.
5. Combining the group theory and semi-classical computation, we can identify the representation of the operator.

An Example to untangle the representation of operator

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, JHEP **12** 204 (2021), arXiv:2102.04390 [hep-th].)

- We are interested in $SU(3) \times SU(3)$ and considering operators $\mathcal{O}$ with classical scaling dimension $\bar{Q} = 4$
- Since $H \sim (\mathbf{F}_L, \bar{\mathbf{F}}_R)$, $H^\dagger \sim (\bar{\mathbf{F}}_L, \mathbf{F}_R)$, thus the operators belongs to (Γ_L, Γ_R). Γ_L and Γ_R are respectively in $(\mathbf{Adj}_L)^{\bar{Q}/2}$ and $(\mathbf{Adj}_R)^{\bar{Q}/2}$
- Operators live in the decomposition of the tensor product $\mathbf{8} \otimes \mathbf{8}$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus 2(\mathbf{8}) \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}\,.$$

- We can only construct two different charge configuration matrices:

$$\mathcal{Q}^{(4)}_{3A} = \mathrm{diag}\,(1, -1, 0)\,, \qquad \mathcal{Q}^{(4)}_{3\mathrm{B}} = \mathrm{diag}\,(1, -1/2, -1/2)$$

- The weight which corresponds to $\mathcal{Q}^{(4)}_{3A}$ is $(4, -2)$ only appearing in $\mathbf{27}$ while the weight corresponds to $\mathcal{Q}^{(4)}_{3B}$ reads $(3, 0)$ appearing in both $\mathbf{27}$ and $\mathbf{10}$. Thus, we have the following correspondence

$$\mathcal{Q}^{(4)}_{3A} : (\mathbf{27}, \mathbf{27})\,, \quad \mathcal{Q}^{(4)}_{3B} : \begin{cases} (\mathbf{27}, \mathbf{27}) \\ (\mathbf{10}, \overline{\mathbf{10}}) \end{cases} .$$

An Example

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, JHEP **12** 204 (2021), arXiv:2102.04390 [hep-th].)

- For $N > \sqrt{3}$, the scalar couplings at the WF fixed points are complex and thus the scaling dimensions at the fixed points are also complex.
- We choose the real part of the scaling dimensions.

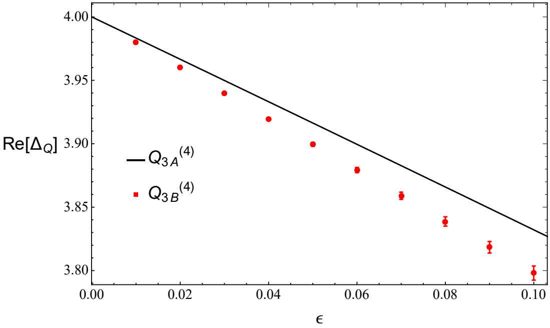


图: The real part of the scaling dimension at the fixed point for the $U(3) \times U(3)$ operators with CSD $\bar{Q} = 4$, carrying the charges $\mathcal{Q}_{3A}^{(4)}$ (black line) and $\mathcal{Q}_{3B}^{(4)}$ (red dots) as a function of ϵ. The error bars encode the numerical error in evaluating Δ_0 for $\mathcal{Q}_{3B}^{(4)}$.

Dire. 3: Large Charge Method and Higgs Explosion

- Can we address the problematic multi-Higgs boson production processes at future colliders?
  Answer: we can calculate the LO and NLO (but to all order in couplings) scaling dimensions of $O(N)$ and $U(N) \times U(M)$ models around 4D for a family of fixed-charge operators by using the semi-classical method and state-operator correspondence.

