

Extending EFT of inflation/dark energy to arbitrary background with timelike scalar profile

Shinji Mukohyama
(YITP, Kyoto U)

Collaborators



V. Yingcharoenrat



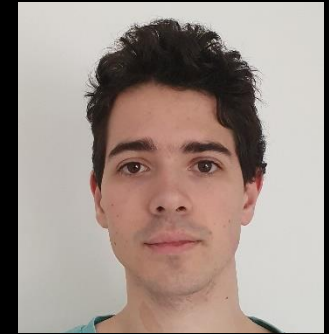
K. Takahashi



K. Tomikawa



K. Aoki



E. Seraille



M. A. Gorji



C. G. A. Barura



H. Kobayashi



N. Oshita

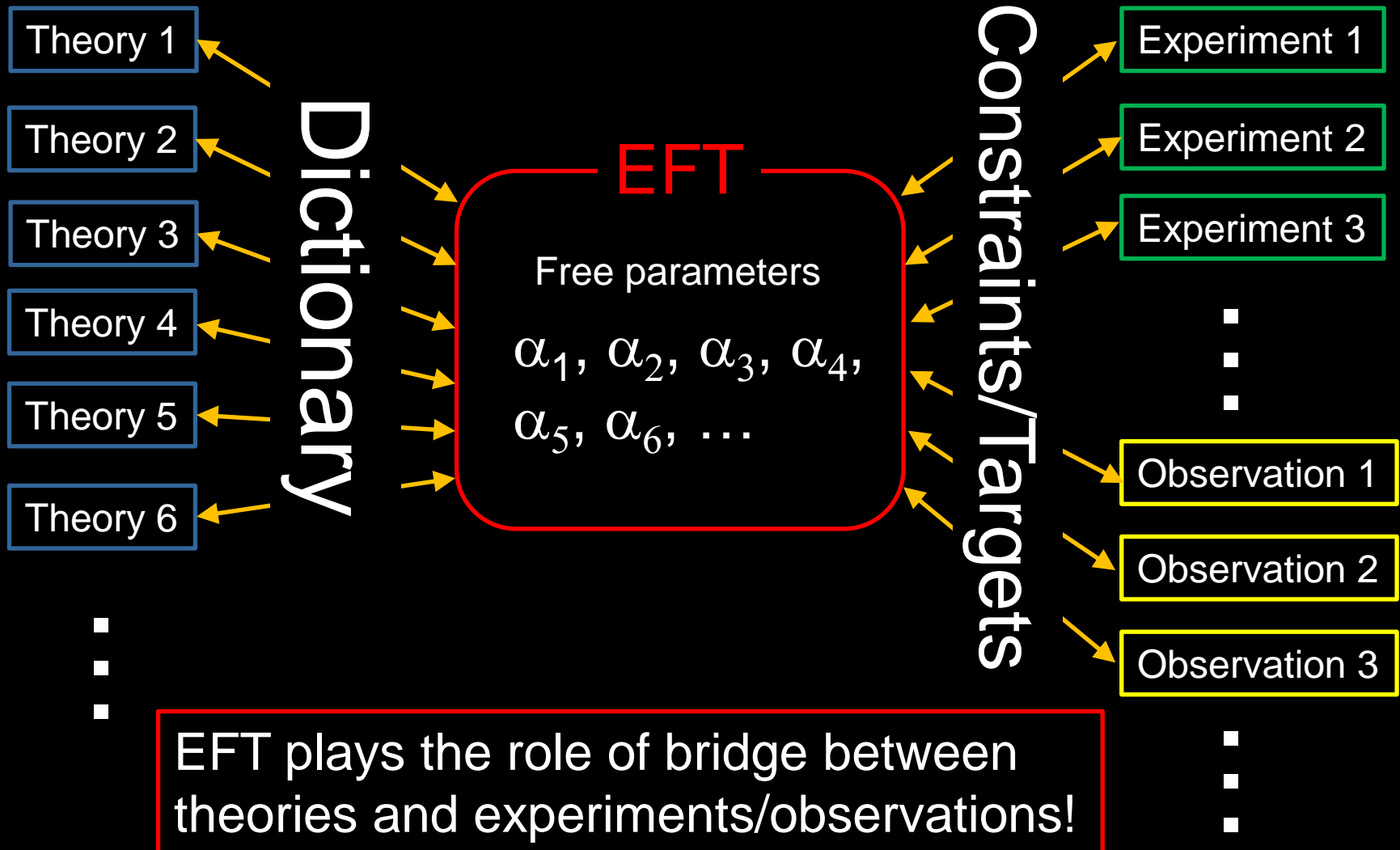
- Ref.
- arXiv: 2204.00228 w/ V. Yingcharoenrat
 - arXiv: 2208.02943 w/ K. Takahashi, V. Yingcharoenrat
 - arXiv: 2304.14304 w/ K. Takahashi, K. Tomikawa, V. Yingcharoenrat
 - arXiv: 2405.10813 w/ C. G. A. Barura, H. Kobayashi, N. Oshita, K. Takahashi, V. Yingcharoenrat
 - arXiv: 2406.04525 w/ N. Oshita and K. Takahashi
 - arXiv: 2407.xxxxx w/ E. Seraille, K. Takahashi, V. Yingcharoenrat
 - arXiv: 2111.08119 w/ K. Aoki, M. A. Gorji, K. Takahashi
 - arXiv: 2311.06767 w/ K. Aoki, M. A. Gorji, K. Takahashi, V. Yingcharoenrat

