

Majorana phases beyond neutrinoless double beta decay

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Neutrinos: some historical perspective

1930: "Neutrino" hypothesis by Pauli to explain missing energy in beta decay (later called neutrinos by Fermi).

1956: **Neutrino Discovery** by C. Cowan and F. Reines.

Reactor neutrinos with subsequent $\bar{\nu}_e p \rightarrow n e^+$, and $e^+e^- \rightarrow \gamma\gamma$

1958: Neutrino oscillation hypothesis by Pontecorvo.

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1968: **Solar Neutrino Problem** (R. Davis and J. N. Bahcall)

1986: Atmospheric Neutrino Anomaly.

1998: **Discovery of Atmospheric Neutrino Oscillations** by Super-Kamiokande in Japan.

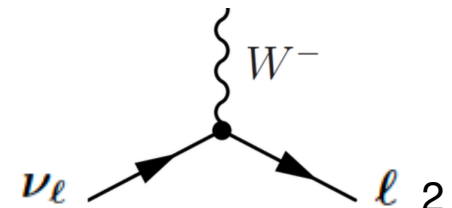
$\nu_\mu + H_2O \rightarrow \mu^- X$, $\nu_e + H_2O \rightarrow e^- X$

2000: **Discovery of Tau Neutrino** by DONUT at Fermilab.

(*) 2001: Confirmation of solar neutrino oscillations by Sudbury Neutrino Observatory.

2002: Rediscovery of the disappearance of (laboratory produced) muon neutrinos by K2K (KEK to Kamioka).

Most of these discoveries were based on
neutrino charged current interactions (with the exception of (*))



The neutrino un-answered questions

aka, why are neutrinos interesting

- * What is the origin of neutrino masses?
- * **Are neutrinos Dirac or Majorana? i.e. are neutrinos their own antiparticle?**
- * **Do neutrino interactions violate the CP symmetry?**
- * What are the values of the neutrino masses and mixing angles? Inverted or normal hierarchy?
- * Baryon asymmetry of the Universe – explained through leptogenesis?
- * Is Dark Matter a new yet-to-be-discovered neutrino (keV sterile neutrinos)?

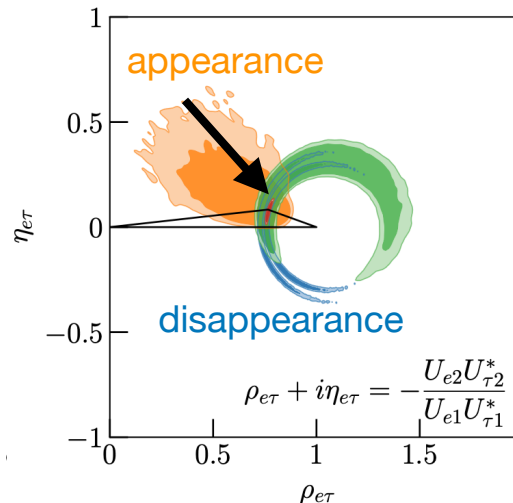
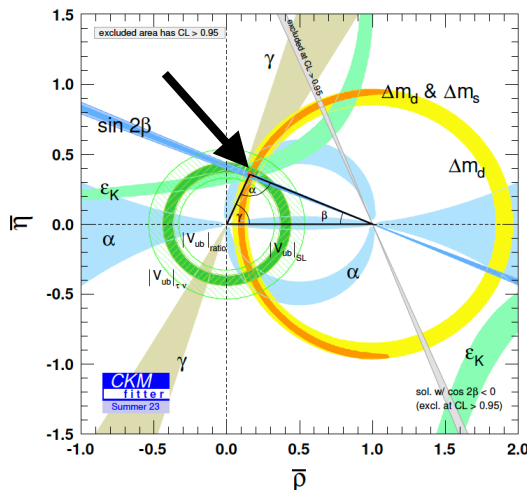
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In general, neutrinos are among the least known particles

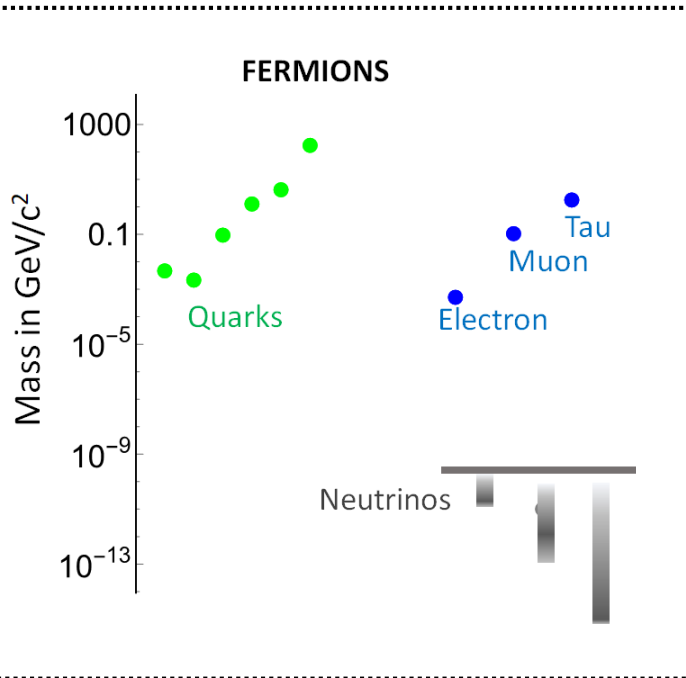
Quark sector
CKM unitarity



Lepton sector
PMNS unitarity

Ellis et al.,
2004.13719

Determining the origin of neutrino masses

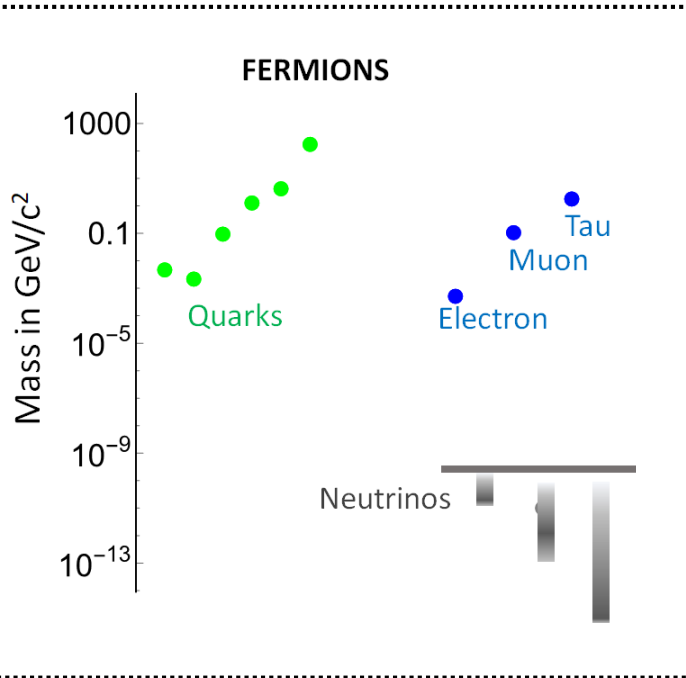


Why are neutrinos so light?

