INFLATION AND THE HUBBLE TENSION

THE DARK SIDE OF THE UNIVERSE - DSU2024 Corfu, 14th September 2024

Image: Planck's View of BICEP2/Keck Array Field. Credit: Jet Propulsion Laboratory, NASA and Caltech

WILLIAM GIARÈ

w.giare@sheffield.ac.uk www.williamgiare.com

Research Associate in Theoretical Cosmology

The University of Sheffield School of Mathematics & Statistics



Based on:

WG, PRD 109 (2024) 12, 12354 • arXiv: 2404.12779

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We obtain inflation from a single scalar field minimally coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Slow-Roll Conditions

$$V(\phi) \gg \dot{\phi} \qquad \frac{V_{\phi}^2}{V} \ll H^2 \qquad |V_{\phi\phi}| \ll H^2$$

Slow-Roll Parameters

$$\epsilon \doteq \frac{M_{\rm pl}^2}{2} \left(\frac{V_{\phi}^2}{V^2} \right) \ll 1 \quad |\eta| \doteq \left| M_{\rm pl}^2 \left(\frac{V_{\phi\phi}}{V} \right) \right| \ll 1$$







PRIMORDIAL PERTURBATIONS

Primordial Scalar Modes

Quantum fluctuations of the Inflaton field can source irregularities in the CMB

$$\mathscr{P}_{s}(k) = A_{s}\left(\frac{k}{k_{*}}\right)^{n_{s}-1} \qquad n_{s}-1 = \frac{d\ln\mathscr{P}_{s}}{d\ln k}\bigg|_{k=k_{*}} = 2\eta - 6\epsilon$$

Primordial Tensor Modes

Quantum fluctuations in the metric could source a stochastic background of Primordial Gravitational Waves, imprinting the CMB

$$\mathscr{P}_T(k) = rA_s \left(\frac{k}{k_*}\right)^{n_T} \qquad n_T = \frac{d\ln\mathscr{P}_T}{d\ln k} \bigg|_{k=k_*} = -\frac{r}{8} = -2\epsilon$$



Planck 2018 1807.06209









PLANCK 2018

1807.06209



 $30 < \ell \leq 2500$ in the TT Spectrum

 $30 < \ell \leq 2000$ in the EE Spectrum

100

80

60

40

20



The low-TE data show excess of variance compared to simulations at low multipoles, for reasons that are not understood

High-multipole TE data

 $30 < \ell \leq 2000$ in the TE Spectrum







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2110.00483

Scalar and Tensor modes contribution to CMB spectra:

TT spectrum: Scalar > Tensor at any ℓ

TE spectrum: Scalar > Tensor at any ℓ

EE spectrum: Scalar > Tensor at any ℓ

BB spectrum: Tensor > Scalar at $\ell \leq 100$ (i.e., at large scales)

B-Modes Polarization

To constrain primordial tensor modes we need large-scale B-mode polarization

Many experiments have been (and will be) collecting data

BICEP/KEK-2018 most precise data so far

Note: $\ell \propto 1/\theta \propto 1/R$



Figure inspired by Gorbunov & Rubakov "Cosmological Perturbations and Inflationary Theory", Chapter 10 See also A. Challinor arXiv:astro-ph/0606548





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JOINT PLANCK-BICEP/KEK ANALYSIS

Inflationary spectrum parameters:

1) $n_s \neq 1$ at 8.5 σ : $n_s = 0.9678 \pm 0.0036$ (at 68% CL)

2) No detection of tensor modes: r < 0.035 (at 95%CL)

Slow-roll parameters:

1) η measured to $\eta = -0.0130^{+0.0024}_{-0.0029}$ (at 68% CL)

2) upper limit $\epsilon < 0.0022$ (at 95%CL)

3) Slow-roll hierarchy $1 \gg |\eta| \gg \epsilon$







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JOINT PLANCK-BICEP/KEK ANALYSIS

"All models are equal, but some models are more equal than others"

Starobinsky Inflation

Inflation is controlled by the squared Ricci scalar in the effective action

$$S = \frac{1}{2M_{\rm Pl}^2} \int d^4x \sqrt{-g} \left(R + \frac{R^2}{6M^2} \right)$$

It gives **predictions** for n_s and r

$$n_s \simeq 1 - \frac{2}{\mathcal{N}} \qquad r \simeq \frac{12}{\mathcal{N}^2} \qquad 50 \leq \mathcal{N} \leq 70$$

Model in perfect agreement with Planck and BICEP/KECK



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THE HUBBLE TENSION

5 σ tension in the value of the Hubble parameter H_0

Direct Measurement

SH0ES: $H_0 = 73 \pm 1 \text{ km/s/Mpc}$

Model-independent, based on Type-Ia Supernovae

Indirect Measurement

Planck: $H_0 = 67.4 \pm 0.5$ km/s/Mpc

Model-dependent, inferred from CMB measurement (in ΛCDM)

Tension confirmed by many other independent probes

Snowmass 2021 – 2203.06142





THE HUBBLE TENSION

How do we measure H_0 from the CMB?