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Scalar-tensor gravity

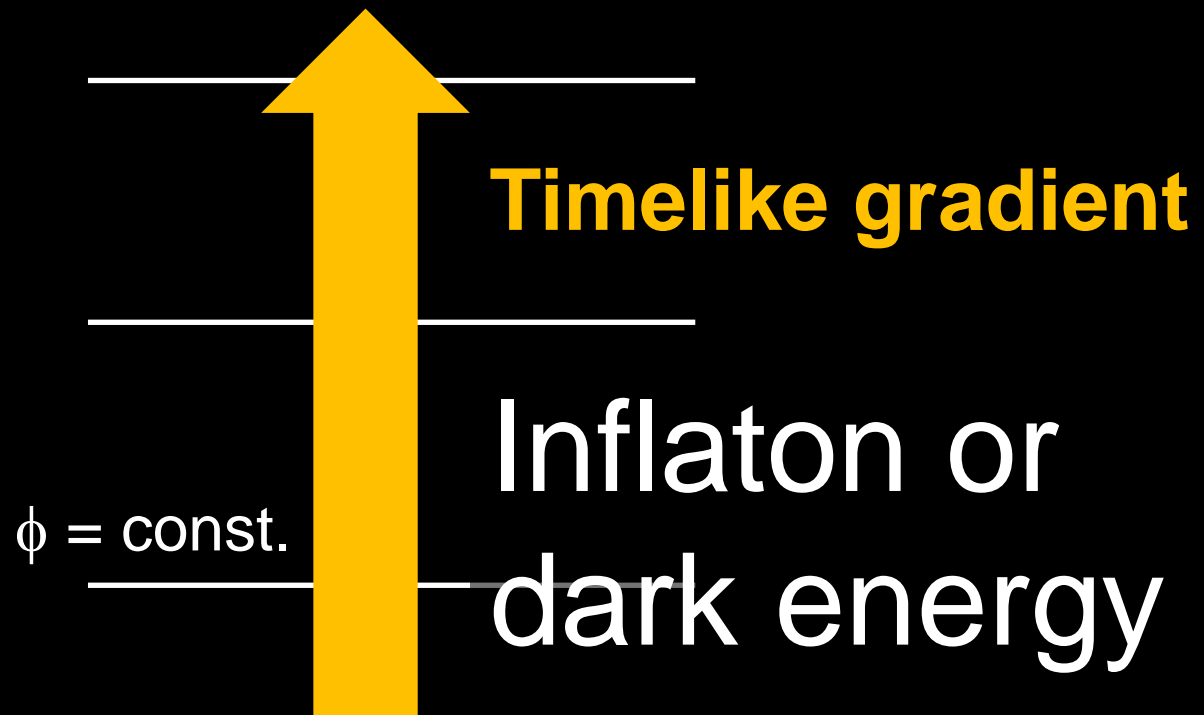
- Contains majority of inflation & dark energy models
- Contains GR + a scalar field as a special case
- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), ...
- Most general scalar-tensor theory of gravity with 2nd order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- U-DHOST theories beyond DHOST: DeFelice, Langlois, Mukohyama, Noui & Wang (2018)
- All of them (and more) are universally described by an effective field theory (EFT)

Effective field theory (EFT) approach



EFT of scalar-tensor gravity with timelike scalar profile

- **Inflaton/dark energy has timelike derivative**
- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**



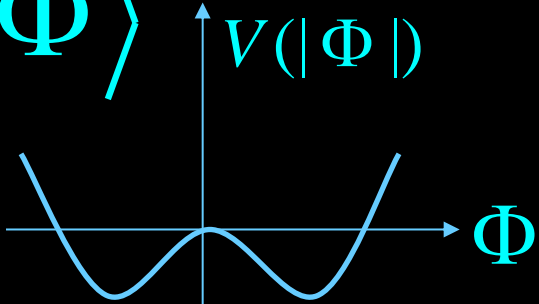
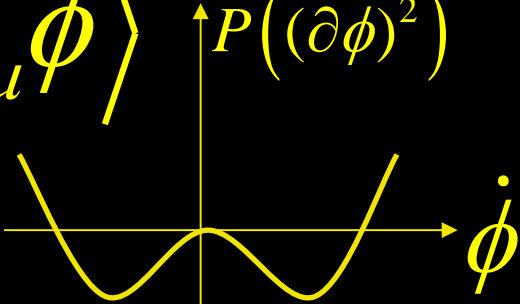
EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT on Minkowski
background

= ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

	Higgs mechanism	Ghost condensate Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2 \Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	$V'=0, V''>0$	$P'=0, P''>0$
Broken symmetry	Gauge symmetry	Time diffeomorphism
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

EFT of ghost condensation = EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

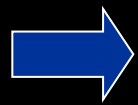
Backgrounds characterized by

✧ $\langle \partial_\mu \phi \rangle = \text{const} \neq 0$ and timelike

✧ Minkowski metric

$t \rightarrow t + \text{const}$ & $t \rightarrow -t$ unbroken

up to $\phi \rightarrow \phi + \text{const}$ & $\phi \rightarrow -\phi$



$$L_{\text{eff}} = L_{\text{EH}} + M^4 \left\{ \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \dots \right\}$$

Gauge choice: $\phi(t, \vec{x}) = t$. $\pi \equiv \delta\phi = 0$
(Unitary gauge)

Residual symmetry: $\vec{x} \rightarrow \vec{x}'(t, \vec{x})$

→ Write down most general action invariant under this residual symmetry.

(→ Action for π : undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

Beginning at quadratic order, since we are assuming flat space is good background.

$$\left\{ \begin{array}{l} (h_{00})^2 \quad \text{OK} \\ \cancel{(h_{0i})^2} \\ K^2, K^{ij} K_{ij} \quad \text{OK} \end{array} \right.$$

$$K_{ij} = \frac{1}{2} (\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j})$$

$$\Rightarrow L_{eff} = L_{EH} + M^4 \left\{ (h_{00})^2 - \frac{\alpha_1}{M^2} K^2 - \frac{\alpha_2}{M^2} K^{ij} K_{ij} + \dots \right\}$$

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Action for π

$$\xi^0 = \pi \left\{ \begin{array}{l} h_{00} \rightarrow h_{00} - 2\partial_0 \pi \\ K_{ij} \rightarrow K_{ij} + \partial_i \partial_j \pi \end{array} \right.$$

$$\Rightarrow L_{eff} = L_{EH} + M^4 \left\{ (h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2} (K + \vec{\nabla}^2 \pi)^2 - \frac{\alpha_2}{M^2} (K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi) (K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi) + \dots \right\}$$

$$E \rightarrow rE$$

$$dt \rightarrow r^{-1} dt$$

$$dx \rightarrow r^{-1/2} dx$$

$$\pi \rightarrow r^{1/4} \pi$$

Make
invariant

$$\rightarrow \int dt d^3x \left[\frac{1}{2} \dot{\pi}^2 - \frac{\alpha (\vec{\nabla}^2 \pi)^2}{M^2} + \dots \right]$$

Leading nonlinear operator in infrared $\int dt d^3x \frac{\dot{\pi} (\nabla \pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

⇒ **Good low-E effective theory**
Robust prediction

e.g. Ghost inflation [Arkani-hamed, Creminelli, Mukohyama, Zaldarriaga 2004]

EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

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EFT on cosmological
background

= EFT of inflation

Creminelli, Luty, Nicolis and Senatore 2006

Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007

Extension to FLRW background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under $x^i \rightarrow x^i(t, x)$

- Ingredients

$$g_{\mu\nu}, g^{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu,$$

t & its derivatives

- 1st derivative of t

$$\partial_\mu t = \delta_\mu^0 \quad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu} \partial_\mu t \partial_\nu t}} = \frac{\delta_\mu^0}{\sqrt{-g^{00}}}$$
$$g^{00} \quad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2nd derivative of t

$$K_{\mu\nu} \equiv h_\mu^\rho \nabla_\rho n_\nu$$

Unitary gauge action

$$I = \int d^4x \sqrt{-g} L(t, \delta_\mu^0, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})$$



derivative & perturbative expansions

$$I = M_{Pl}^2 \int dx^4 \sqrt{-g} \left[\frac{1}{2} R + c_1(t) + c_2(t) g^{00} \right. \\ \left. + L^{(2)}(\tilde{\delta} g^{00}, \tilde{\delta} K_{\mu\nu}, \tilde{\delta} R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu) \right]$$

$$L^{(2)} = \lambda_1(t) (\tilde{\delta} g^{00})^2 + \lambda_2(t) (\tilde{\delta} g^{00})^3 + \lambda_3(t) \tilde{\delta} g^{00} \tilde{\delta} K_\mu^\mu \\ + \lambda_4(t) (\tilde{\delta} K_\mu^\mu)^2 + \lambda_5(t) \tilde{\delta} K_\nu^\mu \tilde{\delta} K_\mu^\nu + \dots$$

$$\tilde{\delta} g^{00} \equiv g^{00} + 1 \quad \tilde{\delta} K_{\mu\nu} \equiv K_{\mu\nu} - H \gamma_{\mu\nu}$$

$$\tilde{\delta} R_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - 2(H^2 + \mathfrak{K}/a^2) \gamma_{\mu[\rho} \gamma_{\sigma]\nu} + (\dot{H} + H^2) (\gamma_{\mu\rho} \delta_\nu^0 \delta_\sigma^0 + (3\text{perm.}))$$