Do they receive mass through the [Higgs mechanism](#) or is there [another mechanism](#) responsible for their masses?

- * In general, the Standard Model neutrinos cannot have a mass if we do not add any extra ingredient
- * Neutrino oscillations tell us that neutrinos have a mass and that lepton flavor number L_α is not a good symmetry

Determining the origin of neutrino masses



Why are neutrinos so light?

Do they receive mass through the **Higgs mechanism** or is there **another mechanism** responsible for their masses?

* Dirac neutrinos: (Lepton number **conserving** model)

$$\mathcal{L}_\nu = Y_\nu \bar{L} H N_R + \text{h.c.}, \quad Y_\nu = \mathcal{O}(10^{-11})$$

* Majorana neutrinos: (Lepton number **violating** model)

$$\mathcal{L}_w = \frac{c}{\Lambda} (LH)(LH) + \text{h.c.}, \quad \frac{c}{\Lambda} \sim 10^{-15} \text{ GeV}^{-1}$$

for example, in Type-I see saw:

Majorana condition $\mathcal{L}_\nu = (Y_\nu \bar{L} H N_R + \text{h.c.}) + M N_R N_R, \quad m_\nu = \mathcal{O}\left(\frac{Y_\nu^2 v^2}{M}\right)$

$\nu_R = \nu_L$ particle and antiparticle are the same

* In general, the Standard Model neutrinos cannot have a mass if we do not add any extra ingredient

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Neutrino masses and mixings, Dirac neutrinos

Different neutrino flavor, f, mix:

$$|\nu_\alpha^f\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} (U_{\beta\alpha}^* \bar{\nu}_\alpha \gamma^\rho \ell_\beta) W_\rho + \text{h.c.}$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ & & 1 \end{pmatrix}$$

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We do not know if the PMNS matrix, U , contains a **new source of CP violation**

Parameter	Value from PDG	
	NO	IO
θ_{12}	$(33.41_{-0.72}^{+0.75})^\circ$	same
θ_{23}	$(49.1_{-1.3}^{+1.0})^\circ$	$(49.5_{-1.2}^{+0.9})^\circ$
θ_{13}	$(8.54_{-0.12}^{+0.11})^\circ$	$(8.57_{-0.11}^{+0.12})^\circ$

Future prospects on δ :
 $\delta = (0 \pm 7)^\circ$, $\delta = (90 \pm 22)^\circ$
 after 10 years of Hyper-K run

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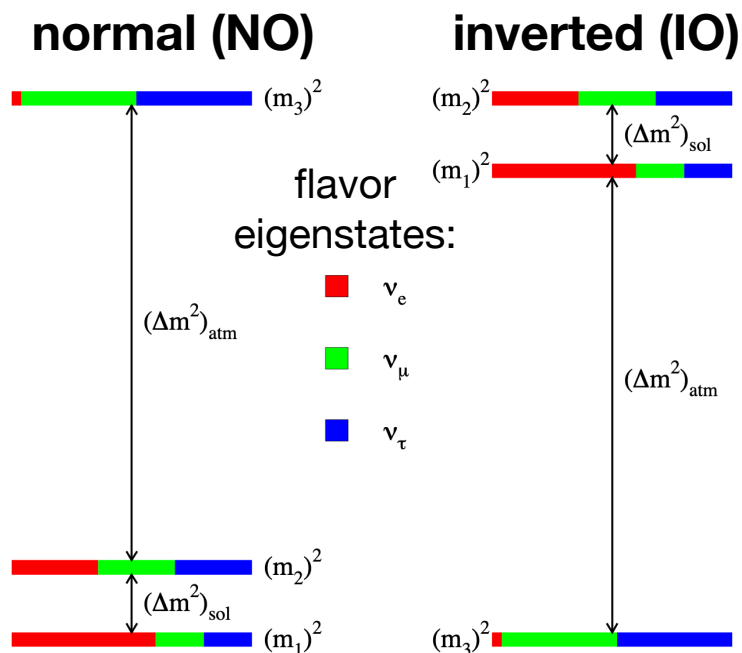
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We do not know if **normal or inverted ordering**:



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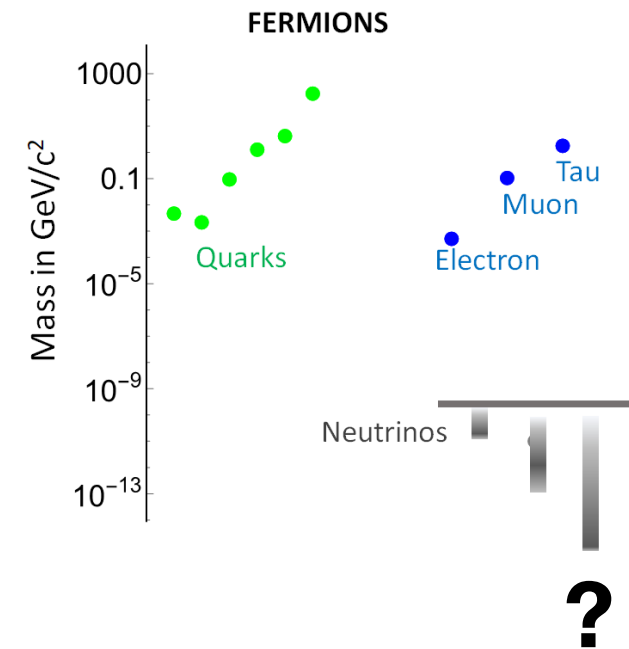
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$\Delta m_{21}^2 / (10^{-5} \text{ eV}^2)$	$7.41^{+0.21}_{-0.20}$	same
$\Delta m_{32}^2 / (10^{-3} \text{ eV}^2)$	$2.437^{+0.028}_{-0.027}$	$-2.498^{+0.032}_{-0.025}$

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The absolute scale of neutrino masses

Neutrino oscillations give information on the neutrino mass splitting, but not on the absolute scale:

$$\sum_i m_i > 0.1(0.059) \text{ eV}, \text{ IO(NO)}$$



The absolute scale of neutrino masses

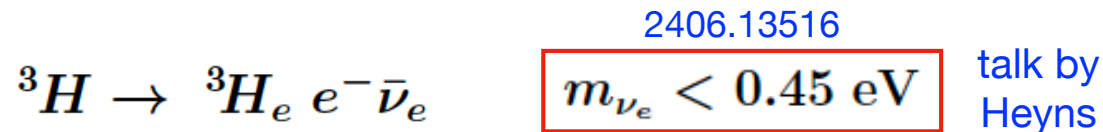
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The absolute scale receives constraints from:

Laboratory experiments:

KATRIN has produced the tightest constraints to date, from measuring the endpoint of the tritium β -decay spectrum.



