

Recent developments in low dimensional holography

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I. Introduction

Low dimensional holography plays a very important role in **black hole physics**, as well as in the study of **conformal defects**

The high dimensionality of the internal space implies a rich structure of geometries and topologies (and supersymmetries)

Therefore, classifying AdS3 and AdS2 spaces is especially challenging

For **AdS3**:

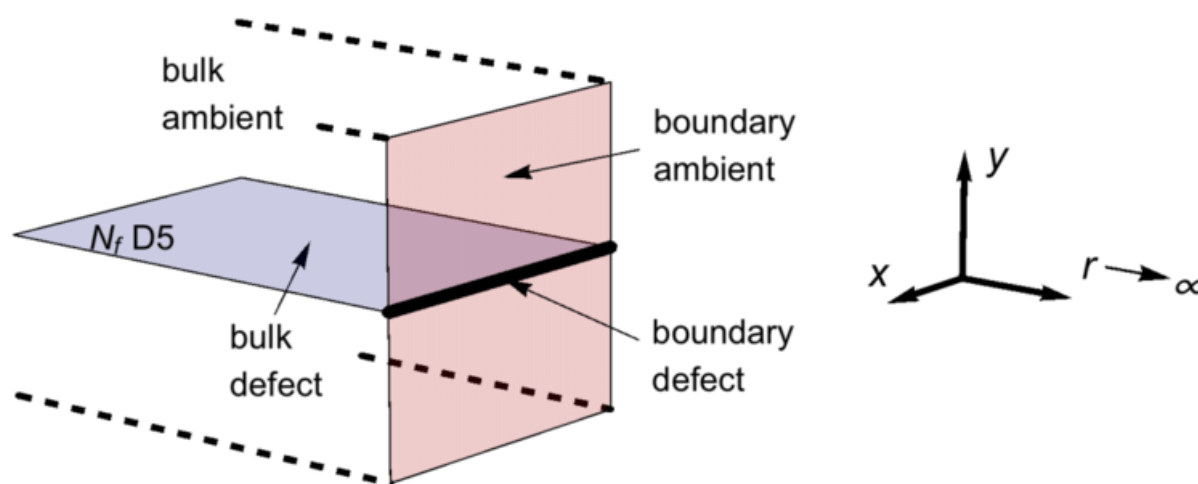
Partial classifications of $(0,8)$, $(4,4)$, $(0,4)$, $(3,3)$, $(0,3)$, $(2,2)$, $(0,2)$, $(1,1)$...

Bachas, Chiodaroli, Couzens, D'Hoker, Dibitetto, Estes, Gauntlett, Gutperle, Kim, Krym, Legramandi, Lo Monaco, Lawrie, Lozano, Macpherson, Martelli, Nunez, O Colgain, Passias, Petri, Ramirez, Schafer-Nameki, Tomasiello, Waldram..

More recently: $(0,5)$ and $(0,6)$: Macpherson, Ramirez'23

In this talk we will focus on the CFT interpretation of some classes of solutions to massive IIA and their possible relation to **conformal defects**

These are defects that preserve a subgroup of the superconformal group of the system where they are embedded



Thus, holography provides a very powerful tool for their study

In recent years many low dimensional AdS solutions have been proposed as dual to defect CFTs

In some of these realisations it has been possible to embed the defect CFT within the higher d theory through explicit quiver-like constructions

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In some of these realisations it has been possible to embed the defect CFT within the higher d theory through explicit quiver-like constructions

In this talk we will focus on **AdS3 solutions with (0,6) and (0,4) supersymmetries**

Based on Y.L., Macpherson, Petri, Ramirez'24

Conti, Dibitetto, Y.L., Petri, Ramirez'24

Outline

1. The class of $(0,6)$ $AdS_3 \times CP^3 \times I$ solutions to massive IIA:

Surface defects in ABJM/ABJ

2. Two classes of $(0,4)$ $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA:

Deconstruction and surface defects in 6d $(1,0)$ CFTs

2. New AdS_3 solutions to massive IIA with (0,6) SUSY

(Macpherson, Ramirez'23)

$AdS_3 \times CP^3 \times I$ solutions to massive IIA with (0,6) SUSY
and $OSp(6|2)$ superconformal group \rightarrow $SO(6)$ R-symmetry

Can be regarded as an **extension of ABJM/ABJ to the massive case**, in which one of the external directions becomes an energy scale, and generates a flow towards an AdS_3 space

$$ds^2 = \frac{h}{\sqrt{2hh'' - h'^2}} ds^2_{AdS_3} + \sqrt{2hh'' - h'^2} \left(\frac{1}{h} dr^2 + \frac{1}{h''} ds^2_{CP^3} \right)$$

$$e^\phi = \frac{(2hh'' - h'^2)^{1/4}}{(h'')^{3/2}} \quad B_2 = 4\pi \left(-(r - l) + \frac{h'}{h''} \right) J$$

