(Λ-)BMS symmetries in the Carroll and Galilei limits

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Outline:

1. Introduction
2. Symmetry algebras and their interrelations
   - Kinematical algebras – example of 2+1 dimensions
   - (Λ-)BMS_3 algebras and their subalgebras
3. Carrollian and Galilean contractions of the (Λ-)BMS algebras
   - Contractions of (Λ-)BMS_3
   - BMS_4 and its contractions
4. Conclusions and prospects

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Non-Lorentzian kinematics and gravity

Carrollian structures:

- Associated with the Carroll (“ultrarelativistic”) limit $c \to 0$
  - Ultralocality – trivial dynamics of free particles
  - Two Carroll limits of GR: “electric” and “magnetic”
  - Strong-gravity expansion, BKL conjecture, asymptotic silence[a]
- Null hypersurfaces of Lorentzian geometries
  - Black-hole horizons; plane gravitational waves
  - BMS group $\cong$ a conformal extension of Carroll group

[a] Mielczarek & **T. T.**, PRD **96**, 024012 (2017)

Galilean structures:

- Associated with the Galilei (“nonrelativistic”) limit $c \to \infty$
  - Weak-gravity expansion, gravitational waves
  - GR in this regime is extended to non-zero torsion
- Null reductions of Lorentzian geometries

Asymptotic (BMS) symmetries

Their variants and manifestations:

- standard BMS algebra was originally discovered at null infinity of asymptotically flat spacetimes
- relaxed boundary conditions lead to an extension of the algebra
- tweaking the conditions allows it to arise also at spatial infinity
- BMS can be generalized to the case of $\Lambda \neq 0$ (algebroid in 4d!)
- applications – memory effects, soft theorems, holography...

Their Carrollian and Galilean contraction limits:

- recently derived for the standard BMS algebra[a]
- obtained either algebraically or out of asymptotic conditions
- **our aim** was to investigate the (Barnich-Troessaert) BMS algebra

[a] A. Pérez, JHEP **12**, 173 (2021); O. Fuentealba et al., PRD **106**, 104047 (2022)

The origin of BMS algebra – example of 4d

Let us consider a spacetime metric (in retarded time u and spherical coordinates $r, x^2 = \varphi, x^3 = \theta$)

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}dudr + g_{AB}\left(U^A du - dx^A\right)\left(U^B du - dx^B\right), \quad (1)$$

where β, V, U^A and g_{AB} are arbitrary functions of the coordinates. Then, we assume the asymptotic flatness at null infinity, i.e. that

$$\beta = O(r^{-2})\,, \qquad U^A = O(r^{-2})\,, \qquad V/r = \ldots + O(r^{-1})\,, \qquad (2)$$

and g_{AB} is conformally spherical up to $O(r)$. These conditions are preserved by Killing vectors. If we allow the latter to be well defined only locally, they can be expanded on the sphere ($z, \bar{z} \in \mathbb{C}$) via

$$L_n = -z^{n+1}\partial_z\,, \quad \bar{L}_n = -\bar{z}^{n+1}\partial_{\bar{z}}\,, \quad T_{nm} = z^n\bar{z}^m\,, \qquad n, m \in \mathbb{Z}\,, \quad (3)$$

which generate (Barnich-Troessaert) BMS algebra[a] **(see later)**.

[a] G. Barnich, C. Troessaert, PRL **105**, 111103 (2010)

Lorentz, Carroll and Galilei (in 3d)

The brackets of Poincaré and (anti-)de Sitter algebras in (2+1)d can be written in a unified fashion (with $\Lambda = 0$, $\Lambda < 0$ or $\Lambda > 0$):

$$[J, K_a] = \epsilon_a{}^b K_b\,, \quad [K_1, K_2] = -J\,, \quad [J, P_a] = \epsilon_a{}^b P_b\,, \quad [J, P_0] = 0\,,$$
$$[K_a, P_b] = \delta_{ab} P_0\,, \quad [K_a, P_0] = P_a\,, \quad [P_1, P_2] = \Lambda\, J\,, \quad [P_0, P_a] = -\Lambda\, K_a\,. \quad (4)$$

