Workshop on Noncommutative and Generalized Geometry in String theory, Gauge theory and Related Physical Models

Conformal defects and RG flows

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2211.16501, 2205.01647, 2312.13283

also with Marco Bianchi, 2409.xxxxx

Defects in Gauge Theories

Defect are classified as

- order-type defects described by the insertion in the path integral of non-local operators supporting dynamical degrees of freedom which dress the vacuum non-trivially.

  Ex: Wilson lines/loops

$$\langle\langle \cdots \rangle\rangle = \int [d\phi] e^{iS(\phi)}\, W \ldots\,, \qquad W = \mathcal{P} e^{-i \int_\Gamma \mathcal{L}(\phi)}$$

- disorder-type defects defined as non-trivial boundary conditions in the path integral

$$\langle\langle \cdots \rangle\rangle = \int [d\phi]_{\phi(\Gamma)=\phi_0}\, e^{-S(\phi)} \ldots$$

  Ex: 't Hooft lines/loops

- kinematical defects: Submanifolds supporting protected sectors of operators of the bulk theory.

  Ex: The topological line

We are interested in studying Wilson loops

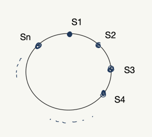
Wilson loops as one-dimensional dCFT

- In a CFT W preserves the 1d conformal algebra $su(1,1) \sim sl(2,\mathbb{R})$

- Addition of supersymmetry leads to dSCFT preserving a fraction $\#$ of supercharges ($\#$-BPS WL)

- Observables
  - Free energy of the dSCFT $\quad \mathcal{F} = \log\langle 0|W|0\rangle \qquad (\langle 0|W|0\rangle \equiv \langle\langle 0|0\rangle\rangle)$
  - 1d correlation functions of local operators in unitary irr representations of the superconformal algebra on the defect

$$\langle\langle \mathcal{O}(s_1)\mathcal{O}(s_2)\ldots\mathcal{O}(s_n)\rangle\rangle \equiv \frac{\langle \mathrm{Tr}\left[W_{(0,s_1)}\mathcal{O}(s_1)W_{(s_1,s_2)}\mathcal{O}(s_2)\ldots W_{(s_{n-1},s_n)}\mathcal{O}(s_n)W_{(s_n,2\pi)}\right]\rangle}{\langle 0|W_{(0,2\pi)}|0\rangle}$$



- Classifying all the 1d superconformal defects means classifying all the unitary irr representations of the superconformal algebra preserved by the defect.

- Conformal data: 1-pt functions, anomalous dimensions, 3-pt function coefficients (OPE).
  They are computable using Boostrap, SUSY localization, Integrability, AdS/CFT correspondence

- Interaction with the bulk theory
  - Bulk-boundary correlators (bulk-to-defect OPE gives another set of conformal data)
  - Bulk physical observables obtained from defect data.

Main motivation

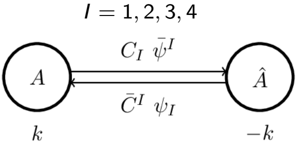
A central problem in theoretical physics is the identification of the space of consistent QFTs (*theory space*). We attempt a classification of dSCFTs (fixed points and RG flows).

3D supersymmetric Chern-Simons-matter theories, in particular ABJM theory

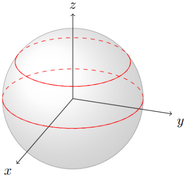
Plan of the talk

- Review on the classification of (BPS) Wilson loops in ABJM theory
- Study of RG flows connecting dCFTs: ordinary and enriched flows
- Irreversibility of the flows
- Conclusions and perspectives

MWLs in ABJM



$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint_{\mathcal{C}}\left(A_\mu \dot{x}^\mu + \text{matter}\right)\,dt\right]$$



- 1-node loops (bosonic, at most 1/6 BPS)

$$W^{\mathrm{bos}} = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint\left(A + \xi\, C\bar{C}\right)\,dt\right] \qquad \hat{W}^{\mathrm{bos}} = \ldots$$

- 2-node loops (fermionic, at most 1/2 BPS)

$$W^{\mathrm{fer}} = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint\begin{pmatrix} A + \xi_1\, C\bar{C} & \xi_2\,\bar{\psi} \\ \psi\,\xi_3 & \hat{A} + \xi_4\,\bar{C}C \end{pmatrix} dt\right]$$

dSCFT classification

ABJM is a SCFT with supergroup Osp(6|4) $\to$ 12 + 12 supercharges
SU(4) R-symmetry