Specified by $h(r)$, that satisfies the Bianchi identity:

$$h''' = -2\pi F_0 \quad \text{with } F_0 \text{ the Romans mass}$$

Construct global solutions by glueing local solutions with D8-branes:

$$h_l(r) = Q_2^l - Q_4^l(r - l) + \frac{1}{2}Q_6^l(r - l)^2 - \frac{1}{6}Q_8^l(r - l)^3$$

with Q_p^l the Page charges at $r \in [l, l + 1]$

NS5-branes located at $r = l$

Hanany-Witten brane set-up:

$\otimes N_8^{[0,1]D8}$ $N_2^{[0,1]D2}$	$\otimes N_8^{[1,2]D8}$ $N_2^{[1,2]D2}$	⋯
$N_6^{[0,1]D6}$ $\otimes N_4^{[0,1]D4}$	$N_6^{[1,2]D6}$ $\otimes N_4^{[1,2]D4}$	

The presence of the CP^3 makes it more complicated

Massless limit:

A change of variables $\sinh \mu = \frac{Q_6 r - Q_4}{\sqrt{2Q_2 Q_6 - Q_4^2}} \rightarrow$ **ABJM**

with $ds_{AdS_4}^2 = d\mu^2 + \cosh^2 \mu ds_{AdS_3}^2$

$B_2 = -4\pi \frac{Q_4}{Q_6} J$ **Discrete holonomy of ABJ**

Brane set-up:

branes	x^0	x^1	r	x^3	x^4	x^5	ψ	x^7	x^8	x^9
$N D3$	×	×	×	—	—	—	×	—	—	—
$NS5'$	×	×	×	×	×	×	—	—	—	—
$(1, k)5'$	×	×	×	$\cos \theta$	$\cos \theta$	$\cos \theta$	—	$\sin \theta$	$\sin \theta$	$\sin \theta$

When one takes into account the Freed-Witten anomaly and the higher curvature terms (Aharony, Hashimoto, Hirano, Ouyang'09):

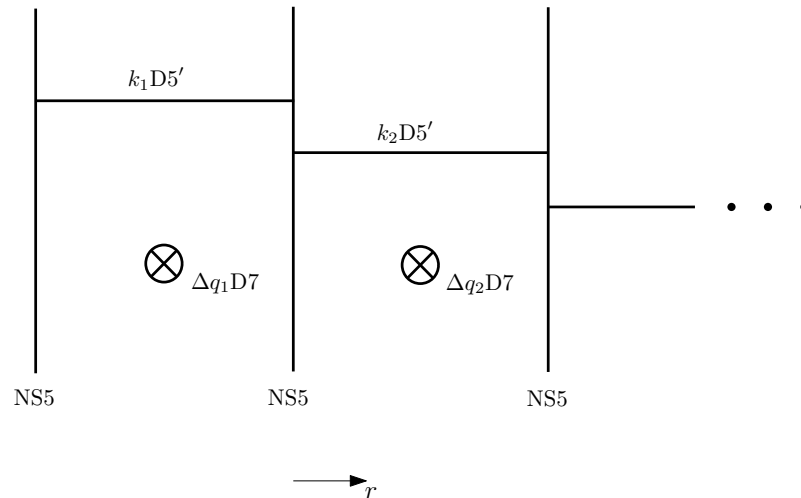
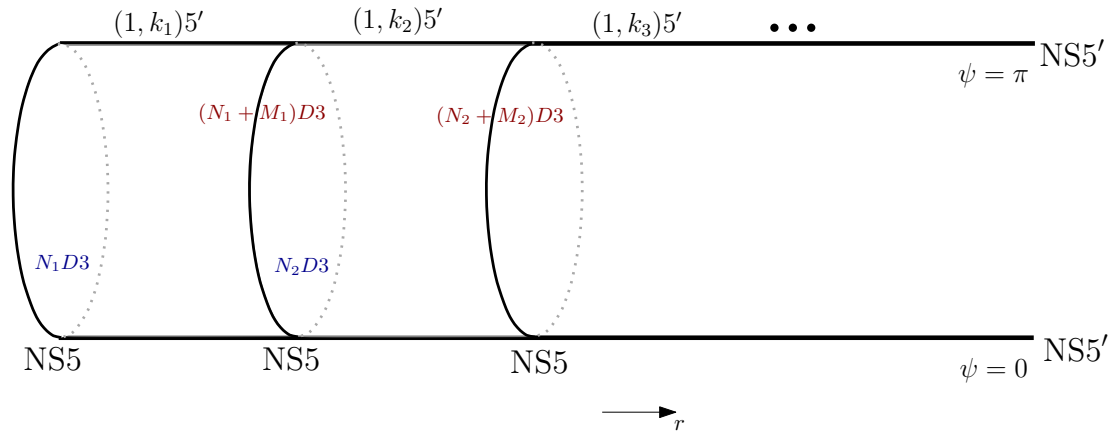
$$Q_2 = N + \frac{k}{12} \quad Q_4 = M - \frac{k}{2} \quad Q_6 = k$$

