Asymptotic Safety and cosmologically coupled Black Holes

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Relevant papers

“Avoidance of singularities in asymptotically safe Quantum Einstein Gravity”
Georgios Kofinas, Vasilios Zarikas
JCAP 1510 (2015) no.10069

“Asymptotic Safe gravity and non-singular inflationary Big Bang with vacuum birth”
Georgios Kofinas, Vasilios Zarikas.
Published in Phys. Rev D 2016
Physical Review D - Particles, Fields, Gravitation and Cosmology, Vol. 94, 10, 2016, 103514

“Effective field equations and scale-dependent couplings in gravity,“
A.Bonanno, G.Kofinas and V. Zarikas
Phys. Rev. D {103}, no.10, 104025 (2021)

G. Kofinas and Vasilios Zarikas

"A solution of the dark energy and its coincidence problem based on local antigravity sources without fine-tuning or new scales"

Phys.Rev. D97 (2018) no.12, 123542

Anagnostopoulos, F. K., Bonanno, A., Mitra, A., & Zarikas, V. (2022).

Swiss-cheese cosmologies with variable G and Λ from the renormalization group. Physical Review D, 105(8) doi:10.1103/PhysRevD.105.083532

A. Bonanno, K F. Dialektopoulos, V. Zarikas, (2024). A new approach to renormalization group improved gravitational action e-Print 2407.18883 [gr-qc]

Outline of the talk

I. Brief required description of Asymptotic Safety framework for Quantum Gravity
II. Novel dark energy proposal based on astrophysical black holes
III. Cosmological coupling of BHs and The case of Asymptotic Safety BHs
- 1st approach cand The case of Asymptotic Safety BHs, Misner-Sharp Mass
- 2nd approach Embedding of BHs in cosmological background

IV. Conclusions

I. Asymptotic Safety Paradigm for Quantum Gravity

Quantum Gravity

- Several attempts of quantum gravity try to address the description of gravity in the Ultraviolet and its interaction with other quantum fields

- String-branes theory / M theory
- Loop gravity-- Spin foam models for quantum gravity
- Asymptotic Safety program (and similar non perturbative RG approaches)

The most minimal framework

- **Asymptotic safety (AS)** works in 4-dim

  Minimal proposal keeps the same symmetries and fields of Quantum Field Theory(i.e. SM) and General Relativity (GR)
- It was able to indicate that GR and extensions of it (with and without SM fields) can be

  a non-perturbatively renormalizable theory
- Not a complete framework
- Continuous spacetime
- Background independent approach

Steps of AS

Theory

Field contents (e.g. graviton)+

+symmetries (e.g. coordinate trns)

Action

Specific interactions of fields

that respect symmetries (e.g. $\sqrt{g}R$)

Theory space

Space containing all actions

with “coordinates” coupling constants (e.g. G, Λ)

Functional Renormalization group flow:

Connects physics at different scales k,

G, Λ running coupling constants

AS and UV completion

- In quantum field theory, observable quantities such as decay rates and cross sections can be expressed as functions of the couplings.
- Generically, if the couplings are finite, also the observable quantities will be finite. So, a way of ensuring that our description of the world has a good ultraviolet limit is to require that it lies on a renormalization group trajectory for which all couplings remain finite when the energy goes to infinity.
- The simplest way of achieving this is to demand that the trajectory flows towards a non-gaussian fixed point.

- Based on RG ideas Weinberg realized in 1976 that perturbative renormalizability is not the only way for a theory to remain meaningful at high energies. It is sufficient to fix, at UV energies, only a finite number of parameters. And none of these parameters should become infinite itself in that limit.
- These two requirements: A finite number of finite parameters that determine the theory at high energies are what make a theory asymptotically safe.

Gravity is a gauge theory

- This then raises the question of whether quantum gravity, though pertubatively non-renormalizable, might be asymptotically safe and meaningful after all. This motivation initiated the Asymptotic Safety, (AS) program.

- While the general idea has been around for many years, it has only been in the late 1990s, following works by Wetterich and Reuter, that asymptotically safe gravity has been formulated.

Asymptotic Safety

- The mathematical technique that gave boost to the asymptotic safety scenario is the functional renormalization group equation for gravity [M. Reuter 1998] which enabled the detailed analysis of the gravitational RG flow at the non-perturbative level.