- ξ_a = special values $\Rightarrow$ SUSY and/or conformality partially preserved
  - non-BPS $W^{\pm} \longrightarrow$ Conformal but not SUSY
  - 1/6-BPS $W_{1/6} \longrightarrow$ SU(1,1|1) dSCFT with SU(2) $\times$ SU(2) R-symmetry
  - 1/2-BPS $W^{\pm}_{1/2} \longrightarrow$ SU(1,1|3) dSCFT with SU(3) R-symmetry

RG Flows

- 1/12, 1/24 BPS W $\longrightarrow$ supersymmetric but not conformal
- ξ_a = generic $\longrightarrow$ generically ~~conformality~~ (and/or ~~SUSY~~)

Bosonic RG flows

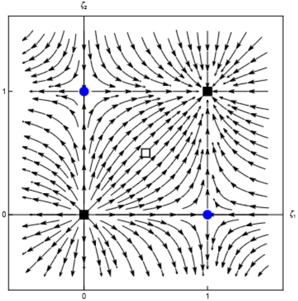
- Perturb around the "ordinary" Wilson Loops (dSCFT)

$$W^{\pm} = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint\left(A_\mu \dot{x}^\mu \pm C_I \bar{C}^I\right)dt\right]$$

$$\downarrow$$

$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint\left(A_\mu \dot{x}^\mu - C_I \bar{C}^I + \Delta M^I_J(\xi_a)\, C_I \bar{C}^J\right)dt\right] \qquad \Delta M^I_J(\xi_a = 0) = 0$$

- $\langle W \rangle = F(\xi_a)$ with non-trivial $\beta(\xi_a)$ s.t. $\beta(\xi_a = 0) = 0$



Arrows go from UV to IR

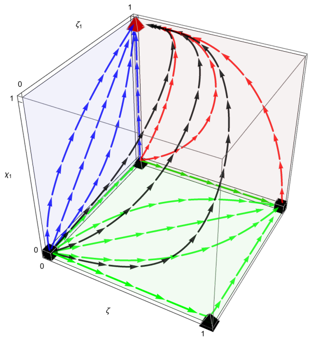
■	"Ordinary" W^+	$\Delta M = 2\mathbb{1}_4$
□	Only gauge	$\Delta M = \mathbb{1}_4$
●	1/6 BPS	$\Delta M = \pm\mathrm{diag}(0,0,2,2)$

Fermionic RG flows

$$W^- = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint\begin{pmatrix} A - C_I\bar{C}^I & 0 \\ 0 & \hat{A} - \bar{C}^I C_I \end{pmatrix} dt\right]$$

$$\downarrow$$

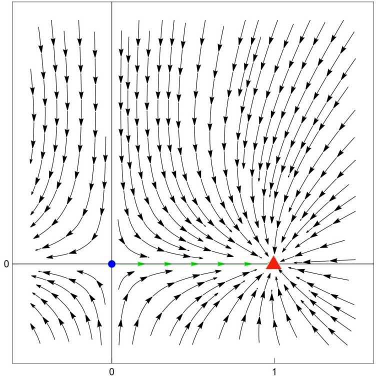
$$W = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint\begin{pmatrix} A - C_I\bar{C}^I + \Delta M_I^J C_I\bar{C}^J & \xi_2\bar{\psi} \\ \psi\,\xi_3 & \hat{A} - \bar{C}^I C_I + \Delta M_J^I \bar{C}^J C_I \end{pmatrix} dt\right]$$



- ■ "Ordinary" W^+ $\quad \Delta M = 2\mathbb{1}_4,\ \xi_{2,3} = 0$
- ▲ SU(3) bosonic $\quad \Delta M = \mathrm{diag}(0,2,2,2),\ \xi_{2,3} = 0$
- ▲ $W^+_{1/2}$ $\quad \begin{cases} \Delta M = \mathrm{diag}(0,2,2,2) \\ \xi_{2\,I}^{\alpha} = \delta_I^1\,(e^{it/2}, -ie^{-it/2})^{\alpha} \\ \xi_{3\,\alpha}^{I} = \delta_1^I \begin{pmatrix} ie^{-it/2} \\ -e^{it/2} \end{pmatrix}_{\alpha} \end{cases}$