- Angular size of the sound horizon (θ_s)
- Baryon density ($\Omega_b h^2$)
- Cold dark matter density ($\Omega_c h^2$)

$$\operatorname{Figure}_{z_{*}} \operatorname{Higher}_{z_{*}} \operatorname{Higher}_$$

- Sound horizon $r_s(z_*)$
- Angular diameter distance from the CMB, $D_A(z_*) = r_s(z_*)/\theta_s$

 $D_{A}(z_{*}) = \int_{0}^{z_{*}} dz H(z)^{-1} H^{2}(z) = H_{0}^{2} \left[\Omega_{m}(1+z)^{3} + \Omega_{\text{DE}}(z) + \dots\right]$

• Hubble Parameter (H_0)

S. Galli "The H₀ debate from a CMB prospective"





EARLY TIME SOLUTIONS

If some New Physics reduces $r_s(z_*)$, H_0 should increase to keep θ_s fixed

$$\theta_{s} = \frac{r_{s}(z_{*})}{D_{A}(z_{*})} - \frac{r_{s}(z_{*})}{D_{A}(z_{*})} \simeq \frac{1}{H_{0}} \int_{0}^{z_{*}} \frac{dz}{\left[\Omega_{m}(1+z)^{3} + \Omega_{\Lambda}\right]^{1/2}}$$

How can we decrease $r_s(z_*)$?

1) Working on the Baryon-Photon fluid sound speed $c_s(z)$ before recombination

2) Increasing the expansion rate of the Universe H(z) before recombination:

$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 \right]^{1/2}$$

Increasing radiation: $\Omega_r = \Omega_{\gamma} \left(1 + 0.23 N_{\text{eff}} \right) N_{\text{eff}} \rightarrow 3.04 + \Delta N_{\text{eff}}$



Knox and Millea – 1908.03663



What happens increasing radiation in the early Universe?



Reducing r_{s} we shift to larger H_{0}

WG – PRD 109 (2024) 12, 12354 • arXiv: 2404.12779

Larger H_0 implies $n_s \rightarrow 1$



Planck 2018 1807.06209



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Constraints on *r* do not change



What happens increasing radiation in the early Universe?



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<u>Upper bounds ϵ do not change</u>







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Constraints on η shift significantly



How Much Dark Radiation is allowed?

- To reducce the H0-tension to ~2 σ we need $\Delta N_{\rm eff}\gtrsim 0.4$, Srongly Disfavoured compared to $\Lambda \rm CDM$ [1]
- Models with $0.2 \leq \Delta N_{\rm eff} \leq 0.3$ can reduce the H0-tension to ~3.5 σ while being "only" weakly disfavoured compared to Λ CDM [1]

To what extent are constraints on inflation sensitive?

• Models with $0.2 \leq \Delta N_{\rm eff} \leq 0.3$ already require a change in perspective for Inflation: Starobinsky-like models are no longer supported

$ \ln B_0 $	Odds	Probability	Strength of evidence
< 0.1	$\lesssim 3:1$	< 0.750	Inconclusive
1	$\sim 3:1$	0.750	Weak
2.5	$\sim 12:1$	0.923	Moderate
5	$\sim 150:1$	0.993	Strong

[1] We refer to the following scale for the strength of evidence:



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Early Dark Energy

A light scalar field behaves similarly to a cosmological constant, increasing the expansion rate in the early Universe. Then it must decay faster than matter.

Effects quantified by the maximal fractional contribution to the total energy density

$$f_{\text{EDE}} = \max_{z} \left(\frac{\rho_{\text{EDE}}(z)}{\rho_c(z)} \right)$$

What if $f_{\rm EDE} \neq 0$?