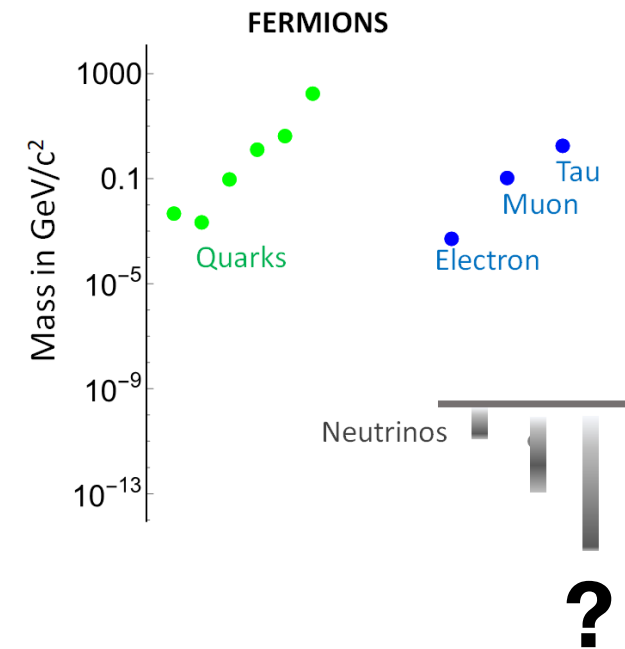
Limits on muon and tau neutrino masses are much weaker (~190keV and ~18.2 MeV from pion and tau decays)

Cosmology:

for the Λ CDM model, DESI BAO + CMB:

$$\sum_i m_i < 0.072 \text{ eV}$$

2404.03002
see, however,
Green, Meyers, 2407.07878



Majorana neutrinos? Neutrinoless double beta decay

$$\mathcal{L}_\nu = (Y_\nu^{ij} \bar{L}_i H N_{R,j} + \text{h.c.}) + M N_{R,i} N_{R,i}$$

hypothesis: no appreciable mixing with sterile neutrinos

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6 degrees of freedom: 3 angles and 3 phases

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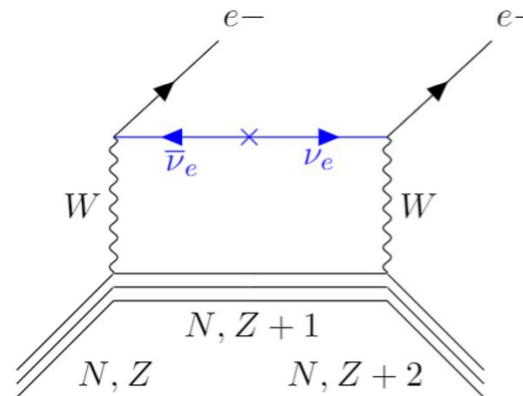
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If we define $m_{ee} \equiv \left| \sum_{i=1}^3 m_i U_{ei}^2 \right|$

Neutrino less double beta decay:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

lepton number violating (LNV) process



$$\text{rate} = 1/T_{1/2} \propto (m_{ee})^2$$

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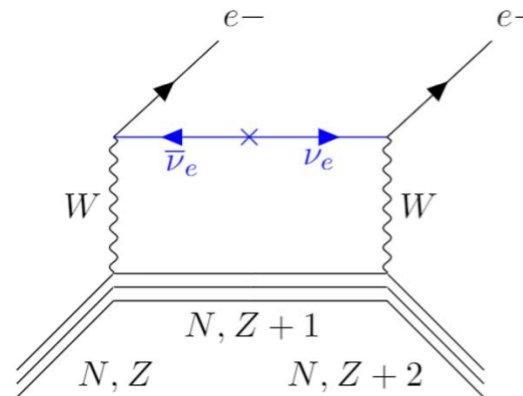
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* Current experimental bound: $m_{ee} < (28 - 122) \text{ meV}$

(uncertainty coming from nuclear matrix element)

corresponding to half-time, $T_{1/2}$: **3.8×10^{26} years!** [KamLAND-Zen, 2406.11438](#)

* Future prospects: half-time \sim **$O(10^{28})$ years** \rightarrow (5-20) meV in \sim 10 years