- This technique uses a Wilsonian RG flow on a space of all theories that consists of all difeomorphism invariant functionals of the metric $g_{\mu\nu}$

- The framework emerging from this construction is called Asymptotic Safety or Quantum Einstein Gravity (QEG).

The problem

In Einstein's theory, the strength of the gravitational coupling is the number $g=G\,k^2$, where G is Newton's constant and k is some momentum scale of the process being considered. The reason why the energy scale k appears is that gravity couples to mass, and energy is mass; the higher the energy of a particle the stronger its gravitational coupling, if G is constant.

The non-perturbative renormalization of Einstein's gravity in simple physical terms.

Perhaps the most illuminating discussion in this context has been presented by Polyakov, who noticed that as gravity is always attractive and therefore a larger cloud of virtual particles implies a stronger gravitational force, Newton's constant G should be anti-screened at small distances. The implication of this behaviour suggests that the dimensionless coupling constant tends to a finite non-zero limit at small distances

The non-perturbative renormalization of Einstein's gravity in simple physical terms.

A positive cosmological constant term is always repulsive therefore a larger cloud of virtual particles implies a less repulsive force, cosmological constant Λ should be larger at small distances. The implication of this behaviour suggests that the dimensionless coupling constant $\lambda = \Lambda k^{-2}$ tends to a finite non-zero limit at small distances

Solution

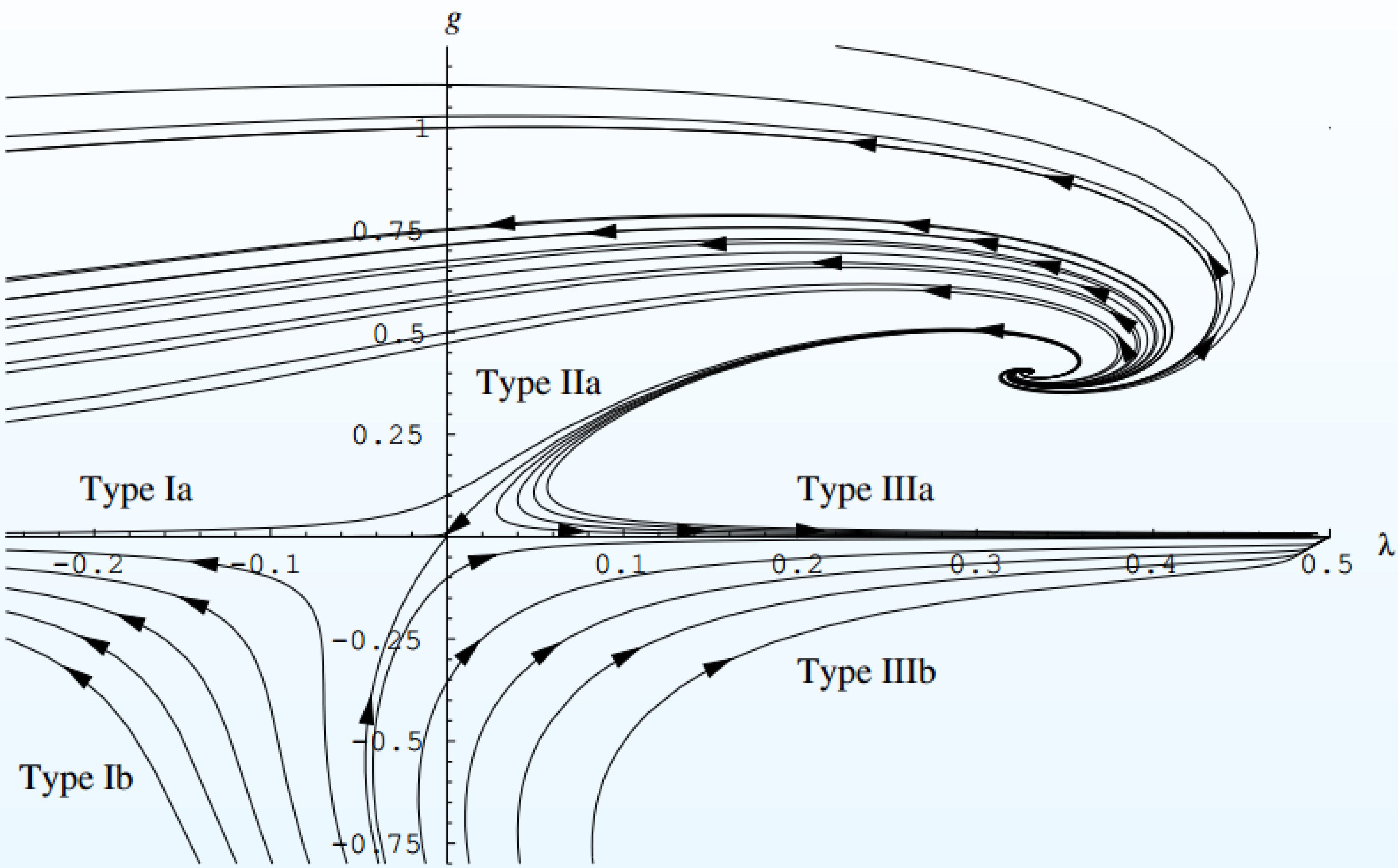
Newton's constant becomes a running coupling ***G(k)*** and it is conceivable (RG flow proof) that for large ***k*** it behaves like $\boldsymbol{k^{-2}}$ and $\boldsymbol{\Lambda(k)}$ behaves like $\boldsymbol{k^{2}}$, then *the dimensionless* **g** *and* $\boldsymbol{\lambda}$ would tend to a constant. This is what is meant by a fixed point for Newton's constant and cosmological constant.

