CORFU 2024, SEPTEMBER 20 TH

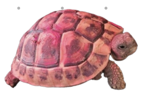
NONCOMMUTATIVE & GENERALIZED GEOMERY IN STRING TH., GAUGE TH. & RELATED PHYSICS

UTILISING THE INTEGRABILITY OF ALL YANG-BAXTER MODELS

SIBYLLE DRIEZEN

BASED ON 2406.09811 W. NIRANJAN KAMATH

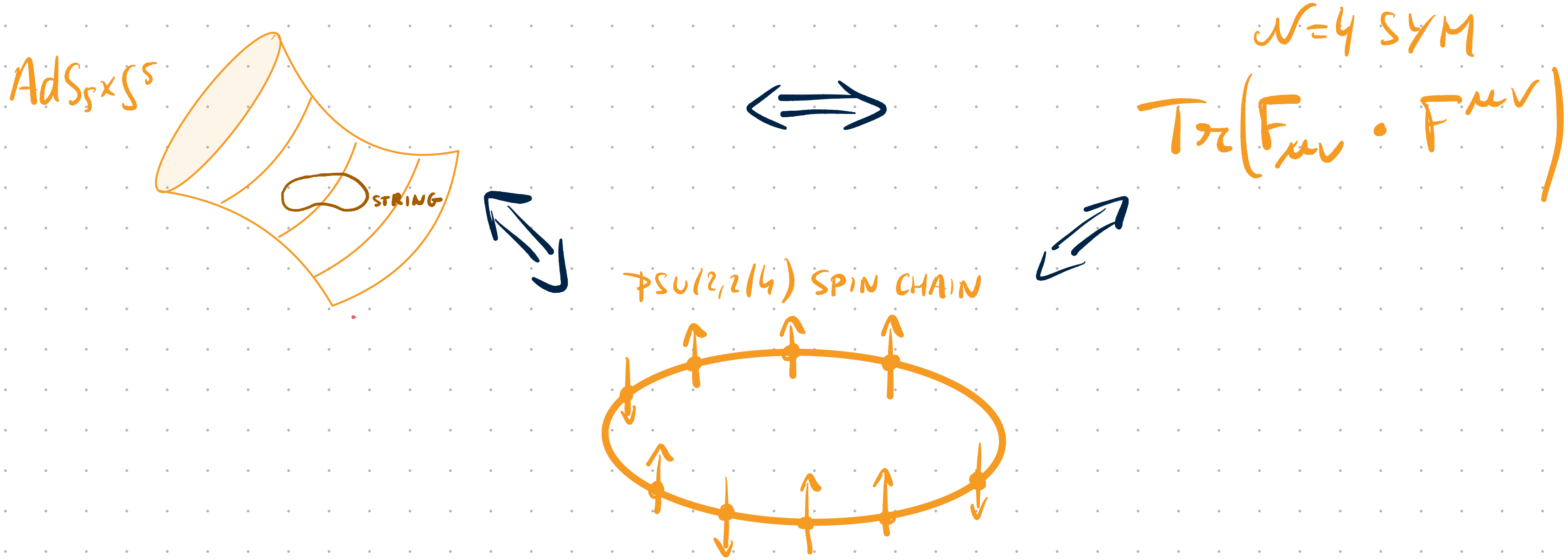
AND EARLIER WORK W. RICCARDO BORSATO



ETH zürich

INTEGRABLE DEFORMATIONS OF AdS/CFT?

UNDEFORMED AdS/CFT : $AdS_5 \times S^5 \sim N=4$ SYM

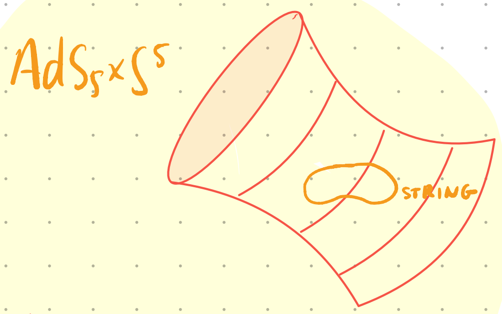


THERE IS AN UNDERLYING INTEGRABILITY (→ VERY SUCCESFUL FOR STRONGLY-COUPLED QFTS, STRINGS & AdS/CFT!)

CAN WE DEFORM IT (& STUDY MODELS W. BROKEN SYMMETRIES)?

INTEGRABLE DEFORMATIONS OF AdS/CFT?

RECENT YEARS: MUCH PROGRESS W. YANG-BAXTER MODELS



YANG-BAXTER STRINGS
(R-MATRIX)

[FROLOV '05 – BORSATO, SD, MIRAMONTES '21]

$\Longleftrightarrow$

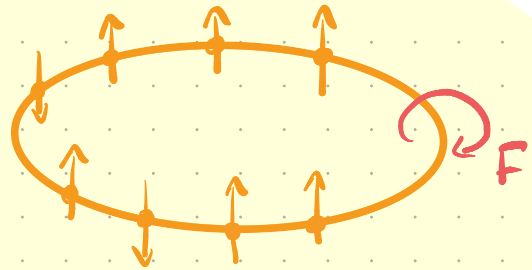
$\mathcal{N}=4$ SYM

$\mathrm{Tr}\left(F_{\mu\nu} * F^{\mu\nu}\right)$

NON-COMMUTATIVE
GAUGE THEORIES
(DRINFELD TWIST)

[BEISERT, ROIBAN '05 – GUICA, LEVKOVICH-MASLYUK, ZAREMBO '17 – VAN TONGEREN, MEIER '23]

PSU(2,2|4) SPIN CHAIN



DRINFELD TWISTED SPIN CHAINS

THERE IS AN UNDERLYING INTEGRABILITY ($\rightarrow$ VERY SUCCESFUL FOR STRONGLY-COUPLED QFTS, STRINGS & AdS/CFT!)

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$AdS_5 \times S^5$

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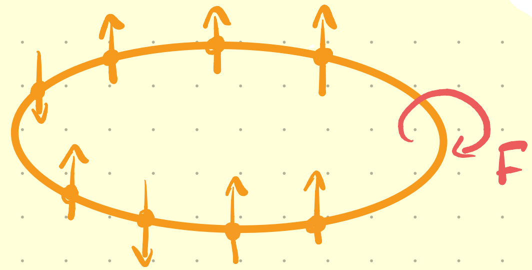


DRINFELD TWISTED SPIN CHAINS

THERE IS AN UNDERLYING INTEGRABILITY (→ VERY SUCCESFUL FOR STRONGLY-COUPLED QFTS, STRINGS & AdS/CFT!)