NG boson

- Undo unitary gauge $t \rightarrow \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$H(t) \rightarrow H(t + \pi), \quad \dot{H}(t) \rightarrow \dot{H}(t + \pi),$$

$$\lambda_i(t) \rightarrow \lambda_i(t + \pi), \quad a(t) \rightarrow a(t + \pi),$$

$$\delta_\mu^0 \rightarrow (1 + \dot{\pi})\delta_\mu^0 + \delta_\mu^i \partial_i \pi,$$

- NG boson in decoupling (subhorizon) limit

$$I_\pi = M_{Pl}^2 \int dt d^3 \vec{x} a^3 \left\{ -\frac{\dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \dot{H} \left(\frac{1}{c_s^2} - 1 \right) \left(\frac{c_3}{c_s^2} \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) + O(\pi^4, \tilde{\epsilon}^2) + L_{\tilde{\delta}K, \tilde{\delta}R}^{(2)} \right\}$$

$$\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8c_s^2 \lambda_2}{-\dot{H}} \left(\frac{1}{c_s^2} - 1 \right)^{-1}$$

- Sound speed

c_s : speed of propagation for modes with $\omega \gg H$

$$\omega^2 \simeq c_s^2 \frac{k^2}{a^2} \text{ for } \pi \sim A(t) \exp(-i \int \omega dt + i \vec{k} \cdot \vec{x})$$

Application: non-Gaussianity of inflationary perturbation $\zeta = -H\pi$

$$I_\pi = M_{Pl}^2 \int dt d^3\vec{x} a^3 \left\{ -\frac{\dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \dot{H} \left(\frac{1}{c_s^2} - 1 \right) \left(\frac{c_3}{c_s^2} \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) + O(\pi^4, \tilde{\epsilon}^2) + L_{\tilde{\delta}K, \tilde{\delta}R}^{(2)} \right\}$$

power spectrum $P_\zeta(\vec{k}) = \frac{\Delta}{k^3}, \quad \Delta = \frac{H^4}{-4M_{Pl}^2 \dot{H} c_s} \Big|_{c_s k \simeq aH}$

non-Gaussianity $\langle \zeta_{\vec{k}_1}(t) \zeta_{\vec{k}_2}(t) \zeta_{\vec{k}_3}(t) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta$

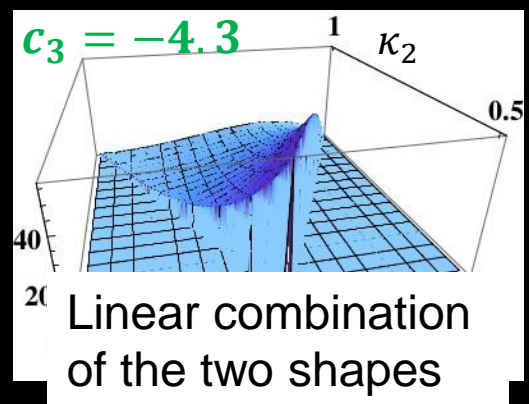
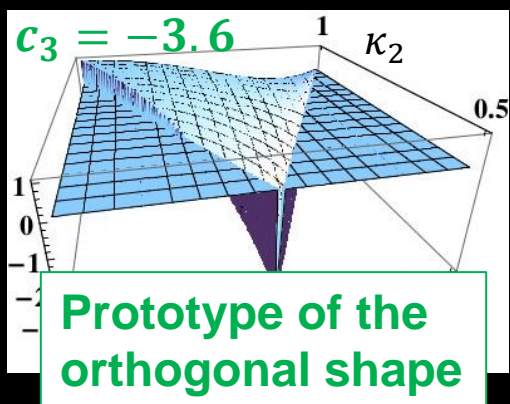
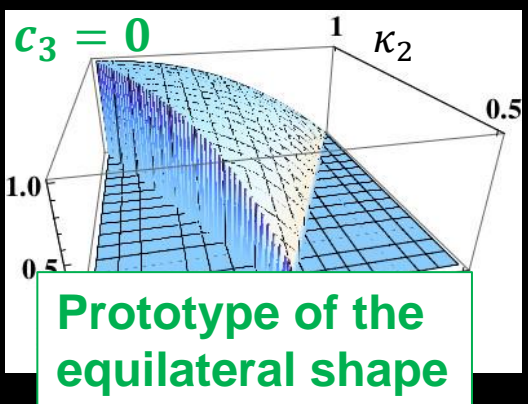
2 types of 3-point interactions