Denoting $R := J\,, \mathcal{T}_a := P_a$ and rescaling $Q_a := c\, K_a\,, \mathcal{T}_0 := c\, P_0$, we take the limit $c \to 0$ to obtain Carroll / (anti-)de Sitter-Carroll algebra:

$$[R, Q_a] = \epsilon_a{}^b Q_b\,, \quad [Q_1, Q_2] = 0\,, \quad [R, \mathcal{T}_a] = \epsilon_a{}^b \mathcal{T}_b\,, \quad [R, \mathcal{T}_0] = 0\,,$$
$$[Q_a, \mathcal{T}_b] = \delta_{ab} \mathcal{T}_0\,, \quad [Q_a, \mathcal{T}_0] = 0\,, \quad [\mathcal{T}_1, \mathcal{T}_2] = \Lambda\, R\,, \quad [\mathcal{T}_a, \mathcal{T}_0] = \Lambda\, Q_a\,. \quad (5)$$

If we denote $R := J\,, \mathcal{T}_0 := P_0$ and rescale $Q_a := c^{-1} K_a\,, \mathcal{T}_a := c^{-1} P_a$, the limit $c \to \infty$ leads to Galilei / (anti-)de Sitter-Galilei algebra:

$$[R, Q_a] = \epsilon_a{}^b Q_b\,, \quad [Q_1, Q_2] = 0\,, \quad [R, \mathcal{T}_a] = \epsilon_a{}^b \mathcal{T}_b\,, \quad [R, \mathcal{T}_0] = 0\,,$$
$$[Q_a, \mathcal{T}_b] = 0\,, \quad [Q_a, \mathcal{T}_0] = \mathcal{T}_a\,, \quad [\mathcal{T}_1, \mathcal{T}_2] = 0\,, \quad [\mathcal{T}_a, \mathcal{T}_0] = \Lambda\, Q_a \quad (6)$$

(for $\Lambda \neq 0$, also known as the "expanding/oscillating Newton-Hooke").

Carroll and Galilei not from contractions

At the abstract level, Poincaré algebra is related by the isomorphism

$$K_a \mapsto |\Lambda|^{-1/2}\mathcal{T}_a\,, \quad P_a \mapsto -|\Lambda|^{1/2}Q_a\,, \quad J \mapsto R\,, \quad P_0 \mapsto \mathcal{T}_0 \qquad (7)$$

with anti-de Sitter-Carroll algebra, hence called the "para-Poincaré". Similarly, (inhomogeneous) Euclidean algebra:

$$\begin{aligned} &[J, K_a] = \epsilon_a{}^b K_b\,, \quad [K_1, K_2] = J\,, \quad [J, P_a] = \epsilon_a{}^b P_b\,, \quad [J, P_3] = 0\,, \\ &[K_a, P_b] = -\delta_{ab} P_3\,, \quad [K_a, P_3] = P_a\,, \quad [P_1, P_2] = 0\,, \quad [P_3, P_a] = 0 \end{aligned} \qquad (8)$$

describes different kinematics but is related by the isomorphism

$$K_a \mapsto \Lambda^{-1/2}\mathcal{T}_a\,, \quad P_a \mapsto \Lambda^{1/2}Q_a\,, \quad J \mapsto R\,, \quad P_3 \mapsto \mathcal{T}_0 \qquad (9)$$

with de Sitter-Carroll algebra, hence called the "para-Euclidean".

Both isomorphisms extend to $d > 2+1$. Other generic features are that Carroll algebra is a subalgebra of Poincaré one dimension higher, while Galilei algebra is a quotient subalgebra of the latter.