BUT THIS IS OFTEN MISTAKEN BY "SOLVABILITY"... (FOLKLORE)

— HOW TO APPLY OR EXTEND OUR INTEGRABILITY METHODS? —

INTEGRABILITY OF 2D SIGMA-MODELS

2D SIGMA-MODEL: FIELD THEORY ON 2D (WORLDSHEET) Σ
OF MAPS $g : \underset{(\tau;\sigma)}{\Sigma} \to$ (TARGET SPACE) $\underset{(x^\mu)}{G}$

CLASSICAL INTEGRABILITY ("weak"): i) LAX CONNECTION ONE-FORM $\mathcal{L}(g;z)$ on Σ s.t.

$$\text{EOMs}[g] \doteq d\mathcal{L}(g;z) + \mathcal{L}(g;z)\wedge\mathcal{L}(g;z) = 0$$

FOR ANY $z \in \mathbb{C}$. ("necessary" & sufficient)

ii) PERIODIC BOUNDARY CONDITIONS OF FIELDS g (sufficient)

$\Rightarrow$ CONSERVED CHARGES GENERATED FROM $M(z) = P\exp\left(-\oint \mathcal{L}(g;z)\right)$

$$\underbrace{\partial_\tau \left(\text{Tr}\, M(z)^n\right) = 0}_{(i)\,\&\,(ii)} \; ; \; \begin{matrix}\forall n \in \mathbb{N}\\ \forall z \in \mathbb{C}\end{matrix} \quad \Rightarrow \quad \partial_\tau \lambda(z) = 0 \; ; \; \forall z \in \mathbb{C}$$

$\hookrightarrow$ EIGENVALUES ENCODE CHARGES BY EXPANDING IN z

INTEGRABILITY OF 2D SIGMA-MODELS: EXAMPLE

ON COSETS G/H (= SYMMETRIC SPACE SIGMA-MODEL; LIE GROUPS G, H)

- $S[g] = \int d\tau d\sigma \, \mathrm{Tr}(j_\pm \cdot P j_\pm)$; $j_\pm = g^{-1}\partial_\pm g$; $g \in G$ (GLOBAL SYMM. $g \to g_0 g$)
- EOMs$[g]$: $d * (g P j g^{-1}) = 0$ (CONSERVATION EQ. ~ NOETHER CHARGES)
- Lax : $\mathcal{L}_\pm(g; z) = (\mathbb{1} - P) j_\pm + z^{\pm 2} P j_\pm$ is flat $\forall z \in \mathbb{C}$ upon EOM (EASY!)

(equiv. $\mathcal{L}^g(g; z) = g \mathcal{L}(g; z) g^{-1} + g d g^{-1}$)

$z \sim z_*$

$$\mathcal{L}_\sigma(g, z) \sim L_0 + (z - z_*) L_1 + \dots$$

$$M(z) \sim 1 + \int L_0 + \int L_0 \int L_0 + \dots + O(z - z_*)$$

$z \sim 1$

$$\mathcal{L}^g_\sigma(g; z) \sim \varepsilon (g P j_\tau g^{-1}) + \dots$$

$$M(z) \sim 1 - \varepsilon \int d\sigma \, g P j_\tau g^{-1} + \dots$$

AFTER DIAGONALISING $M(z) \to \lambda(z) \mathbb{1}$:

LOCAL NOETHER CARTAN CHARGES (GLOBAL SYMMS)

@ ASYMPTOTICS AROUND ZEROES OF LAX

&

INFINITE SETS OF "HIGHER" NON-LOCAL CHARGES ("HIDDEN" SYMMS)

@ OTHER ORDERS

CLASSICAL SPECTRAL CURVE

SPECTRAL PROPERTIES OF $S[g]$ ARE ENCODED IN SPECTRAL PROPERTIES OF $M(z)$

CLASSICAL SPECTRAL CURVE

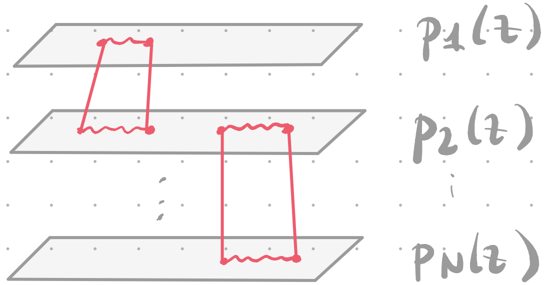
SPECTRAL PROPERTIES OF $S[g]$ ARE ENCODED IN SPECTRAL PROPERTIES OF $M(z)$ DEFINED BY THE ALGEBRAIC CURVE

$$\Gamma(\lambda, z) = \det\left(M(z) - \lambda(z)\mathbb{1} \right) = 0$$ → N×N MATRICES

→ DEGREE N POLYNOMIAL FOR $\lambda(z)$

$$\prod_{I=1}^{N} \left(\lambda - \lambda_I(z)\right) = 0$$

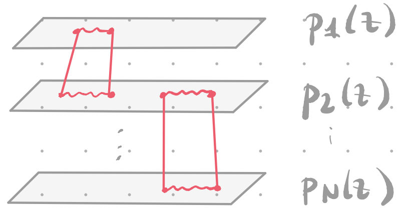
→ N-SHEETED RIEMANN SURFACE $\lambda_I(z) = e^{i p_I(z)}$; QUASIMOMENTA $p_I(z)$



WITH CUTS DEPENDING ON THE SOLUTION OF EOMs$[g]$

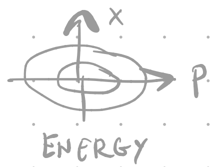
↳ $p_I(z_0) = p_J(z_0)$

CLASSICAL SPECTRAL CURVE



WHEN $M(z)$ IS DIAGONALISABLE

SOLUTIONS OF EOMs [g] :: LABELLED BY CONSERVED CHARGES [E.G. HARMONIC OSCILLATOR]



:: LABELLED BY ANALYTIC DATA (NUMBER, LOCATION OF BRANCH POINTS) & NOETHER CHARGES IN CARTAN (LOCAL ASYMPTOTICS)

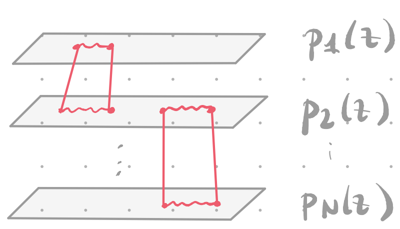
↗ RH

CAN UNIQUELY RECONSTRUCT $p_I(z)$ on $\mathbb{C}$ $\forall$ SOL
[DOREY, VICEDO '06; KRICHEVER, PHONG '96]

