

Are primordial black holes formed during reheating after R^2 inflation?

Yuki Watanabe
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Based on Work with H. Jeong, R. Jinno, J. Yokoyama

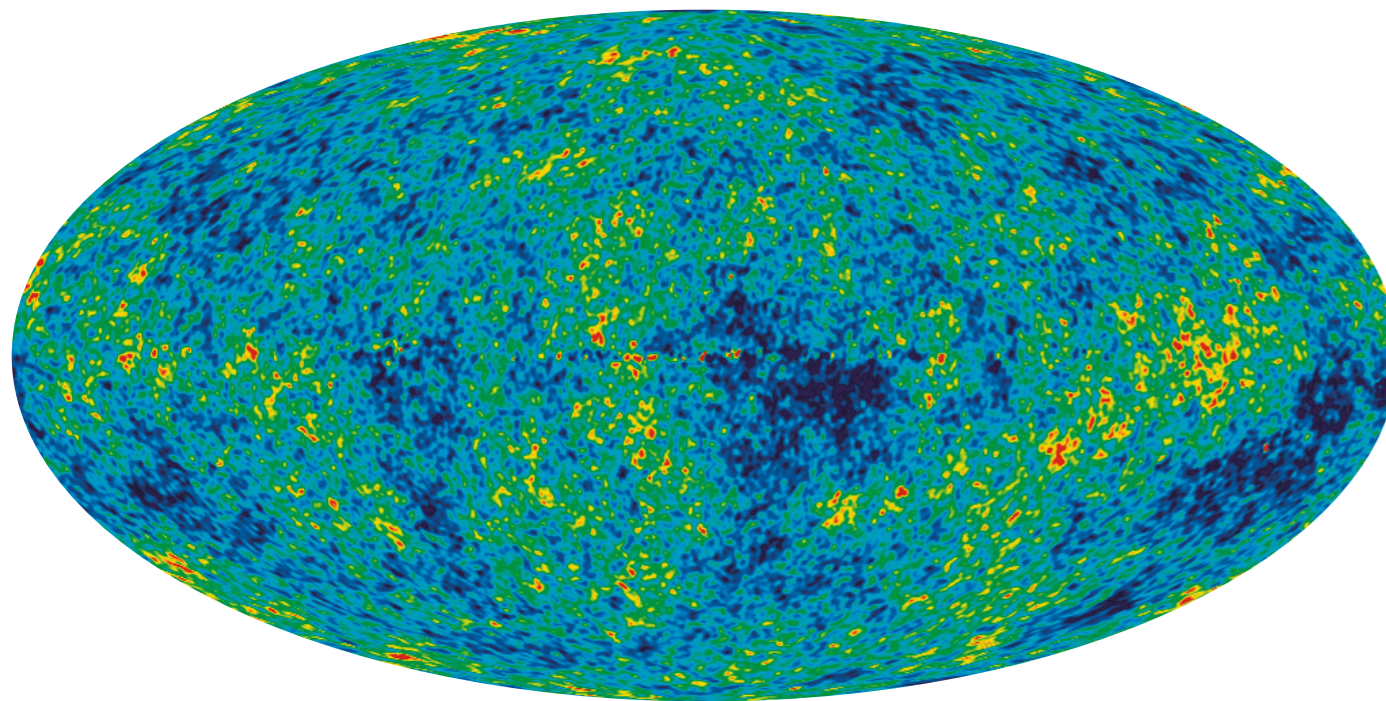


The Dark Side of the Universe (DSU 2024)

Mon-Repos, Corfu, Greece
Sep. 11, 2024

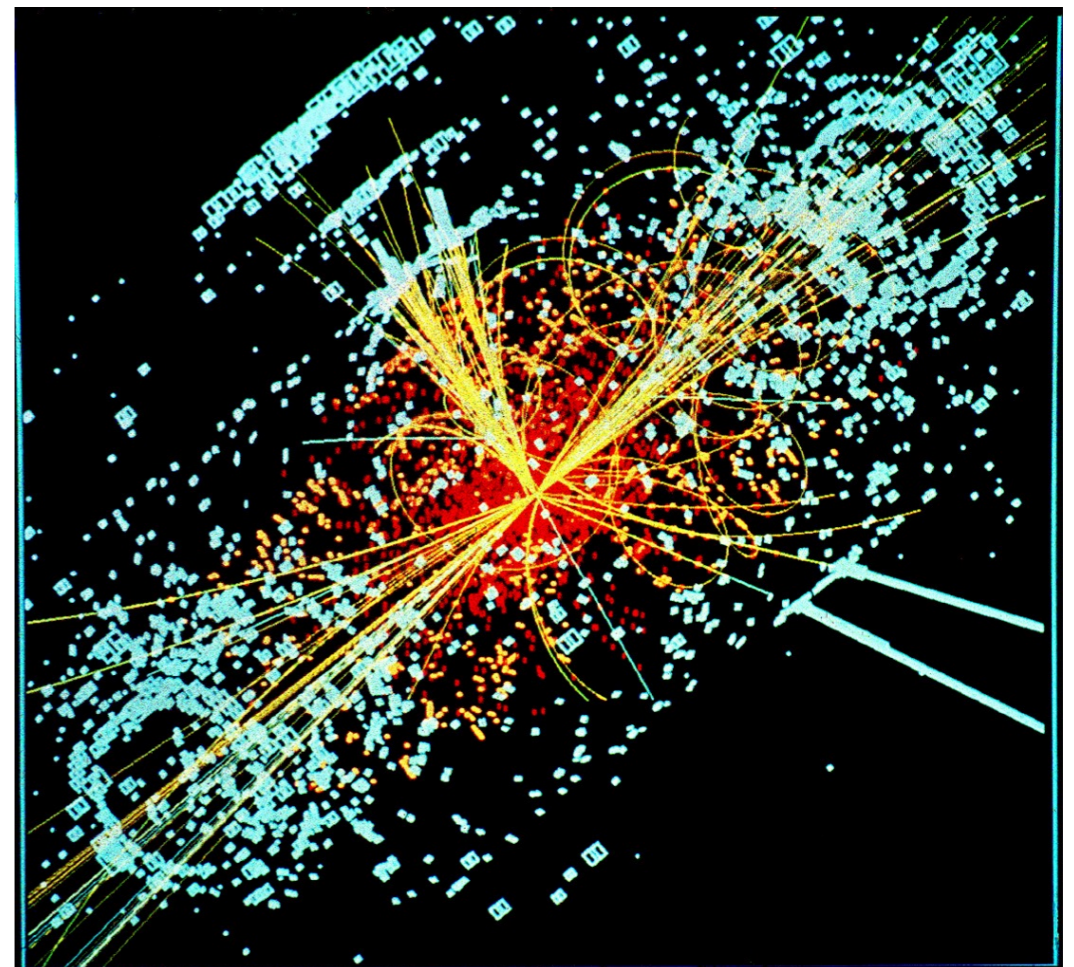
CMB observations and BSM physics

- (n_s, r) precision measurements from CMB
- No signal of physics beyond the Standard Model (BSM) at the LHC



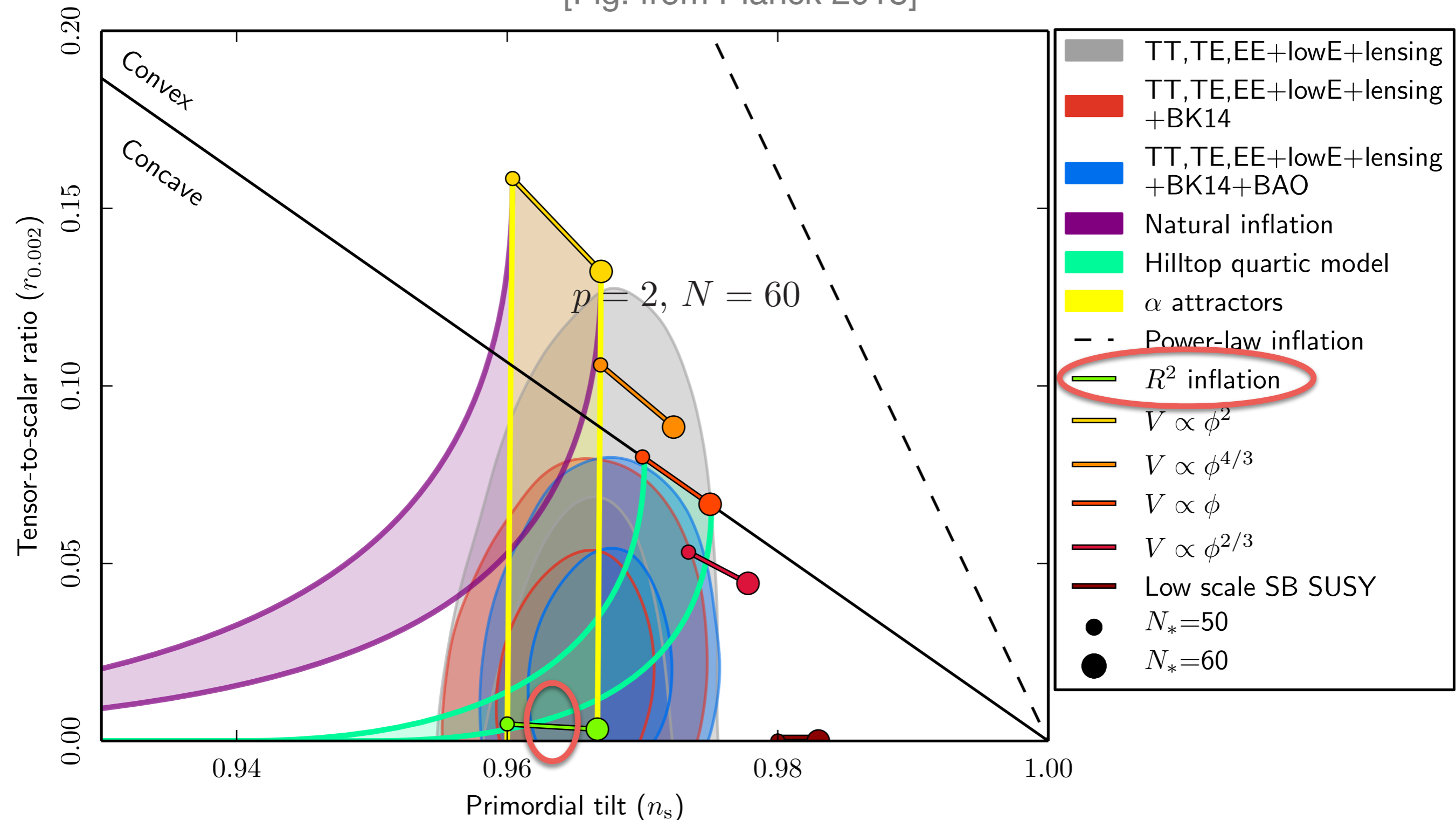
-200 T(μ K) +200 WMAP 5-year

credit: NASA



CMB constraint on inflation models

[Fig. from Planck 2018]



- Monomial potentials ($p \geq 2$) in GR are disfavored.
- What if we could nail down to further precision?

Starobinsky R^2 Inflation

[Starobinsky 1980; Mukhanov & Chibisov 1981]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R + \frac{R^2}{6M^2} \right) + S_m$$

$$S_m = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\nabla\sigma)^2 - V(\sigma) \right] \leftarrow \text{Higgs}$$

+ minimally coupled SM, RHN

+ “*desert*” or BSM

- One of the oldest models of Inflation, before models of Sato and Guth
- A single parameter **M** characterizes the model.

R² Inflation as scalar-tensor theory

[Whitt 1984; Maeda 1988]

$$S_J = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\hat{g}} \left(\hat{R} + \frac{\hat{R}^2}{6M^2} \right) + S_m$$

$$S_m = \int d^4x \sqrt{-\hat{g}} \left[-\frac{1}{2} (\hat{\nabla} \hat{\sigma})^2 - V(\hat{\sigma}) \right]$$

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Jordan frame $\hat{g}_{\mu\nu}$



Einstein frame $g_{\mu\nu}$

$$g_{\mu\nu} = \hat{g}_{\mu\nu} \Omega^2 \quad \Omega^2 = 2\kappa^2 \left| \frac{\partial \mathcal{L}_J}{\partial \hat{R}} \right| = 1 + \frac{\hat{R}}{3M^2} \equiv e^{\sqrt{\frac{2}{3}} \kappa \varphi}$$

$$\hat{R} = \Omega^2 \left[R + 3\Box(\ln \Omega^2) - \frac{3}{2} g^{\mu\nu} \partial_\mu(\ln \Omega^2) \partial_\nu(\ln \Omega^2) \right]$$

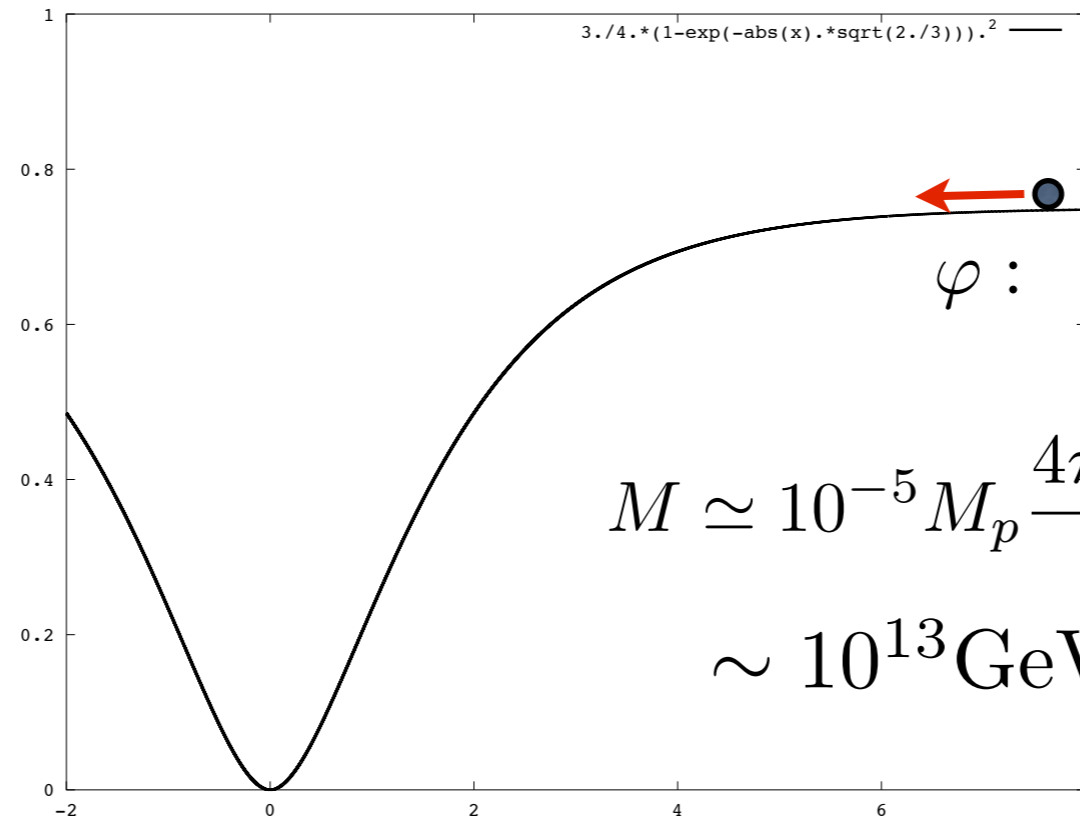
$$S_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla \varphi)^2 - U(\varphi) - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \kappa \varphi} (\nabla \hat{\sigma})^2 - e^{-\sqrt{\frac{8}{3}} \kappa \varphi} V(\hat{\sigma}) \right]$$

$$U(\varphi) = \frac{3}{4} M^2 M_p^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \kappa \varphi} \right)^2 = \begin{cases} \frac{3}{4} M^2 M_p^2 & \text{for } \varphi \gg \varphi_f \\ \frac{1}{2} M^2 \varphi^2 & \text{for } \varphi \ll \varphi_f \end{cases}$$

φ : Scalaron = Inflaton

Starobinsky R² Inflation

[Starobinsky 1980; Mukhanov & Chibisov 1981]



φ : Scalaron = Inflaton

$$M \simeq 10^{-5} M_p \frac{4\pi\sqrt{30}}{N_*} \left(\frac{\mathcal{P}_\zeta(k_*)}{2 \times 10^{-9}} \right)^{1/2}$$