Real forms of a complex algebra – $\mathfrak{o}(4;\mathbb{C})$

The algebra $\mathfrak{o}(4;\mathbb{C}) \cong \mathfrak{o}(3;\mathbb{C}) \oplus \mathfrak{o}(3;\mathbb{C})$ in Cartan-Weyl basis becomes:

$$[H, E_\pm] = \pm E_\pm\,, \qquad [E_+, E_-] = 2H\,,$$
$$[\bar{H}, \bar{E}_\pm] = \pm \bar{E}_\pm\,, \qquad [\bar{E}_+, \bar{E}_-] = 2\bar{H}\,. \tag{10}$$

A real form of a complex Lie algebra $\mathfrak{g}$ is the pair $(\mathfrak{g}, \divideontimes)$, where $\divideontimes$ is an antilinear conjugation. There exist 7 real forms of $\mathfrak{o}(4;\mathbb{C})$, including:

- $\mathfrak{so}(3,1)$, with $H^* = -\bar{H}\,,\ E_\pm^* = -\bar{E}_\pm\,,\ \bar{H}^* = -H\,,\ \bar{E}_\pm^* = -E_\pm\,,$
- $\mathfrak{sl}(2;\mathbb{R}) \oplus \bar{\mathfrak{sl}}(2;\mathbb{R})$, with
  $H^* = -H\,,\ E_\pm^* = -E_\pm\,,\ \bar{H}^* = -\bar{H}\,,\ \bar{E}_\pm^* = -\bar{E}_\pm\,,$
- $\mathfrak{su}(1,1) \oplus \bar{\mathfrak{su}}(1,1)$, with
  $H^* = H\,,\ E_\pm^* = -E_\mp\,,\ \bar{H}^* = \bar{H}\,,\ \bar{E}_\pm^* = -\bar{E}_\mp\,,$
- $\mathfrak{su}(1,1) \oplus \bar{\mathfrak{su}}(1,1)$, with
  $H^* = H\,,\ E_\pm^* = -E_\mp\,,\ \bar{H}^* = -\bar{H}\,,\ \bar{E}_\pm^* = -\bar{E}_\pm\,.$

$\mathfrak{so}(3,1)$ is the (2+1)d de Sitter (or (3+1)d Lorentz) algebra, while the other three are isomorphic to (2+1)d anti-de Sitter algebra $\mathfrak{so}(2,2)$.

BMS algebra for (2+1)d spacetime

In terms of the generators of superrotations l_n and supertranslations T_n, BMS algebra in 2+1 dims (BMS$_3$) is given by:

$$[l_n, l_m] = (n - m)\, l_{n+m}\,, \quad [l_n, T_m] = (n - m)\, T_{n+m}\,, \tag{11}$$

where $n, m \in \mathbb{Z}$ and T_n are commutative. This complex algebra has four real forms, inherited from $\mathfrak{o}(3, \mathbb{C})$:

- type $\mathfrak{sl}(2, \mathbb{R})$, with $l_n^* = -l_n\,,\ T_n^* = -T_n\,,$
- type $\mathfrak{su}(1, 1)$, with $l_n^* = l_{-n}\,,\ T_n^* = T_{-n}\,,$
- type $\mathfrak{so}(2, 1)$, with $l_n^* = (-1)^{n+1} l_n\,,\ T_n^* = (-1)^{n+1} T_n\,,$
- type $\mathfrak{su}(1, 1)|\mathfrak{su}(2)$, with $l_n^* = (-1)^n l_{-n}\,,\ T_n^* = (-1)^n T_{-n}\,.$

The latter two cases will be skipped here.

Types $\mathfrak{sl}(2, \mathbb{R})$ and $\mathfrak{su}(1, 1)$ are isomorphic. However, the distinction can still matter, cf. quantum-group deformations of $\mathfrak{o}(4; \mathbb{C})$[a].