→ SIMILAR "ANALYTIC POWER" USEABLE TO RECONSTRUCT SEMICLASSICAL EXCITATIONS OF CHARGES FOR GIVEN SOLUTION
[GROMOV, SCHAFER-NAMEKI, VIEIRA, ... ~'08]

⇒ VERY EFFICIENT CALCs OF SEMI-CL. EXCITATIONS ω_E & ONE-LOOP SHIFT ΔE

CLASSICAL SPECTRAL CURVE



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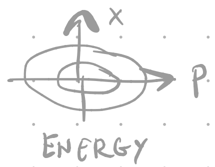

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[GROMOV, SCHAFER-NAMEKI, VIEIRA, ... ~'08]

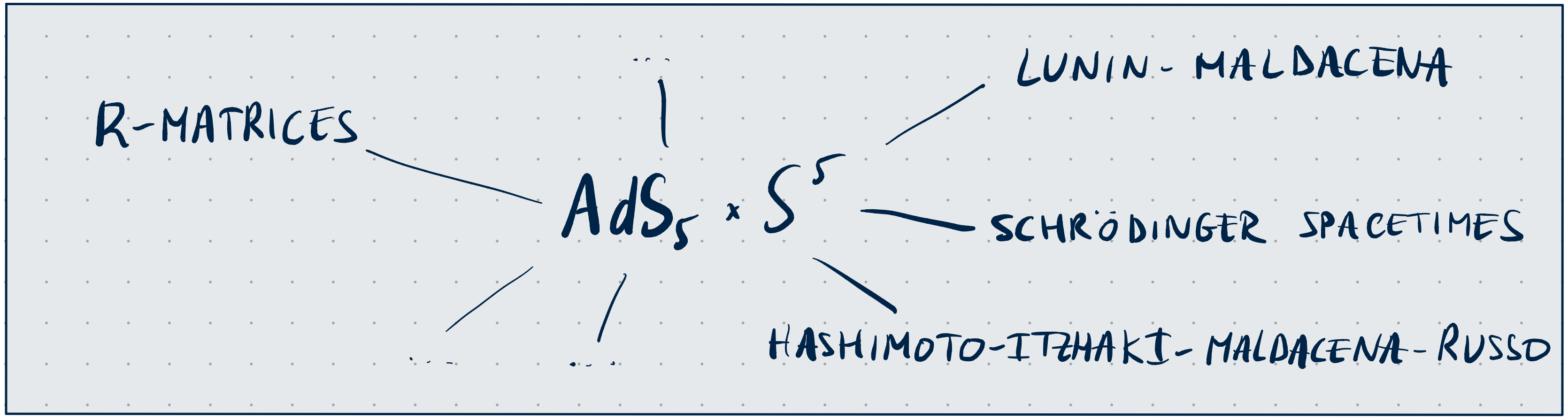
⇒ VERY EFFICIENT CALCS OF SEMI-CL. EXCITATIONS ω_E & ONE-LOOP SHIFT ΔE

⇝ GREAT FOR FIRST CHECKS OF ADS/CFT DUALS!
[BETHE ANSATZ IN (LARGE) SPIN CHAIN PICTURE OF $N=4$ SYM]

YANG-BAXTER MODELS : DEFINITION
(HOMOGENEOUS)

INTEGRABLE DEFORMATIONS OF 2D SIGMA-MODELS W. ISOMETRIES G

$\Rightarrow$ APPLICABLE TO STRINGS $\rightsquigarrow$ IN E.G. $AdS_5 \times S^5$



$R : \mathfrak{g} \to \mathfrak{g}$ WITH $\mathfrak{g} = psu(2,2|4)$

- $[Rx, Ry] - R([Rx, y] - [x, Ry]) = 0, \quad \forall\, x, y \in \mathfrak{g}$ (CYBE)
- $R^T = -R$ (ANTISYMMETRIC)
- $R^{AB}[T_A, T_B] = 0$ (UNIMODULAR)

INTEGRABLE [KLIMCIK '02, DELDUC, MAGRO, VICEDO '13]

+ WEYL-INV. (SUGRA) [BORSATO, WULFF '16]

Three Main Classes

...

R-matrices

Lunin-Maldacena

$AdS_5 \times S^5$

Schrödinger spacetimes

... ...

Hashimoto-Itzhaki-Maldacena-Russo

Three Main Classes

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R-MATRICES

LUNIN-MALDACENA

$\in$ DIAGONAL-TsT YB-DEF^{S}

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SCHRÖDINGER SPACETIMES

... ...

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THREE MAIN CLASSES

R-MATRICES

...

$AdS_5 \times S^5$

LUNIN-MALDACENA
$\in$ DIAGONAL-TST YB-DEFS

SCHRÖDINGER SPACETIMES
$\in$ NON-DIAGONAL-TST YB-DEFS

HASHIMOTO-ITZHAKI-MALDACENA-RUSSO
$\in$ NON-DIAGONAL-TST YB-DEFS

... ...

THREE MAIN CLASSES

$AdS_5 \times S^5$

R-MATRICES

JORDANIAN YB-DEFORMATIONS

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THE SUBALGEBRA (CYBE) $\mathrm{IM}(R) \subset \mathfrak{g}$ IS MIN. 2D, $\mathrm{IM}(R) = \mathrm{SPAN}(T_1, T_2)$, AND

I. DIAGONAL T(DUALITY)-S(HIFT)-T(DUALITY) TRANSFORMATIONS

ABELIAN, $[T_1, T_2] = 0$; BOTH T_i DIAGONALISABLE (CARTAN)