The challenge of neutrinoless double beta decay

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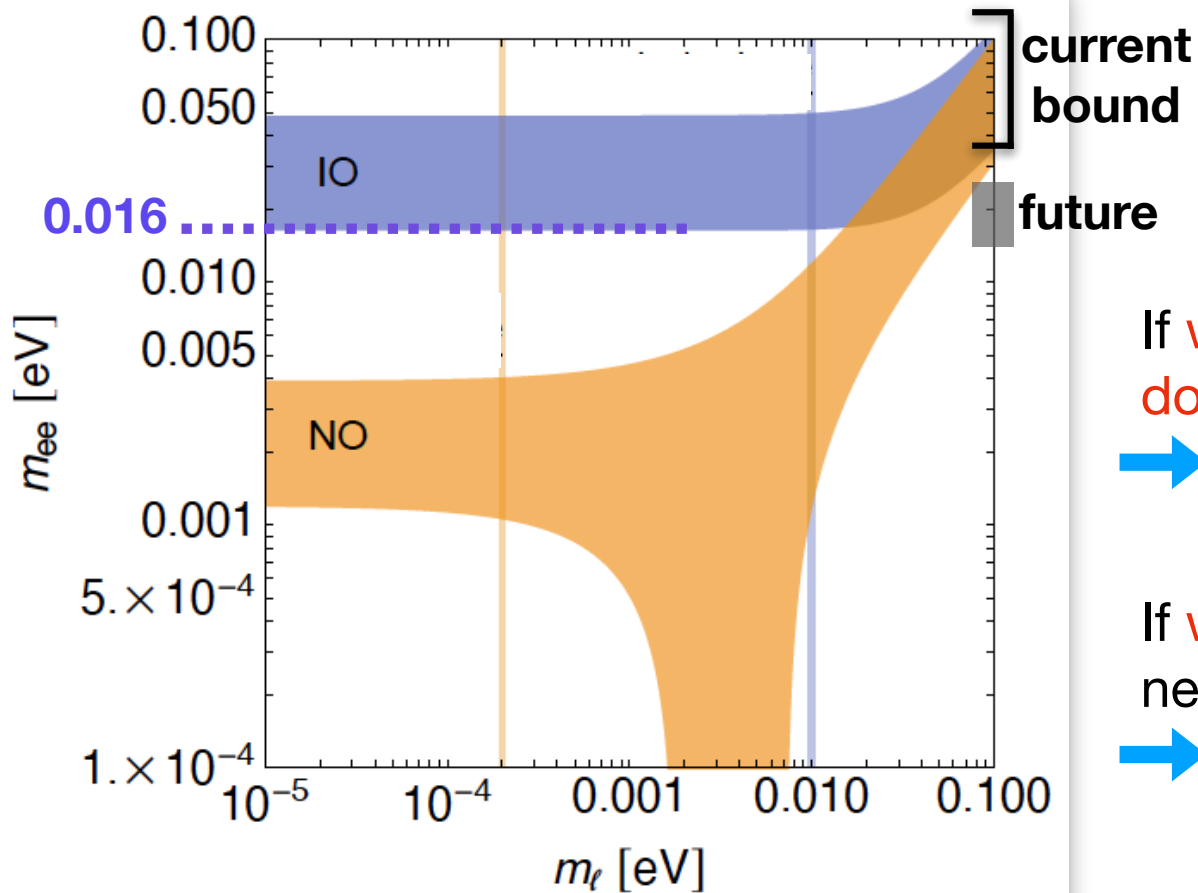
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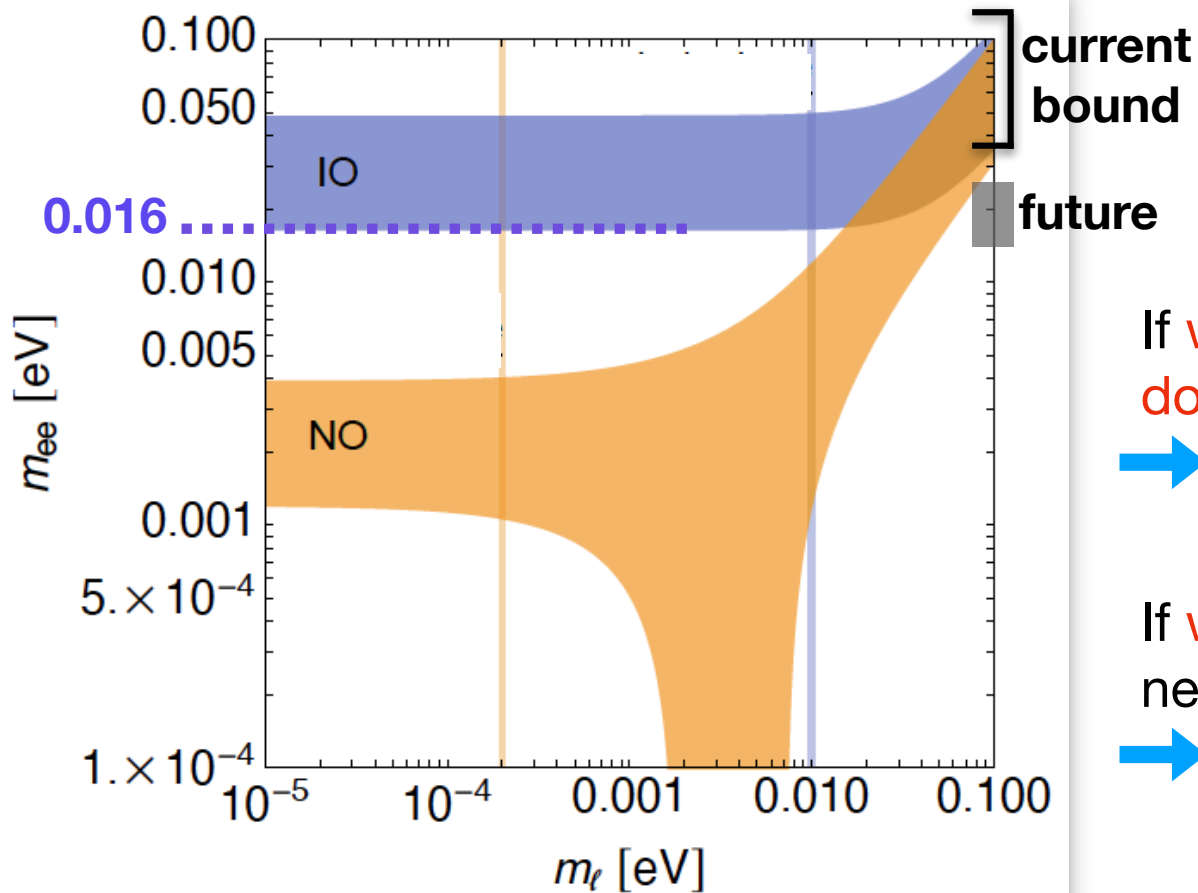
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Regions obtained scanning over the 9 free parameters in their experimental range

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Other ways to determine Majorana vs. Dirac?

Invariants

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This quantities are NOT basis invariant

Dirac phase = CPV phase that can be measured in **lepton number conserving** processes.

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We can define the **basis invariant** quantities:

$$t_{\alpha i \beta j} = U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*, \quad s_{\alpha i j} = U_{\alpha i} U_{\alpha j}^*$$

* 4 (3+1) invariants in the case of **Dirac neutrinos**
 $\{|t_{e2e3}|, |t_{e3e3}|, |t_{\mu 2e3}|, \Psi_D \equiv \arg(t_{\mu 2e3})\}$

* +2 phases in the case of **Majorana neutrinos**
 $\{\Phi_{12}, \Phi_{23}\} \equiv \{\Phi_{12}^e, \Phi_{23}^e\}, \quad \Phi_{ij}^\alpha \equiv \arg(s_{\alpha i j})$

Total number of physical parameters:
3 angles and 3 phases (1+2)

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In terms of the original phases:

$$\begin{cases} \Psi_D &= \arg(c_{12}c_{23}e^{-i\delta} - s_{12}s_{23}s_{13}) \\ \Phi_{12} &= \eta_1 - \eta_2 \\ \Phi_{23} &= \eta_2 + \delta \end{cases}$$

Observation: area of lepton unitary triangles = $-|t_{\mu2e3}| \sin \Psi_D$

Generalizing m_{ee}

$$m_{\alpha\beta} = \left| \sum_{i=1}^3 m_i U_{\alpha i} U_{\beta i} \right|, \quad \alpha, \beta \in \{e, \mu, \tau\}$$

$$\delta_{ij}^{\alpha\beta} \equiv \arg(t_{\alpha i \beta j})$$

$$\Phi_{ij} \equiv \Phi_{ij}^e \equiv \arg(s_{eij})$$

$$= \sum_i m_i^2 |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2 \sum_{i < j} m_i m_j |U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j}| \cos(2\Phi_{ij} + \delta_{ij}^{\alpha e} + \delta_{ij}^{\beta e})$$