1) H(z) increases before recombination, reducing r_{drag} and increasing H_0

2) We move towards $n_s \rightarrow 1$

3) $0.04 \leq f_{\rm EDE} \leq 0.06$ already not compatible with Staribinsky-like models



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Implications for Starobinsky inflation

- 1) Perfect agreement with Planck+BICEP/KEK assuming ACDM
- 2) Can be in agreement with Planck+BICEP/KEK for negligible f_{EDE}
- 3) **NOT** in agreement with Planck+BICEP/KEK if EDE solves the H_0 tension





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CONCLUSIONS

<u>Widespread consensus in the cosmology community</u>

- 1) **Robust constraints** on Inflation from Planck and BICEP/KEK data: $n_s = 0.9678 \pm 0.0036$ and r < 0.035
- 2) **Starobinsky Inflation leading model**

Important caveats surrounding these results

- 1) Any constraint on the inflationary parameters is intrinsically model-dependent (can we rely on ACDM?)
- 2) Early time solutions of the Hubble Tension can shift Planck and BICEP/KEK-2018 results towards $n_s \rightarrow 1$
- 3) ACT small-scale CMB data point towards $n_s \sim 1$ (in disagreement with Planck and Starobinsky Inflation)

Possible implications

- 1) We might need to rethink inflation. Too early to say!
- 2) Doing model selection is premature and not completely safe without understanding the nature of the H_0 tension



BACKUP SLIDES

PRIMORDIAL PERTURBATIONS





(note: assuming that parity is conserved)

1) Angular power spectrum of temperature anisotropies C_{ℓ}^{TT} (TT spectrum)

2) Temperature and E-mode cross-spectrum C_{ℓ}^{TE} (TE spectrum)

3) Angular power spectrum of E-mode polarisation C_{ℓ}^{EE} (EE spectrum)

4) Angular power spectrum of B-mode polarisation C_{ℓ}^{BB} (**BB** spectrum)







$$\begin{bmatrix} C_{\ell}^{XY} \end{bmatrix}_{\text{scalar}} = \frac{2\pi}{\ell(\ell+1)} \int_{0}^{\infty} d\ln k \ T_{\ell}^{X}(k) T_{\ell}^{Y}(k) \ \mathscr{P}_{s}(k)$$

Scalar Transfer functions
Scalar sp

$$\begin{bmatrix} C_{\ell}^{XY} \end{bmatrix}_{\text{tensor}} = \frac{2\pi}{\ell(\ell+1)} \int_{0}^{\infty} d\ln k \ T_{\ell}^{X}(k) T_{\ell}^{Y}(k) \ \mathcal{P}_{t}(k)$$

Tensor Transfer functions
Tensor Stress



Transfer Functions:

- Scalar and Tensor transfer functions are different
- $C_{\ell}^{\text{tot}} = [C_{\ell}]_{\text{scalar}} + [C_{\ell}]_{\text{tensor}}$ $\ln [C_{\ell}^{XY}]_{\text{scalar}}$ we have: $X, Y = \{T, E\}$
- In $\left[C_{\ell}^{XY}\right]_{\text{tensor}}$ we have: $X, Y = \{T, E, B\}$
- Transfer functions are different for T, E, B





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Figure inspired by Gorbunov & Rubakov "Cosmological Perturbations and Inflationary Theory", Chapter 10 See also A. Challinor arXiv:astro-ph/0606548



Joint analysis of Planck and BICPE/KEK:

1) $n_s \neq 1$ at 8.5 σ : $n_s = 0.9678 \pm 0.0036$ (at 68% CL)

2) No detection of tensor modes: r < 0.035 (at 95%CL)

Slow-roll parameters:

1) η measured to $\eta = -0.0130^{+0.0024}_{-0.0029}$ (at 68% CL)

2) **upper limit** $\epsilon < 0.0022$ (at 95%CL)

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2007.07288



High-multipole temperature data

 $600 < \ell \leq 4200$ in the TT Spectrum



High-multipole EE Polarization data

 $350 < \ell \leq 4200$ in the EE Spectrum

High-multipole TE data

 $350 < \ell \leq 42000$ in the TE Spectrum

Note: Planck probes $\ell \in [2,2000]$





2007.07288

ACT shows a preference for $n_s \simeq 1$ (in 3σ disagreement with Planck)

Dataset	Scalar Spectral Index (n_s)	
	ΛСDΜ	
ACT	1.009 ± 0.015	
ACT ($\tau = 0.0544 \pm 0.0070$)	1.007 ± 0.015	
ACT + Planck low E	1.001 ± 0.011	
ACT+BAO (DR12)	1.006 ± 0.013	
ACT+BAO (DR16)	1.006 ± 0.014	
ACT+DES	1.007 ± 0.013	
ACT+SPT+BAO (DR16)	0.997 ± 0.013	
ACT+SPT+BAO (DR12)	0.996 ± 0.012	
Planck	0.9649 ± 0.0044	
Planck+BAO (DR12)	0.9668 ± 0.0038	
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Planck ($2 \le \ell \le 650$)	0.9655 ± 0.0043	
Planck ($\ell > 650$)	0.9634 ± 0.0085	