$$g = G\,k^2 \,, \qquad \lambda = \Lambda k^{-2}$$

$$g_k \equiv k^2 G_k \quad , \quad \lambda_k \equiv k^{-2} \Lambda_k \, ,$$

Einstein-Hilbert-truncation: the phase diagram

M. Reuter and F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054]



II. Novel dark energy proposal based on astrophysical black holes

Structure formation is crucial

Idea: The recent emergence of acceleration is not a coincidence but an outcome of the recent formation of structure.

Before the appearance of structure and the emergence of sufficient repulsive effects, the conventional deceleration scenario is realised.

Motivation

We have to explain "naturally"

- A. Recent passage from deceleration to acceleration
- B. Why now? Coincidence problem

To the best of my knowledge all papers that explain the first problem have a fine tuning problem exactly like ΛCDM and they don't explain convincingly the second problem, **with the exception** of 3 types of papers that connect the recent passage to large scale structure inhomogeneities'.

1. Papers considering structure inhomogeneities. However, these works that consider the inhomogeneities to explain both these problems fail to provide enough acceleration (so only partially they solve the first problem).

Motivation

2. Asymptotic Safety Swiss Cheese model

A solution of the dark energy and its coincidence problem based on local antigravity sources without fine-tuning or new scales, Georgios Kofinas, Vasilios Zarikas. Phy.Rev D 2018. e-Print: arXiv:1706.08779 [gr-qc]

3. D. Farrah et al., *Observational Evidence for Cosmological Coupling of Black Holes and its Implications for an Astrophysical Source of Dark Energy*, *Astrophys. J. Lett.* **944** (2023) L31 [arXiv:2302.07878]

D. Farrah et al.

Recently, Farrah et al. , following previous proposals and building on recent experimental results, found observational evidence for cosmologically-coupled mass growth in supermassive black holes (SMBHs) at the centre of elliptical galaxies at different redshift.

$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k , \qquad \text{with} \quad a \geq a_i$$

- The authors found a preference consistent with $k \sim 3$ at 90% confidence level(C.L.), pointing therefore to nonzero coupling. This would also imply that the density of these objects in a cosmological volume $\propto a^3$ is constant, thus leading to the conclusion that BHs with repulsive forces in the interior (vacuum energy interiors) could be responsible for the observed accelerated expansion of the universe

- *However, the connection of mass growth and the generation of dark energy like cosmological effect has been criticized from various points of view, (i) the theoretical averaging of an action describing inhomogeneities, (ii) from observational point of view and (iii) some other of minor importance arguments.*

1 st approach

- To circumvent this problem authors in M. Cadoni, R. Murgia, M. Pitzalis, and A. P. Sanna, JCAP 03, 026 (2024), arXiv:2309.16444 [gr-qc]
- Proposed to focus on general, model-independent properties, which should characterize the cosmological coupling.
- Previously, the debate was biased by the use of the nonlocal Arnowitt-Deser-Misner (ADM) mass to quantify the energy pertaining to local objects.