[a]Kowalski-Glikman, Lukierski & **T. T.**, JHEP **09**, 096 (2020)
T. T., JHEP **02**, 200 (2024)

Asymptotic (anti-)de Sitter symmetries

BMS$_3$ can be generalized to the (complex) algebra Λ-BMS$_3$:

$$[l_n, l_m] = (n-m)\, l_{n+m}\,, \qquad [l_n, T_m] = (n-m)\, T_{n+m}\,,$$
$$[T_n, T_m] = -\Lambda\, (n-m)\, l_{n+m}\,. \tag{12}$$

where $n, m \in \mathbb{Z}$ and BMS$_3$ is recovered for $\Lambda \to 0$. If we change the basis to $L_n/\bar{L}_n := \frac{1}{2}\left(l_n \pm \frac{1}{\sqrt{-\Lambda}}\, T_n\right)$, so that the brackets (12) become

$$[L_n, L_m] = (n-m)\, L_{n+m}\,, \qquad [\bar{L}_n, \bar{L}_m] = (n-m)\, \bar{L}_{n+m}\,, \tag{13}$$

the real forms of Λ-BMS$_3$, inherited from $\mathfrak{o}(4,\mathbb{C})$, are as follows:

- type $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$, with $L_n^* = -L_n\,,\ \bar{L}_n^* = -\bar{L}_n\,,\ \Lambda < 0\,,$
- type $\mathfrak{su}(1,1) \oplus \mathfrak{su}(1,1)$, with $L_n^* = L_{-n}\,,\ \bar{L}_n^* = \bar{L}_{-n}\,,\ \Lambda < 0\,,$
- type $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{su}(1,1)$, with $L_n^* = -L_n\,,\ \bar{L}_n^* = \bar{L}_{-n}\,,\ \Lambda < 0\,,$
- type $\mathfrak{so}(3,1)_{\mathrm{a}}$, with $L_n^* = -\bar{L}_n\,,\ \Lambda > 0\,,$
- type $\mathfrak{so}(3,1)_{\mathrm{b}}$, with $L_n^* = \bar{L}_{-n}\,,\ \Lambda > 0\,.$

Embedding kinematical algebras into (Λ-)BMS$_3$

(Λ-)BMS$_3$ contains a family of subalgebras $\mathrm{span}\{l_0, l_{\pm n}, T_0, T_{\pm n}\}$, for $n \in \mathbb{N}$, isomorphic to $\mathfrak{o}(4;\mathbb{C})$ or inhomogeneous $\mathfrak{o}(3;\mathbb{C})$, respectively.

If we impose the (Λ-)BMS$_3$ real form of type: $\mathfrak{su}(1,1)$, $\mathfrak{su}(1,1) \oplus \mathfrak{su}(1,1)$ or $\mathfrak{so}(3,1)_{\mathrm{b}}$, it allows to define embeddings of Poincaré algebra $\mathfrak{iso}(2,1)$ into BMS$_3$ or (a)dS algebra ($\mathfrak{so}(2,2)$) $\mathfrak{so}(3,1)$ into Λ-BMS$_3$:

$$J = i\, l_0\,, \qquad K_{1(n)} = \tfrac{1}{2}\,(l_n - l_{-n})\,, \qquad K_{2(n)} = -\tfrac{i}{2}\,(l_n + l_{-n})\,,$$
$$P_0 = i\, T_0\,, \qquad P_{1(n)} = \tfrac{i}{2}\,(T_n + T_{-n})\,, \qquad P_{2(n)} = \tfrac{1}{2}\,(T_n - T_{-n})\,, \quad (14)$$

so that the generators are anti-Hermitian. Similarly, in the case of the (Λ-)BMS$_3$ real form of type: $\mathfrak{sl}(2,\mathbb{R})$, $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$ or $\mathfrak{so}(3,1)_{\mathrm{a}}$, there exists an alternative family of embeddings:

$$J_{(n)} = \tfrac{1}{2}\,(l_n + l_{-n})\,, \qquad K_{1(n)} = \tfrac{1}{2}\,(l_n - l_{-n})\,, \qquad K_2 = -l_0\,,$$
$$P_{0(n)} = \tfrac{1}{2}\,(T_n + T_{-n})\,, \qquad P_{2(n)} = \tfrac{1}{2}\,(T_n - T_{-n})\,, \qquad P_1 = T_0\,. \quad (15)$$