WG et al. – 2210.09018







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WG, *et al.* – *MNRAS* 521 (2023) • arXiv: 2210.09018





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2007.07288

Implications for Starobinsky inflation:

- 1) Perfect agreement with Planck+BICEP/KEK: $\mathcal{N} = 64 \pm 9$ at 68% CL
- 2) Strong disagreement with ACT+BICEP/KEK: $\mathcal{N} > 100$ at 95% CL

Large and small scale CMB data DO NOT agree on the inflationary potential

WG, et al. – JCAP 09 (2023) 019 • arXiv: 2305.15378





Implications for Starobinsky Inflation

- 1) Starobinsky inflation gives *predictions* for $n_s = 1 2/N$ and $r = 12/N^2$
- 2) Increasing ΔN_{eff} decreases r_s and increases H_0 thereby shifting $n_s \rightarrow 1$.
- 3) In Starobinsky Inflation this would require $\mathcal{N} \to \infty$
- 4) It can be no longer supported when considering new physics





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Domino effect in the CMB fit at different scales

$$\frac{\delta H_0}{H_0} \simeq -\frac{\delta D_A}{D_A} \simeq \frac{\delta k_D}{k_D} \simeq \frac{1}{2} \frac{\delta \omega_{cdm}}{\omega_{cdm}} \simeq \frac{\delta \omega_b}{\omega_b} \simeq \frac{\delta n_s}{0.4}$$

(See also Gen Ye et. al. - 2303.09729)











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Implications for slow-roll parameters

- 1) $|\eta| \gg \epsilon$ assuming Λ CDM
- 2) $|\eta| \gtrsim \epsilon$ for negligible f_{EDE}
- 3) $|\eta| \sim \epsilon$ if EDE solves the H_0 tension



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INFLATION AND EARLY DARK ENERGY

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(See also Gen Ye et. al. – 2303.09729)





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Hints of New Physics in small-scale CMB data?

- 1) ACT small-scale CMB data give $n_s \sim 1$
- 2) ACT small-scale CMB data give $f_{\rm EDE} \neq 0$

3) Assuming new physics, both large and small CMB data prefer larger n_s



Atacama Cosmology Telescope

2007.07288

е	

Parameter	EDE $(n = 3)$ Best-Fit	EDE $(n = 3)$ Marg.
$\log(10^{10}A_{ m s})$	3.083	3.067 ± 0.034
$n_{ m s}$	1.064	$0.987^{+0.027}_{-0.047}$
$100 heta_{ m s}$	1.04279	1.04247 ± 0.00079
$\Omega_{ m b}h^2$	0.02214	$0.02141\substack{+0.00044\\-0.00065}$
$\Omega_{ m c}h^2$	0.1425	$0.1307\substack{+0.0054\\-0.0120}$
$oldsymbol{ au}_{ ext{reio}}$	0.061	0.065 ± 0.015
$oldsymbol{y_p}$	0.9951	1.0037 ± 0.0070
$f_{ m EDE}$	0.241	$0.142\substack{+0.039 \\ -0.072}$
$\log_{10}(z_c)$	3.72	< 3.70
$\boldsymbol{\theta_i}$	2.97	> 0.24
$H_0 \mathrm{[km/s/Mpc]}$	77.6	$74.5^{+2.5}_{-4.4}$
$\Omega_{ m m}$	0.274	$0.276\substack{+0.020 \\ -0.023}$
σ_8	0.883	$0.831\substack{+0.027 \\ -0.043}$
S_8	0.844	0.796 ± 0.049
$\log_{10}(f/eV)$	26.65	$27.17\substack{+0.34\\-0.55}$
$\log_{10}(m/{ m eV})$	-26.90	$-27.52\substack{+0.26\\-0.72}$

Colin Hill et. al. (ACT) - 2109.04451



INFLATION AND LATE TIME SOLUTIONS

Late Time Solutions in a Nutshell

If some New Physics decreases the late-time expansion rate while leaving $r_s(z_*)$ fixed, H_0 should increase to keep θ_s fixed

$$\theta_{s} = \frac{r_{s}(z_{\text{CMB}})}{D_{A}(z_{\text{CMB}})} \int_{D_{A}(z_{*})} = \frac{1}{H_{0}} \int_{0}^{z_{*}} \frac{dz}{\left[\Omega_{m}(1+z)^{3} + \Omega_{\text{DE}}(1+z)^{3(1+w)}\right]^{1/2}}$$

How?

A naive way to decrease the late-time expansion rate would be to consider a phantom Dark Energy equation of state w < -1

$$H(z) \simeq H_0 \left[\Omega_m (1+z)^3 + \Omega_{de} (1+z)^{3(1+w)} \right]^{1/2}$$





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