ADM mass

- For isolated objects in asymptotically-flat spacetimes, the ADM mass, a nonlocal quantity defined in terms of a surface integral at spatial infinity can be used.
- For BHs not in vacuum, like nonsingular BHs, this identification is less straightforward, even if the manifold is asymptotically flat.

- In these cases, there is a different definition which better encapsulates the local properties that the energy of a gravitational system should satisfy: the Hawking-Hayward quasi-local mass, which, for spherically-symmetric spacetimes, reduces to the Misner-Sharp (MS) mass.

$$M_{\mathrm{HH}} := \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}}\int_S \mu\left(\mathcal{R} + \theta_+\theta_- - \frac{1}{2}\sigma^+_{ab}\sigma_-^{ab} - 2\omega_a\omega^a\right),$$

where $\mathcal{R}$ is the induced Ricci scalar on S, $\theta_\pm$ and $\sigma^\pm_{ab}$ are the expansion and shear tensors of a pair of null geodesic congruences (outgoing and ingoing from the surface S), ω^a is the projection onto S of the commutator of the null normal vectors to S, μ is the volume two-form on the surface S, and A is the area of S.

MS mass

- The identification of the quasi-local Misner-Sharp (MS) mass is the most appropriate quantity to determine the energy of local compact objects, and to investigate their cosmological coupling.
- The MS mass is covariantly defined, and it reduces to the ADM mass at asymptotically flat infinity. Therefore, it can be identified as an ideal tool for making theoretical predictions to compare with astrophysical measurements.

MS mass

The quasi-local MS mass represents a quite natural definition, as it emerges naturally from Einstein's equations and is directly related to astrophysical observations of the internal energy of a spherically-symmetric, virialized system.

For a spherically-symmetric spacetime with a metric of the general form. One system is given by Lemaître coordinates (t, r, θ, φ)

$$ds^2 = -e^{\alpha(t,r)}dt^2 + e^{\beta(t,r)}dr^2 + R(t,r)^2 d\Omega^2 \,,$$

the MS mass reads as

$$M_{\rm MS} = \frac{R}{2G}\left(1 + \dot{R}^2 e^{-\alpha} - R'^2 e^{-\beta}\right)$$

III. Cosmological coupling of BHs

The simplest known example of an embedding of a compact object (mass-particle) in a cosmological background is the Schwarzschild-de Sitter metric, which is a vacuum solution of Einstein's equations with a positive cosmological constant.

$$\mathrm{d}s^2 = -\left(1 - \frac{2Gm}{r} - H^2 r^2\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - \frac{2Gm}{r} - H^2 r^2} + r^2 \mathrm{d}\Omega^2 \,.$$

$$M_{\mathrm{MS}} = m + \frac{H^2}{2G} r^3 \,.$$

The Sultana-Dyer solution

The SD solution is another exact solution describing a BH embedded in a spatially flat FLRW.

$$\mathrm{d}s^2 = -\frac{\left(1-\frac{Gm_0}{2r}\right)^2}{\left(1+\frac{Gm_0}{2r}\right)^2}\mathrm{d}t^2 + a^2\left(1+\frac{Gm_0}{2r}\right)^4\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right) .$$

$$M_{\mathrm{MS}} = a\, m_0 + \frac{H^2 R^3}{2G\left(1-\frac{2Gam_0}{R}\right)} .$$