$c_s^2 \rightarrow$ size of non-Gaussianity

$$f_{NL}^{\dot{\pi}(\partial_i \pi)^2} = \frac{85}{324} \left(1 - \frac{1}{c_s^2} \right) \quad f_{NL}^{\dot{\pi}^3} = \frac{5c_3}{81} \left(1 - \frac{1}{c_s^2} \right) \propto \frac{1}{c_s^2} \text{ for small } c_s^2$$

$c_3 \rightarrow$ shape of non-Gaussianity

plots of $B_\zeta(k, \kappa_2 k, \kappa_3 k) / B_\zeta(k, k, k)$



Parametrization suitable for DE

→ EFT of DE

Gubitosi, Piazza, Vernizzi 2012

Gleyzes, Langlois, Piazza, Vernizzi 2013

- Matter (in addition to DE) needs to be added
→ Jordan frame description is convenient
- In Jordan frame the coefficient of the 4d Ricci scalar is not constant.

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[\boxed{M_*^2 f R} - \rho_D + p_D - M_*^2 (5H\dot{f} + \ddot{f}) - \left(\rho_D + p_D + M_*^2 (H\dot{f} - \ddot{f}) \right) g^{00} \right. \\ + M_2^4 (\delta g^{00})^2 - \bar{m}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K^2 - \bar{M}_3^2 \delta K_\mu^\nu \delta K^\mu_\nu + m_2^2 h^{\mu\nu} \partial_\mu g^{00} \partial_\nu g^{00} \\ + \lambda_1 \delta R^2 + \lambda_2 \delta R_{\mu\nu} \delta R^{\mu\nu} + \mu_1^2 \delta g^{00} \delta R + \gamma_1 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + \gamma_2 \epsilon^{\mu\nu\rho\sigma} C_{\mu\nu}^{\kappa\lambda} C_{\rho\sigma\kappa\lambda} \\ \left. + \frac{M_3^4}{3} (\delta g^{00})^3 - \bar{m}_2^3 (\delta g^{00})^2 \delta K + \dots \right],$$

EFT of scalar-tensor gravity with timelike scalar profile

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Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

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Creminelli, Luty, Nicolis and Senatore 2006; Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007; Gubitosi, Piazza, Vernizzi 2012; Gleyzes, Langlois, Piazza, Vernizzi 2013

EFT on arbitrary
background

= **Main subject of this talk**

Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

It is not straightforward...

- General action in the unitary gauge ($\phi = \tau$)

$$S = \int d^4x \sqrt{-g} F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$$

- Taylor expansion around the background

$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \dots \right]$$

- The whole action is invariant under 3d diffeo but **each term is not...**
- Each coefficient is a function of (τ, x^i) but cannot be promoted to an arbitrary function.

Solution: consistency relations

- The chain rule

$$\left[\begin{array}{l} \frac{d}{dx^i} \bar{F} = \bar{F}_{g^{\tau\tau}} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \frac{\partial \bar{K}}{\partial x^i} + \dots \\ \frac{d}{dx^i} \bar{F}_{g^{\tau\tau}} = \bar{F}_{g^{\tau\tau} g^{\tau\tau}} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{g^{\tau\tau} K} \frac{\partial \bar{K}}{\partial x^i} + \dots \\ \frac{d}{dx^i} \bar{F}_K = \bar{F}_{g^{\tau\tau} K} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{KK} \frac{\partial \bar{K}}{\partial x^i} + \dots \end{array} \right.$$

relates x^i -derivatives of an EFT coefficient to other EFT coefficients, and **leads to consistency relations.**

- **The consistency relations ensure the spatial diffeo invariance.**
- Taylor coefficients should satisfy the consistency relations but are otherwise arbitrary.
- (No consistency relation for τ -derivatives.)