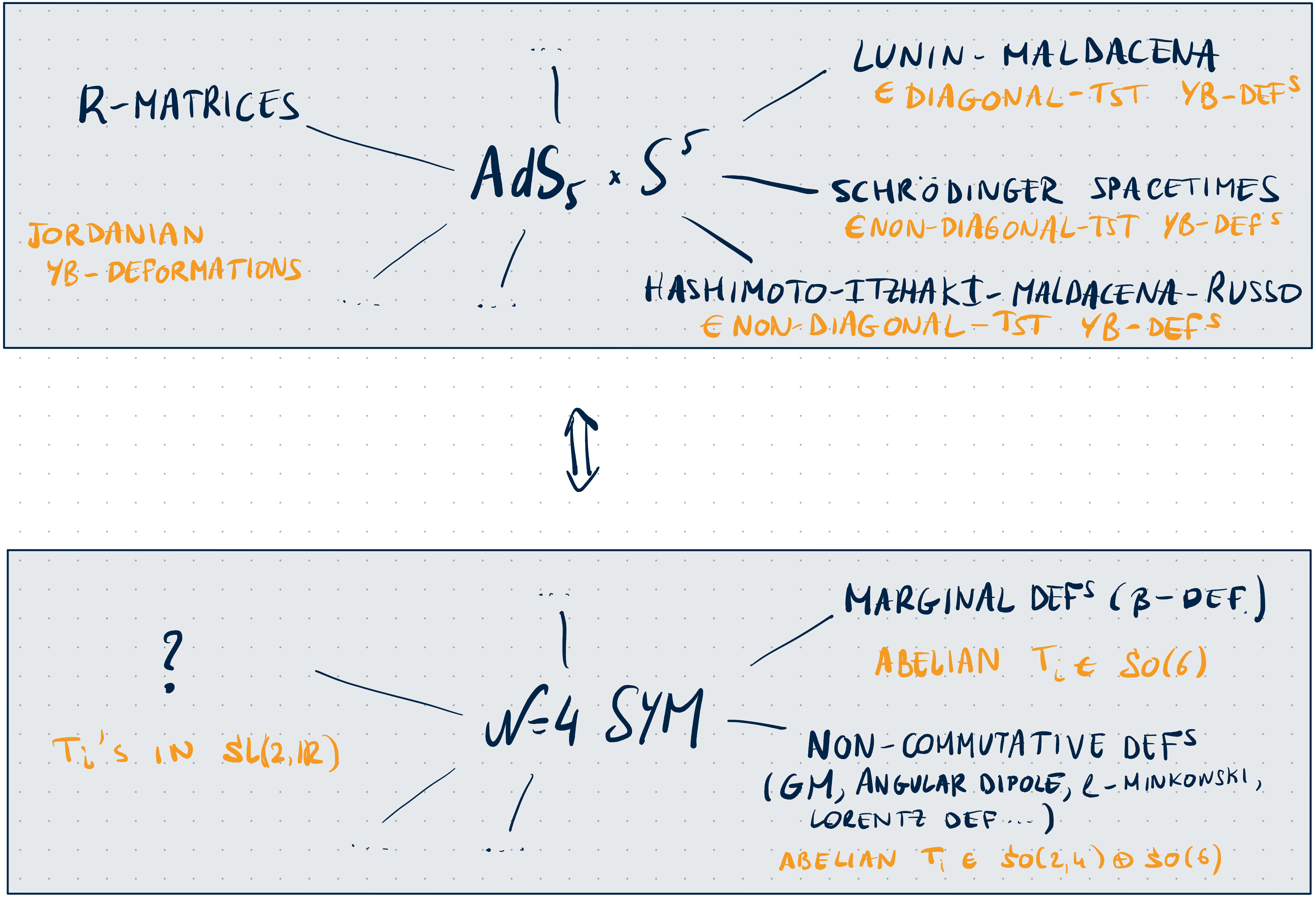
II. NON-DIAGONAL TST TRANSFORMATIONS

ABELIAN, $[T_1, T_2] = 0$; ONE OR BOTH T_i NON-DIAGONALISABLE

III. JORDANIAN DEFORMATIONS

NON-ABELIAN ; $\mathrm{IM}(R) = \mathfrak{sl}(2, \mathbb{R})$; $[\underset{\hookrightarrow \text{CARTAN}}{T_1}, T_2] = T_2$

[OSTEN, KAWAGUCHI, MATSUMOTO, VAN TONGEREN, YOSHIDA, ...]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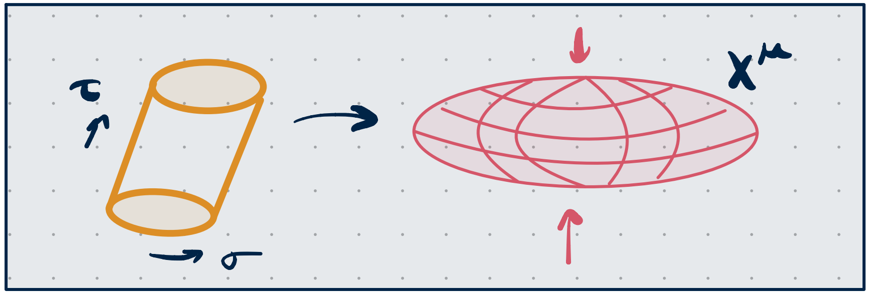
∈ NON-DIAGONAL-TsT YB-DEF^S
JORDANIAN YB-DEFORMATIONS
?
T_i's IN $\mathfrak{sl}(2,\mathbb{R})$
$\mathcal{N}=4$ SYM
MARGINAL DEF^S (β-DEF.)
ABELIAN $T_i \in \mathfrak{so}(6)$
NON-COMMUTATIVE DEF^S
(GM, ANGULAR DIPOLE, ℓ-MINKOWSKI,
LORENTZ DEF ...)
ABELIAN $T_i \in \mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$

TWO INTEGRABILITY DESCRIPTIONS

[FROLOV '05 (TsT); ...; BORSATO SD, MIRAMONTES '21 (HYB)]

PERIODIC WORLDSHEET, DEFORMED TARGET SPACE



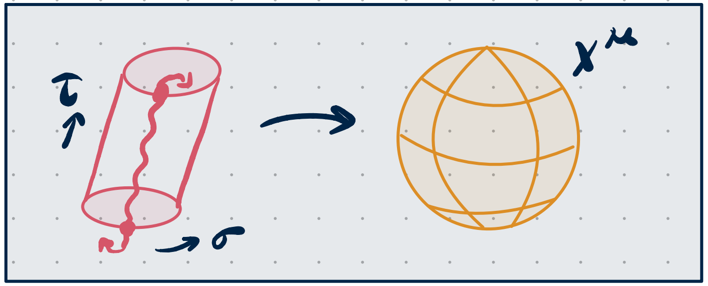
$$g[X^\mu(\tau, \sigma+2\pi)] = g[X^\mu(\tau,\sigma)]$$

$$G'_{\mu\nu} - B'_{\mu\nu} = (G_0^{-1} + \eta R)^{-1}_{\mu\nu}$$

OR

$\approx$

TWISTED WORLDSHEET, UNDEFORMED TARGET SPACE



$$g[X^\mu(\tau,\sigma+2\pi)] = e^{\eta R(Q)} g[X^\mu(\tau,\sigma)]$$

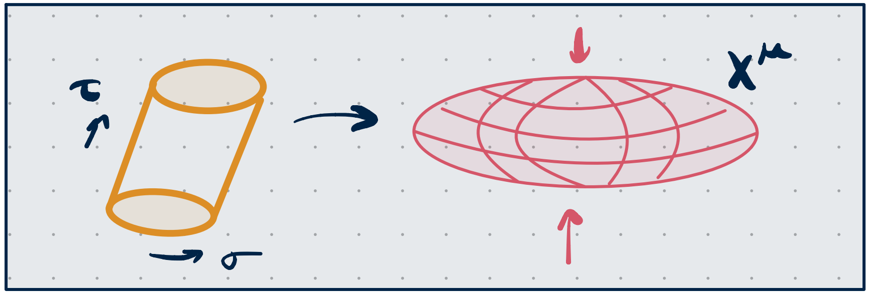
$$\delta G_{\mu\nu}(X) = 0$$

"twist" charge

Two Integrability Descriptions

[Frolov '05 (TsT); …; Borsato, SD, Miramontes '21 (HYB)]