The case of Asymptotic Safety BHs

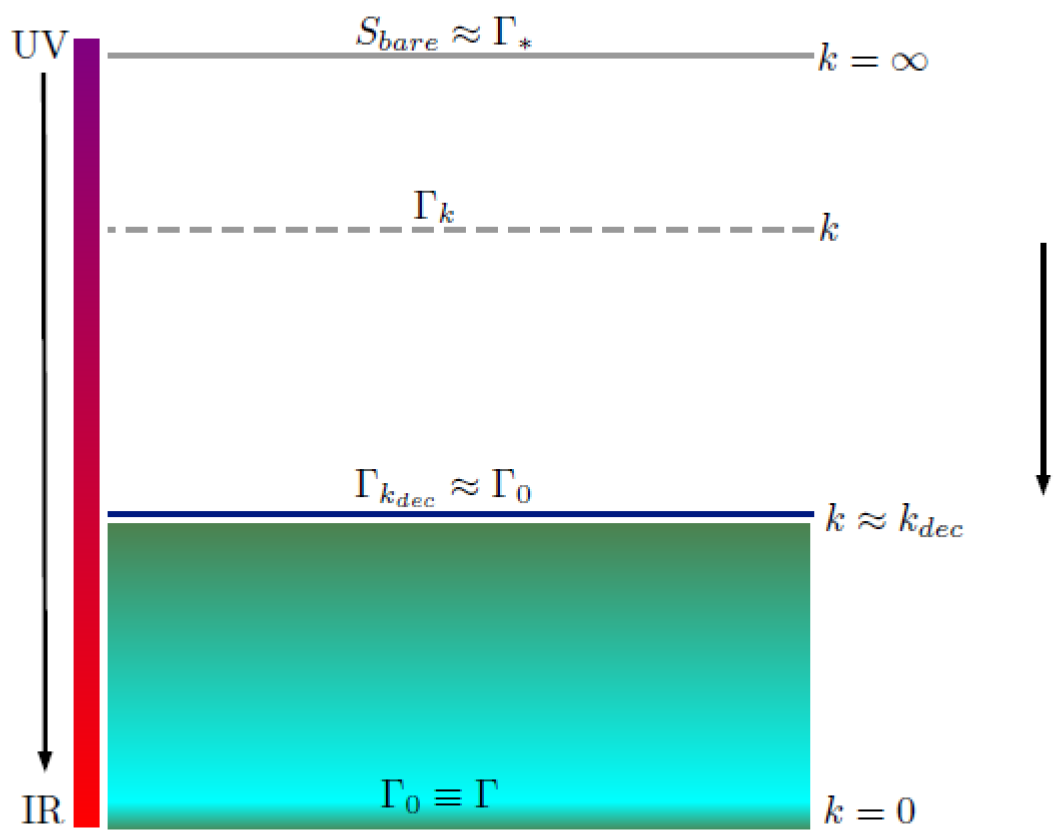
RG improved BHs, Koch and F. Saueressig BH

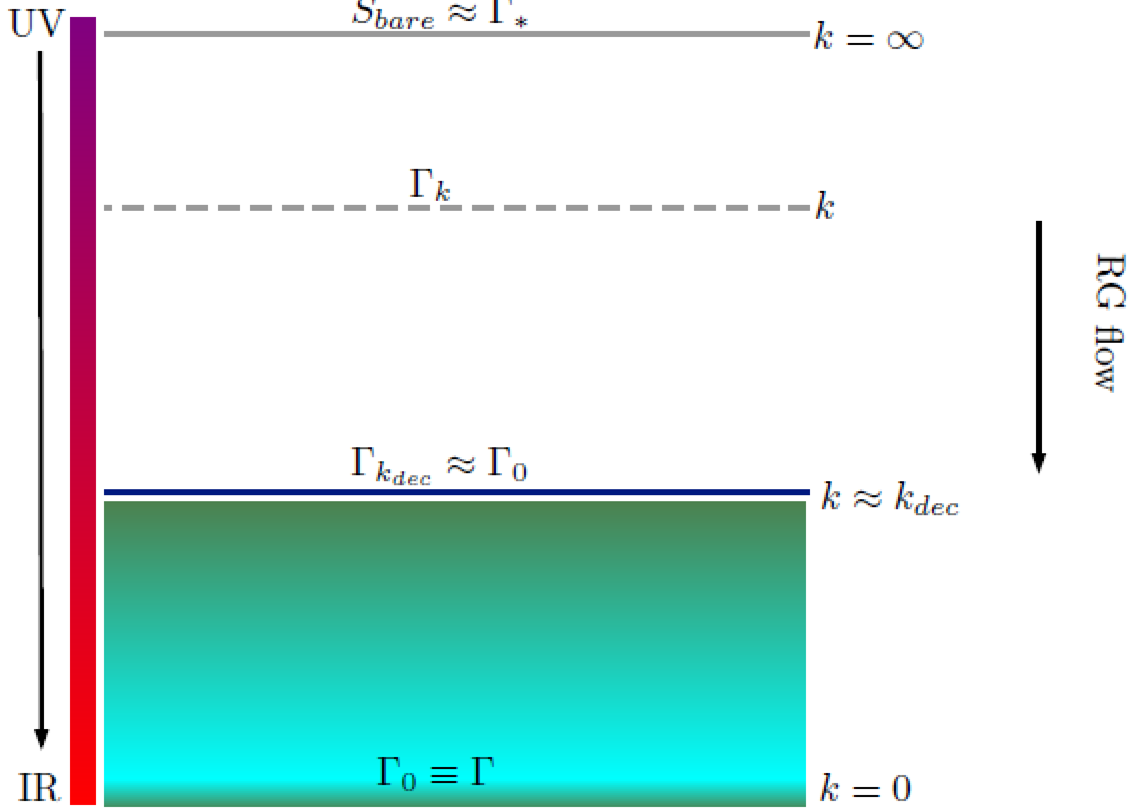
$$ds^2 = -\left(1 - \frac{2G_k M}{R} - \frac{1}{3}\Lambda_k R^2\right) dT^2 + \frac{dR^2}{1 - \frac{2G_k M}{R} - \frac{1}{3}\Lambda_k R^2} + R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