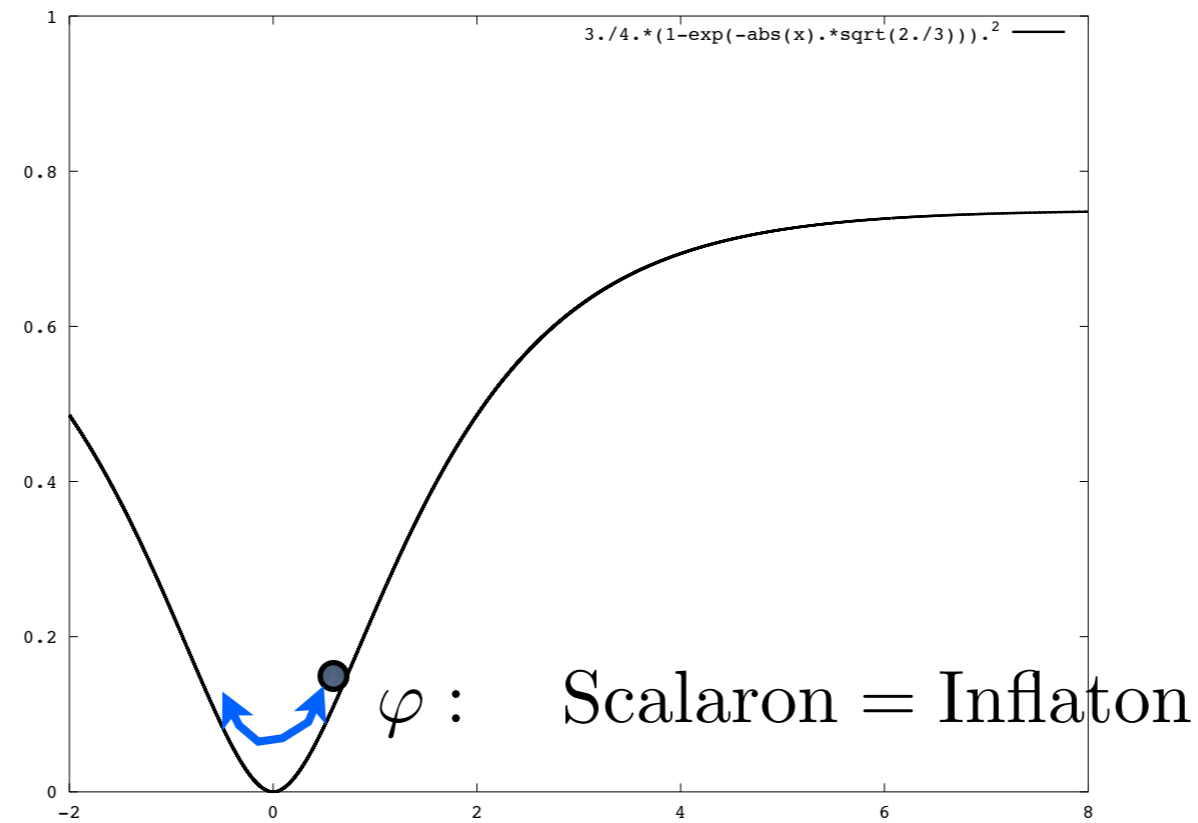
$$\sim 10^{13} \text{ GeV}$$

Scalaron mass M is fixed by CMB temp. anisotropy

$$U(\varphi) = \frac{3}{4} M^2 M_p^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \kappa \varphi} \right)^2 = \left\{ \begin{array}{l} \frac{3}{4} M^2 M_p^2 \text{ for } \varphi \gg \varphi_f \\ \frac{1}{2} M^2 \varphi^2 \text{ for } \varphi \ll \varphi_f \end{array} \right\}$$

Starobinsky R² Inflation

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Gravitational reheating by scalaron decay

[YW & Komatsu gr-qc/0612120; YW 1011.3348; YW & White 1503.08430]

$$\sigma \equiv e^{-\frac{\kappa}{\sqrt{6}}\varphi} \hat{\sigma}$$

$$\mathcal{L}_{\text{scalar}} = -\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{\kappa\sigma}{\sqrt{6}}\partial_{\mu}\sigma\partial^{\mu}\varphi - \frac{\kappa^2\sigma^2}{12}\partial_{\mu}\varphi\partial^{\mu}\varphi - \frac{m_{\sigma}^2}{2}e^{-\frac{2}{\sqrt{6}}\kappa\varphi}\sigma^2$$

$$\psi \equiv e^{-\frac{3\kappa}{2\sqrt{6}}\varphi} \hat{\psi}$$

$$\mathcal{L}_{\text{fermion}} = -\bar{\psi}\not{D}\psi - e^{-\frac{1}{\sqrt{6}}\kappa\varphi}m_{\psi}\bar{\psi}\psi$$

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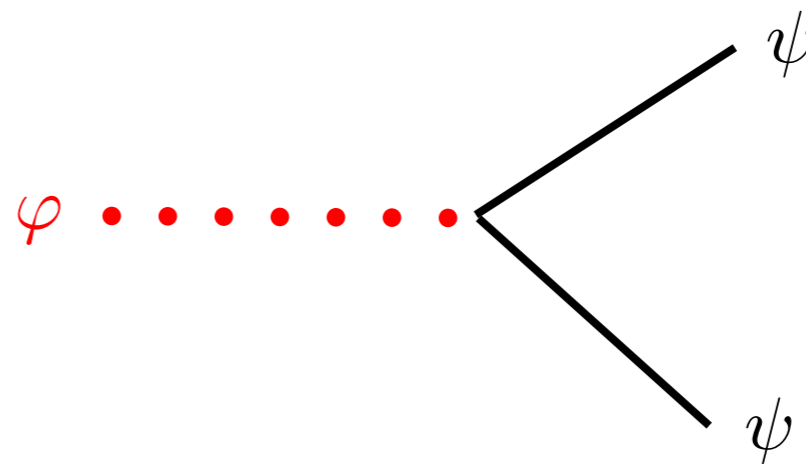
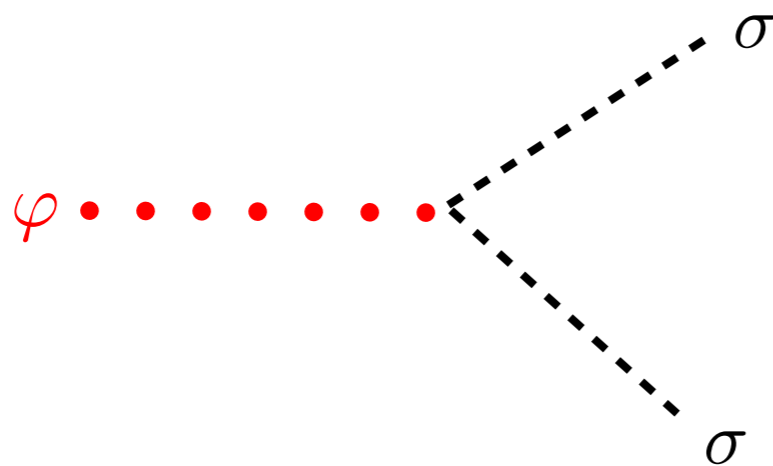
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$$\mathcal{L}_{\text{3leg}} = \frac{1}{\sqrt{6}M_{\text{Pl}}}\varphi\partial^{\mu}\sigma\partial_{\mu}\sigma + \frac{2m_{\sigma}^2}{\sqrt{6}M_{\text{Pl}}}\varphi\sigma^2 + \frac{m_{\psi}^2}{\sqrt{6}M_{\text{Pl}}}\varphi\bar{\psi}\psi$$



Gravitational reheating by scalaron decay

[YW & Komatsu gr-qc/0612120; YW 1011.3348; YW & White 1503.08430]

$$\Gamma(\varphi \rightarrow \sigma\sigma) = \frac{\mathcal{N}_\sigma (M^2 + 2m_\sigma^2)^2}{192\pi M_{\text{Pl}}^2 M}$$

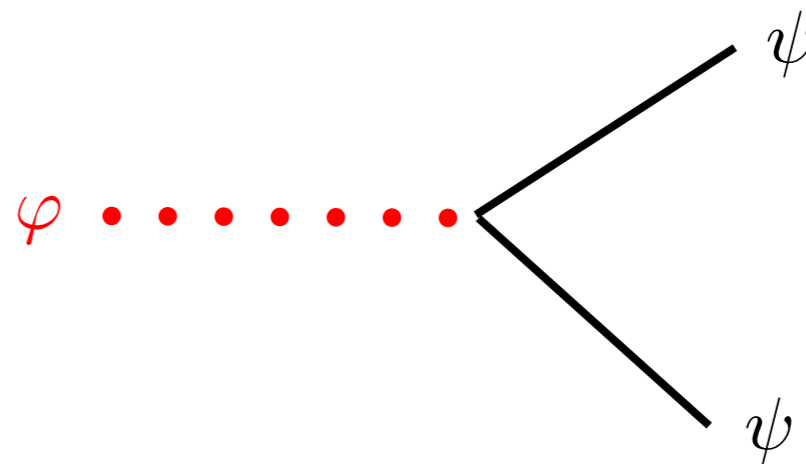
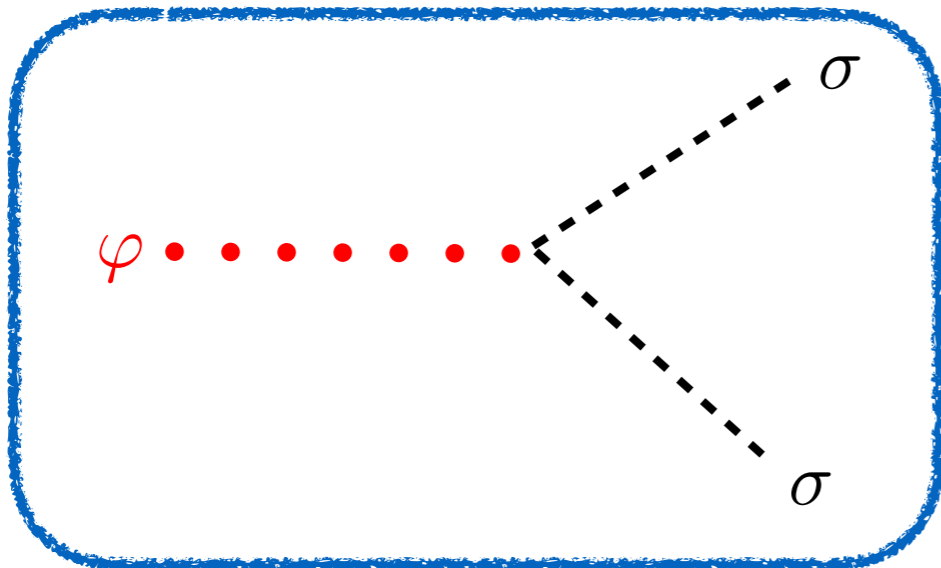
$$\simeq \frac{\mathcal{N}_\sigma M^3}{192\pi M_{\text{Pl}}^2} + \frac{\mathcal{N}_\sigma m_\sigma^2 M}{48\pi M_{\text{Pl}}^2}$$

Leading term

$$\Gamma(\varphi \rightarrow \bar{\psi}\psi) = \frac{\mathcal{N}_\psi m_\psi^2 M}{48\pi M_{\text{Pl}}^2}$$

$H_{\text{rh}} = \Gamma$

$$T_{\text{rh}} \simeq 0.1 \sqrt{\Gamma_{\text{tot}} M_p} \left(\frac{\mathcal{N}_{\text{tot}}}{100} \right)^{-1/4}$$



Gravitational reheating by scalaron decay


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Leading term

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$$T_{\text{rh}} \simeq 0.1 \sqrt{\Gamma_{\text{tot}} M_p} \left(\frac{\mathcal{N}_{\text{tot}}}{100} \right)^{-1/4} \sim 10^{-9} M_p,$$

$$N_* \simeq 54 + \frac{1}{3} \ln \left(\frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right),$$

If we know the matter sector (e.g. SM minimally coupled to gravity),
inflationary predictions can be made without uncertainty.

Predictions depend on reheating temperature

e-folds of inflation

$$N_* \simeq 54 + \frac{1}{3} \ln \left(\frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right),$$

scalaron mass

$$M \simeq 10^{-5} M_p \frac{4\pi\sqrt{30}}{N_*} \left(\frac{\mathcal{P}_\zeta(k_*)}{2 \times 10^{-9}} \right)^{1/2} \\ \sim 10^{13} \text{ GeV}$$

grav. waves

$$r = \frac{\mathcal{P}_\gamma(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon \simeq \frac{12}{N_*^2}.$$

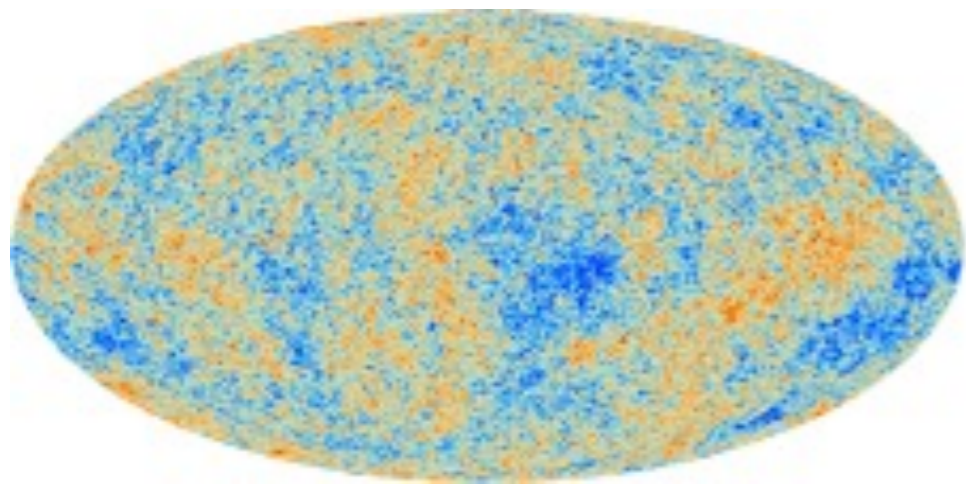
tilt and running of spectra

$$n_s - 1 = \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k} \simeq -6\epsilon_V + 2\eta_V \simeq -\frac{2}{N_*},$$

$$n_t = \frac{d \ln \mathcal{P}_\gamma(k)}{d \ln k} \simeq -2\epsilon_V \simeq -\frac{3}{2N_*^2},$$

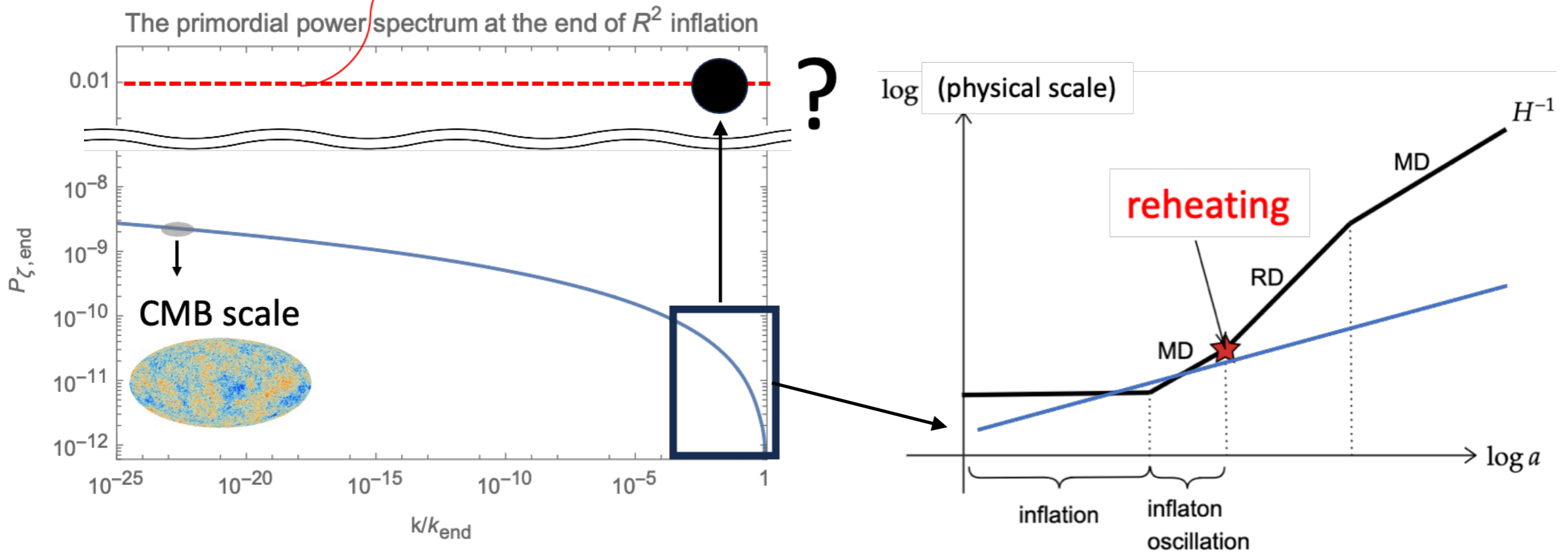
$$\frac{dn_s}{d \ln k} \simeq 16\epsilon_V \eta_V - 24\epsilon_V^2 - 2\xi_V^2 \simeq -\frac{2}{N_*^2},$$

$$\frac{dn_t}{d \ln k} \simeq 4\epsilon_V \eta_V - 8\epsilon_V^2 \simeq -\frac{3}{N_*^3},$$



PBH formation after R^2 inflation?