Periodic worldsheet, deformed target space or **Twisted worldsheet, undeformed target space**



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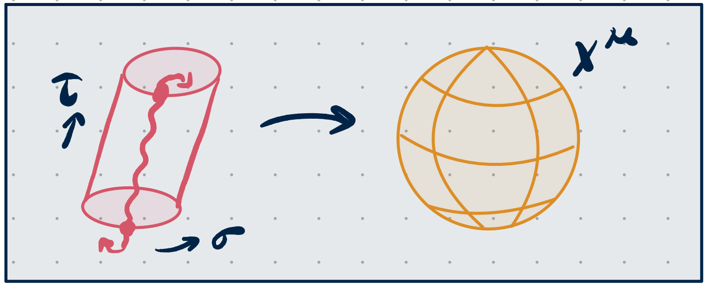


Lax: $\mathcal{L}'_\pm(g;z) = (\mathbb{1}-P)j'_\pm + z^{\pm 2} P j'_\pm$;

$$j'_\pm = (1 \pm \eta R_g P)^{-1} j_\pm$$

CSC? $\mathcal{L}'$ (nor $\mathcal{L}'^g$) has no zeroes…

⇒ $M'(z)$ has no local asymptotics (with e.g. energy)

⇒ CSC in the paper bin

Lax: $\mathcal{L}_\pm(g;z) = (\mathbb{1}-P)j_\pm + z^{\pm 2} P j_\pm$

CSC? $\mathcal{L}^g_\pm$ has zeroes

$$M(z) = e^{-\eta R(Q)}\left(\mathbb{1} + \varepsilon \int g P j_\pm g^{-1} + \dots\right)$$

⇒ CSC + corrs. possible

[Borsato, SD, Nieto-García, Wyss '22 ; SD, Kamath '24]

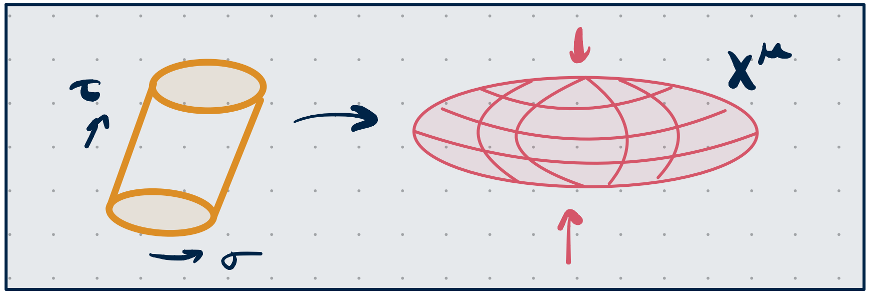
TWO INTEGRABILITY DESCRIPTIONS

[FROLOV '05 (TsT); ...; BORSATO SD, MIRAMONTES '21 (HYB)]

PERIODIC WORLDSHEET,
DEFORMED TARGET SPACE

OR

TWISTED WORLDSHEET,
UNDEFORMED TARGET SPACE



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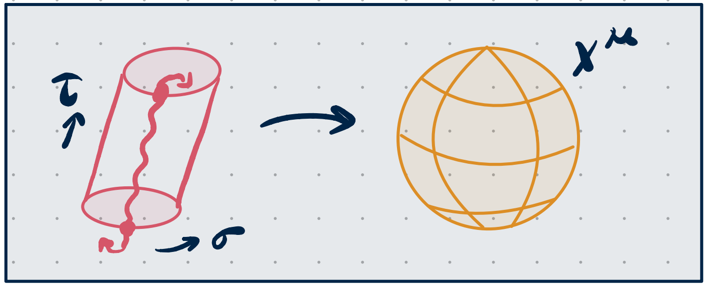


LAX: $\mathcal{L}_\pm(g;z) = (\mathbb{1}-P)j_\pm + z^{\pm 2} P j_\pm$

CSC? $\mathcal{L}_\pm^g$ HAS ZEROES

$$M(z) = e^{-\eta R(Q)}\left(\mathbb{1} + \varepsilon \int g P j_\pm g^{-1} + \dots\right)$$

$\Rightarrow$ CSC + CORRS. POSSIBLE

[BORSATO, SD, NIETO-GARCIA, WYSS '22; SD, KAMATH '24]

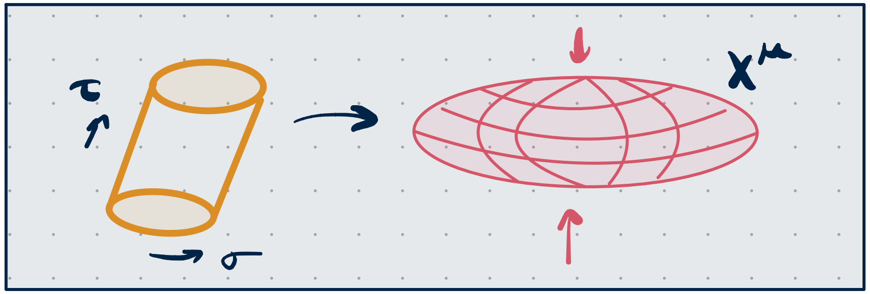
TWO INTEGRABILITY DESCRIPTIONS

[FROLOV '05 (TsT); …; BORSATO SD, MIRAMONTES '21 (HYB)]