In the massive case:

branes	x^0	x^1	r	x^3	x^4	x^5	ψ	x^7	x^8	x^9
$D3$	×	×	×	—	—	—	×	—	—	—
$NS5'$	×	×	×	×	×	×	—	—	—	—
$(1, k)5'$	×	×	×	$\cos \theta$	$\cos \theta$	$\cos \theta$	—	$\sin \theta$	$\sin \theta$	$\sin \theta$
$D7$	×	×	—	×	×	×	—	×	×	×
$NS5$	×	×	—	—	—	—	×	×	×	×

Extra $D7$ - $NS5$, or $D8$ - $NS5$ *defect* branes render the field theory two dimensional and $(0,4)$ supersymmetric (in IIB) or $(0,6)$ (in IIA)

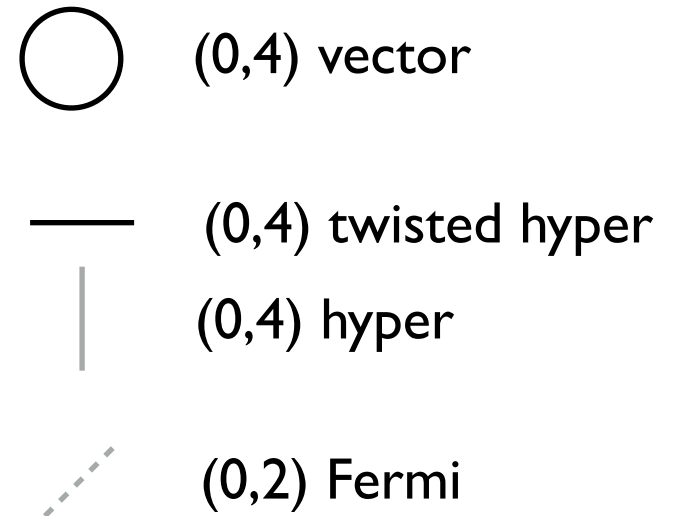
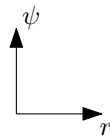
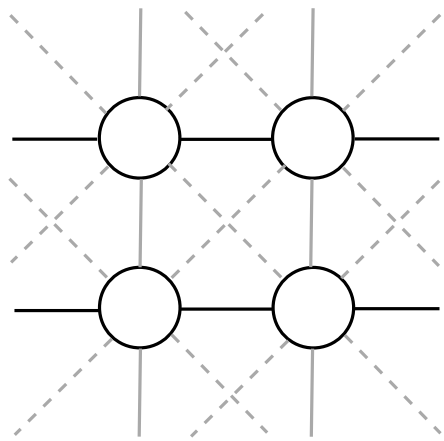
The brane box is $(0,3)$ SUSY, due to the rotations of the NS5'. This should be enhanced to $(0,4)$ in the IR, as happens in ABJM