Necessary for the PBH formation



These modes **re-enter** the horizon
& **grow** during the long inflaton-oscillation epoch

Perturbations during the inflaton-oscillation era

[Finelli, Brandenberger 1999; Jedamzik, Lemoine, Martin 2010]

Background

$$\phi(t) \propto a(t)^{-3/2} \sin Mt$$

→ Effective MD

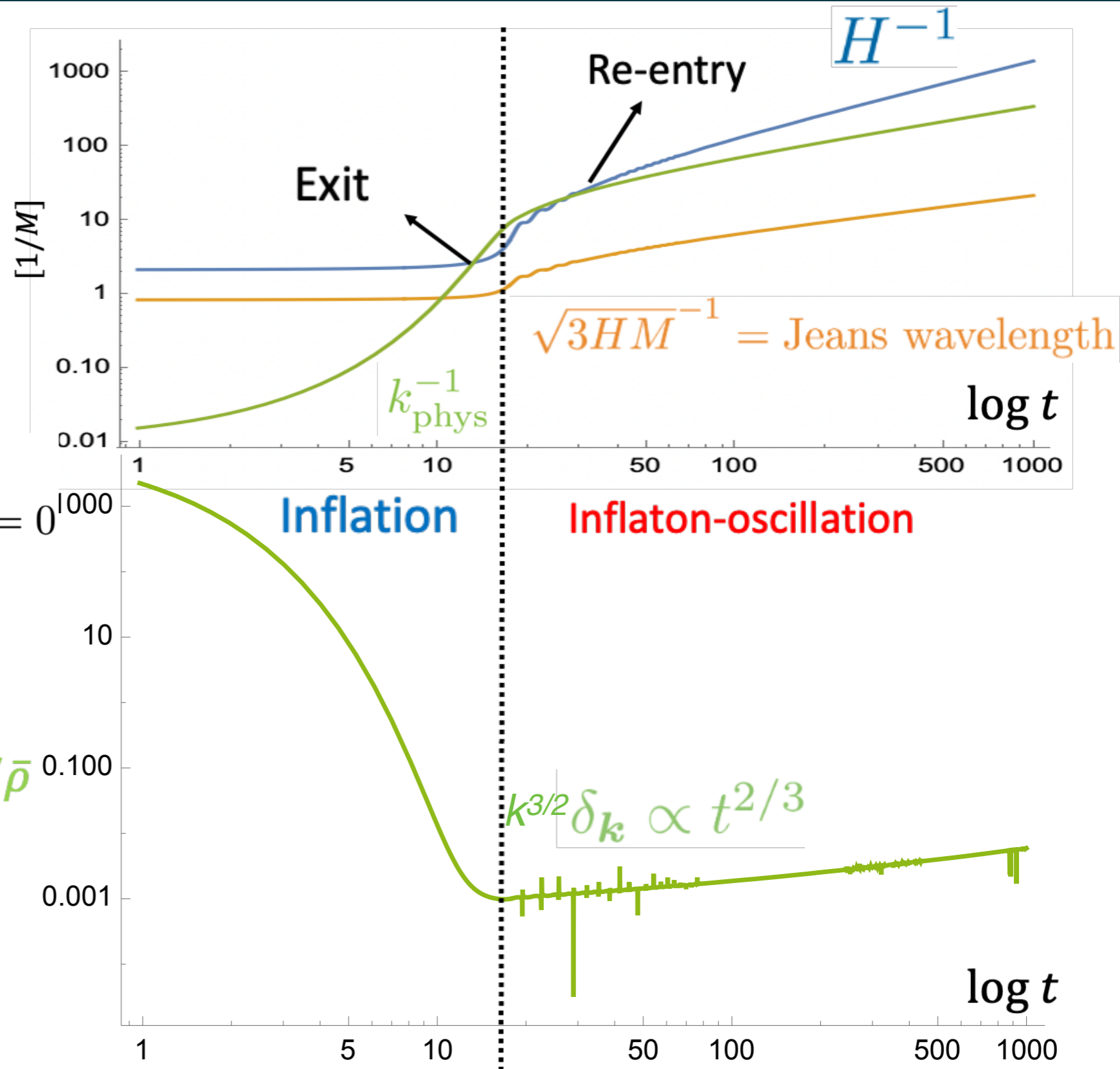
Perturbation

Mukhanov-Sasaki Eq.

$$v_k'' + \left(k^2 - \frac{\left(a\sqrt{\epsilon}[\phi(t)] \right)''}{a\sqrt{\epsilon}[\phi(t)]} \right) v_k = 0$$

→ Mathieu Eq.

Linear growth of $\delta = (\rho - \bar{\rho})/\bar{\rho}$
due to **Jeans instability**



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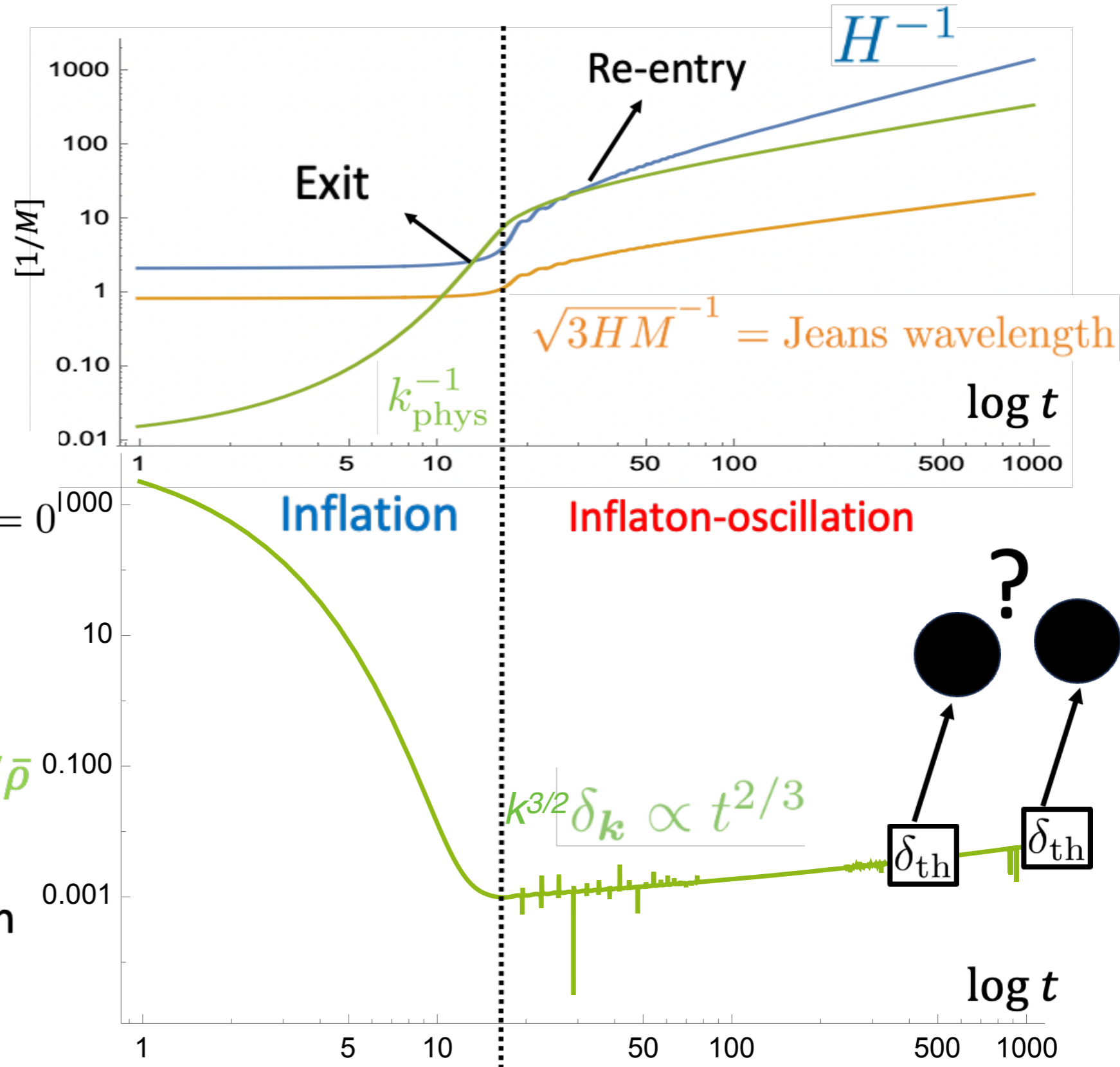
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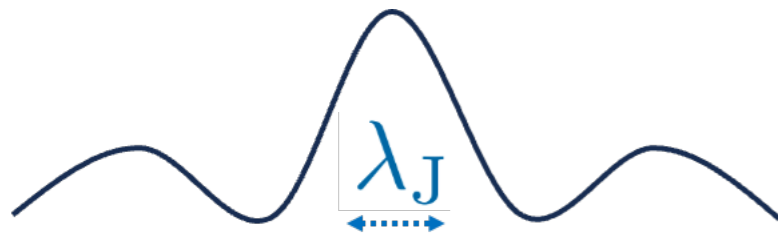
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Long inflaton-oscillation epoch
⇒ δ grows large ⇒ PBH?

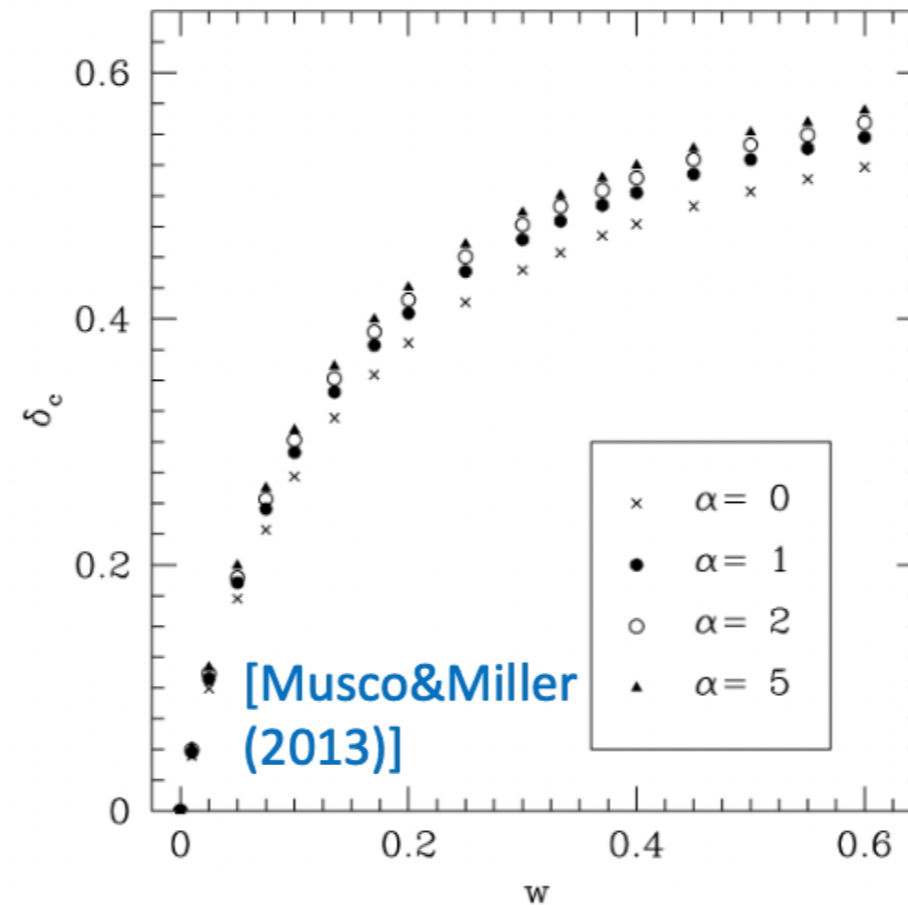


PBH formation condition: Jeans criterion $\delta > \delta_{\text{th}}$

RD($w = 1/3$)



$1/3 > w > 0$

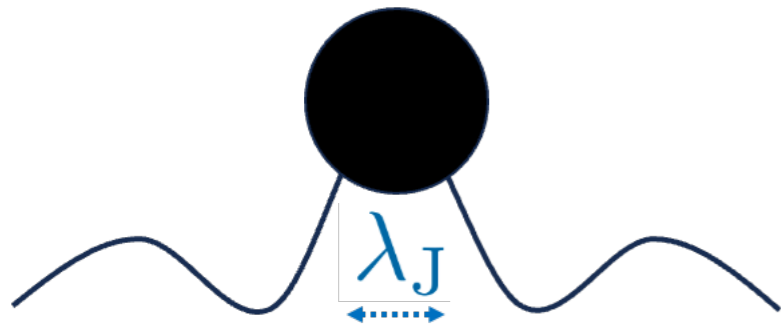


MD($w = 0$)



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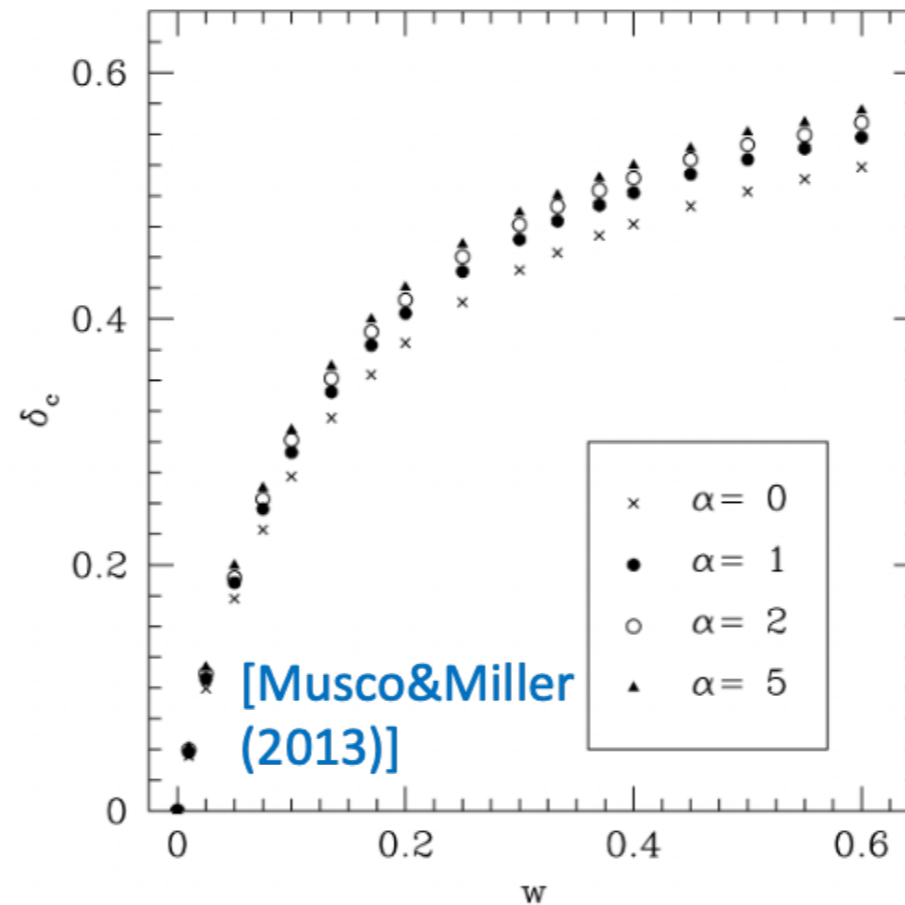
Jeans condition

$$\lambda > \lambda_J$$

$$\Rightarrow \delta_{\text{th}} \simeq 0.33$$

[B. J. Carr 1975]