PERIODIC WORLDSHEET, DEFORMED TARGET SPACE

OR

TWISTED WORLDSHEET, UNDEFORMED TARGET SPACE



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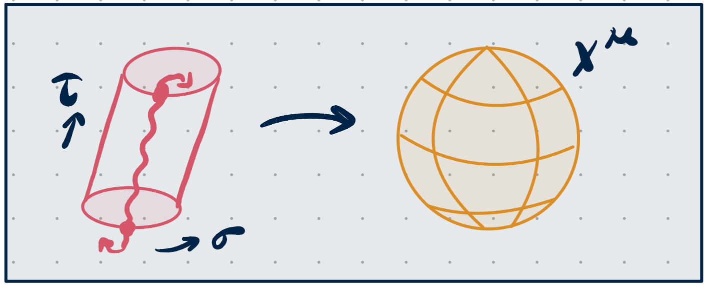


INSTEAD: S-MATRIX INTEGRABILITY (WORLDSHEET SCATTERING)

$$S \rightarrow S' = F_{OP}\, S\, F^{-1}$$

[VAN TONGEREN, ZIMMERMAN '21; BORSATO, SD, HOARE, RETORE, SEIBOLD '23; BORSATO, SD: WIP]

LAX: $\mathcal{L}_\pm(g;z) = (\mathbb{1}-P)j_\pm + z^{\pm 2} P j_\pm$

CSC? $\mathcal{L}_\pm^g$ HAS ZEROES

$$M(z) = e^{-\eta R(Q)}\left(\mathbb{1} + \varepsilon \int g P j_\pm g^{-1} + \dots\right)$$

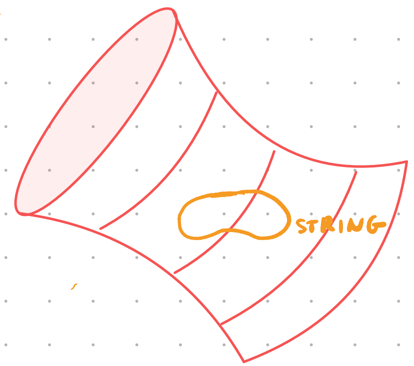
$\Rightarrow$ CSC + CORRS. POSSIBLE

[BORSATO, SD, NIETO-GARCIA, WYSS '22; SD, KAMATH '24]

TWIST-DEFORMED AdS/CFT?

[MINAHAN, ZAREMBO '02, BEISERT '03 ...]
[BEISERT, ROIBAN '05 ..., GUICA, LEVKOVICH-MASLYUK, ZAREMBO '17 ..., VAN TONGEREN, MEIER '23]

$AdS_5 \times S^5$



DEFORMED SUPERGRAVITY (YR)

$\Longleftrightarrow$

$\mathcal{N}=4$ SYM

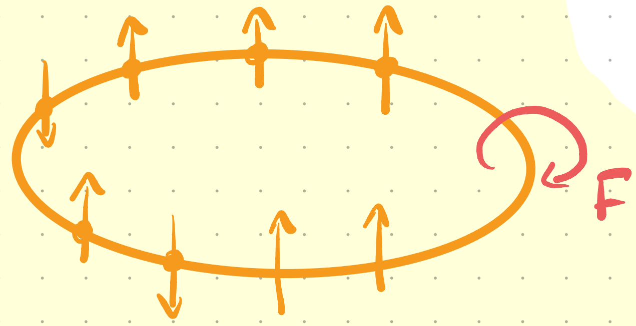
$$\mathrm{Tr}\left(F_{\mu\nu} \star F^{\mu\nu}\right)$$

<u>TWISTED</u> <u>FIELD PRODUCTS</u> (GAUGE TH.)

$\rightsquigarrow$ $\star$ :: DRINFELD TWISTS $\mathcal{F}$

$\mathcal{F} \sim e^{\gamma R} \sim e^{\gamma (T_1 \wedge T_2)}$

PSU(2,2|4) SPIN CHAIN



<u>TWISTED</u> <u>BOUND. CONDS</u> (CHAIN)

$\rightsquigarrow F \sim e^{\gamma R[Q]}$ FROM DRINFELD TWISTING EACH SITE

$$\mathcal{R}_{ab} \to \mathcal{F}_{ba}^{-1} \mathcal{R}_{ab} \mathcal{F}_{ab} \Rightarrow T(z) = F \begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix} \Rightarrow \text{BETHE ANSATZ POSSIBLE}$$

TWIST-DEFORMED AdS/CFT?

[MINAHAN, ZAREMBO '02, BEISERT '03...]
[BEISERT, ROIBAN '05..., GUICA, LEVKOVICH-MASLYUK, ZAREMBO '17..., VAN TONGEREN, MEIER '23]

$AdS_5 \times S^5$

TWISTED BOUND. CONDS (STRING)

$\rightsquigarrow W = e^{\gamma R[Q]}$

$\Longleftrightarrow$

$\mathcal{N}=4$ SYM

$$\mathrm{Tr}\left(F_{\mu\nu} * F^{\mu\nu}\right)$$

TWISTED FIELD PRODUCTS (GAUGE TH.)

$\rightsquigarrow$ $*$: DRINFELD TWISTS $\mathcal{F}$

$\mathcal{F} \sim e^{\gamma R} \sim e^{\gamma(T_1 \wedge T_2)}$

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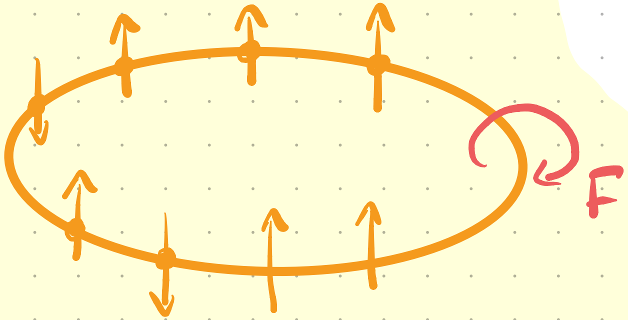

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$\mathcal{R}_{ab} \to \mathcal{F}_{ba}^{-1} \mathcal{R}_{ab} \mathcal{F}_{ab} \Rightarrow T(z) = F \begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix} \Rightarrow$ BETHE ANSATZ POSSIBLE

TWISTED YB-MODELS

$$M(z) = e^{-gR(Q)}\left(\mathbb{1} + \varepsilon \int g P j_\tau g^{-1} + \dots\right)$$

$$W = e^{gR(Q)} = e^{g \underline{Q_1} T_2 - g \underline{Q_2} T_1}$$

[BORSATO, SD, MIRAMONTES '21, BORSATO, SD, NIETO-GARCIA, WYSS '22]

$$W = \mathcal{F}^{-1}(\tau, 2\pi)\,\mathcal{F}(\tau, 0)\ ; \quad \mathcal{F} = e^{-RX}\ ; \quad \int P_{\mathrm{IM}(R)^*}(g P j g^{-1}) = j^{-1} P_{\mathrm{IM}(R)^*} \frac{1 - e^{-\mathrm{ad}_{RX}}}{\mathrm{ad}_{RX}} X$$