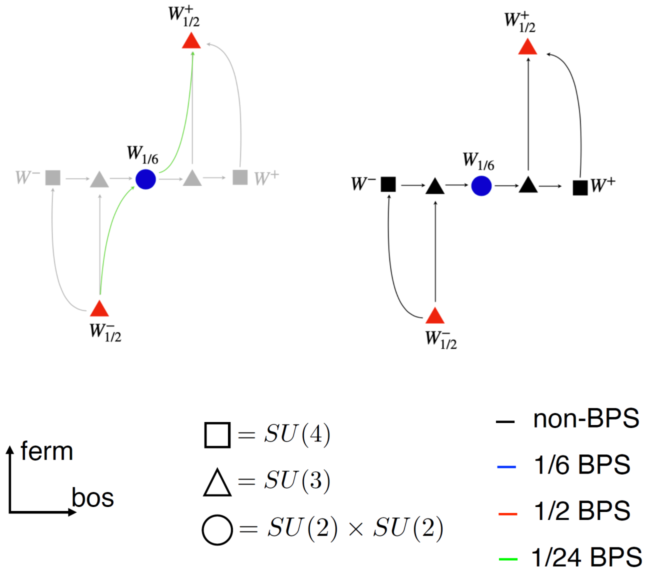
Enriched RG flows

ξ_a constrained & susy preserved $\rightarrow$ **Enriched flows**



- ● 1/6 BPS bosonic
- – 1/24 BPS
- – 1/6 BPS fermionic
- ▲ 1/2 BPS

Irreversibility of RG flows



Irreversibility of RG flows

- Anomalous dimension of perturbations $\mathcal{O}_i$

$$\mu \frac{\partial \mathcal{O}_i}{\partial \mu} = \Delta_i^{\ j} \mathcal{O}_j = (\delta_i^{\ j} + \Gamma_i^{\ j}) \mathcal{O}_j$$

Defect stress-energy tensor $T_D = \beta_i \mathcal{O}_i$, with $\Delta(T_D) = 1$ implies

$$\boxed{\Gamma_i^{\ j} = \frac{\partial \beta^j}{\partial \xi_i}}$$

Close to UV unstable fixed points anomalous dimensions are negative

marginally relevant deformations

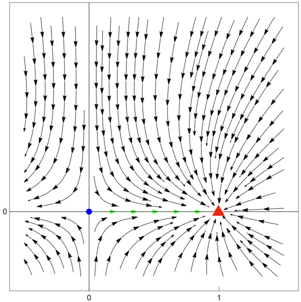
- g-theorem [Affleck-Ludwig 1991; Cuomo-Komargodski-Raviv Moshe, '21]

Defect free energy: $\mathcal{F} = \log \langle W \rangle$

g-theorem states that for reflection positive (unitary) defects, $\boxed{\mathcal{F}_{\text{UV}} > \mathcal{F}_{\text{IR}}}$

Our RG flows confirm this theorem, except $W^{-}_{1/2}{}^{\blacktriangle} \longrightarrow \blacktriangle$

Caveat: Cohomological anomaly



● 1/6 BPS bosonic ($W^{\text{bos}}_{1/6}$)

– 1/24 BPS

– 1/6 BPS fermionic

▲ 1/2 BPS

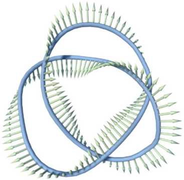
$$W = W^{\text{bos}}_{1/6} + \mathcal{Q}V, \quad \mathcal{Q} \text{ mutually preserved}$$

[Drukker-SP et al, '19]
[Drukker-Tenser-Trancanelli, '20]

- VEVs localize to the same matrix model
- However, we have seen that $\langle W \rangle = F(\xi_a)$

  $\mathcal{Q}$ broken at quantum level

Framing



$$\langle W^{\text{CS}} \rangle_f \to \int_{\mathcal{C}} dx_1^{\mu} \int_{\mathcal{C}} dx_2^{\nu} \langle A_\mu(x_1) A_\nu(x_2) \rangle$$

$$\langle W^{\text{CS}} \rangle_f = \exp\left(\frac{i\pi N}{k} f\right) \langle W^{\text{CS}} \rangle_{f=0}$$

$$f = \frac{1}{4\pi} \int_{\mathcal{C}} dx_1^{\mu} \int_{\mathcal{C}_f} dx_2^{\nu} \epsilon_{\mu\nu\rho} \frac{(x_1 - x_2)^{\rho}}{|x_1 - x_2|^3}$$

Linking number

- Ordinary perturbation theory corresponds to $f=0$
- Exact result (MM) holds at $f=1$

Cohomological anomaly must be canceled by non-trivial framing

Conclusions and perspectives

- We have found RG flows driven by non-trivial β-functions for the deforming parameters.
  β-functions computed perturbatively in the bulk couplings, but *exactly* in the parameters.

- RG flows are irreversible. Therefore, in the moduli space of WLs we managed to determine the IR stable ones. They correspond to 1d strongly coupled dSCFTs.

- Is the spectrum of fixed points stable against higher order corrections in the bulk coupling?
  One possible way to answer is to look at the holographic description of the flows.

- Cohomological anomaly and framing presently under investigation [2409.xxxxx]

- Generalizations: WLs on different contours, WL in $\mathcal{N} = 4$ CS-matter theories, higher-dimensional defects.