They do not process a singularity for some values of θ1, θ2

$$g_k = g_* + g_1\left(\frac{k}{M_\mathrm{P}}\right)^{-\theta_1} + g_2\left(\frac{k}{M_\mathrm{P}}\right)^{-\theta_2}$$

$$\lambda_k = \lambda_* + \lambda_1\left(\frac{k}{M_\mathrm{P}}\right)^{-\theta_1} + \lambda_2\left(\frac{k}{M_\mathrm{P}}\right)^{-\theta_2}$$





The RG improvement
was introduced in the context of gauge and matter field theories as a short-cut to access some of terms in the effective action and, in some cases, determine leading-order modifications to the solutions to the corresponding effective field equations .

The RG improvement procedure consists of the following steps.

1. First, starting from a classical system, e.g., an action or a solution, one replaces its couplings with their running counterparts, $g_i \rightarrow g_i(k)$. At the level of the action, this step is akin to promoting an ansatz for the action to an Effective Average Action EAA Γ_k.

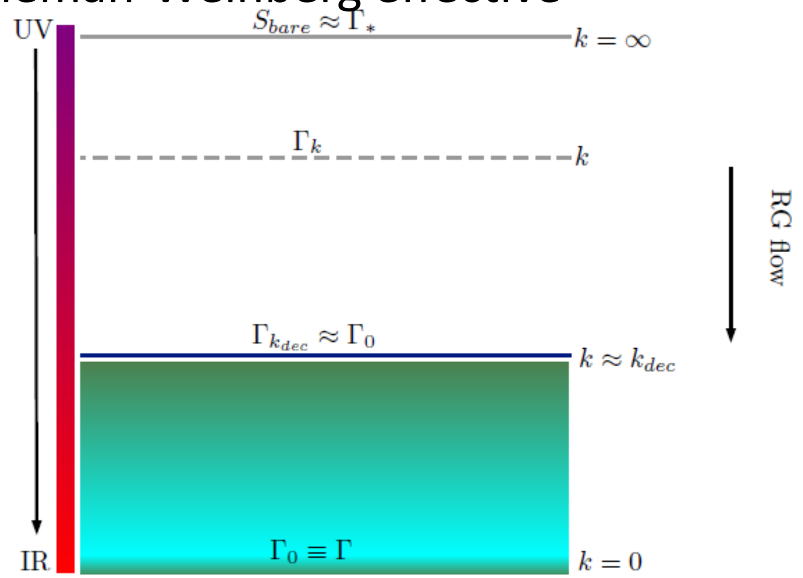
2. Second, the running couplings $g_i(k)$ are replaced with the solutions to the corresponding RG equations, $k\partial_k g_i(k) = \beta_i[g_j(k)]$, complemented by suitable physical initial conditions.

3.Finally, k is identified with a scale of the system which could act as a physical IR cutoff.

The reason why this procedure is supposed to work, at least in simple cases, lies in the so-called decoupling mechanism :

if in the flow of Γ_k there are physical IR scales (e.g., masses, curvature, or interactions terms) that prevail over the unphysical regulator R_k below a certain threshold scale k_{dec} , then as a result, the EAA at the decoupling scale approximates the full effective action Γ_0.

This idea is illustrated in Figure. In particular, the decoupling mechanism can give access to some of the interaction terms in the effective action that were not considered in the initial truncation. This is the case for instance in scalar electrodynamics, where the decoupling condition together with the RG improvement can be used to determine the Coleman-Weinberg effective potential .