I. DIAGONAL TsT → W IS DIAGONALISABLE

$Q_i = \int d\sigma\, \mathrm{Tr}[T_i \cdot g P j_\tau g^{-1}]$ ARE NOETHER CHARGES (CARTAN)

II. NON-DIAGONAL TsT → W IS NON-DIAGONALISABLE

$Q_i = \int d\sigma\, \mathrm{Tr}[T_i \cdot g P j_\tau g^{-1}]$ ARE NOETHER CHARGES (NOT CARTAN)

III. JORDANIAN DEFS → W IS DIAGONALIBLE

$Q_1 = \mathcal{Q}[Y_2]\, q[Y_i]$, $Q_2 = \mathcal{Q}[Y_2]$ ARE NOT NOETHER & CARTAN BUT THEY ARE LOCAL + CONSERVED

TWISTED YB-MODELS

$$M(z) = e^{-\eta R(Q)}\left(\mathbb{1} + \varepsilon \int g P j_{\tau} g^{-1} + \dots\right)$$

$$W = e^{\eta R(Q)} = e^{\eta \underline{Q_1} T_2 - \eta \underline{Q_2} T_1}$$

[BORSATO, SD, MIRAMONTES '21, BORSATO, SD, NIETO-GARCIA, WYSS '22]

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I. DIAGONAL TsT → W is DIAGONALISABLE

$Q_i = \int d\sigma\, \mathrm{Tr}[T_i \cdot g P j_\tau g^{-1}]$ ARE NOETHER CHARGES

II. NON-DIAGONAL TsT → W is NON-DIAGONALISABLE ⇝ $\underline{\underline{\text{CSC}}}$?

$Q_i = \int d\sigma\, \mathrm{Tr}[T_i \cdot g P j_\tau g^{-1}]$ ARE NOETHER CHARGES

III. JORDANIAN DEFS → W is DIAGONALIBLE

$Q_1 = Q[Y_2]\, q[Y_i]$, $Q_2 = Q[Y_2]$ ARE NOT <u>NOETHER</u> CHARGES BUT THEY ARE LOCAL + CONSERVED

NON-DIAGONAL TWISTS

- $M(z) = W\left(\mathbb{1} + \varepsilon \int g P_{\mathfrak{J}} g^{-1} + \dots\right)$ WITH W NON-DIAGONALISABLE

CSC? THE ANALYTIC DATA OF THE SPECTRAL CURVE BECOMES ILL-DEFINED:

THE POINT OF EXPANSION $z \sim 1 + \varepsilon$ FOR LOCAL CHARGES IS ALSO A BRANCH PT

$$\lambda_\pm(z) \sim \lambda_0 \pm \# \sqrt{z-1} + \dots \quad \rightarrow \text{NON-POLYNOMIAL EXPANSION}$$

⇒ "RECONSTRUCTION" THEOREMS ILL-DEFINED; NOT CLEAR HOW TO USE THEM FOR ALL SOLUTIONS

[⇒ ADDS EXTRA LAYERS OF COMPLEXITY TO THE (ALREADY INVOLVED, BUT POWERFUL) QSC]

- $T(z) = F\begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix}$ WITH F NON-DIAGONALISABLE

BETHE ANSATZ? TO DIAGONALISE SPIN CHAIN HAMILTONIAN $H \sim \partial_z \log \mathrm{Tr}\, T(z)|_{z_*}$

NEEDS "VACUUM" STATE := EIGENSTATE $A_F(z)$ & $D_F(z)$; & $C_F(z)|\rangle = 0$

→ ACTING W. $B_F(z)$ CREATES EIGENSTATES OF $\mathrm{Tr}\, T(z)$ IFF BAE

⇒ ABSENT WHEN F IS NOT DIAGONALISABE

"Regularising" Non-Diagonal TsT

[SD, Kamath '24]

Consider $\mathrm{Im}(R) = \{T_1\, T_2\}$; $[T_1, T_2] = 0$

$\rightarrow$ Take basis $T_2 = \begin{pmatrix} \cancel{\lambda_2} & 1 \\ \cdot & \cancel{\lambda_2} \end{pmatrix} = \cancel{\lambda_2 \mathbb{1}} + \begin{pmatrix} \cdot & 1 \\ \cdot & \cdot \end{pmatrix} \rightsquigarrow T_1 = \begin{pmatrix} \lambda_1 & t \\ \cdot & \lambda_1 \end{pmatrix}$

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→ lift $T_2 = \begin{pmatrix} \cdot & 1 \\ \cdot & \cdot \end{pmatrix}$ to $SL(2, \mathbb{R})$ ⇝ consider $H = \begin{pmatrix} 1 & \cdot \\ \cdot & -1 \end{pmatrix}$

$$\boxed{[H, T_2] = T_2}$$

to define a Jordanian deformation [classified in $psu(2,2|4)$ in Borsato, SD '22]

→ combine the Jordanian with the non-diagonal TsT =: Jordanian'

$$\boxed{[H + \xi T_1, T_2] = [H, T_2] = T_2}$$

"Regularising" Non-Diagonal TsT

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to define a Jordanian deformation [classified in $psu(2,2|4)$ in Borsato, SD '22]

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$$\eta \cdot \boxed{[H + \xi T_1, T_2] = [H, T_2] = T_2}$$

Then in the deformed/twisted model (ηR) one can take the limit

$\eta \to 0$ but $\eta \cdot \xi \equiv \eta_{TsT}$ constant

to obtain back the non-diagonalisable TsT with η_{TsT} $\boxed{[T_1, T_2] = 0}$

"Regularising" Non-Diagonal TsT

⇒ Work with the lifted Jordanian' (= Jord + non diag TsT)

→ has diagonalisable twist

$$W = e^{\eta R[Q]} = e^{\eta [Q(H' + q T_2)]} = \begin{pmatrix} e^{\eta Q} & q \sinh Q \\ & e^{-\eta Q} \end{pmatrix}$$