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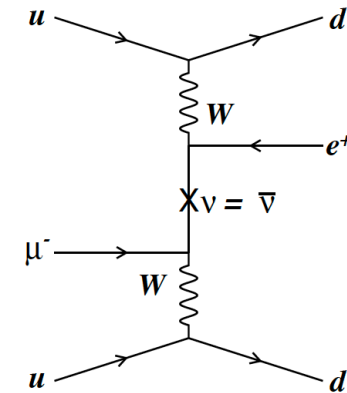
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Observations:

* Any rate of lepton number violating processes is proportional to the corresponding $(m_{\alpha\beta})^2$

For example, the rate for $\mu^- \rightarrow e^+$ is proportional to $(m_{\mu e})^2$



Observables and bounds

Experimental bounds:

Lepton number and flavor violating processes

$$\langle m_{\alpha\beta} \rangle < \begin{pmatrix} 1.2 \times 10^{-10} & 1.7(8.2) \times 10^{-2} & 4.2 \times 10^3 \\ & 50 & 4.4 \times 10^3 \\ & & 2.0 \times 10^4 \end{pmatrix} \text{ GeV}$$

updated from Rodejohann, Zuber, 0011050

Spread of 14 orders
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$\langle m_{ee} \rangle$	$0\nu\beta\beta$	KamLAND-Zen
$\langle m_{e\mu} \rangle$	$\text{Ti} + \mu^- \rightarrow \text{Ca} + e^+$	SINDRUM
$\langle m_{\mu\mu} \rangle$	$K^+ \rightarrow \pi^- \mu^+ \mu^+$	NA62
$\langle m_{e\tau} \rangle$	$e^+ p \rightarrow \bar{\nu}_e e^+ \tau^+ X$	HERA
$\langle m_{\mu\tau} \rangle$	$e^+ p \rightarrow \bar{\nu}_e \mu^+ \tau^+ X$	HERA
$\langle m_{\tau\tau} \rangle$	$e^+ p \rightarrow \bar{\nu}_e \tau^+ \tau^+ X$	HERA

Additional possible tests from other meson decays like

$$B_s \rightarrow M \pi^- \mu^+ \mu^+$$

Mu2e + COMET will improve the bound on the rate by ~4 orders of magnitude.

Rates are small though:

$$\sim 3 \times 10^{-22} \frac{|\langle m_{\mu e} \rangle|^2}{m_e^2} |M|^2$$

nuclear matrix element

Experimental prospects for searching for $0\nu\beta\beta$ -decay are incomparably better because the number of potentially $0\nu\beta\beta$ -decaying nuclei is much much larger than the particles used for setting the other bounds.

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Any correlation between different elements of $m_{\alpha\beta}$?

The single phase limit



Let's define $\{\nu_\ell, \nu_2, \nu_o\}$ such that:

Normal hierarchy

$$\nu_0 = \nu_3 \text{ —————}$$

$$\nu_2 = \nu_2 \text{ —————}$$

$$\nu_1 = \nu_1 \text{ —————}$$

our convention  usual convention 

Inverted hierarchy

$$\nu_2 = \nu_2 \text{ —————}$$

$$\nu_0 = \nu_1 \text{ —————}$$

$$\nu_1 = \nu_3 \text{ —————}$$

in the $m_l \rightarrow 0$ limit:

$$m_{\alpha\beta}^2 \approx m_2^2 |U_{\alpha 2}|^2 |U_{\beta 2}|^2 \left[1 + \frac{m_o^2}{m_2^2} \frac{|U_{\alpha o}|^2 |U_{\beta o}|^2}{|U_{\alpha 2}|^2 |U_{\beta 2}|^2} + 2 \frac{m_o}{m_2} \frac{|U_{\alpha o}| |U_{\beta o}|}{|U_{\alpha 2}| |U_{\beta 2}|} \cos (2\Phi_{2o} + \delta_{o2}^{\alpha e} + \delta_{o2}^{\beta e}) \right]$$

Only one Majorana phase enters independently on the values of $\alpha\beta$

The single phase limit

Let's define $\{\nu_\ell, \nu_2, \nu_o\}$ such that:

Normal hierarchy

$$\nu_0 = \nu_3 \text{ —————}$$

$$\nu_2 = \nu_2 \text{ —————}$$

$$\nu_1 = \nu_1 \text{ —————}$$

our convention — usual convention

Inverted hierarchy

$$\nu_2 = \nu_2 \text{ —————}$$

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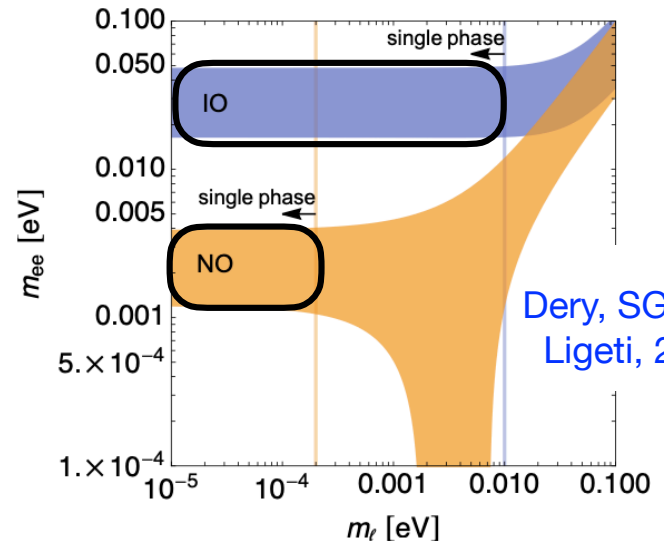
in the $m_1 \rightarrow 0$ limit:

$$m_{\alpha\beta}^2 \approx m_2^2 |U_{\alpha 2}|^2 |U_{\beta 2}|^2 \left[1 + \frac{m_o^2}{m_2^2} \frac{|U_{\alpha o}|^2 |U_{\beta o}|^2}{|U_{\alpha 2}|^2 |U_{\beta 2}|^2} + 2 \frac{m_o}{m_2} \frac{|U_{\alpha o}| |U_{\beta o}|}{|U_{\alpha 2}| |U_{\beta 2}|} \cos(2\Phi_{2o} + \delta_{o2}^{\alpha e} + \delta_{o2}^{\beta e}) \right]$$