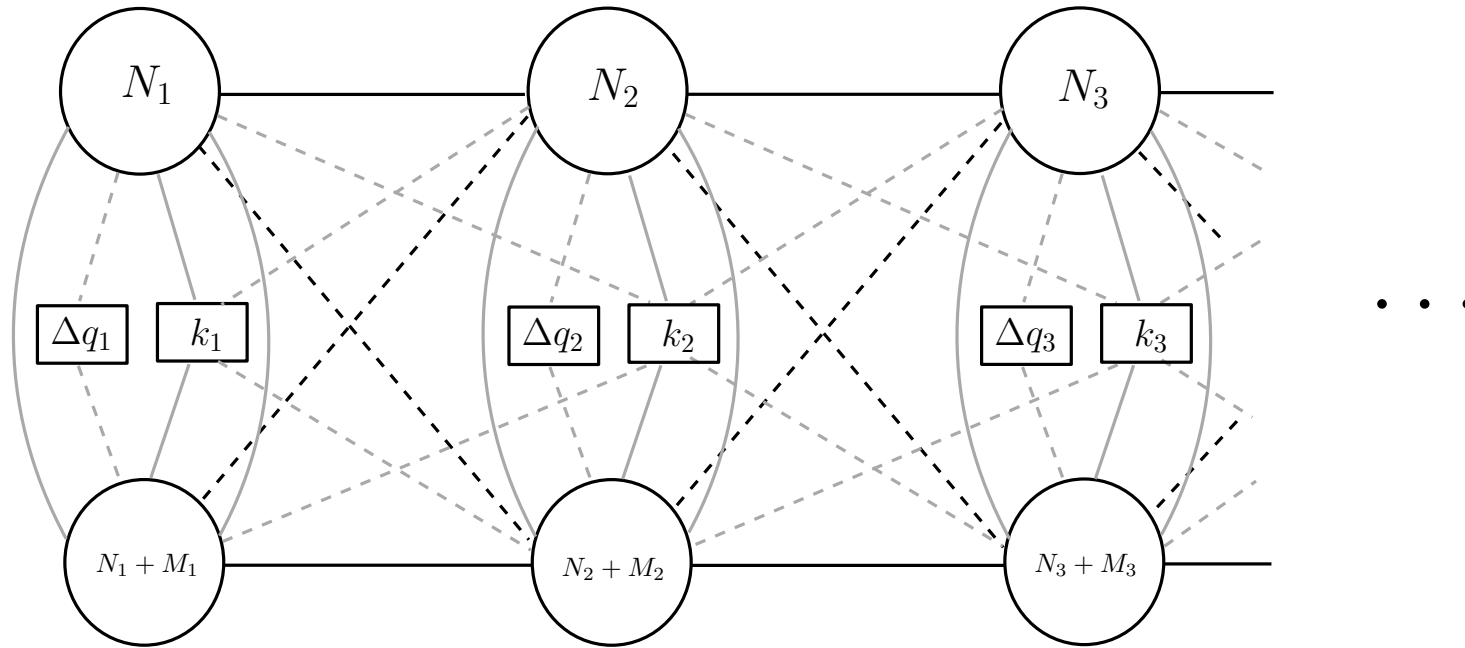


As in 3d, the $(0,3)$ gauge theory living in the D3-branes is expected to have the same field content as that of a $(0,4)$ gauge theory, except for the deformations introduced by the rotations of the 5'-branes, that will render some multiplets massive

The field theory can then be studied using what is known for $(0,4)$ brane box models (Hanany, Okazaki'18):

Building blocks:





Embedding of D8-NS5 defect branes within the ABJM 3d quiver

Gauge anomaly is cancelled with the quantised charges obtained from the solutions

In each interval (Bergman, Lifschytz'10):

$$Q_2 = N + \frac{k}{12} \quad Q_4 = M - \frac{k}{2} + \frac{q}{12} \quad Q_6 = k \quad Q_8 = -q$$

Transformations of charges across intervals (brane creation):

$$N \rightarrow N - M + k \quad M \rightarrow M - k \quad k \rightarrow k + q$$

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$$N \rightarrow N - M + k \quad M \rightarrow M - k \quad k \rightarrow k + q$$

Extension of **Seiberg duality** in ABJ for non-vanishing mass!

$$U(N + M)_k \times U(N)_{-k+q} \rightarrow U(N)_{k+q} \times U(N - M + k)_{-k}$$

Derived in Bergman, Lifschytz'10 in a non-supersymmetric setting

Seiberg duality as a **large gauge transformation** (Benini, Canoura Cremonesi, Nunez, Ramallo'07) albeit in one dimension less

When the **D8-branes** are embedded in the **3d theory** in a supersymmetric way, which implies adding as well NS5-branes, one of the 3d field theory directions turns into an energy scale, and generates a **flow towards a 2d CFT**

Geometrically the backreaction of the D8-NS5 branes gives rise to a (0,6) supersymmetric $AdS_3 \times CP^3$ geometry where r becomes part of the internal space

In this geometric setting it is now possible to perform **large gauge transformations** along the r direction that precisely realise the extension of **Seiberg duality** to the massive case proposed by Bergman, Lifschytz

Prediction for the central charge from holography:

$$c_{hol} = \frac{3R}{2G_3} = \int dr \left(2hh'' - (h')^2 \right)$$

Recover in field theory from (0,3) multiplets? Extremization?

Interesting to address!

Conclusions

Field theory interpretation of AdS3 solutions with (0,6) SUSY, in particular in connection to surface defects

Extension of ABJM/ABJ to the massive case, in which one of the external directions becomes an energy scale, and generates a flow towards an AdS3 space

Explicit quivers realising the embedding of the surface defects in the 3d quiver associated to ABJM/ABJ

Realisation of Seiberg-duality as a large gauge transformation

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THANKS!!