$1/3 > w > 0$

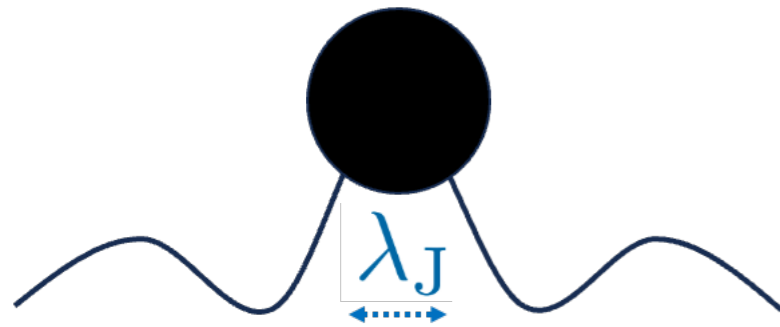


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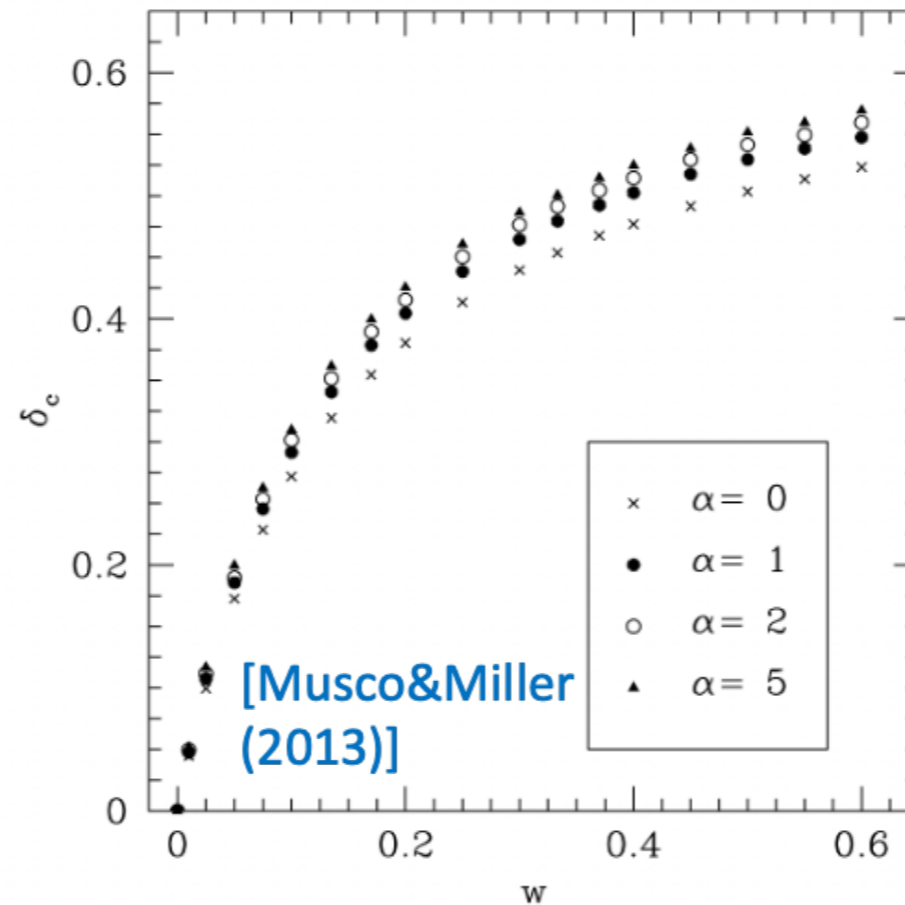
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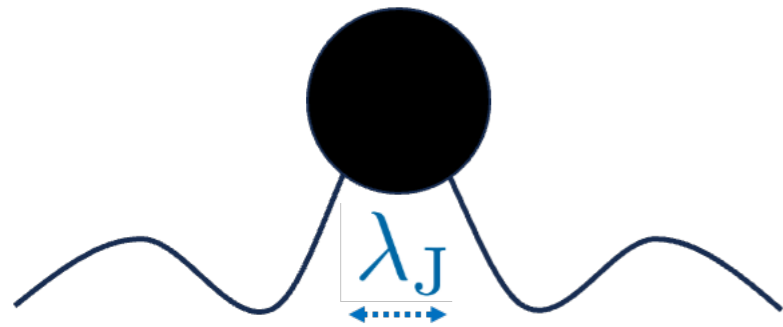


$$\delta_{\text{th}} = \frac{3(1+w)}{5+3w} \sin^2 \left[\frac{\pi\sqrt{w}}{1+3w} \right]$$

[T. Harada et al. 2013]

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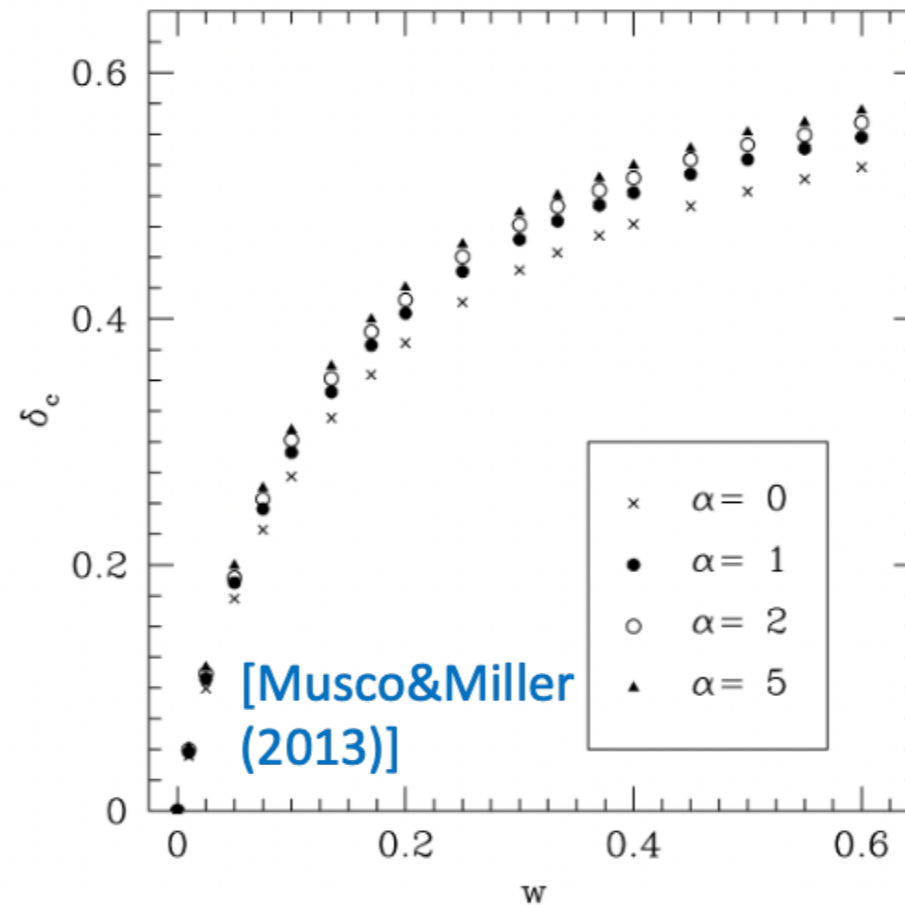
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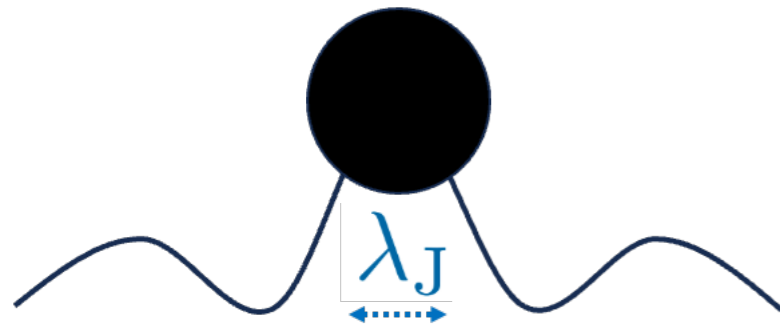
naive

extrapolation

$$\delta_{\text{th}} = 0$$

PBH formation condition: Jeans criterion $\delta > \delta_{\text{th}}$

RD($w = 1/3$)



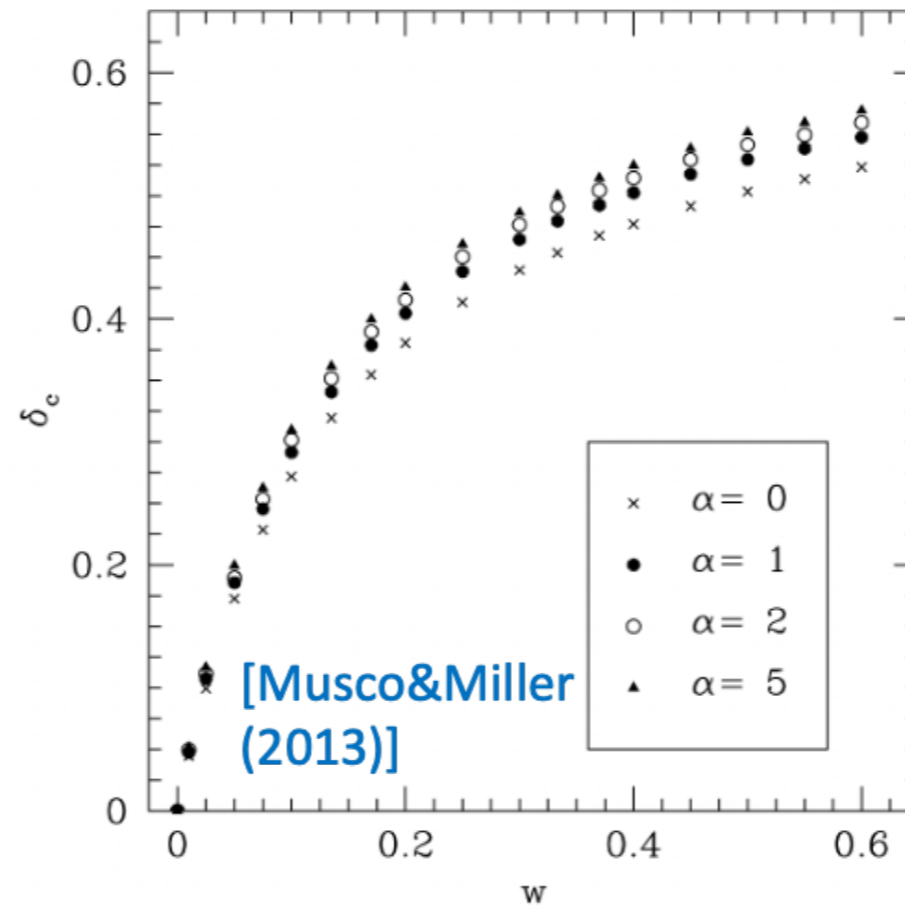
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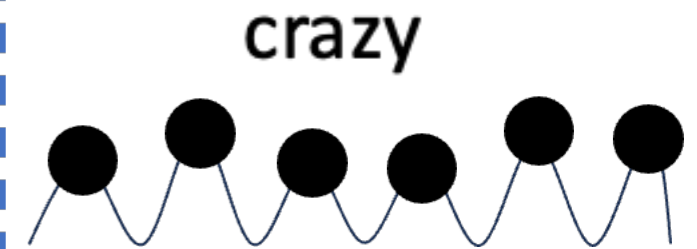


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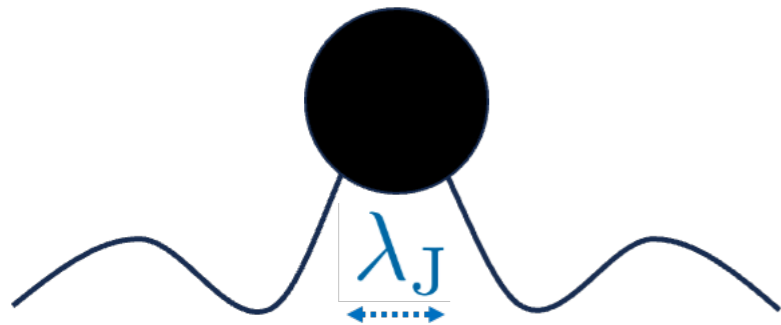
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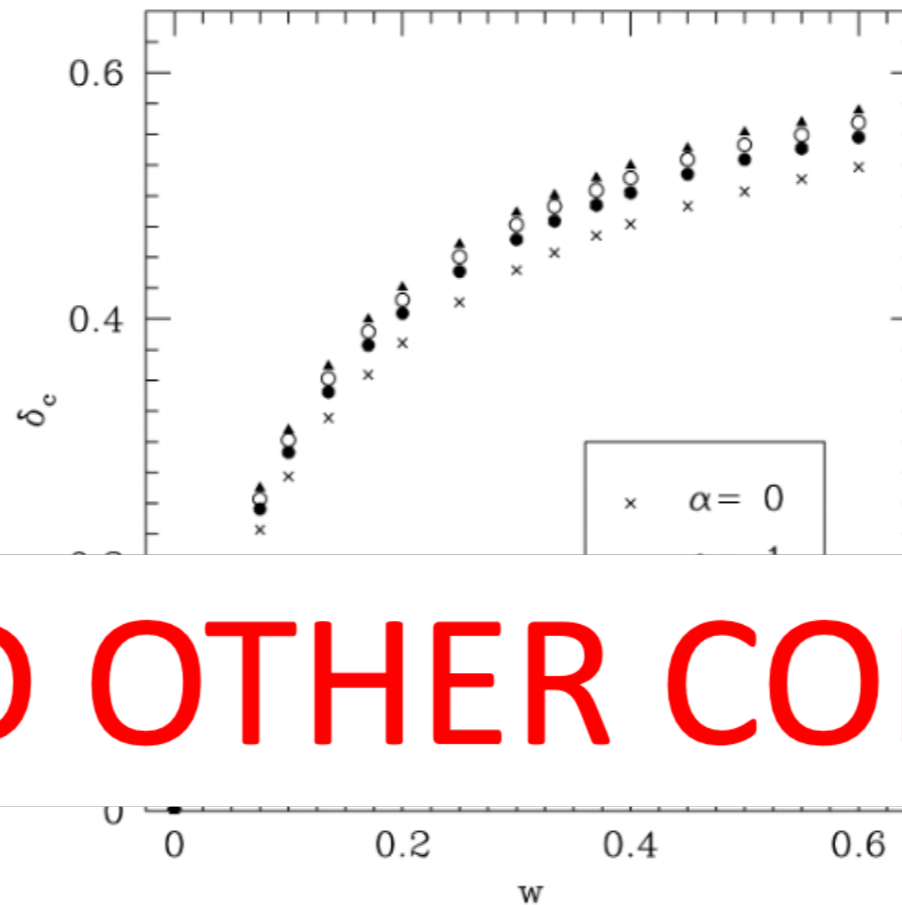
Untrustable

PBH formation condition: Jeans criterion $\delta > \delta_{\text{th}}$

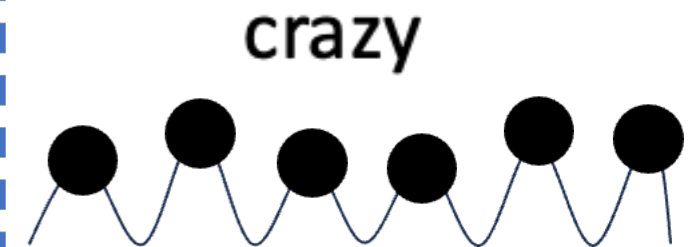
RD($w = 1/3$)



$1/3 > w > 0$



MD($w = 0$)



Jea **WE NEED OTHER CONDITION**

$$\Rightarrow \delta_{\text{th}} \simeq 0.33 \longrightarrow \delta_{\text{th}} = \frac{3(1+w)}{5+3w} \sin^2 \left[\frac{\pi\sqrt{w}}{1+3w} \right] \xrightarrow[\text{extrapolation}]{\text{naive}} \delta_{\text{th}} = 0$$

[B. J. Carr 1975]

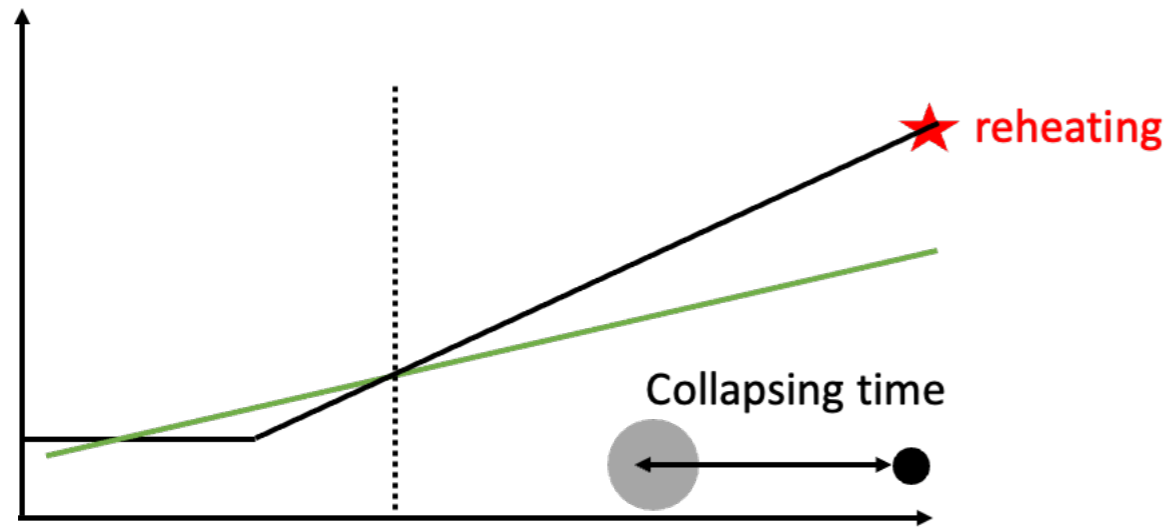
[T. Harada et al. 2013]