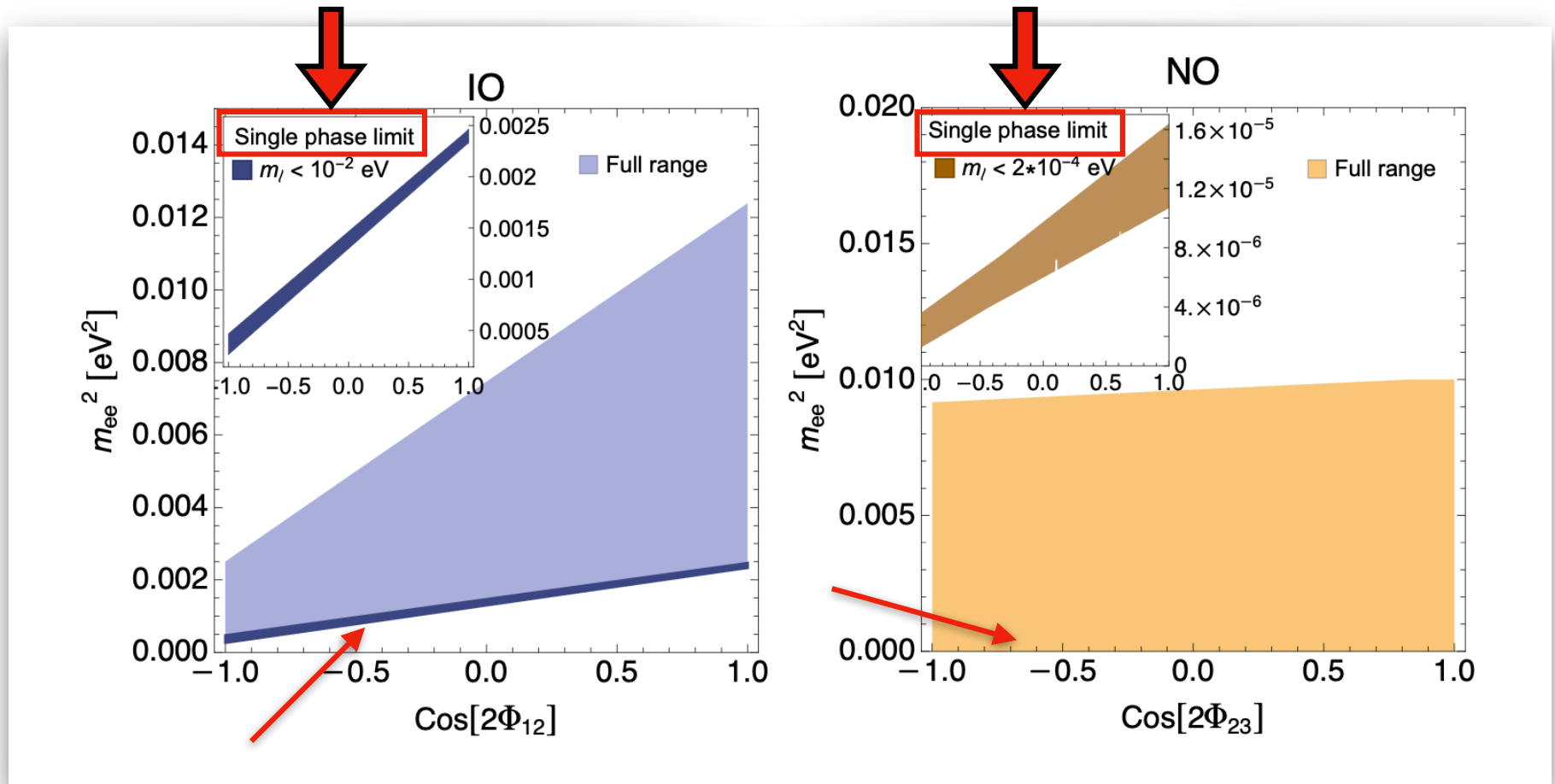
→ all $m_{\alpha\beta}$ are correlated

These relations are **independent on the exact absolute scale** of neutrino masses, as long as $m_1 \ll m_0, m_2$ (Simkovic et al. for IO, 1210.1306)