→ can use the method of the classical spectral curve to compute corrections

In YB-$AdS_5 \times S^5$ example of [SD, Kamath '24]

$$\begin{cases} H' = (D - J_{03}) + \xi J_{12} \\ T_2 = P_0 + P_3 \end{cases}$$

$$\Rightarrow \begin{cases} \hat{p}_1 = Q\xi - 4\pi\varepsilon(E - S) \\ \hat{p}_2 = Q(i - \xi) + 4\pi\varepsilon(2i\xi - 1)S \\ \hat{p}_3 = Q(-i - \xi) + 4\pi\varepsilon(2i\xi + 1)S \\ \hat{p}_4 = Q\xi + 4\pi\varepsilon(E + S) \end{cases} \quad \text{in } AdS_5$$

⇒ corrections around specific solution fixed

$\omega_E(\eta, \xi)$, $\Delta E_{1\text{-loop}}(\eta, \xi)$, $\delta Q = 0$

Take $\eta \to 0$ but $\eta \cdot \xi = \eta_{TsT}$ constant on results (we^{AdS} matches w. effective action)

CONCLUSIONS & OPEN QUESTIONS

FROM THE P.O.V. OF THE STRING SIGMA-MODEL, WE HAVE A WAY TO USE THE LAX-INTEGRABILITY (CLASSICAL SPECTRAL CURVE) TO

* CAPTURE ALL SOLUTIONS EOM[g]
* DERIVE FIRST ORDER CORRECTIONS TO NOETHER CHARGES (E)

OF ALL (HOMOGENEOUS) YANG-BAXTER MODELS

→ TWISTED BOUNDARY CONDITIONS & JORDANIAN REGULARISATION

[BORSATO, SD, MIRAMONTES '21] [SD, KAMATH '24]

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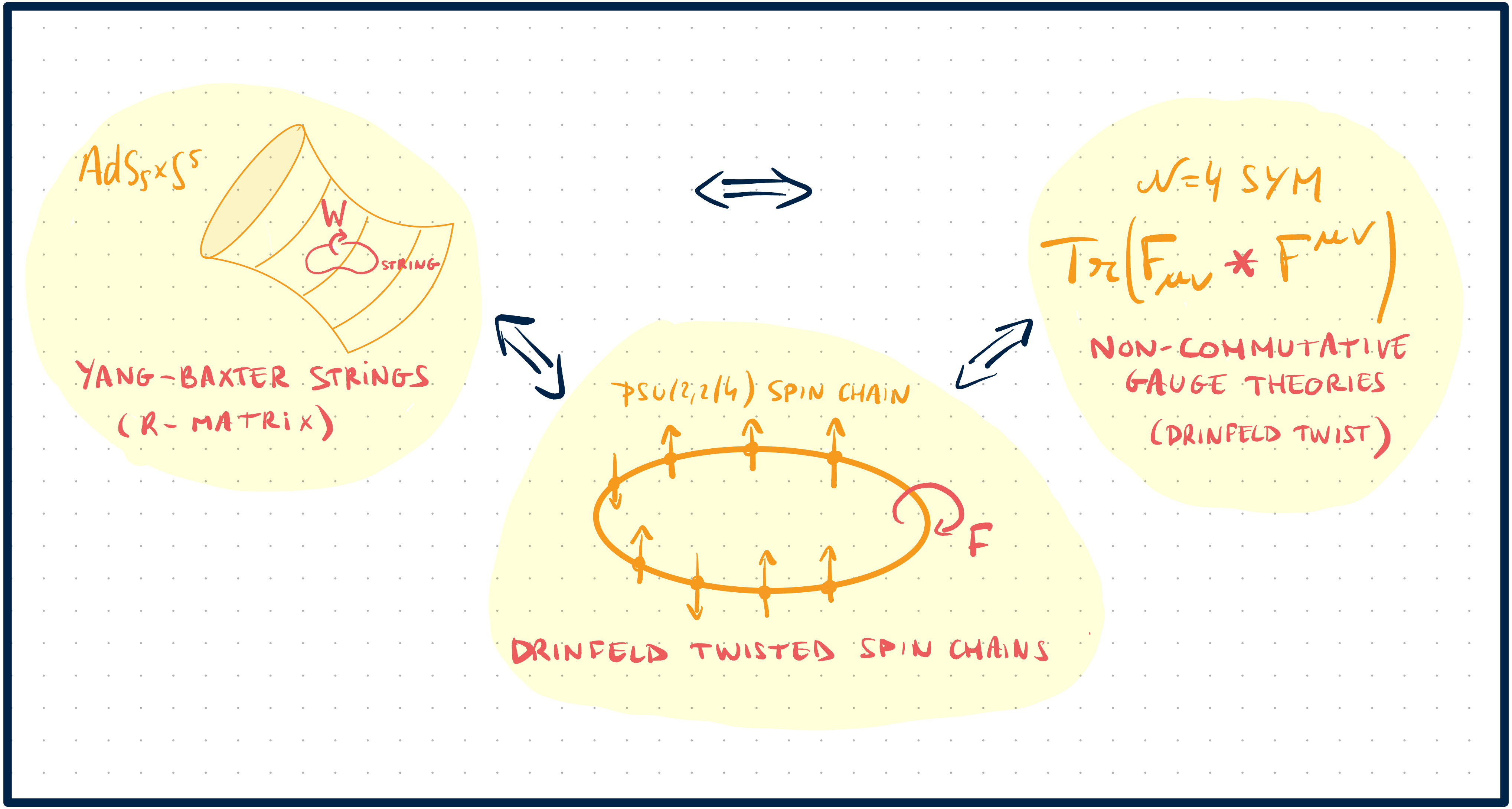
[BORSATO, SD, MIRAMONTES '21] [SD, KAMATH '24]

FOR DEFORMED AdS/CFT, HOWEVER, USABILITY OF JORDANIAN TWISTS VERY MUCH UNCHARTED TERRAIN ⇝ NON-ABELIAN NATURE MAKES IT HARD

* JORDANIAN TWISTED SPIN CHAIN? CLOSED FORM FOR TWIST?
* JORDANIAN TWISTED STAR PRODUCTS / NC THEORIES? GAUGE-INVARIANT OPERATORS?
* JORDANIAN TWISTED ALL-LOOP S-MATRIX? [BORSATO, SD, HOARE, RETORE, SEIBOLD '23; BORSATO, SD: WIP]
* YB-TWISTED QUANTUM SPECTRAL CURVE?



$AdS_5 \times S^5$
W
STRING
YANG-BAXTER STRINGS
(R-MATRIX)
$\mathcal{N}=4$ SYM
$Tr(F_{\mu\nu} * F^{\mu\nu})$
NON-COMMUTATIVE GAUGE THEORIES
(DRINFELD TWIST)
PSU(2,2|4) SPIN CHAIN
F
DRINFELD TWISTED SPIN CHAINS

THANK YOU !