Untrustable

Overproduction of PBHs during oscillation era

[Martin, Papanikolaou, Venin 2020]

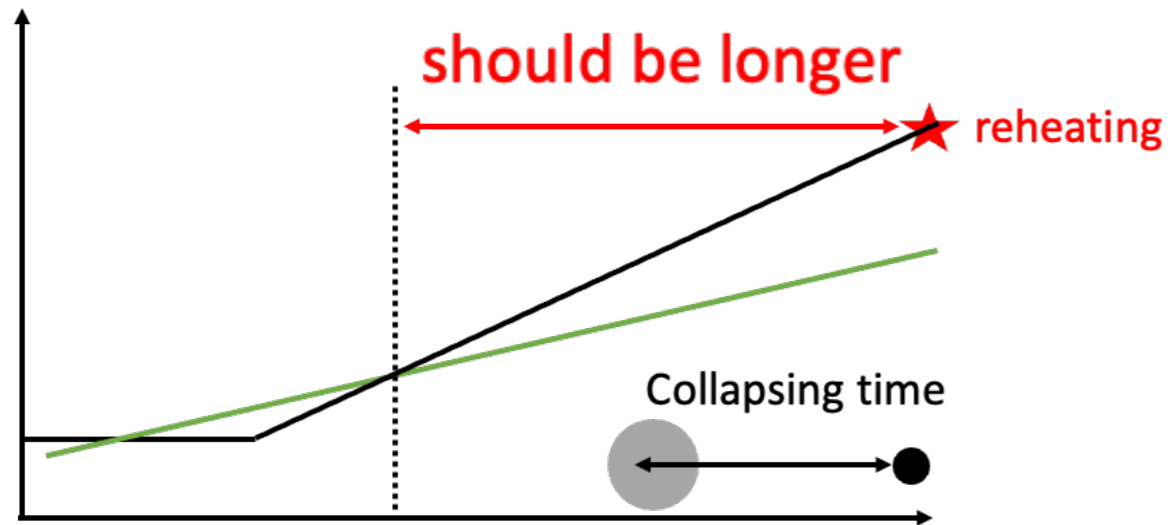
Another condition



Overproduction of PBHs during oscillation era

[Martin, Papanikolaou, Venin 2020]

Another condition



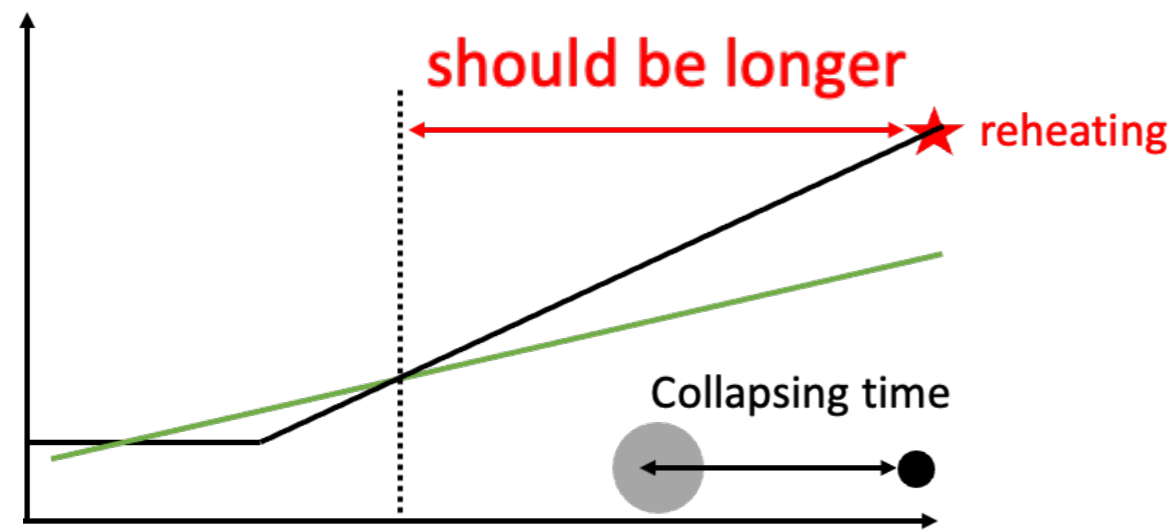
Mass fraction of PBHs

$$\beta(k(M)) = 2 \int_{\delta_{\text{th}}}^{\infty} P(\delta) d\delta$$

Overproduction of PBHs during oscillation era

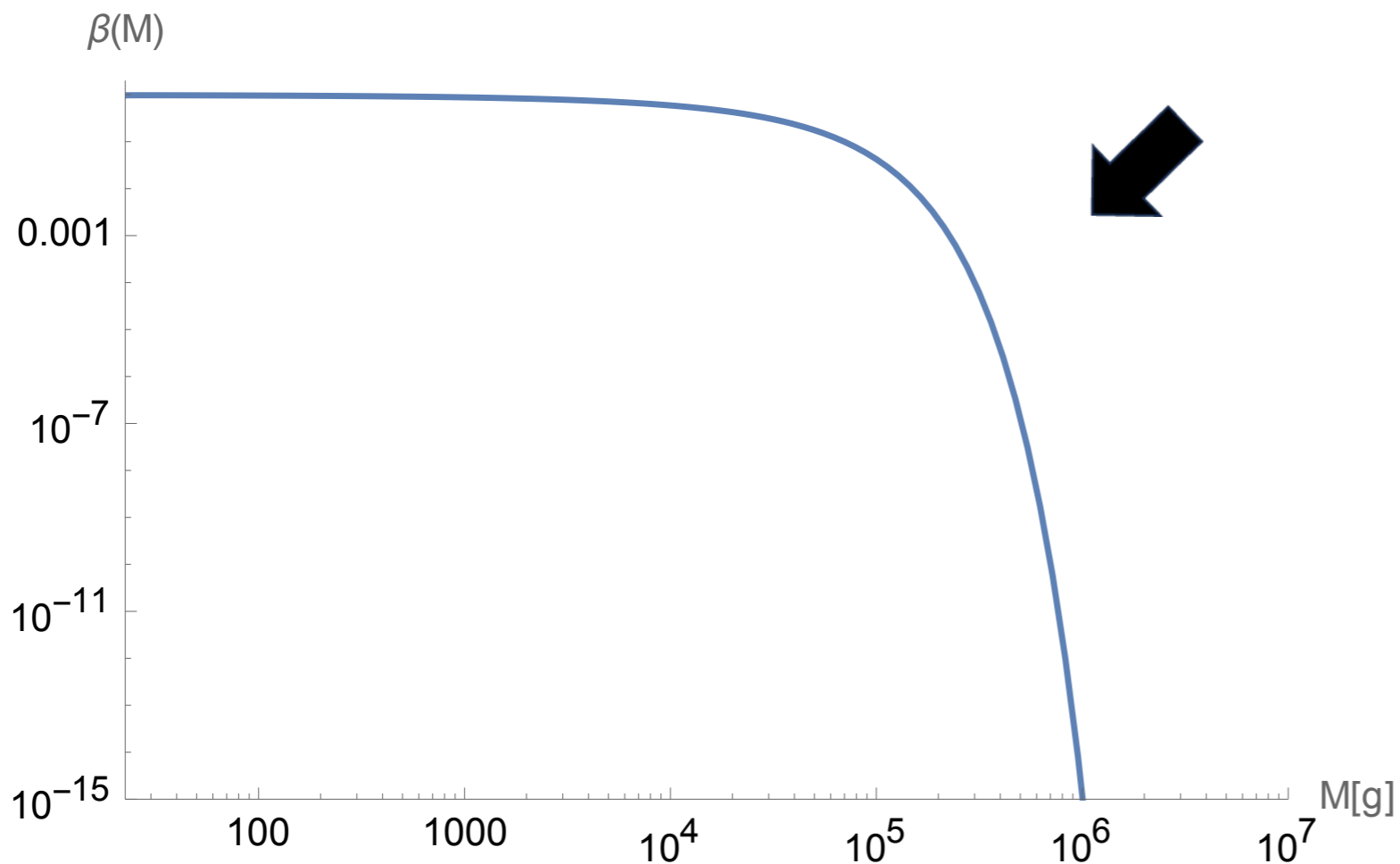
[Martin, Papanikolaou, Venin 2020]

Another condition



Mass fraction of PBHs

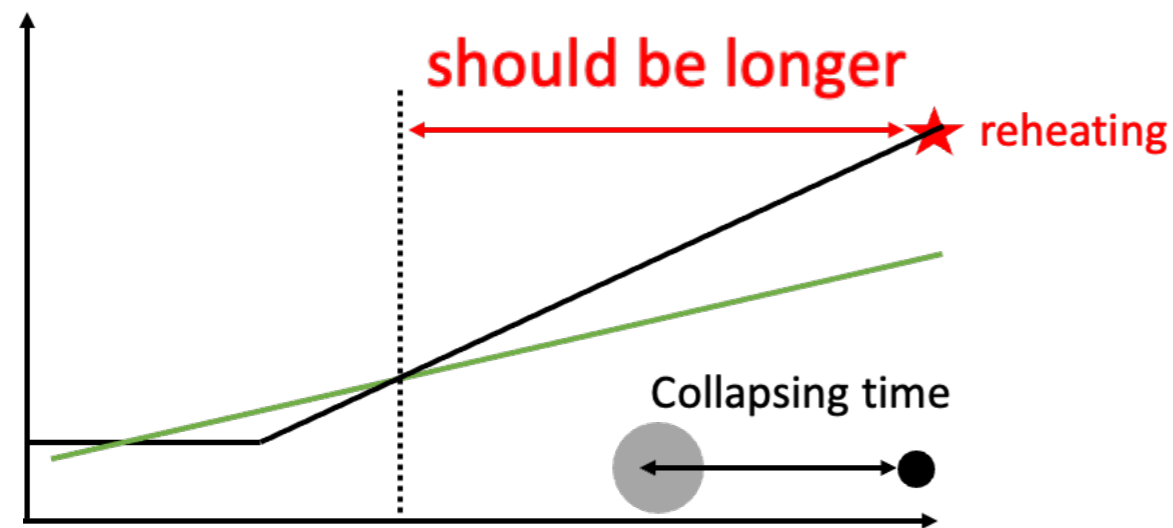
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Overproduction of PBHs during oscillation era

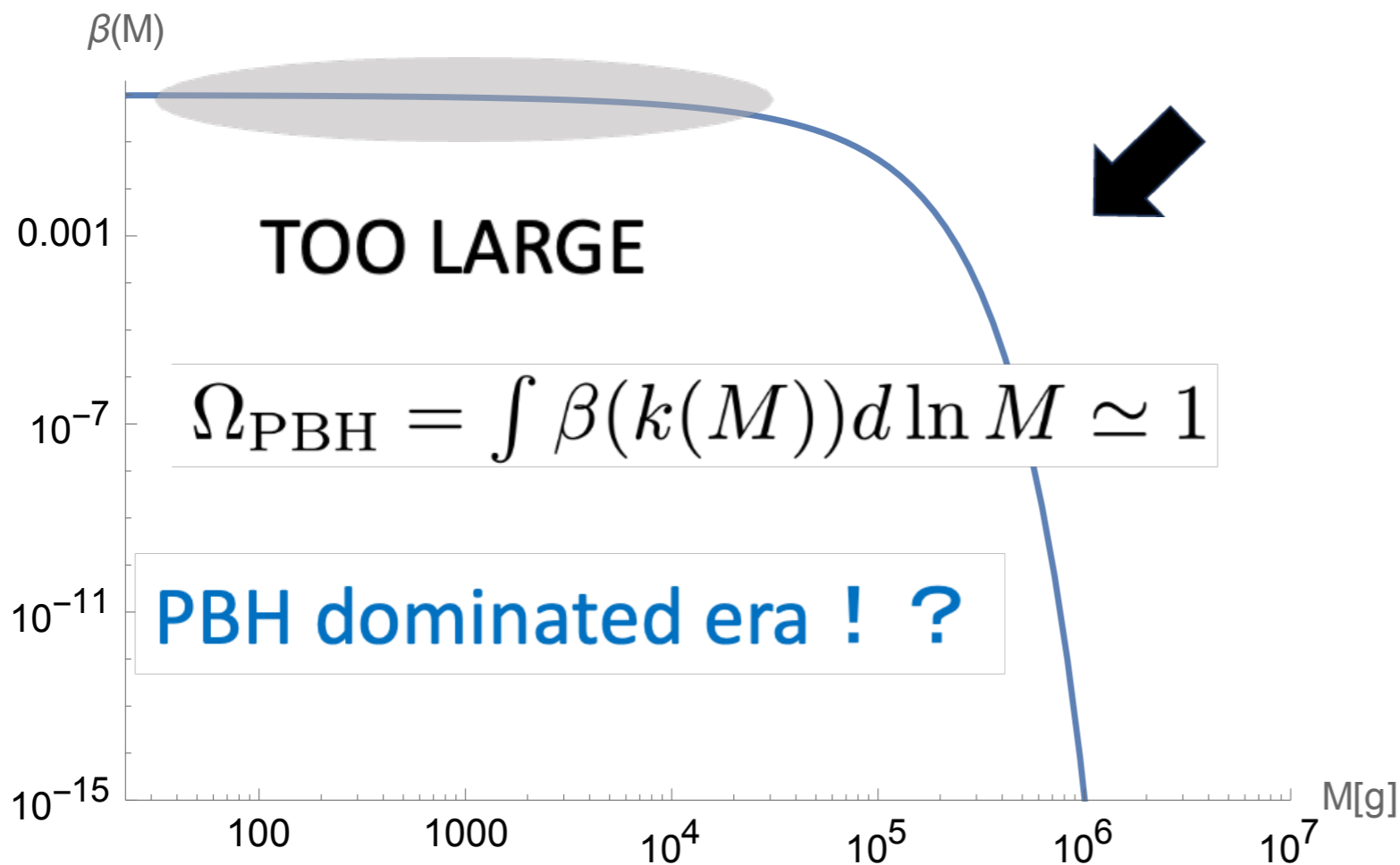
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Mass fraction of PBHs

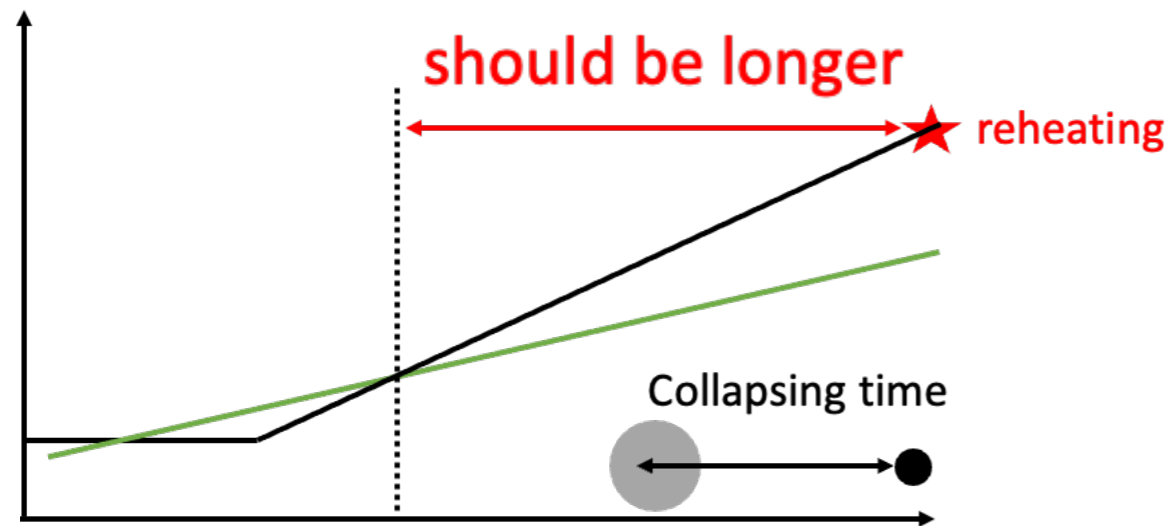
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Overproduction of PBHs during oscillation era

