Complex scalar field in κ-Minkowski noncommutative spacetime

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Motivation

- Due to a lack of experimental data, the quantum theory of gravity remains a distant future
- An attractive idea is thus an intermediate theory - quantum effects in the flat limit - e.g. spacetime noncommutativity
- One of the most well-studied models is the so-called κ-Minkowski spacetime[1][2]:

$$[\hat{x}^0, \hat{x}^j] = \frac{i}{\kappa}\hat{x}^j, \quad [\hat{x}^i, \hat{x}^j] = 0$$

- The deformation parameter $1/\kappa$ has the dimension of length and is expected to be of the order of Planck length

[1] J. Lukierski, H. Ruegg and W. J. Zakrzewski, "Classical quantum mechanics of free kappa relativistic systems," *Annals Phys.* **243** 90 (1995) [hep-th/9312153]

[2] S. Majid and H. Ruegg, "Bicrossproduct structure of kappa Poincaré group and noncommutative geometry," *Phys. Lett. B* **334** 348 (1994) [hep-th/9405107].

κ-Minkowski

- In the complementary picture of deformed spacetime symmetries (κ-Poincaré), the square of $1/\kappa$ can be regarded as the curvature of momentum space, which turns out to be a submanifold of dS_4[3].
- This is most apparent in the so-called classical basis:

$$-p_0^2 + \mathbf{p}^2 + p_4^2 = \kappa^2$$

- In this basis, the algebra sector of spacetime symmetries remains undeformed. The deformation affects the coalgebra, resulting in non-linearly modified composition rules for momenta:

Undeformed:

- $(p \oplus q)_0 = p_0 + q_0$
- $(p \oplus q)_j = p_j + q_j$

Deformed:

- $(p \oplus q)_0 = \frac{q_0+q_4}{\kappa} p_0 + \frac{\mathbf{p}\cdot\mathbf{q}}{p_0+p_4} + \frac{\kappa}{p_0+p_4} q_0$
- $(p \oplus q)_j = \frac{q_0+q_4}{\kappa} p_j + q_j$

[3] J. Kowalski-Glikman and S. Nowak, "Doubly special relativity and de Sitter space," *Class. Quant. Grav.* **20** (2003), [arXiv:hep-th/0304101 [hep-th]].

κ-Minkowski

- These deformed composition rules constitute the backbone of all operations on plane waves. We make use of the following Weyl map:

$$\mathcal{W}(e^{i\mathbf{p}\cdot\hat{\mathbf{x}}}e^{ip_0\hat{x}^0}) = e^{ipx}$$

which gives rise to the star product:

$$e^{ipx} \star e^{iqx} = \mathcal{W}(e^{i\mathbf{p}\cdot\hat{\mathbf{x}}}e^{ip_0\hat{x}^0}e^{i\mathbf{q}\cdot\hat{\mathbf{x}}}e^{iq_0\hat{x}^0}) = e^{i(p\oplus q)x}$$

- They also result in the antipodal map which replaces conventional plane wave conjugation:

$$\left(e^{ipx}\right)^\dagger = e^{iS(p)x}$$

where

$$S(p_0) = -p_0 + \frac{\mathbf{p}^2}{p_0 + p_4},$$

$$S(p_j) = -\frac{\kappa}{p_0 + p_4}p_j$$

Motivation

- Since we are interested in phenomenology, we need to apply the mathematical structure to a physical object, hence - field theory
- There have been many attempts at this, notably the 2008 paper[4] by Freidel, Kowalski-Glikman and Nowak
- In our present research, we are interested in how κ-deformation affects discrete symmetries, which could be a potential window into experimental probing. For the $\mathcal{C}$ transformation to be non-trivial, we begin with the complex scalar field

[4] L. Freidel, J. Kowalski-Glikman and S. Nowak, "Field theory on kappa-Minkowski space revisited: Noether charges and breaking of Lorentz symmetry," *Int. J. Mod. Phys. A* **23** (2008), [arXiv:0706.3658 [hep-th]].

Complex scalar field

- With the help of the Weyl map and noncommutative Fourier transforms, we can perform all calculations on commutative x variables but using the deformed composition rules. We define the on-shell complex scalar field as[5]

$$\phi(x) = \int \frac{d^3p}{2\omega_{\mathbf{p}} p_4/\kappa} \tilde{\phi}(p) e^{ipx} + \int \frac{d^3S(p)}{2\omega_{S(\mathbf{p})} p_4/\kappa} \tilde{\phi}(S(p)) e^{iS(p)x},$$

  which, with the appropriate choice of $a_{\mathbf{p}}$ and $b_{\mathbf{p}}$, becomes

$$\phi(x) = \int \frac{d^3p}{\sqrt{2\omega_{\mathbf{p}}}} \left(a_{\mathbf{p}} e^{ipx} + b^\dagger_{\mathbf{p}} e^{i(S(p))x} \right)$$

  in full analogy with the undeformed case (where $S(p) = -p$)

[5] here ipx is shorthand for $i(\omega_{\mathbf{p}} t - \mathbf{x} \cdot \mathbf{p})$

The action

- In order to obey $\mathcal{C}$-invariance, the action has to be

$$S = \frac{1}{2}(S_1 + S_2)$$

where

$$S_1 = \frac{1}{2}\int d^4x\,(\partial_\mu\phi)^\dagger \star \partial^\mu\phi - m^2\phi^\dagger \star \phi$$

$$S_2 = \frac{1}{2}\int d^4x\partial^\mu\phi \star (\partial_\mu\phi)^\dagger - m^2\phi \star \phi^\dagger$$