Swiss cheese with Koch and F. Saueressig BH

$$ds^2 = -\left(1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2\right) dT^2 + \frac{dR^2}{1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2} + R^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

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R[t] = a[t] * r
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$$M_{MS} = m + \frac{r^3\, a[t]^3\, \Lambda[r,t]}{6\, G} - \frac{3\, r^4\, a[t]^2\, a'[t]^2}{2\, G\, \left(6\, G\, m - 3\, r\, a[t] + r^3\, a[t]^3\, \Lambda[r,t]\right)}$$

Assuming $G_k = G_N, \qquad \Lambda_k = \gamma k^b$

the k function of

$$k_S = \frac{\xi}{D_S},$$

the proper distance Ds

$$D_S = \int_{R_1}^{R_S} \frac{dR}{\sqrt{F(R)}}$$

The varying Λ becomes function of the Shucking radius rs :

$$rs = \left(\frac{3 * MM_{MS}}{\rho[t] * 4 * \pi * a[t]^3} \right) ^ (1 / 3)$$

$$\gamma t = G^{1-\frac{b}{2}} \gamma$$

$$\Lambda[r, t] := \gamma t * \xi ^ b \left(\frac{1}{G} * \left(\frac{\sqrt{G}}{rs} \right) ^ b \right)$$

Misner Sharp mass

Then,

$$M_{MS} = m + 2^{-1+\frac{2b}{3}} \times 3^{-1-\frac{b}{3}} G^{-2+\frac{b}{2}} M_{MS}^{-b/3} \pi^{b/3} r^3 \gamma t \xi^b a[t]^{3+b} \rho[t]^{b/3} - \frac{4 G m\, M_{MS}^{-1+\frac{b}{3}} \pi r^4 a[t]^4 \rho[t]}{3 G M_{MS}^{b/3} (2 G m - r a[t]) + 2^{2b/3} G^{b/2} \left(\frac{\pi}{3}\right)^{b/3} r^3 \gamma t \xi^b a[t]^{3+b} \rho[t]^{b/3}} - \frac{2^{-1+\frac{2b}{3}} \left(\frac{\pi}{3}\right)^{b/3} r^4 \gamma t \xi^b a[t]^4 \rho[t]^{b/3}}{3 G M_{MS}^{b/3} (2 G m - r a[t]) + 2^{2b/3} G^{b/2} \left(\frac{\pi}{3}\right)^{b/3} r^3 \gamma t \xi^b a[t]^{3+b} \rho[t]^{b/3}}$$

where everywhere r is the value of r at the matching surface ie, the Shucking radius rs

2^{nd} Approach

- A solid approach to tackle the cosmological coupling problem is to use a differential geometry framework that matches two spacetime solutions. Along the lines of the Einstein Strauss (swiss cheese model). (alternatively, along the reasoning of work on the subject by McVittie) .
- A general cosmological embedding of local objects is needed to give a general description of the coupling between local structures/ inhomogeneities and the cosmological background.

AS corrected Koch and F. Saueressig BH

Our swiss cheese model matches a homogeneous and isotropic cosmological metric with the AS corrected Koch and F. Saueressig metric

$$ds^2 = -\left(1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2\right)dT^2 + \frac{dR^2}{1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2} + R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

We much with a homogeneous and isotropic cosmological metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] ,$$

A Swiss cheese model with spherical symmetry overcomes the difficulty of how to glue a static solution of the theory at hand within a larger time-dependent homogeneous and isotropic spacetime.

The idea is to assume a very large number of local objects homogeneously and isotropically distributed in the universe. The matching of a spatially homogeneous metric as the exterior spacetime to a local interior solution has to be realized across a spherical boundary that stays at a fixed coordinate radius in the cosmological frame while evolves in the interior frame.

$r = r_\Sigma$

- In cosmological metric coordinates, a spherical boundary is defined to have a fixed coordinate radius $r = r_\Sigma$, with r_Σ, constant. Of course, this boundary is seen by a cosmological observer to expand, following the universal expansion.

Two matching conditions

Let us consider a four-dimensional manifold M with metric $g_{\mu\nu}$ and a timelike hypersurface Σ which splits the spacetime M into two parts.

Continuity of the spacetime across the hypersurface Σ implies that h_{ij} (induced metric) is continuous on Σ, which means that h_{ij} is the same when computed on either side of Σ.

Two matching conditions

If we consider Einstein gravity with a regular spacetime matter content and vanishing distributional energy-momentum tensor on Σ, then the Israel-Darmois matching conditions imply that the sum of the two extrinsic curvatures computed on the two sides of Σ is zero.