3. New $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA with (0,4) SUSY (small)

$$ds^2 = \sqrt{\frac{\alpha}{-\alpha''}} \left(x^2 (ds_{AdS_3}^2 + ds_{S^3}^2) + \frac{dx^2}{\sqrt{c+x^4}} \right) + \frac{\sqrt{c+x^4}}{x^2} \sqrt{\frac{-\alpha''}{\alpha}} \left(dz^2 + \frac{\alpha^2 x^4}{\Delta} ds_{S^2}^2 \right)$$

$$\Delta = x^4 (\alpha'^2 - 2\alpha\alpha'') - 2c\alpha\alpha''$$

$\alpha(z)$ satisfies the Bianchi identity $\alpha(z)''' = -162\pi^3 F_0$

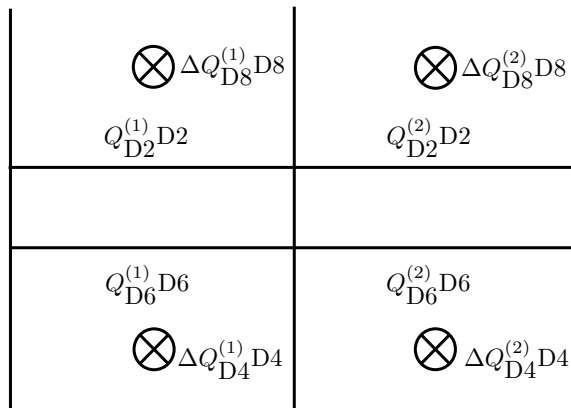
Non-vanishing B_2 such that: one NS5 at $z = l$

	x^0	x^1	ρ	θ^1	θ^2	θ^3	z	ζ	φ^1	φ^2
D2	x	x					x			
D4	x	x						x	x	x
NS5	x	x	x	x	x	x				
D6	x	x	x	x	x	x	x			
D8	x	x	x	x	x	x		x	x	x

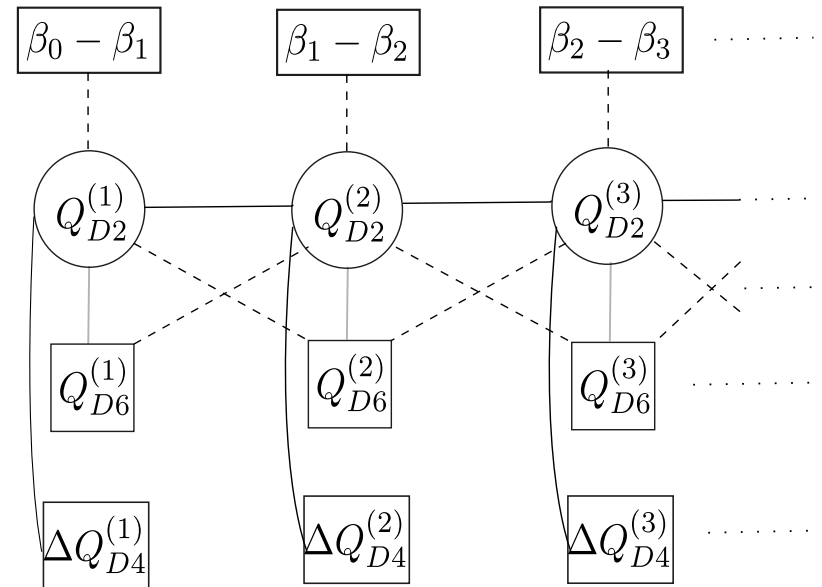
D2-D4 defect branes in 6d (1,0) theories living in NS5-D6-D8 brane intersections?

The solutions asymptote locally to $AdS_7 \times S^2 \times I$

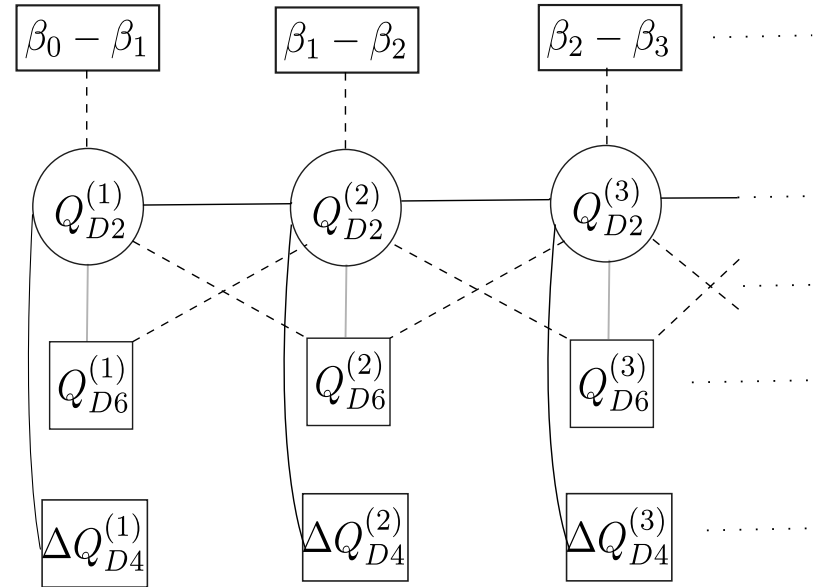
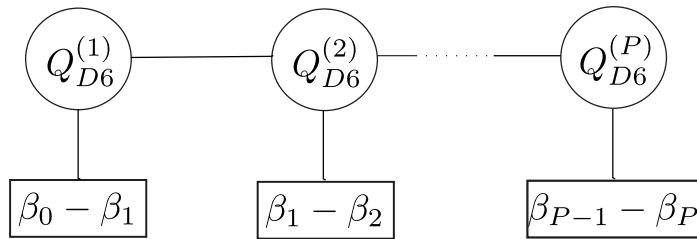
Hanany-Witten brane set-up and associated quiver:



.....



Explicit embedding of D2-D4 branes in 6d quivers:



However, computation of central charge:

$$c_{hol}^{2d} = \int dz (-\alpha \alpha'') \int_0^{\tilde{\Lambda}} dx x^3 \quad \text{and} \quad c_{hol}^{6d} = \int dz (-\alpha \alpha'') \int_{\Lambda}^{\frac{\pi}{2}} d\beta \frac{\cot^3 \beta}{\sin^2 \beta}$$

with
$$ds_{AdS_7}^2 = \frac{1}{\sin^2 \beta} \left(ds_{AdS_3}^2 + \cos^2 \beta ds_{S^3}^2 + d\beta^2 \right)$$

exactly agree with $\cot \Lambda = \tilde{\Lambda}$

Indeed,

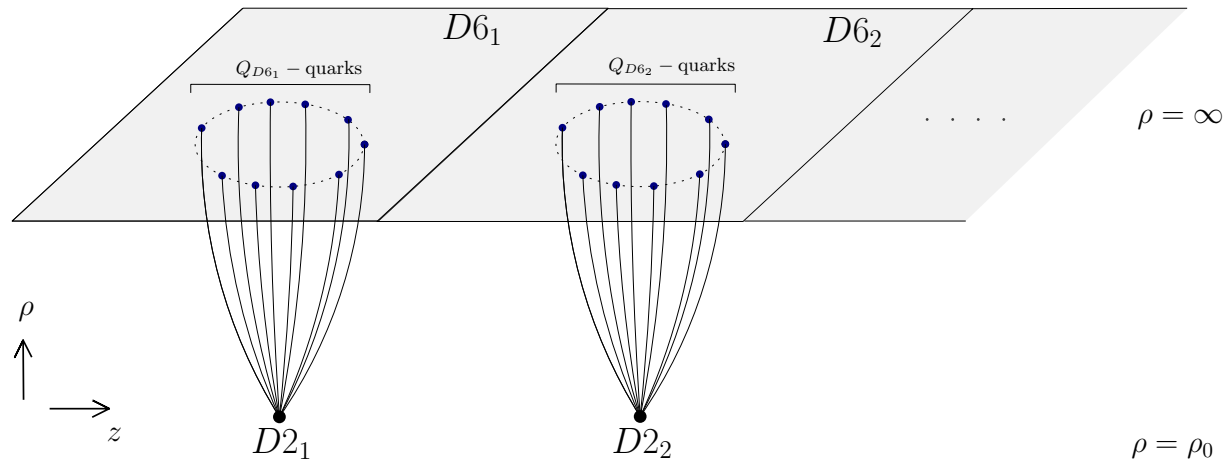
$$ds_7^2 = x^2 \left(\frac{dx_{1,1}^2 + du^2 + u^2 ds_{S^3}^2}{u^2} \right) + \frac{dx^2}{\sqrt{c + x^4}} = x^2 \frac{dx_{1,5}^2}{u^2} + \frac{dx^2}{\sqrt{c + x^4}},$$

⇒ Localised 6d theory

Analyse carefully the contribution of the defects using the prescription in Estes et al'14; Gentle, Gutperle, Marasinou'15;

Study other configurations, such as baryon vertices and giant gravitons

Baryon vertex in the 6d theory:



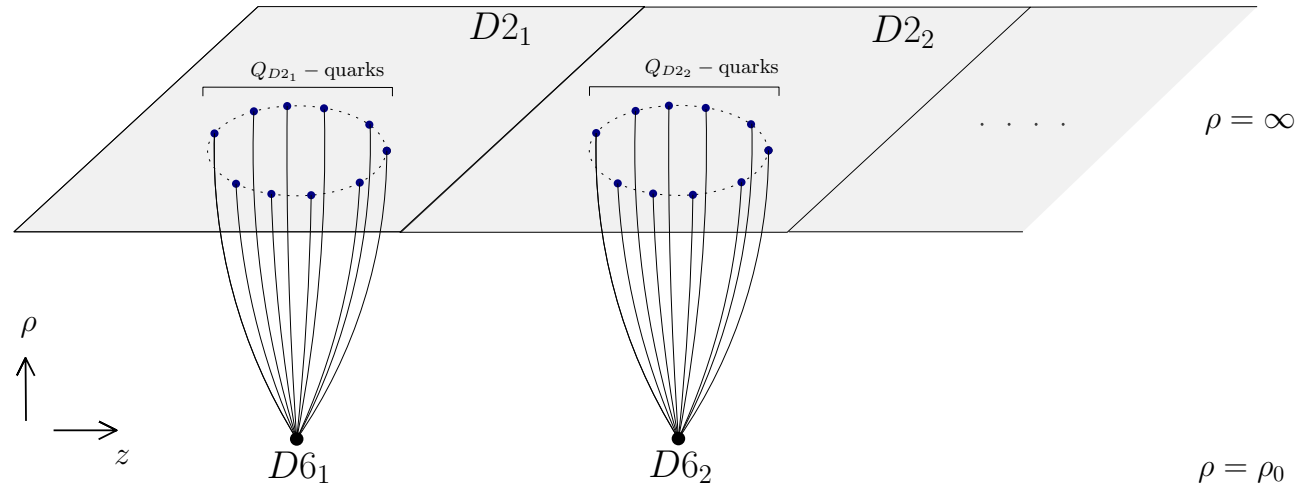
$$\frac{\rho^4}{\sqrt{\dot{\rho}^2 + \rho^4}} = \rho_0^2 \sqrt{\frac{15}{16}} \equiv \rho_0^2 \beta$$

Size: $\ell = \frac{\beta}{3\rho_0} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; \beta^2\right)$

Energy: $E_{\text{bin(string)}} = -\frac{f_k}{\ell}$

$$f_k = \frac{2^{5/2}}{3} \beta \sqrt{\frac{\alpha_k}{-\alpha_k''}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; \beta^2\right) \left({}_2F_1\left(\frac{1}{2}, -\frac{1}{4}, \frac{3}{4}; \beta^2\right) - \frac{1}{4} \right) = 1.26776 \sqrt{\frac{\alpha_k}{-\alpha_k''}}$$

In the 2d theory:



$$\frac{\rho^4}{\sqrt{\dot{\rho}^2 + \rho^4}} = \rho_0^2 \sqrt{1 - \frac{1}{16} \left(1 - \frac{c}{\tilde{\Lambda}^2 \sqrt{c + \tilde{\Lambda}^4}} \operatorname{arcsinh}\left(\frac{\tilde{\Lambda}^2}{\sqrt{c}}\right) \right)^2} \equiv \rho_0^2 \beta$$

$\tilde{\Lambda} \rightarrow \infty \rightarrow$ Same size and energy per string

Our interpretation:

The AdS₃ solution describes 6d (1,0) CFTs deconstructed in terms of 2d (0,4) degrees of freedom.

The 2d quivers are explicit realisations of this

In order to describe 2d defects we need a genuine 2d theory

4. New $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA with (0,4) SUSY (large)

$$ds^2 = \sqrt{\frac{\alpha}{-\alpha''}} X^{-1/2} ds_7^2 + X^{5/2} \sqrt{\frac{-\alpha''}{\alpha}} \left(dz^2 + \frac{\alpha^2}{\alpha'^2 - 2\alpha\alpha'' X^5} ds_{S^2}^2 \right)$$

$$ds_7^2 = \frac{1}{\sin^2 \beta} \left(\frac{1}{(1-\lambda)^2 X^2} ds_{AdS_3}^2 + \frac{\cos^2 \beta}{(1+\lambda)^2 X^2} ds_{S^3}^2 + X^8 d\beta^2 \right)$$

$$X = (1 + \lambda \sin^2 \beta)^{-1/5}$$

	x^0	x^1	ρ	θ^1	θ^2	θ^3	z	ζ	φ^1	φ^2
D2	x	x					x			
D4	x	x						x	x	x
NS5	x	x	x	x	x	x				
D6	x	x	x	x	x	x	x			
D8	x	x	x	x	x	x		x	x	x

Asymptote locally to $AdS_7 \times S^2 \times I$ but

$$ds_7^2 \sim \frac{dx_{1,1}^2 + du^2 + u^2(1-\lambda)^2(1+\lambda)^{-2} \cos^2 \beta ds_{S^3}^2}{u^2} + d\beta^2$$

Conical singularity in the \mathbb{R}^4 parametrised by u and S^3

Non-vanishing contribution of the defects to the central charge

Baryon vertex configuration with size and energy λ dependent

5. Conclusions

Field theory interpretation of AdS3 solutions with 6 and 4 supercharges, in particular in connection to surface defects

- With **(0,6) supersymmetry**: **Extension of ABJM/ABJ to the massive case**, in which one of the external directions becomes an energy scale, and generates a flow towards an AdS3 space

Realisation of Seiberg-duality as a large gauge transformation

- With **(0,4) supersymmetry**: **Defect interpretation** linked to the existence of a non-compact direction that becomes part of AdS_7 when it approaches infinity

With small susy: No contribution from defects to the degrees of freedom \rightarrow **Deconstructed 6d (1,0) theories**

THANKS!!