Only one Majorana phase enters independently on the values of $\alpha\beta$



Determining the Majorana phase through m_{ee}

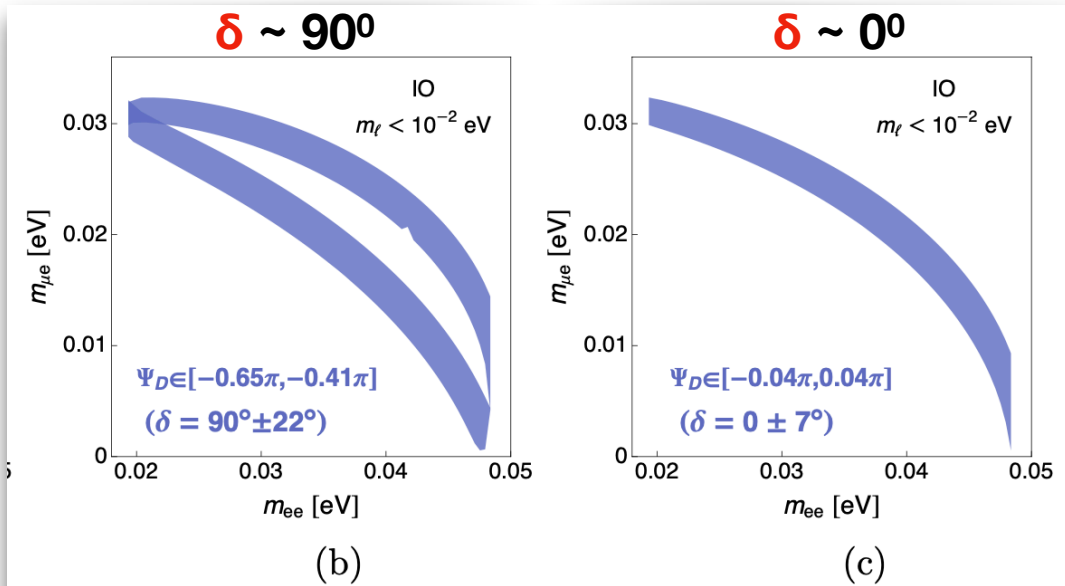


Dery, SG, Grossman, Ligeti, 2406.18647

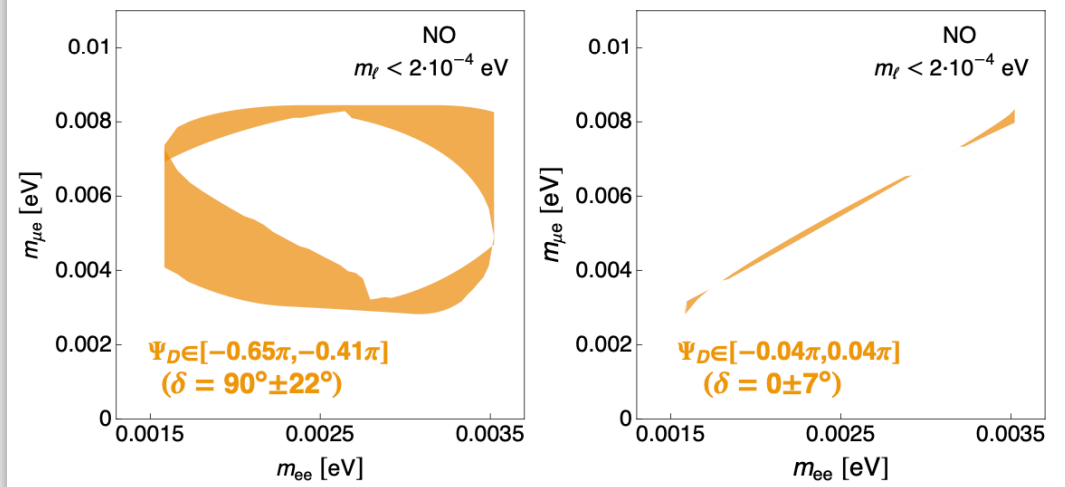
From here we conclude that there must be a correlation between m_{ee} and all the other elements $m_{\alpha\beta}$

Predicting $m_{\mu e}$ from m_{ee}

inverted hierarchy



normal hierarchy

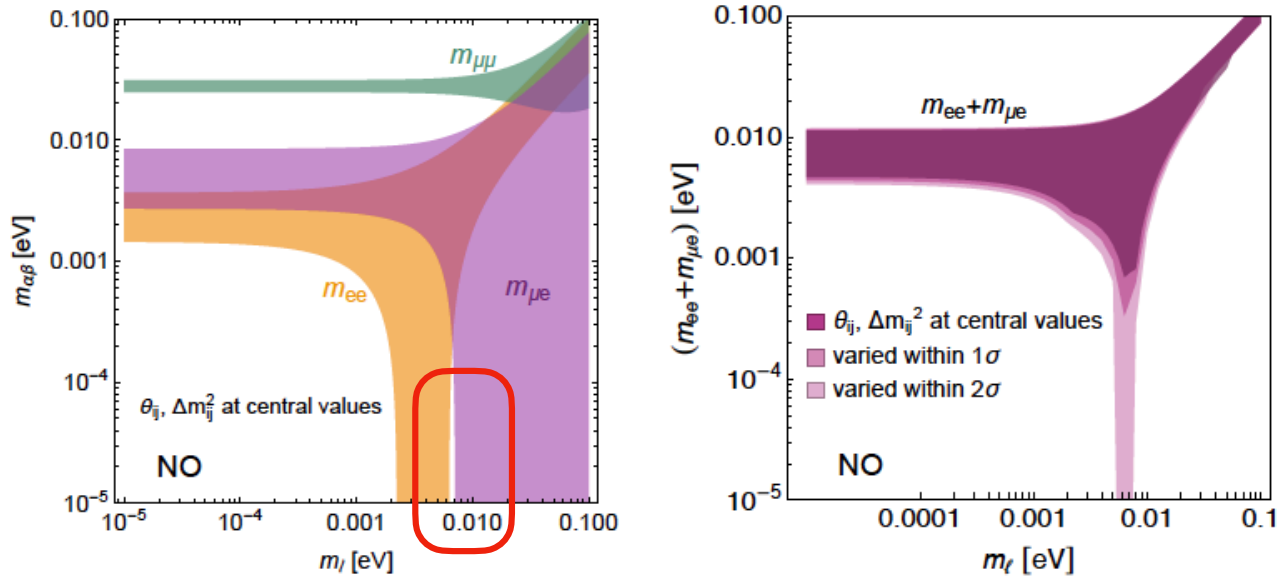


Regions obtained scanning over the 9 free parameters in their experimental range and considering two possible measurements for δ at Huper-K or DUNE:
 $\delta = (0 \pm 7)^\circ$, $\delta = (90 \pm 22)^\circ$

Dery, SG, Grossman, Ligeti, 2406.18647

A no-loose theorem?

Dery, SG, Grossman, Ligeti, 2406.18647

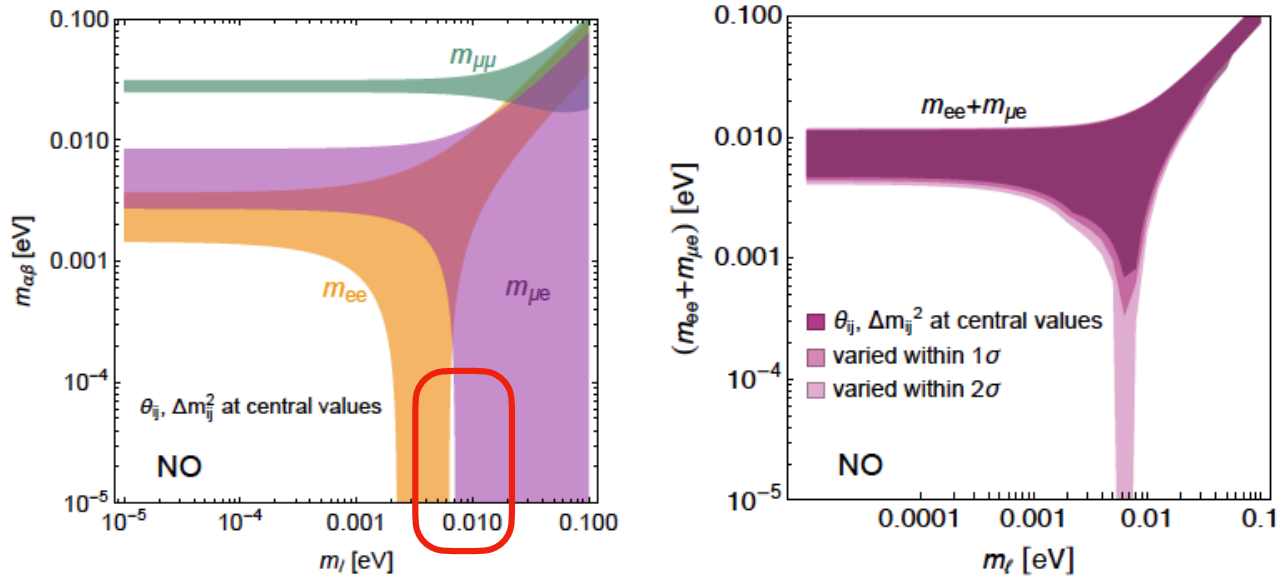


With the present status of measurement of the parameters of the PMNS matrix + neutrino mass splitting, $m_{\mu e}$ and m_{ee} cannot be simultaneously = 0 (even in the case of normal hierarchy)