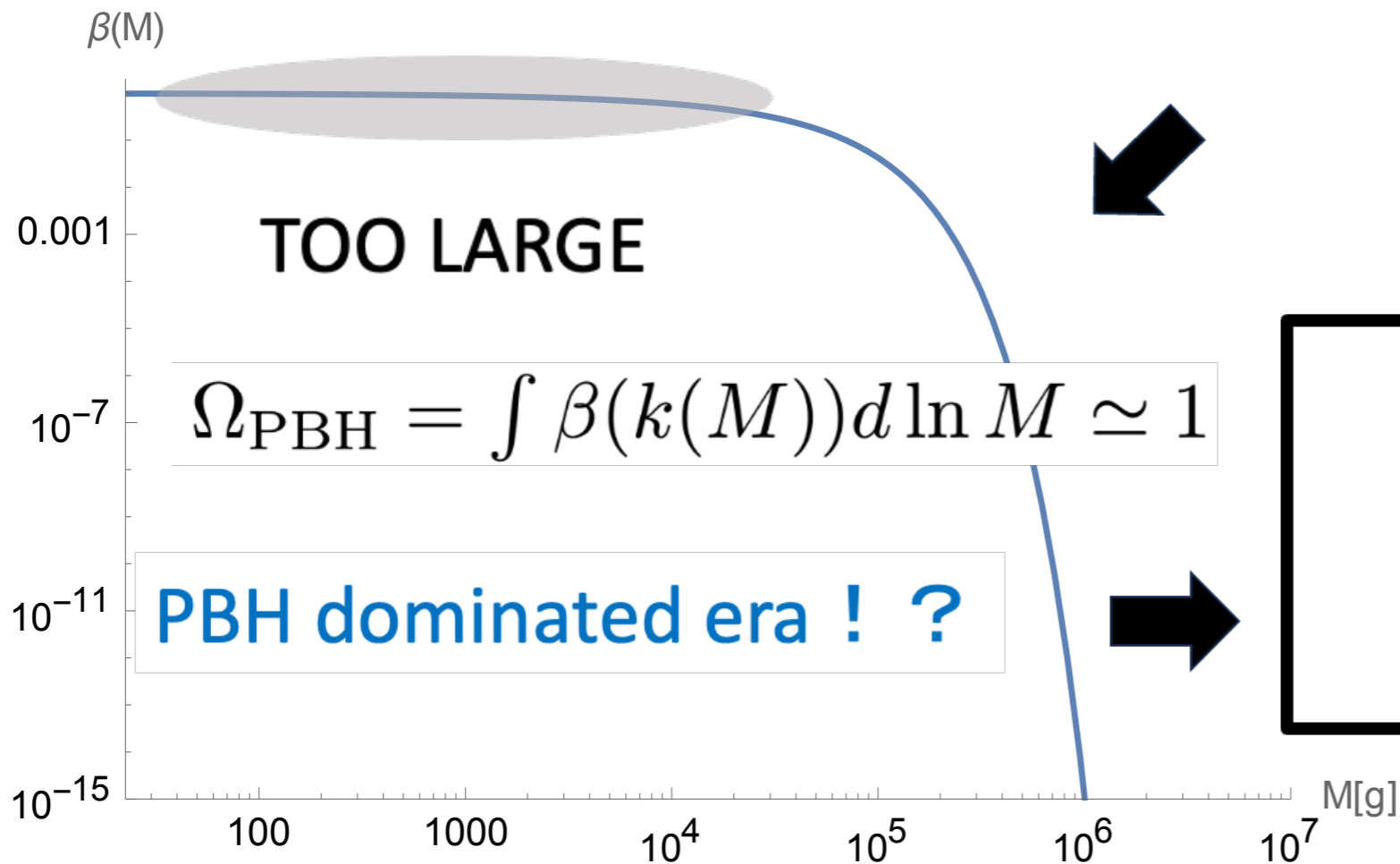
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Mass fraction of PBHs

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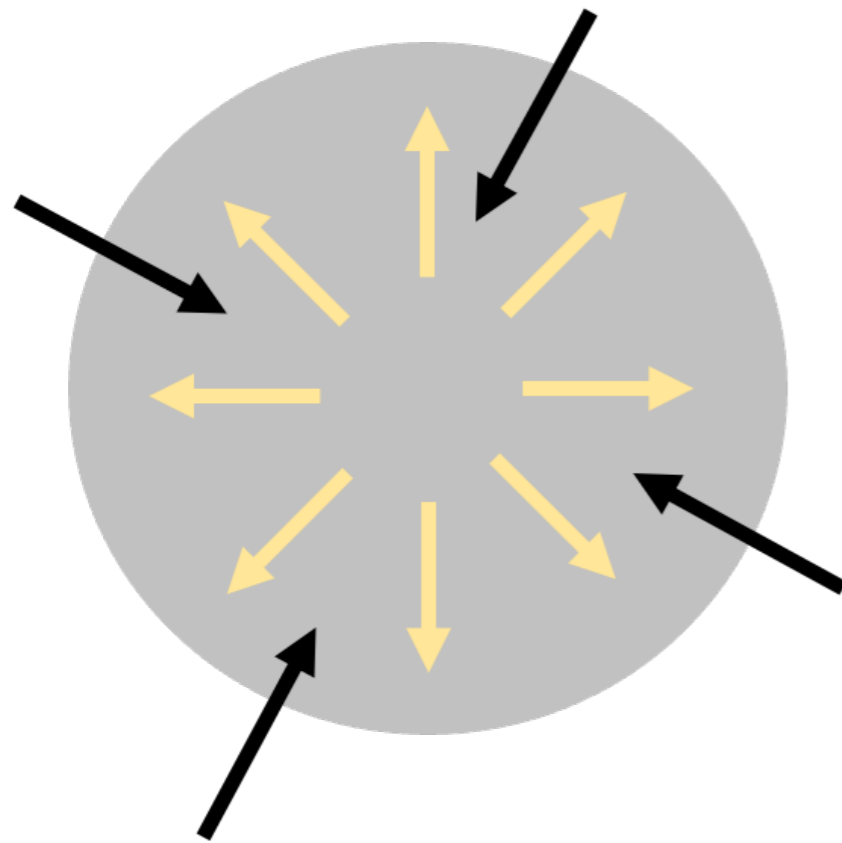


Other important effects:
Pancake collapse effect
 &
Spin effect

Our viewpoint

The evolution of the anisotropy

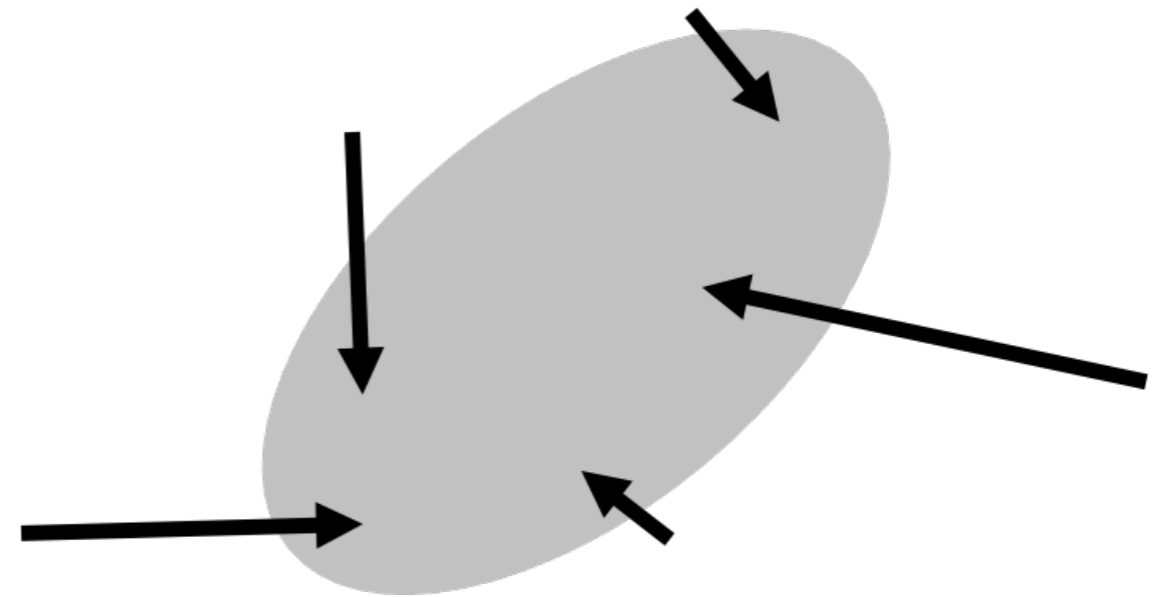
Radiation Dominated



Spherical Collapse

Matter Dominated

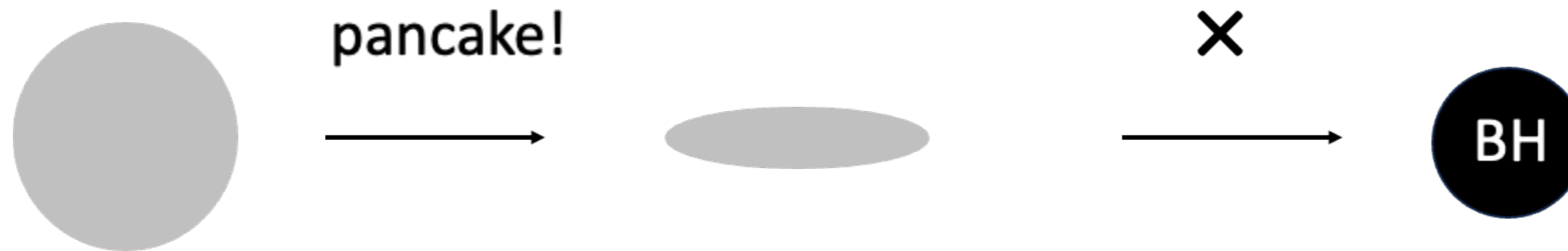
w/o Pressure gradient force



Non-Spherical Collapse

Pancake Collapse Effect

[Khlopov, Polnarev 1980; Harada, Yoo, Kohri, Jhingan 2016]



Hoop Conjecture [K. Thorne 1972]

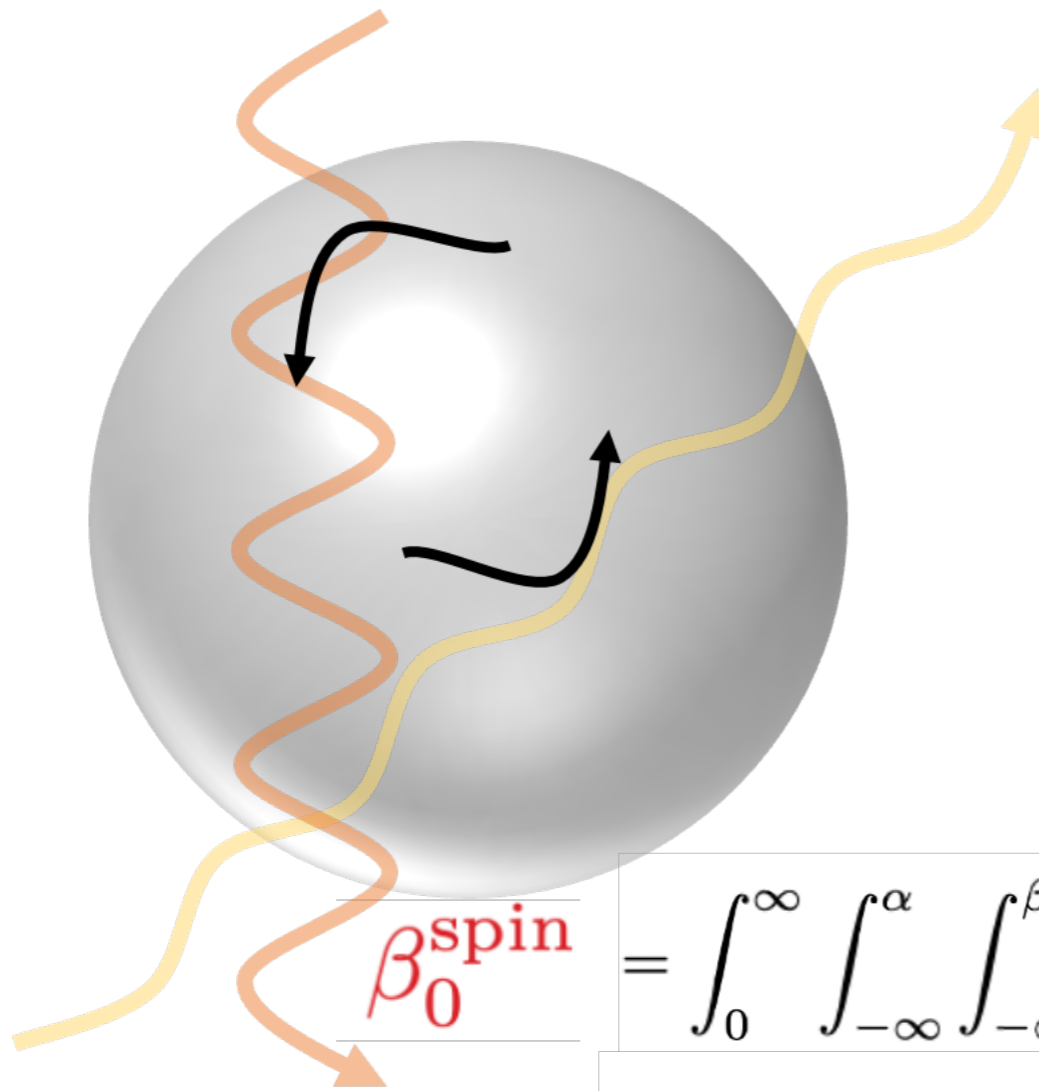
If \exists perimeter $> 2\pi \times$ (Schwarzschild radius) \rightarrow cannot collapse

$$\beta_0^{\text{pancake}} = \int_0^\infty d\alpha \int_{-\infty}^\alpha d\beta \int_{-\infty}^\beta d\gamma \theta(1 - h(\alpha, \beta, \gamma)) w(\alpha, \beta, \gamma)$$
$$\simeq 0.071 \sigma_H^5$$

$$\sigma_H = \sqrt{\mathcal{P}_\delta}$$

The Spin Effect

[Harada, Yoo, Kohri, Nakao 2017]



Two modes couples to produce the torque

Centrifugal force prevents the collapsing

Kerr parameter ≤ 1 determines the **threshold**

β_0^{spin}

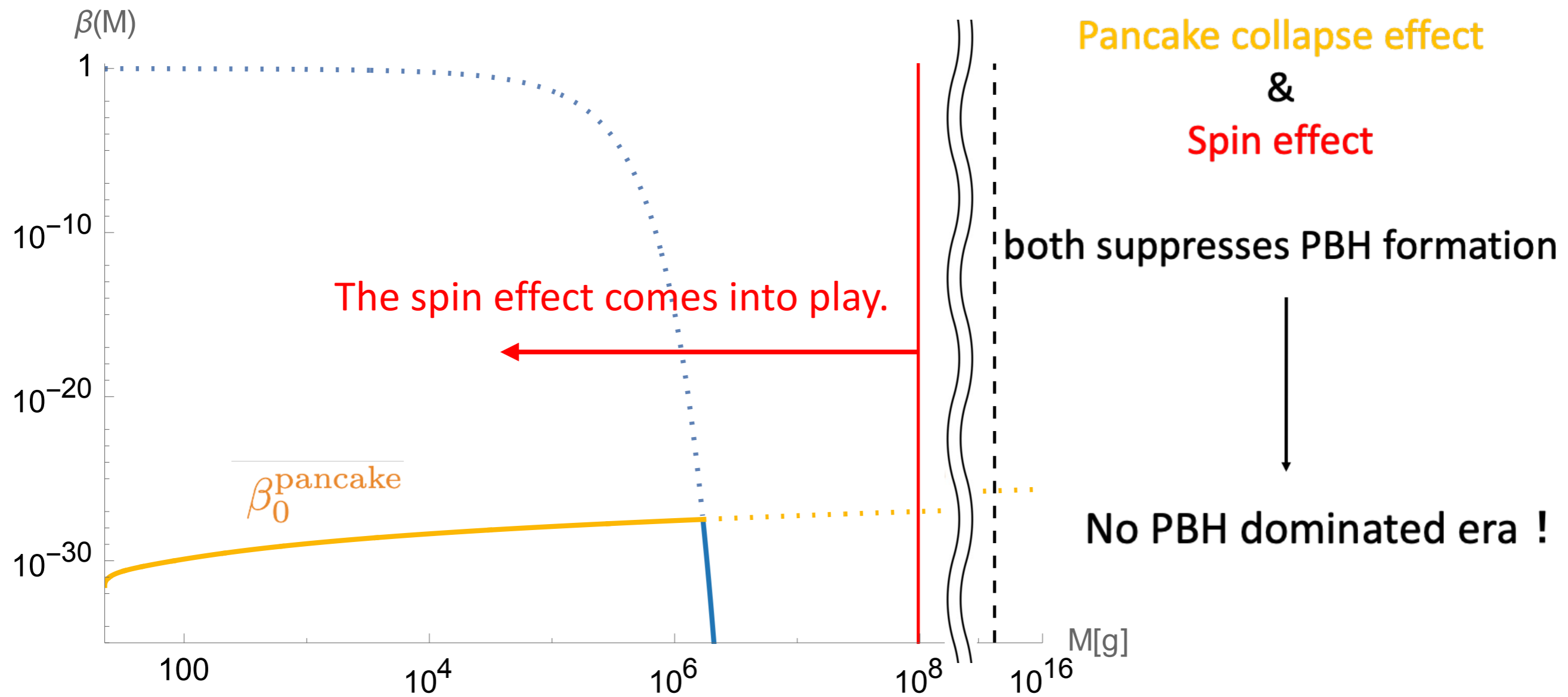
$$= \int_0^\infty \int_{-\infty}^\alpha \int_{-\infty}^\beta \theta(\delta_H(\alpha, \beta, \gamma) - \delta_{\text{th}}) \times \theta(1 - h(\alpha, \beta, \gamma)) w(\alpha, \beta, \gamma)$$

$$\simeq 1.921 \times 10^{-7} f_q(q_c) \mathcal{I}^6 \sigma_H^2 \times \exp \left[-0.1474 \times \frac{\mathcal{I}^{4/3}}{\sigma_H^{2/3}} \right]$$

$$\sigma_H \sim 10^{-5}$$

PBH production is suppressed very very very much!!!