Defect conformal field theories can be engineered in terms of brane intersections of **defect branes** and **background branes**, where, in the latter, a higher dimensional CFT lives

Holographically this is described by low dimensional AdS spaces with non-compact internal manifolds, that reproduce a higher dim AdS geometry *asymptotically locally* (D'Hoker, Gutperle, Dibitetto, Petri..)

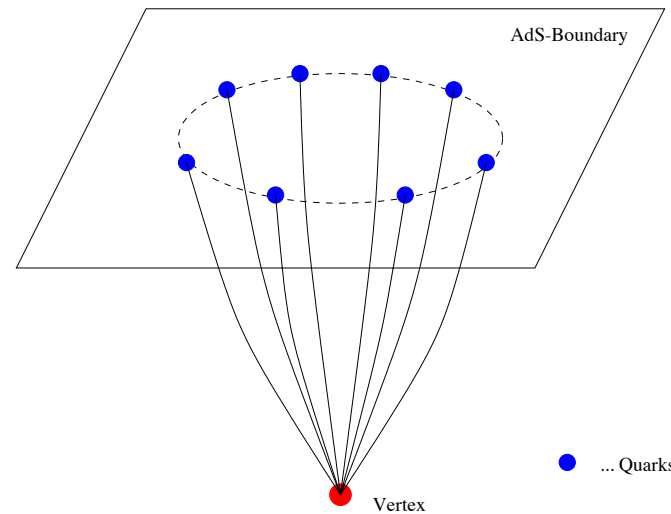
This is **different from an RG flow**, where the low dim AdS would only arise in the IR and the high dim AdS would not have extra fluxes

The presence of the non-compact direction renders the defect field theory ill-defined, but this is interpreted as the need to complete the CFT by the higher dim one away from the defects

The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS



D5-brane:

$$S_{CS} = 2\pi T_5 \int_{\mathbb{R} \times S^5} P[F_5] \wedge A = N T_{F1} \int_{\mathbb{R}} dt A_t$$

Cancel this charge with the charge induced by the endpoints of N open F-strings stretching between the D5 and the boundary

N F-strings connecting the D5-brane to the boundary of AdS behave as fermions

Dual configuration on the CFT side: N Wilson lines ending on an epsilon tensor \rightarrow Bound state of N quarks

Stability in the AdS direction (Brandhuber, Itzhaki, Sonnenschein, Yankielowicz'98):

Use the probe brane approximation

Consider: $S = S_{D5} + S_{NF1}$

$$S_{NF1} = -NT_{F1} \int dt dy \sqrt{\frac{16\rho^4}{L^4} + \dot{\rho}^2}$$

$\rho = \rho(y)$: position in AdS

Bulk equation of motion:
$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \dot{\rho}^2}} = a$$

Boundary equation of motion:
$$\frac{\dot{\rho}}{\sqrt{\frac{16\rho^4}{L^4} + \dot{\rho}^2}} = \frac{2T_5}{LNT_{F1}}$$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \dot{\rho}^2}} = \frac{1}{4} \beta \rho_0^2 L^2 \quad \text{where} \quad \beta = \sqrt{\frac{15}{16}}$$

Integrating: **Size of the configuration:**

$$\ell = \int_0^\ell dy = \frac{L^2}{4\rho_0} \int_1^\infty d\hat{\rho} \frac{\beta}{\hat{\rho}^2 \sqrt{\hat{\rho}^4 - \beta^2}}$$

On-shell energy:

$$E = E_{D5} + E_{NF1} = NT_{F1}\rho_0 \left(\sqrt{1 - \beta^2} + \int_1^\infty d\hat{\rho} \frac{\hat{\rho}^2}{\sqrt{\hat{\rho}^4 - \beta^2}} \right)$$

Subtracting the energy of the constituents (when the brane is located at $\rho_0 = 0$ the strings become radial and correspond to free quarks) \rightarrow Binding energy:

$$E_{\text{bin}} = NT_{F1}\rho_0 \left(\sqrt{1 - \beta^2} + \int_1^\infty d\hat{\rho} \left[\frac{\hat{\rho}^2}{\sqrt{\hat{\rho}^4 - \beta^2}} - 1 \right] - 1 \right) = -\frac{f}{l}$$

$f_{\text{string}} = 0.035 > 0 \Rightarrow$ The configuration is stable