$$\Delta_{T_{\bar{Q}}} = \bar{Q} + \left(-\frac{\bar{Q}}{2} + \frac{\bar{Q}(\bar{Q}-1)}{8+N}\right)\epsilon - \left[\frac{184+N(14-3N)}{4(8+N)^3}\bar{Q} + \frac{(N-22)(N+6)}{2(8+N)^3}\bar{Q}^2 + \frac{2}{(8+N)^2}\bar{Q}^3\right]\epsilon^2 + \left[\frac{8}{(8+N)^3}\bar{Q}^4\right.$$
$$+\frac{-456-64N+N^2+2(8+N)(14+N)\zeta(3)}{(8+N)^4}\bar{Q}^3 - \frac{-31136-8272N-276N^2+56N^3+N^4+24(N+6)(N+8)(N+26)\zeta(3)}{4(N+8)^5}\bar{Q}^2$$
$$\left.+\frac{-65664-8064N+4912N^2+1116N^3+48N^4-N^5+64(N+8)(178+N(37+N))\zeta(3)}{16(N+8)^5}\bar{Q}\right]\epsilon^3 + O\left(\epsilon^4\right).$$

The scaling dimension of $\bar{Q}$-index traceless symmetric tensor $T_{\bar{Q}} \equiv T^{(\bar{Q})}_{i_1\ldots i_{\bar{Q}}}$.

- **Outlook** Theory: add fermions, gauge bosons; explore complex CFT
  Phenomenology: found various interesting calculable EFT operators; address full SM Higgs Explosion

What we have achieved so far

(Antipin, Bersini, Sannino, Wang and Zhang, “Charging the $O(N)$ model,” PRD 102 (2020) 045011.)

- Rubakov’s insight: $\sigma_{1\to n}(E) \propto \exp\left[nF(\lambda n, \epsilon)\right]$
- Rattazzi calculate (leading classical, leading quantum) to the scaling dimension of operator $[\phi^n]$ in $U(1)$ symmetric $\lambda\left(\bar{\phi}\phi\right)^2$ theory.(G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

$$
\begin{aligned}
&Z_{\phi^n}^2 \lambda_0^n \langle \left[\bar{\phi}^n\right](x_f)\left[\phi^n\right](x_i)\rangle \\
=&\lambda_0^n n! \exp\left[\frac{1}{\lambda_0}\Gamma_{-1}(\lambda_0 n, x_{fi}) + \Gamma_0(\lambda_0 n, x_{fi}) + \Gamma_1(\lambda_0 n, x_{fi}) + \cdots\right]
\end{aligned}
$$

- Around the fixed point, the ground state energy of the system corresponds to the scaling dimension of the operator with the lowest classical scaling dimension.
- We did a non-trivial generalization of Rattazi’s work to $O(N)$, $U(N)\times U(N)$, $U(N)\times U(M)$ theory.
- The perturbative expansion of our all order expression can match the existing perturbative loop calculation up to three loop order.

The Effect of PDF

(C. Degrande, V. V. Khoze and O. Mattelaer, Phys. Rev. D **94** (2016), 085031.)

- Question: whether the factorial growth will be completely washed away by PDF suppression?
- For the collider above $50\,\mathrm{TeV}$, the growth of the cross section can not be killed while for lwer energy collider, the PDF are killing the cross-section before reaching the fast growth regime.

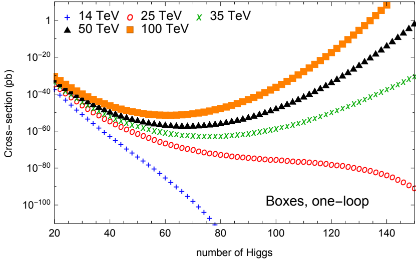


图: Cross-sections for multi-Higgs production (3.6) at proton colliders including the PDFs for different energies of the proton-proton collisions plotted as the function of the Higgs multiplicity. Only the contributions from the boxes are included.

Conclusion

- Asymptotically Safe Standard Model is feasible through GUT embedding with large N_f and $U(1)$ Landau pole problem is addressed.
- Both Safe Pati-Salam model and the Safe Trinification Model can roughly match the SM at IR.
- Strong first order phase transition due to Coleman Weinberg symmetry breaking generates interesting gravitational wave signals within the detection region of near future LIGO Voyager.
- Combining semi-classical computation with CFT, we can calculate the scaling dimensions of a class of fixed charge operator up to NLO in charge expansion but to all order in the couplings.
- We create a recipe to untangle the representations of the fixed charge operator associated with a specific charge configuration.
- Multi-Higgs production is important for the future $100\,\mathrm{TeV}$ collider. Our work is one step towards the “Holy Grail" summation function beyond tree level and has the potential to address the Higgs explosion as a long goal.

The Power of Asymptotic Safety