EFT action

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[\frac{M_*^2}{2} f(y) R - \Lambda(y) - c(y) g^{\tau\tau} - \beta(y) K - \alpha_\nu^\mu(y) \sigma_\mu^\nu - \gamma_\nu^\mu(y) r_\mu^\nu + \frac{1}{2} m_2^4(y) (\delta g^{\tau\tau})^2 \right. \\
 & + \frac{1}{2} M_1^3(y) \delta g^{\tau\tau} \delta K + \frac{1}{2} M_2^2(y) \delta K^2 + \frac{1}{2} M_3^2(y) \delta K_\nu^\mu \delta K_\mu^\nu + \frac{1}{2} M_4(y) \delta K \delta^{(3)} R \\
 & + \frac{1}{2} M_5(y) \delta K_\nu^\mu \delta^{(3)} R_\mu^\nu + \frac{1}{2} \mu_1^2(y) \delta g^{\tau\tau} \delta^{(3)} R + \frac{1}{2} \mu_2(y) \delta^{(3)} R^2 + \frac{1}{2} \mu_3(y) \delta^{(3)} R_\nu^\mu \delta^{(3)} R_\mu^\nu \\
 & + \frac{1}{2} \lambda_1(y)_\mu^\nu \delta g^{\tau\tau} \delta K_\nu^\mu + \frac{1}{2} \lambda_2(y)_\mu^\nu \delta g^{\tau\tau} \delta^{(3)} R_\nu^\mu + \frac{1}{2} \lambda_3(y)_\mu^\nu \delta K \delta K_\nu^\mu + \frac{1}{2} \lambda_4(y)_\mu^\nu \delta K \delta^{(3)} R_\nu^\mu \\
 & \left. + \frac{1}{2} \lambda_5(y)_\mu^\nu \delta^{(3)} R \delta K_\nu^\mu + \frac{1}{2} \lambda_6(y)_\mu^\nu \delta^{(3)} R \delta^{(3)} R_\nu^\mu + \dots \right],
 \end{aligned}$$

- EFT coefficients should satisfy the consistency relations but are otherwise arbitrary
- One can restore 4d diffeo by Stueckelberg trick
- Easy to find dictionary between EFT coefficients and theory parameters
- Can be applied to arbitrary background with timelike scalar profile

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EFT on arbitrary background

= **Main subject of this talk**

Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

Taylor expansion of the general action $S = \int d^4x \sqrt{-g} F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$

$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \dots \right]$$

Consistency relations  S is invariant under spatial diffeo but the background breaks it.

$$\frac{d}{dx^i} \bar{F} = \bar{F}_{g^{\tau\tau}} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \frac{\partial \bar{K}}{\partial x^i} + \dots$$

Conformal/disformal transformation

[arXiv: 2407.15123 w/E.Seraille, K.Takahashi & V.Yingeharoenrat]

- EFT of DE is usually written in **Jordan frame**, to which matter minimally couple
- EFT of BH perturbations is studied mainly in an **almost Einstein frame** (with constant coefficient of 4d Ricci scalar)
- In order to bridge these EFTs, one needs to know how EFT coefficients are mapped under conformal/disformal transformations

$$\hat{g}_{\mu\nu} = f_0(\Phi, X)g_{\mu\nu} + f_1(\Phi, X)\partial_\mu\Phi\partial_\nu\Phi$$

GW speed near BH

[arXiv: 2407.15123 w/E.Seraille, K.Takahashi & V.Yingeharoenrat]

- GW170817 $\rightarrow |c_{\text{GW}} - 1| < 10^{-15}$ @ cosmological scale \rightarrow constraint on DE/MG models
- Typically, one requires $c_{\text{GW}}=1$ on FLRW for all $H(t)$ & $\phi(t)$ @ low E
- Does this imply $c_{\text{GW}}=1$ around BH @ low E?
- Yes, in Horndeski theory [$G_{4,\chi}=0=G_5$].
- No, in general, e.g. in cubic HOST theories.
- In EFT, the following operator does the job.

$$M_6(y) \bar{\sigma}_\nu^\mu \delta K_\alpha^\nu \delta K_\mu^\alpha$$

$\bar{\sigma}_\nu^\mu$ traceless part of background K_ν^μ

Stealth BH with $\phi = qt + \psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzschild-dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- Fortunately, Scordatura (= detuning of degeneracy condition) solves the strong coupling problem (Motohashi & Mukohyama 2019), if and only if the scalar profile is timelike.
- EFT of ghost condensation already includes scordatura (Arkani-Hamed & Cheng & Luty & Mukohyama 2004)
- Approximate Schwarzschild in ghost condensation (Mukohyama 2005). Also in quadratic HOST (DeFelice & Mukohyama & Takahashi, JCAP 03 (2023) 050).

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
[arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
→ Generalized Regge-Wheeler equation
[arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
[see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]
→ Quasi-normal mode
[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