Extending the contractions to BMS_3

The assumption: a given contraction of BMS_3 should turn its Poincaré subalgebras into Carroll/Galilei subalgebras of the contracted BMS_3. This can be consistently implemented for the $\mathfrak{su}(1,1)$-type real form. The case of type $\mathfrak{sl}(2,\mathbb{R})$ – **wait for a discussion of BMS_4.**

In particular, rescaling $\forall n \neq 0 : l_n \mapsto c\, l_n = \tilde{l}_n$, $T_0 \mapsto c\, T_0 = \widetilde{T}_0$ and taking the limit $c \to 0$, we obtain the Carroll-BMS_3 algebra:

$$[\tilde{l}_n, \tilde{l}_m] = 0\,, \qquad [l_0, \tilde{l}_n] = -n\,\tilde{l}_n\,, \qquad [l_0, T_n] = -n\, T_n\,,$$
$$[l_0, \widetilde{T}_0] = 0\,, \qquad [\tilde{l}_n, \widetilde{T}_0] = 0\,, \qquad [\tilde{l}_n, T_m] = 2\delta_{m,-n} n\, \widetilde{T}_0\,. \qquad (16)$$

Similarly, rescaling $\forall n \neq 0 : l_n \mapsto c^{-1} l_n = \hat{l}_n$, $T_n \mapsto c^{-1} T_n = \widehat{T}_n$ and taking the limit $c \to \infty$ leads to the Galilei-BMS_3 algebra:

$$[\hat{l}_n, \hat{l}_m] = 0\,, \qquad [l_0, \hat{l}_n] = -n\,\hat{l}_n\,, \qquad [l_0, \widehat{T}_n] = -n\, \widehat{T}_n\,,$$
$$[l_0, T_0] = 0\,, \qquad [\hat{l}_n, T_0] = n\, \widehat{T}_n\,, \qquad [\hat{l}_n, \widehat{T}_m] = 0\,. \qquad (17)$$

Applying the same approach to Λ-BMS_3

In the Carrollian case, it leads to the brackets of Carroll-BMS_3 and

$$\left[T_n, \widetilde{T}_0\right] = -\Lambda\, n\, \widetilde{l}_n\,, \qquad \left[T_n, T_{-n}\right] = -2\Lambda\, n\, l_0\,, \qquad \left[T_n, T_m\right] \to \infty\,, \quad (18)$$

which means that the contraction limit does not exist!
In the Galilean case, we obtain Galilei-Λ-BMS_3 algebra, which differs from Galilei-BMS_3 by the additional non-zero bracket:

$$\left[\widehat{T}_n, T_0\right] = -\Lambda\, n\, \widehat{l}_n\,. \qquad (19)$$

The interpretation of contractions again depends on a real form.

Moreover, extending the isomorphism (7) of Poincaré and adS-Carroll algebras to BMS_3 and (the alleged) Carroll-Λ-BMS_3, we recover (18) (without the divergence) but miss one bracket of Carroll-BMS_3. This may be compared with the (3+1)d case[a]

[a] A. Pérez, JHEP **09**, 044 (2022)

BMS algebra for (3+1)d spacetime

The Barnich-Troessaert (extended) BMS algebra in 3+1 dims, or BMS_4, has the brackets

$$\begin{aligned}
[L_n, L_m] &= (n-m)\, L_{n+m}\,, & [L_k, T_{nm}] &= \left(\tfrac{k+1}{2} - n\right) T_{n+k,m}\,,\\
[\bar{L}_n, \bar{L}_m] &= (n-m)\, \bar{L}_{n+m}\,, & [\bar{L}_k, T_{nm}] &= \left(\tfrac{k+1}{2} - m\right) T_{n,m+k}\,,\\
[L_n, \bar{L}_m] &= 0\,, & [T_{nm}, T_{n'm'}] &= 0
\end{aligned} \qquad (20)$$

for the generators of superrotations L_n, $\bar{L}_n$ and supertranslations T_{nm}, where $n, m \in \mathbb{Z}$. Its subalgebra $\mathrm{span}\{L_n, \bar{L}_n\,; n \in \mathbb{Z}\} \cong \Lambda\text{-BMS}_3$ and hence one can define the following real forms of BMS_4:

- type $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$, i.e. $L_n^* = -L_n\,,\ \bar{L}_n^* = -\bar{L}_n\,,\ T_{nm}^* = -T_{nm}\,,$
- type $\mathfrak{su}(1,1) \oplus \mathfrak{su}(1,1)$, i.e. $L_n^* = L_{-n}\,,\ \bar{L}_n^* = \bar{L}_{-n}\,,\ T_{nm}^* = -T_{1-n,1-m}\,,$
- type $\mathfrak{so}(3,1)_{\mathrm{a}}$, i.e. $L_n^* = -\bar{L}_n\,,\ T_{nm}^* = T_{mn}\,,$
- type $\mathfrak{so}(3,1)_{\mathrm{b}}$, i.e. $L_n^* = \bar{L}_{-n}\,,\ T_{nm}^* = T_{1-m,1-n}\,.$

The latter two (with Lorentzian-like superrotations) are of interest here.

Embedding Poincaré algebra into BMS_4

If we consider the real form of type $\mathfrak{so}(3,1)_{\mathrm{a}}$, the basis

$$R_n := L_n + \bar{L}_n\,, \qquad \bar{R}_n := -i\,(L_n - \bar{L}_n)\,,$$
$$S_{nm} := i\,\tfrac{1}{2}(T_{nm} + T_{mn})\,, \qquad A_{nm} := \tfrac{1}{2}(T_{nm} - T_{mn}) \qquad (21)$$

is anti-Hermitian and allows to define a family of embeddings of Poincaré algebra $\mathfrak{iso}(3,1)$ into BMS_4 for $n \in 2\mathbb{N}-1$:

$$J_1^{(n)} = \tfrac{1}{2}(\bar{R}_{-n} - \bar{R}_n)\,, \qquad J_2^{(n)} = \tfrac{1}{2}(R_n + R_{-n})\,, \qquad J_3 = -\bar{R}_0\,,$$
$$K_1^{(n)} = \tfrac{1}{2}(R_n - R_{-n})\,, \qquad K_2^{(n)} = \tfrac{1}{2}(\bar{R}_n + \bar{R}_{-n})\,, \qquad K_3 = R_0\,,$$
$$P_{0/3}^{(n)} = \tfrac{1}{2}(S_{qq} \pm S_{pp})\,, \qquad P_1^{(n)} = S_{pq}\,, \qquad P_2^{(n)} = A_{pq}\,, \qquad (22)$$

where $p = \frac{1+n}{2}$, $q = \frac{1-n}{2}$. Therefore, the embeddings do not cover the whole BMS_4 and do not provide a framework for the contractions.

Our work on the real form of type $\mathfrak{so}(3,1)_{\mathrm{b}}$ is not yet complete.

Using the analogy of structures of BMS_4 and $\mathfrak{iso}(3,1)$

We define the quasi-Carrollian contraction as rescaling $\bar{R}_n \mapsto c\,\bar{R}_n$, $A_{pq} \mapsto c\,A_{pq}$, $\forall n,p,q \in \mathbb{Z}$ and taking the limit $c \to 0$, which gives:

$$
\begin{aligned}
&[R_n, R_m] = (n-m)\,R_{n+m}\,, \qquad [R_n, \bar{R}_m] = (n-m)\,\bar{R}_{n+m}\,,\\
&[R_n, S_{pq}] = \left(\tfrac{1}{2}(n+1)-p\right) S_{p+n,q} + \left(\tfrac{1}{2}(n+1)-q\right) S_{p,q+n}\,,\\
&[R_n, A_{pq}] = \left(\tfrac{1}{2}(n+1)-p\right) A_{p+n,q} + \left(\tfrac{1}{2}(n+1)-q\right) A_{p,q+n}\,,\\
&[\bar{R}_n, S_{pq}] = \left(\tfrac{1}{2}(n+1)-p\right) A_{p+n,q} - \left(\tfrac{1}{2}(n+1)-q\right) A_{p,q+n}\,.
\end{aligned} \tag{23}
$$