With metric representing a static spherically symmetric spacetime in Schwarzschild-like coordinates

The interior metric $r<r_\Sigma$ is replaced by the following metric

$$ds^2 = -J(R)F(R)dT^2 + \frac{dR^2}{F(R)} + R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) ,$$

First condition

$$JF\left(\frac{dT_S}{dt}\right)^2 - \frac{1}{F}\left(\frac{dR_S}{dt}\right)^2 = 1$$

$$R_S = ar_\Sigma\,.$$

Second condition

$$\frac{d^2T_S}{dt^2} = -\epsilon\sqrt{1-\kappa r_\Sigma^2}\,\frac{(F\sqrt{J})'}{F^2J}\,\frac{dR_S}{dt}$$

$$\frac{d^2R_S}{dt^2} = -\frac{F'}{2}\,.$$

If the black hole is described by the classical Schwarzschild solution we recover the dust FRW

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} - \frac{\kappa}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{G_N M}{r_\Sigma^3 a^3},$$

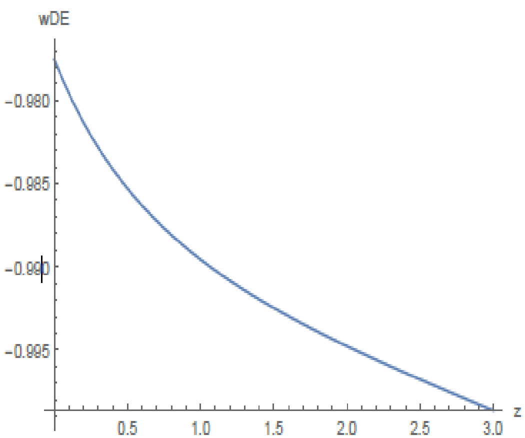
Scaling

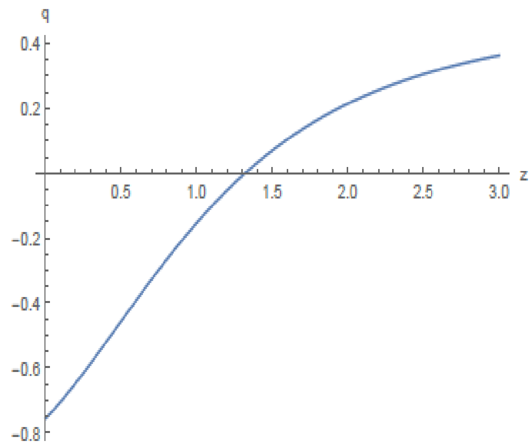
$$k_S = \frac{\xi}{R_S} \, .$$

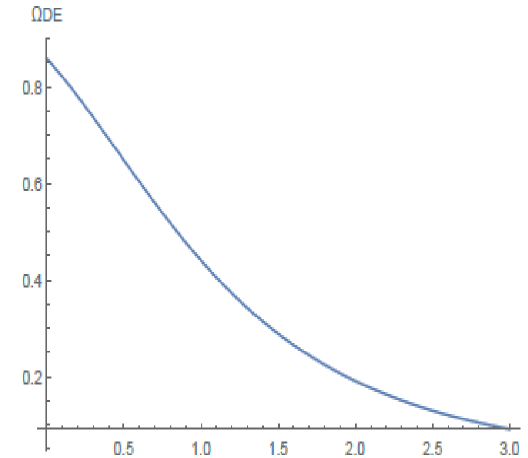
$$k_S = \frac{\xi}{D_S} \, ,$$

$$D_S = \int_{R_1}^{R_S} \frac{dR}{\sqrt{F(R)}}$$

Scaling as k=ξ/Ds







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What about fine tuning

The remarkable thing was that the proposed mechanism lacks also of a fine tuning problem!!! Why? Because astrophysical inhomogeneities length scale is connected with the today value of the cosmological constant needed to explain the amount of cosmic acceleration !!!!

Asymptotic safe gravity corrections works!

In the context of asymptotically safe gravity, we have explained the observed amount of dark energy using the galaxy or cluster length scales, and dimensionless order one parameters predicted by the theory, without fine-tuning or extra unproven energy scales.

Relevant papers

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Future work

- Simulations and comparison with observational data
- Extend the present study using Szekeres type models
- Find new spherical solutions using the new AS compatible modified Einstein equations

Thank you

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END