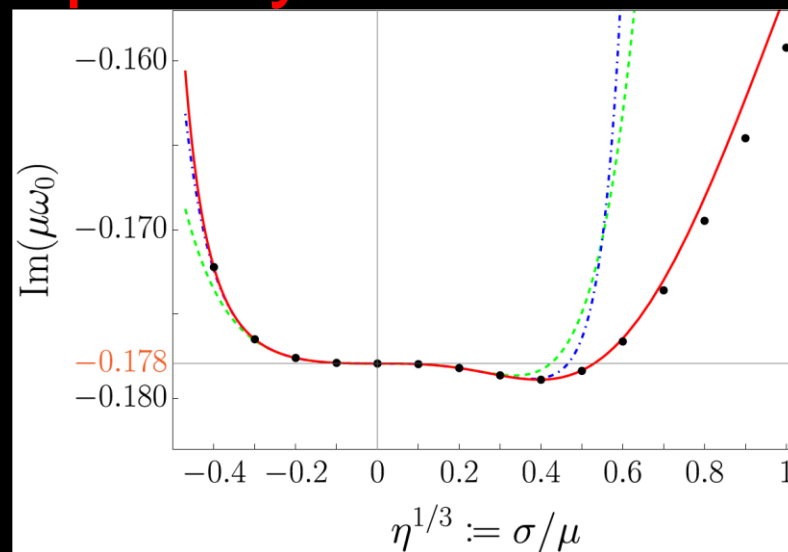
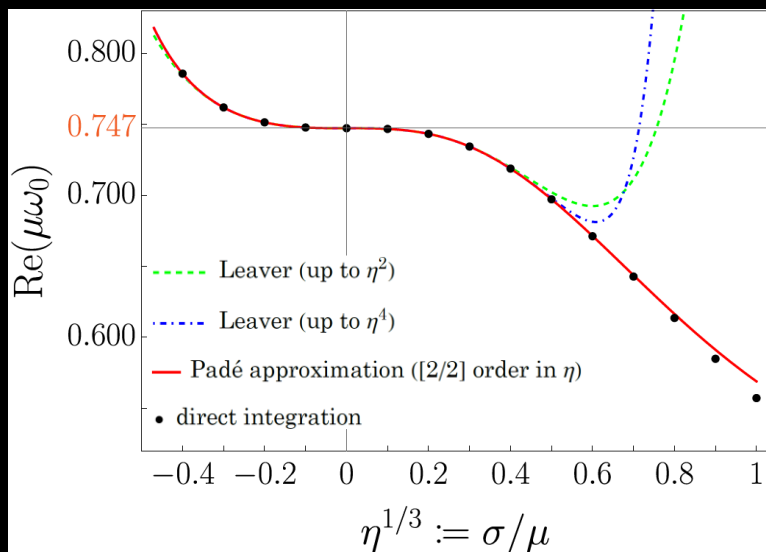
QNM of Hayward BH

[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

- Non-singular BH background

$$A = B = 1 - \frac{\mu r^2}{r^3 + \sigma^3}$$

- Set $p_4 = M_3^2 = 0$ to ensure $c_T^2 = 1$ @ $r \rightarrow \infty$
- Fundamental QNM frequency



- Overtones show more prominent deviations [Konoplya, arxiv: 2310.19205]

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
[arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
 - Generalized Regge-Wheeler equation
[arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
[see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]
 - Quasi-normal modes deviate from GR
[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
 - Static Tidal Love number
[arXiv: 2405.10813 w/C.G.A.Barura, H.Kobayashi, N.Oshita, K.Takahashi, V.Yingcharoenrat]

Tidal Love number of Hayward BH

[arXiv: 2405.10813 w/C.G.A.Barura, H.Kobayashi, N.Oshita, K.Takahashi, V.Yingcharoenrat]

- TLNs \leftarrow regularity @ horizon $x \equiv r/r_g$

$$\tilde{\psi}(x) = x^{\ell+1} [1 + \mathcal{O}(x^{-1})] + K_\ell(\eta) x^{-\ell} [1 + \mathcal{O}(x^{-1})]$$

- Analytic continuation of multipole index ℓ
 \rightarrow Separation of growing & decaying sols.

- Expansion w.r.t. η
 $\eta \equiv \sigma^3 / r_g^3$

$$K_\ell(\eta) = \sum_{k \geq 0} \eta^k K_\ell^{(k)}$$

- Static tidal Love numbers are non-vanishing

$$K_{\ell=2} = \frac{7}{20} \eta^2 - \frac{11}{20} \eta^3 + \frac{2}{5} \eta^4 + \dots$$

$$K_{\ell=3} = \frac{5}{42} \eta + \frac{1417}{504} \eta^2 - \frac{1285}{1008} \eta^3 + \frac{3713}{4032} \eta^4 + \dots$$