Result: No PBH dominant era



Conclusion

MODEL: R^2 inflation model

FOCUS: The density perturbation @ inflaton-oscillation epoch

Previous
work

The possibility of PBH over-production

→ PBH dominated era [J. Martin, Papanikolaou, Venin 2020]

However

This
Talk

Pancake-like collapse effect
& Spin effect

We show the anisotropic effect
suppresses the production of PBH

→ NO PBH dominated era

$$T_{\text{rh}} \sim 10^9 \text{ GeV}$$

Appendix: SUSY Scenarios

Higher derivative SUGRA [Cecotti 1987; Ferrara & Porrati 2014]

R is the supercurvature

$$S = \int d^4x d^4\theta E (N(\mathcal{R}, \bar{\mathcal{R}}) + J(\phi, \bar{\phi} e^{gV})) \phi, V \text{ are the matter sector}$$

$$+ \left[\int d^4x d^2\Theta 2\mathcal{E} \left(F(\mathcal{R}) + P(\phi) + \frac{1}{4} h_{AB}(\phi) W^A W^B \right) + \text{H.c.} \right]$$

↓ duality trans. by T, S (T is the Lagrange multiplier)

$$S = \int d^4x d^2\Theta 2\mathcal{E} \frac{3}{8} (\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) e^{-K/3} + W + \frac{1}{4} h_{AB} W^A W^B + \text{H.c.}$$

Kahler pot: $K = -3 \ln \left(\frac{T + \bar{T} - N(S, \bar{S}) - J(\phi, \bar{\phi} e^{gV})}{3} \right),$

Superpot: $W = 2TS + F(S) + P(\phi).$

Starobinsky SUGRA R2 inflation

[Terada, YW, Yamada, Yokoyama 1411.6746; Dalianis & YW 1801.05736]

$$\mathcal{L} = -3M_P^2 \int d^4\theta E \left[1 - \frac{4}{m_\Phi^2} \mathcal{R}\bar{\mathcal{R}} + \frac{\zeta}{3m_\Phi^4} \mathcal{R}^2\bar{\mathcal{R}}^2 \right]$$

$$N(S, \bar{S}) = -3 + \frac{12}{m_\Phi^2} S\bar{S} - \frac{\zeta}{m_\Phi^4} (S\bar{S})^2$$

↓
S, ImT are stabilized.

$$F(S) = 0,$$

Real part of T becomes the inflaton Φ : $V = \frac{3m_\Phi^2}{4} \left(1 - e^{-\sqrt{2/3}\widehat{\text{Re}T}} \right)^2$

$$S = \int d^4x d^2\Theta 2\mathcal{E} \frac{3}{8} (\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) e^{-K/3} + W + \frac{1}{4} h_{AB} W^A W^B + \text{H.c.}$$

$$K = -3 \ln \left(\frac{T + \bar{T} - N(S, \bar{S}) - J(\phi, \bar{\phi} e^{gV})}{3} \right),$$

$$W = 2TS + F(S) + P(\phi).$$

↓
Grav. coupling to matter ϕ, V

Starobinsky SUGRA R2 inflation

[Terada, YW, Yamada, Yokoyama 1411.6746; Dalianis & YW 1801.05736]

$$\mathcal{L} = -3M_P^2 \int d^4\theta E \left[1 - \frac{4}{m_\Phi^2} \mathcal{R}\bar{\mathcal{R}} + \frac{\zeta}{3m_\Phi^4} \mathcal{R}^2\bar{\mathcal{R}}^2 \right]$$

$$N(S, \bar{S}) = -3 + \frac{12}{m_\Phi^2} S\bar{S} - \frac{\zeta}{m_\Phi^4} (S\bar{S})^2$$

$$F(S) = 0,$$

Real part of T becomes the inflaton Φ : $V = \frac{3m_\Phi^2}{4} \left(1 - e^{-\sqrt{2/3}\widehat{\text{Re}}T} \right)^2$

SUSY breaking field:

$$J(z, \bar{z}) = |z|^2 - \frac{|z|^4}{\Lambda^2},$$

$$P(z) = \mu^2 z + W_0,$$

Z may dominate after inflation.

Inflaton decay after SUGRA R2 inflation

[Terada, YW, Yamada, Yokoyama 1411.6746; Dalianis & YW 1801.05736]

Scalars:
$$\Gamma(T \rightarrow \phi^i \bar{\phi}^{\bar{i}}) = \frac{3m_i^4}{8\pi M_P^2 m_\Phi}, \quad \Gamma(T \rightarrow \phi^i \phi^j) = \frac{m_\Phi^3}{96\pi M_P^2} |J_{ij}|^2.$$

Fermions:
$$\Gamma(T \rightarrow \chi^i \bar{\chi}^{\bar{i}}) = \frac{m_i^2 m_\Phi}{192\pi M_P^2},$$

Gauge fields
& gauginos:

$$\Gamma(T \rightarrow AA) + \Gamma(T \rightarrow \lambda\lambda) \simeq \frac{3N_g \alpha^2 m_\Phi^3}{128\pi^3 M_P^2} \left(T_G - \frac{1}{3} T_R \right)^2$$

Gravitinos: (Φ is inflaton)

$$\Gamma(\Phi_{R\pm} \rightarrow \psi_{3/2} \psi_{3/2}) \simeq \frac{m_\Phi^3}{48\pi M_P^2} \times \begin{cases} 16 \left(\frac{m_{3/2}}{m_\Phi} \right)^2 & (m_z^2 \ll m_\Phi m_{3/2}) \\ \left(\frac{m_z}{m_\Phi} \right)^4 & (3m_\Phi m_{3/2} \ll m_z^2 \ll m_\Phi^2) \\ 1 & (m_\Phi^2 \ll m_z^2) \end{cases}$$

Constraints from gravitino abundance

[Terada, YW, Yamada, Yokoyama 1411.6746]

Gravitinos generated from:

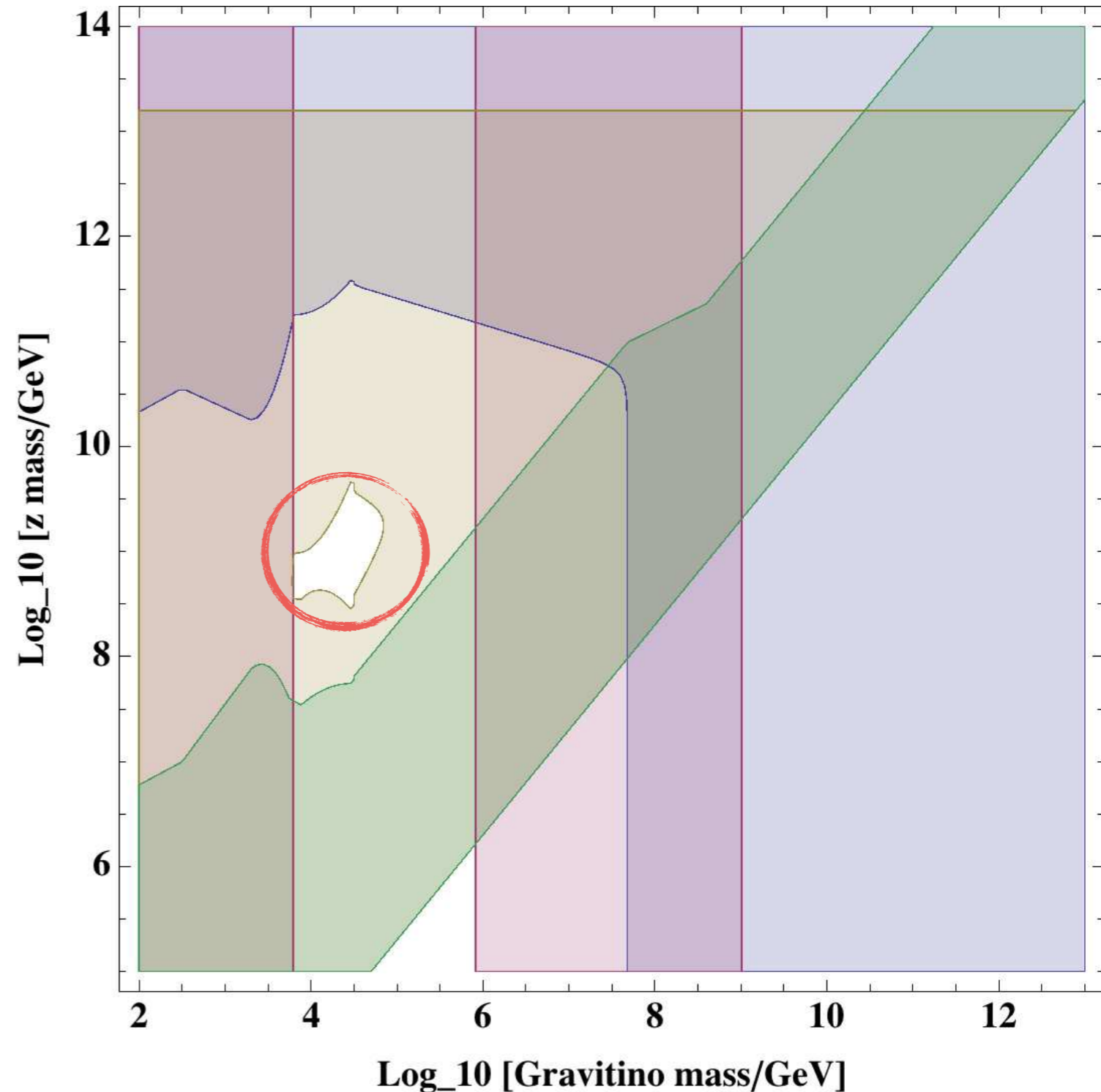
- 1) inflaton decay
- 2) thermal scatterings
- 3) decay of particles
- 4) decay of oscillating Z

Neutralino LSP (\sim TeV WIMP)

is assumed for:

gravitino mass $> 10^{4.5}$ GeV
→ anomaly mediation

gravitino mass $< 10^{4.5}$ GeV
→ gravity mediation

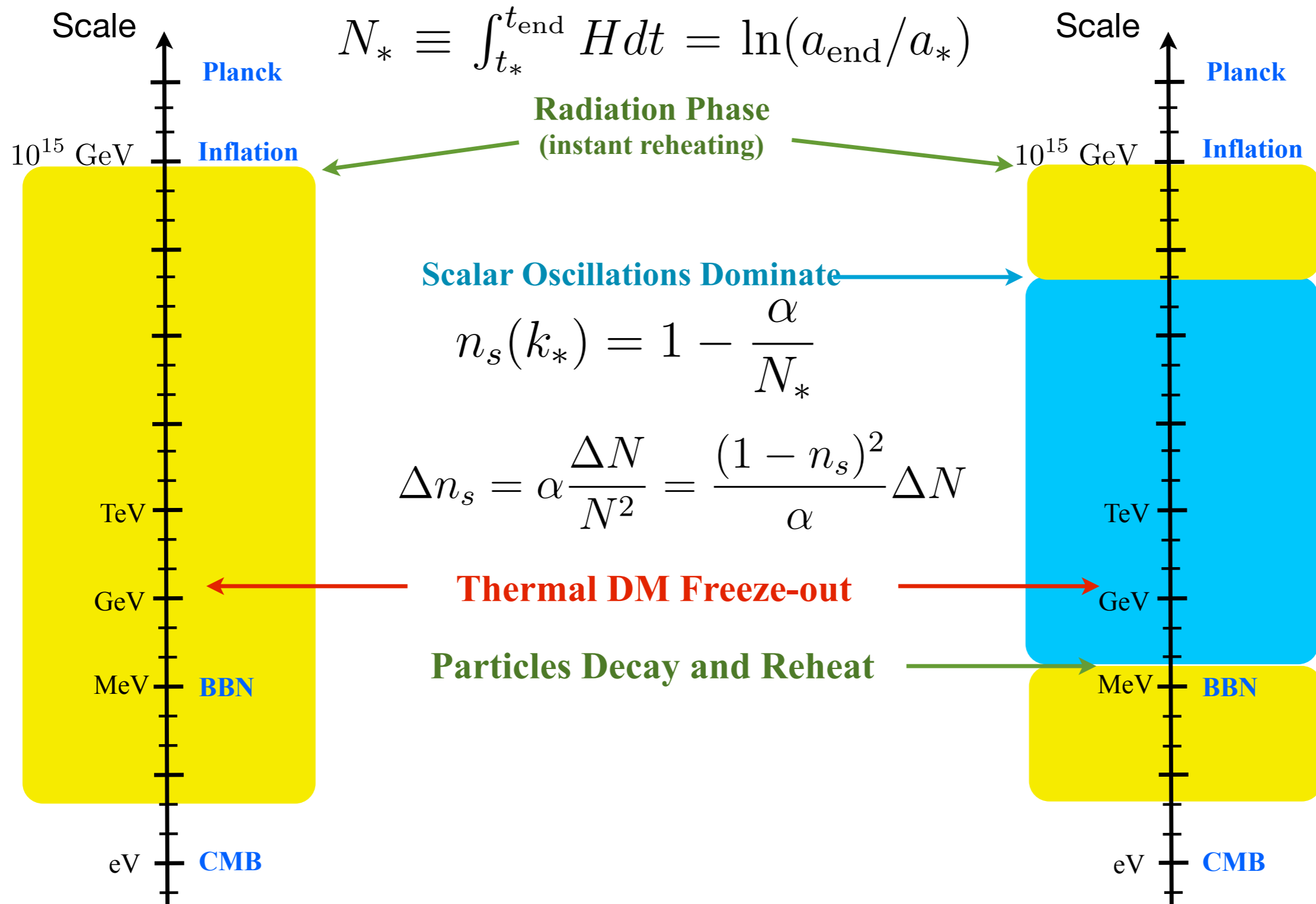


CMB uncertainties from the post-inflationary evolution

[Easter, Galvez, Ozsoy, Watson 2013]