The quasi-Galilean contraction consists in rescaling $\bar{R}_n \mapsto c^{-1}\bar{R}_n$, $S_{pq} \mapsto c^{-1}S_{pq}$, $\forall n,p,q \in \mathbb{Z}$ and taking the limit $c \to \infty$, which gives:

$$
\begin{aligned}
&[R_n, R_m] = (n-m)\,R_{n+m}\,, \qquad [R_n, \bar{R}_m] = (n-m)\,\bar{R}_{n+m}\,,\\
&[R_n, S_{pq}] = \left(\tfrac{1}{2}(n+1)-p\right) S_{p+n,q} + \left(\tfrac{1}{2}(n+1)-q\right) S_{p,q+n}\,,\\
&[R_n, A_{pq}] = \left(\tfrac{1}{2}(n+1)-p\right) A_{p+n,q} + \left(\tfrac{1}{2}(n+1)-q\right) A_{p,q+n}\,,\\
&[\bar{R}_n, A_{pq}] = -\left(\tfrac{1}{2}(n+1)-p\right) S_{p+n,q} + \left(\tfrac{1}{2}(n+1)-q\right) S_{p,q+n}\,.
\end{aligned} \tag{24}
$$

Interpretation of the new type of contractions

In terms of the lightcone basis of Poincaré subalgebras:

$$M_{\pm 1}^{(n)} = \tfrac{1}{\sqrt{2}}\left(\pm J_2^{(n)} - K_1^{(n)}\right), \qquad M_{\pm 2}^{(n)} = \tfrac{1}{\sqrt{2}}\left(\mp J_1^{(n)} - K_2^{(n)}\right),$$
$$P_{\pm}^{(n)} = \tfrac{1}{\sqrt{2}}\left(P_0^{(n)} \pm P_3^{(n)}\right), \tag{25}$$

the quasi-Carrollian contraction involves rescaling of the generators of spatial translations $P_2^{(n)}$, while quasi-Galilean – the generators of null and spatial translations $P_{\pm}^{(n)}$, $P_1^{(n)}$, as well as they both involve rescaling of the generators of null rotations $M_{\pm 2}^{(n)}$ and rotations J_3.

The contractions of (Λ-)BMS_3 for the alternative real forms (type $\mathfrak{sl}(2,\mathbb{R})$, $\mathfrak{sl}(2,\mathbb{R})\oplus\mathfrak{sl}(2,\mathbb{R})$ or $\mathfrak{so}(3,1)_{\mathrm{a}}$) have exactly the same meaning of quasi-Carrollian/Galilean but lead to the Carroll/Galilei contraction limits.

Conclusions and prospects

Summary:

- BMS_3 and Λ-BMS_3 can be covered by (overlapping) subalgebras isomorphic to 3D Poincaré or (anti-)de Sitter, respectively
- we define the Carrollian/Galilean contraction of (Λ-)BMS_3 by demanding that all such subalgebras become Carrollian/Galilean
- the interpretation changes for certain real forms of (Λ-)BMS_3
- Carroll-Λ-BMS_3 turns out to not exist, at least in the expected way
- (extended) BMS_4 is a more tricky case, the work still in progress

Some future directions:

- a more detailed comparison with results for the standard BMS
- what asymptotic conditions lead to our contraction limits of BMS?
- applying the contractions to quantum-group BMS deformations[a]

[a] A. Borowiec et al., JHEP **02**, 084 (2021); JHEP **11**, 103 (2021)