In principle, LNV processes should be detectable, if neutrinos are Majorana

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In principle, LNV processes should be detectable, if neutrinos are Majorana

$$m_{ee} + m_{\mu e} > \begin{cases} 7 \times 10^{-4} \text{ eV}, & \text{central values} \\ 2 \times 10^{-4} \text{ eV}, & 1\sigma \text{ ranges} \end{cases}$$

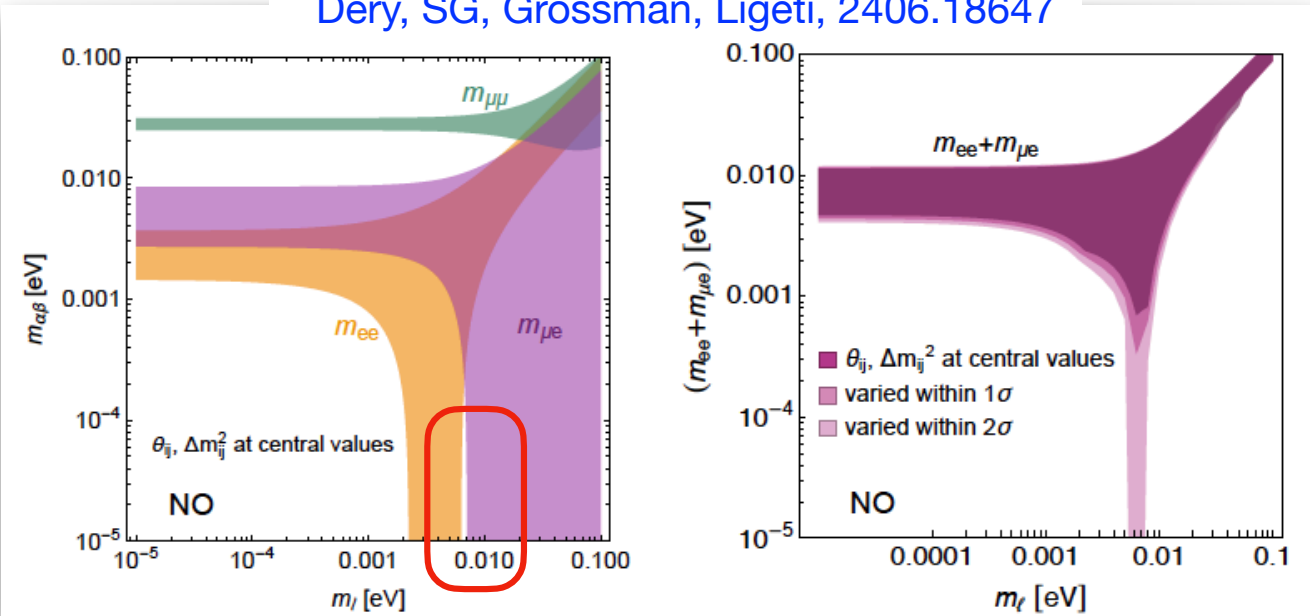
Much larger effects in models with additional interactions (e.g., Graf et al., 2010.15109, Berryman et al., 1611.00032, ...)

Very challenging to test these values, using the bounds we saw earlier.

Astrophysical bounds?
work in progress with Dery, Grossman, Ligeti

A no-loose theorem?

Dery, SG, Grossman, Ligeti, 2406.18647



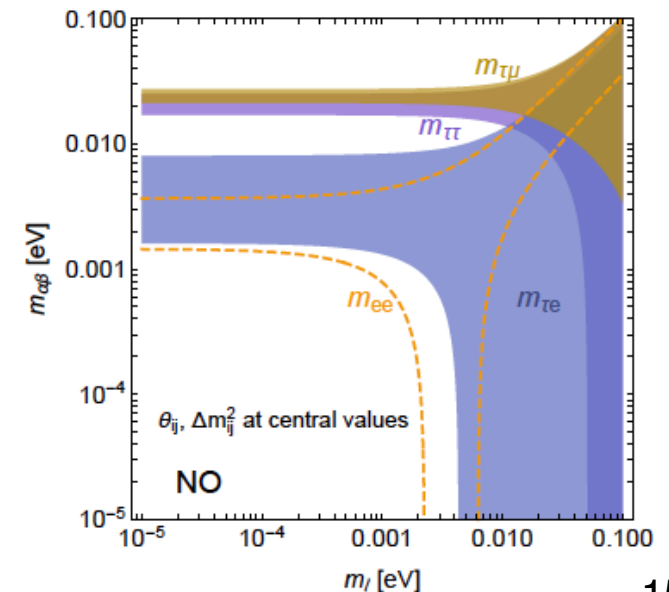
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Similarly, for the other elements:



Conclusions & Outlook

The nature of neutrino masses (Dirac vs. Majorana) is a fundamental open question.

Neutrinoless double beta decay experiments (m_{ee}) test one of the two Majorana phases. They can

- discover the Majorana nature of neutrinos
- rule out the Majorana nature in the particular case of inverted ordering
- be not conclusive in the case of normal ordering if no observation



What about the 2nd Majorana phase?

($m_{\mu e}$ depends on the 2nd phase if m_1 is not too light)



Present neutrino oscillation measurements imply that there are no regions where m_{ee} and $m_{\mu e}$ both vanish. More precise oscillation measurements are needed.

➔ The Majorana nature of neutrinos could in principle be ruled out by the non-observation of both m_{ee} and $m_{\mu e}$ (from $\mu^- \rightarrow e^+$ transitions).

No-loose theorem for the discovery of Majorana neutrinos

Phenomenologically challenging!

Explicit expressions for m_{ee} and $m_{\mu e}$

(single phase limit)

$$m_{ee}^2 \approx \sum_i m_i^2 |U_{ei}|^4 + 2m_2 m_o |U_{e2}|^2 |U_{eo}|^2 \underbrace{\cos 2\Phi_{2o}}_{\substack{\text{4-fold ambiguity} \\ \{\Phi_{2o}, \pi - \Phi_{2o}, -\Phi_{2o}, \Phi_{2o} - \pi\}}}$$

$$m_{\mu e}^2 \approx \sum_i m_i^2 |U_{ei}|^2 |U_{\mu i}|^2 + 2m_2 m_o |U_{e2} U_{eo} U_{\mu 2} U_{\mu o}| \cos (2\Phi_{2o} + \delta_{2o}^{\mu e})$$

2-fold ambiguity

$$\sin 2\Phi_{2o} \sin \delta_{2o}^{\mu e}$$