Thermal History

Alternative History



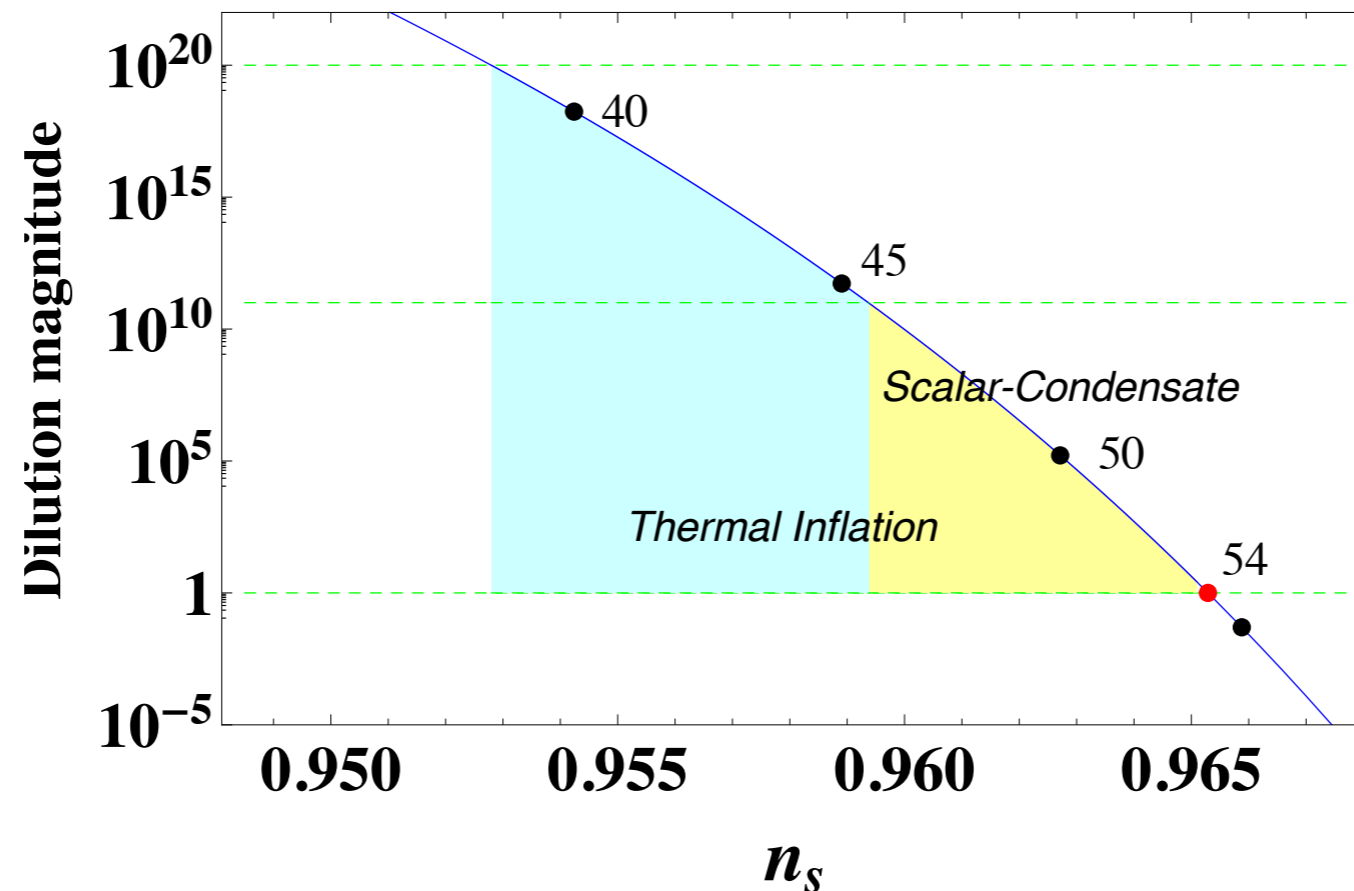
Shift in (n_s , r) due to late entropy production

- After inflaton decay, a diluter field X (modulus, flaton) may dominate the universe until BBN. Decays of X produce **entropy**:

$$\Delta N_X = \frac{1}{3} \ln \left[\left(\frac{g_*(T_X^{\text{dom}})}{g_*(T_X^{\text{dec}})} \right)^{1/4} D_X \right] \equiv \frac{1}{3} \ln \tilde{D}_X$$

$$D_X \equiv \frac{S_{\text{after}}}{S_{\text{before}}} = 1 + \frac{g_s(T_X^{\text{dec}})}{g_*(T_X^{\text{dec}})} \frac{g_*(T_X^{\text{dom}})}{g_s(T_X^{\text{dom}})} \frac{T_X^{\text{dom}}}{T_X^{\text{dec}}} \simeq \frac{T_X^{\text{dom}}}{T_X^{\text{dec}}} \geq 1$$

$$R^2, T_{rh} = 10^9 \text{ GeV}$$



Supersymmetric dark matter scenarios

Merits: Gauge coupling unification, stable dark matter, baryogenesis, stringy UV completion, ...

1. Gravitino LSP

2. Neutralino LSP (WIMP)

- Thermal DM (freeze out): thermal scatterings with the MSSM, messenger fields
- Non-thermal DM (freeze in): decays, thermal scatterings

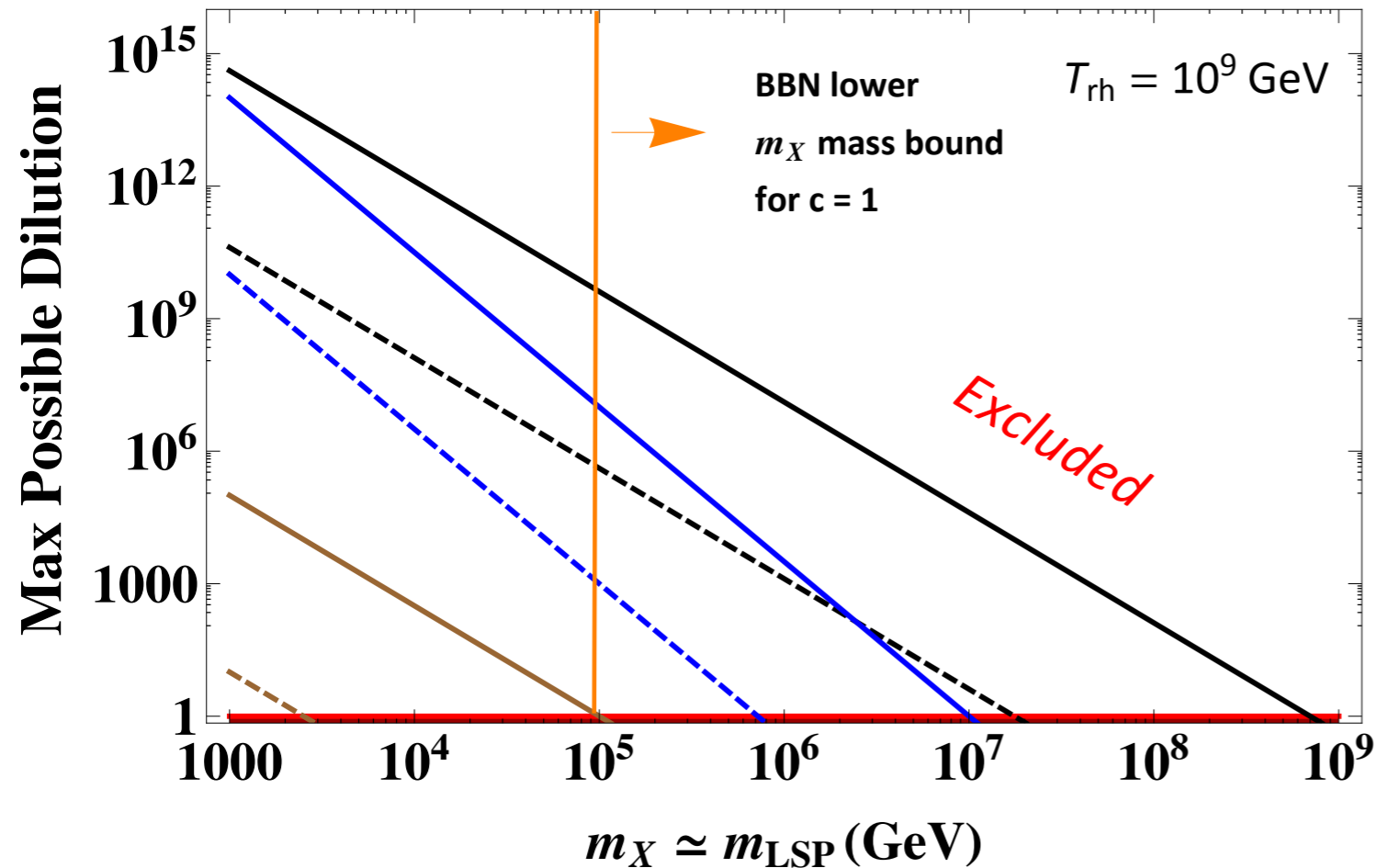
Light WIMP mass is disfavored by the LHC.

$\Omega_{\text{DM}} h^2$ is severely constrained when **sparticle masses increase**:

$$\Omega_{3/2} \propto m_{3/2}^\alpha \left(\frac{m_{\tilde{g}}}{m_{3/2}} \right)^\beta \left(\frac{m_{\tilde{f}}}{m_{3/2}} \right)^\gamma T_{\text{rh}}^\delta, \quad m_{3/2} < m_{\tilde{g}}, m_{\tilde{f}},$$

$$\Omega_{\tilde{\chi}^0} \propto m_{\tilde{\chi}^0}^{\tilde{\alpha}} m_{3/2}^{\tilde{\beta}} \left(\frac{m_{\tilde{f}}}{m_{3/2}} \right)^{\tilde{\gamma}} T_{\text{rh}}^{\tilde{\delta}}, \quad m_{\tilde{\chi}^0} < m_{3/2}, m_{\tilde{f}}$$

Alternative cosmic histories and SUSY



—	Dilution Bound
—	$c=1$, Gravitino LSP
- -	$c=10^8$, $\gg \gg$
—	$c=1$, Thermal Neutralino LSP
- -	$c=10^8$, $\gg \gg$
—	$c=1$, Thermal Gravitino LSP
- -	$c=10^8$, $\gg \gg$

$$\Gamma_X = \frac{c}{4\pi} \frac{m_X^3}{M_{\text{Pl}}^2}$$

★ High reheating temp. generally overproduce light LSP

→ Dilution of DM abundance is necessary: **diluter field X**

• If $D_X = 1$ then $T_{\text{rh}} \lesssim \tilde{m}$ or $\tilde{m} \sim \text{TeV}$

• If $\mathcal{O}(\text{TeV}) < (m_{\text{LSP}}, \tilde{m}) < T_{\text{rh}}$ then $D_X \geq D_X^{\text{min}} \equiv \frac{\Omega_{\text{LSP}}^{\leq}}{0.12 h^{-2}}$

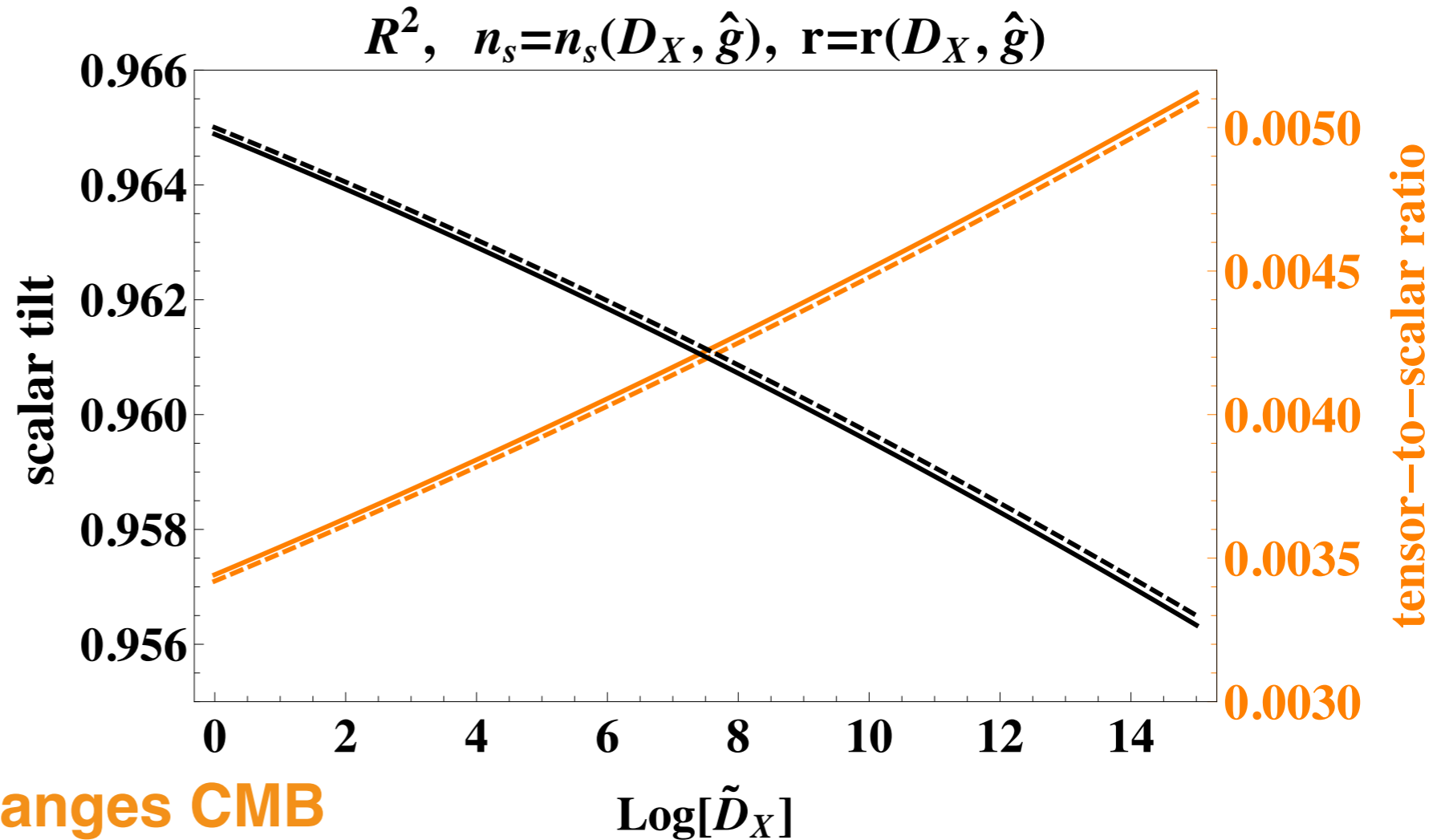
where \tilde{m} the sparticle mass scale.

CMB observables: SUGRA R2 inflation

$$n_s^{(\text{th})} \Big|_{R^2} = 0.965,$$

$$r^{(\text{th})} \Big|_{R^2} = 0.0034$$

$$N^{(\text{th})} = 54$$



★ Dilution factor D_X changes CMB

observables: $\Delta N_X = \frac{1}{3} \ln \tilde{D}_X$

$$N_* \Big|_{R^2} = 55.9 + \frac{1}{4} \ln \epsilon_* + \frac{1}{4} \ln \frac{V_*}{\rho_{\text{end}}} + \frac{1}{12} \ln \left(\frac{g_{*\text{rh}}}{100} \right) + \frac{1}{3} \ln \left(\frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right) - \Delta N_X$$

CMB observables: SUGRA R2 inflation

[Dalianis & YW 1801.05736]

$$\mathcal{L} = -3M_P^2 \int d^4\theta E \left[1 - \frac{4}{m^2} \mathcal{R}\bar{\mathcal{R}} + \frac{\zeta}{3m^4} \mathcal{R}^2\bar{\mathcal{R}}^2 \right] + \text{MSSM, Z, X, (messengers)}$$

Gravitino DM (in GeV units)

#	m_Z	$m_{\tilde{g}}$	$m_{\tilde{f}}$	$m_{3/2}$ (LSP)	D_X	N_*	n_s	r	Origin
Alternative history after reheating									
4	10^3	10^3	10^4	10	1	54	0.965	0.0034	Th

Neutralino (WIMP) DM

#	m_Z	$m_{3/2}$	$m_{\tilde{f}}$	$m_{\tilde{\chi}^0}$ (LSP)	$D_{(X)}$	N_*	n_s	r	Origin
Alternative history after reheating									
4	10^5	10^5	10^5	10^3	1	54	0.965	0.0034	Th

CMB observables: SUGRA R2 inflation

[Dalianis & YW 1801.05736]

$$\mathcal{L} = -3M_P^2 \int d^4\theta E \left[1 - \frac{4}{m^2} \mathcal{R}\bar{\mathcal{R}} + \frac{\zeta}{3m^4} \mathcal{R}^2\bar{\mathcal{R}}^2 \right] + \text{MSSM, Z, X, (messengers)}$$

Gravitino DM (in GeV units)

#	m_Z	$m_{\tilde{g}}$	$m_{\tilde{f}}$	$m_{3/2}$ (LSP)	D_X	N_*	n_s	r	Origin
1	10^4	10^4	10^4	10^2	$10^4 _{\min}$	$51 _{\max}$	0.963 $ _{\max}$	0.0038 $ _{\min}$	Th
2	10^4	10^4	10^5	10^3	$10^{10} _{\min}$	$46 _{\max}$	0.960 $ _{\max}$	0.0044 $ _{\min}$	Th
3	10^6	10^5	10^6	10^4	$10^6 _{\min}$	$49 _{\max}$	0.962 $ _{\max}$	0.0041 $ _{\min}$	Non-th
4	10^3	10^3	10^4	10	1	54	0.965	0.0034	Th

Neutralino (WIMP) DM

#	m_Z	$m_{3/2}$	$m_{\tilde{f}}$	$m_{\tilde{\chi}^0}$ (LSP)	$D_{(X)}$	N_*	n_s	r	Origin
1	10^7	10^6	10^6	10^3	$10^2 _{\min}$	$52 _{\max}$	0.964 $ _{\max}$	0.0036 $ _{\min}$	Non-th
2	10^9	10^8	10^8	10^3	$10^2 _{\min}$	$52 _{\max}$	0.964 $ _{\max}$	0.0036 $ _{\min}$	Th
3	10^8	10^7	10^7	10^5	$10^8 _{\min}$	$48 _{\max}$	0.961 $ _{\max}$	0.0042 $ _{\min}$	Non-th
4	10^5	10^5	10^5	10^3	1	54	0.965	0.0034	Th

CMB observables: SUGRA R2 inflation

[Dalianis & YW 1801.05736]