- The equations of motion turn out to have the same form as in the undeformed case, although arriving at them is a bit more involved

Noether analysis

- Our main interest lies in the Noether charges of κ-Poincaré and discrete symmetries.
- The deformed composition and conjugation of plane waves makes canonical calculations of these quantities much more challenging than expected
- In 2022, the charges were computed[6] using the covariant phase space method, which circumnavigates these difficulties by approaching the problem geometrically:
  the charge Q_ξ associated with the vector field ξ is calculated[7] from the equation

$$-\delta_\xi \lrcorner \Omega = \delta Q_\xi$$

  where δ is the exterior derivative in phase space and Ω is the symplectic form

[6] A. Bevilacqua, J. Kowalski-Glikman and W. Wislicki, "κ-deformed complex scalar field: Conserved charges, symmetries, and their impact on physical observables," *Phys. Rev. D* **105** (2022) [arXiv:2201.10191 [hep-th]].

[7] D. Harlow and J. Q. Wu, "Covariant phase space with boundaries," *JHEP* **10** (2020) [arXiv:1906.08616 [hep-th]].

Noether analysis

- The results, as expected, reduce to the well-known undeformed charges in the $\kappa \to \infty$ limit:

$$\mathcal{P}_\mu = \int d^3p \left(-S(p_\mu) a_\mathbf{p} a_\mathbf{p}^\dagger + p_\mu b_\mathbf{p} b_\mathbf{p}^\dagger \right)$$

$$\mathcal{N}_j = -\frac{1}{2} \int d^3p \left[S(\omega_\mathbf{p}) \left(\frac{\partial a_\mathbf{p}^\dagger}{\partial S(p^j)} a_\mathbf{p} - \frac{\partial a_\mathbf{p}}{\partial S(p^j)} a_\mathbf{p}^\dagger \right) + \omega_\mathbf{p} \left(\frac{\partial b_\mathbf{p}}{\partial p^j} b_\mathbf{p}^\dagger - \frac{\partial b_\mathbf{p}^\dagger}{\partial p^j} b_\mathbf{p} \right) \right]$$

$$\mathcal{M}_i = -\epsilon_i^{\ jk} \frac{1}{8} \int d^3p \left[S(p_j) \left(\frac{\partial a_\mathbf{p}^\dagger}{\partial S(p^k)} a_\mathbf{p} - \frac{\partial a_\mathbf{p}}{\partial S(p^k)} a_\mathbf{p}^\dagger \right) + p_j \left(\frac{\partial b_\mathbf{p}}{\partial p^k} b_\mathbf{p}^\dagger - \frac{\partial b_\mathbf{p}^\dagger}{\partial p^k} b_\mathbf{p} \right) \right]$$

- The charges computed this way indeed satisfy the traditional Poincaré algebra, but they possess a peculiar property

Noether analysis

- The boosts do not commute with the charge conjugation operator $\mathcal{C} = \int d^3p(b^\dagger_{\mathbf{p}} a_{\mathbf{p}} + a^\dagger_{\mathbf{p}} b_{\mathbf{p}})$[8]:

$$[\mathcal{N}_j, \mathcal{C}] =\neq 0$$

- This has very interesting, potentially testable phenomenological consequences:
  while particles and antiparticles at rest have the same (usual) mass-shell relation, charge conjugation applied to a boosted particle should produce an antiparticle of different momentum
- This property could already be used to devise experiments that could put some bounds on the possible value of the deformation parameter $1/\kappa$

[8] A. Bevilacqua, J. Kowalski-Glikman and W. Wislicki, "κ-deformed complex scalar field: Conserved charges, symmetries, and their impact on physical observables," *Phys. Rev. D* **105** (2022) [arXiv:2201.10191 [hep-th]]

Present issues

- Many techniques known from undeformed field theory break down or require careful alterations in κ-Minkowski
- For example, the Jost-Whiteman-Greenberg [9][10][11] theorem has been shown not to hold[12], which frees us to seek CPT violations without breaking Lorentz invariance
- An investigation is currently underway on whether the results obtained from the covariant phase space formalism can be reproduced using the canonical method starting from the variation of the action
- The charges have been reproduced for the real field, however the complex case in κ-Minkowski is significantly more challenging, unlike its counterpart in commutative spacetime

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[9] O. W. Greenberg, "CPT violation implies violation of Lorentz invariance," *Phys. Rev. Lett.* **89** (2002) [arXiv:hep-ph/0201258 [hep-ph]]

[10] R. Jost, "A remark on the C.T.P. theorem," *Helv. Phys. Acta* **30** (1957)

[11] A. Wightman, "Recent Achievements of Axiomatic Field Theory"

[12] A. Bevilacqua, J. Kowalski-Glikman and W. Wislicki, "κ-deformed complex scalar field: Conserved charges, symmetries, and their impact on physical observables," *Phys. Rev. D* **105** (2022) [arXiv:2201.10191 [hep-th]]

Future prospects

- As soon as we can clarify these findings - that is either confirm them or find out how to tweak the method, we will proceed to apply it to higher spin fields, and eventually interacting theories, looking for further phenomenological implications