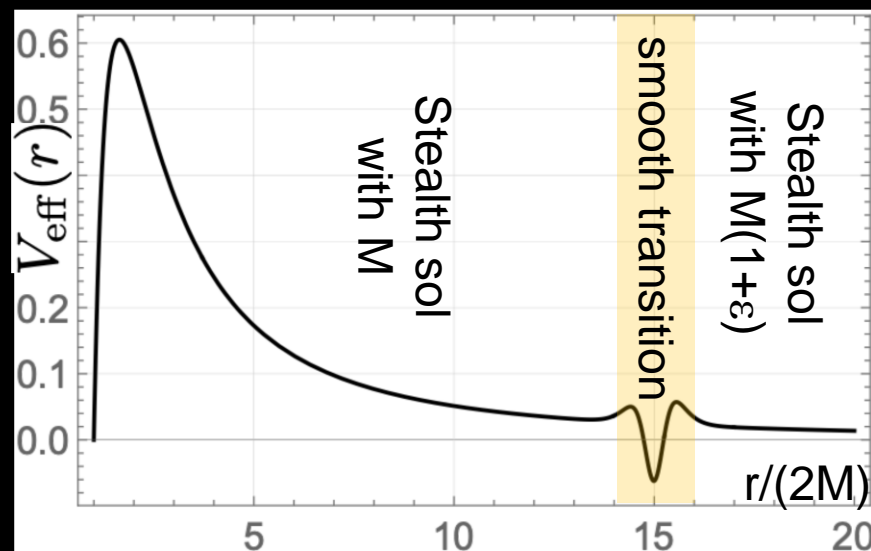
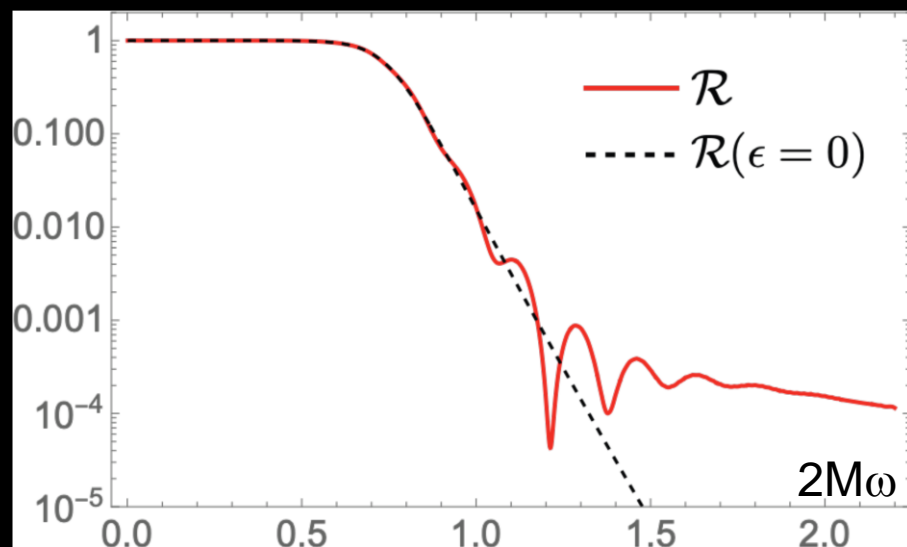
$$K_{\ell=4} = \frac{23}{840} \eta + \left(\frac{110051}{50400} - \frac{24}{25} \log x \right) \eta^2 + \dots \quad \text{logarithmic running}$$

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[arXiv: 2406.04525 w/N.Oshita and K.Takahashi]

(In)stability of greybody factor

[arXiv: 2406.04525 w/N.Oshita and K.Takahashi]



$$\mathcal{R}(\omega) := \left| \frac{A_{\text{out}}}{A_{\text{in}}} \right|^2 = 1 - \Gamma(\omega)$$

Reflectivity Greybody factor

$$\psi_{\text{in}} = \begin{cases} e^{-i\omega r_*} & \text{for } r_* \rightarrow -\infty, \\ A_{\text{out}}(\omega)e^{i\omega r_*} + A_{\text{in}}(\omega)e^{-i\omega r_*} & \text{for } r_* \rightarrow \infty, \end{cases}$$

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- Even-parity perturbation around spherical BH
[work in progress w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
- Rotating BH
[work in progress w/ N.Oshita & K.Takahashi & Z.Wang & V.Yingcharoenrat]

SUMMARY

- Majorities of inflation/DE models are described by scalar-tensor gravity with timelike scalar profile.
- Ghost condensation universally describes all scalar-tensor theories of gravity with timelike scalar profile on Minkowski background respecting time translation / reflection symmetry (up to shift / reflection of the scalar).
- Extension of ghost condensation to FLRW backgrounds results in the EFT of inflation/DE.
- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.

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- **EFT of scalar-tensor gravity with timelike scalar profile on arbitrary background** was developed. Consistency relations among EFT coefficients ensure the spatial diffeo invariance. **Applicable to BHs with scalar field DE.**

EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT on Minkowski background

= ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

EFT on cosmological background

= EFT of inflation/dark energy

Creminelli, Luty, Nicolis and Senatore 2006; Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007; Gubitosi, Piazza, Vernizzi 2012; Gleyzes, Langlois, Piazza, Vernizzi 2013


EFT on arbitrary background

= EFT of BH perturbations

Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

Taylor expansion of the general action $S = \int d^4x \sqrt{-g} F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$

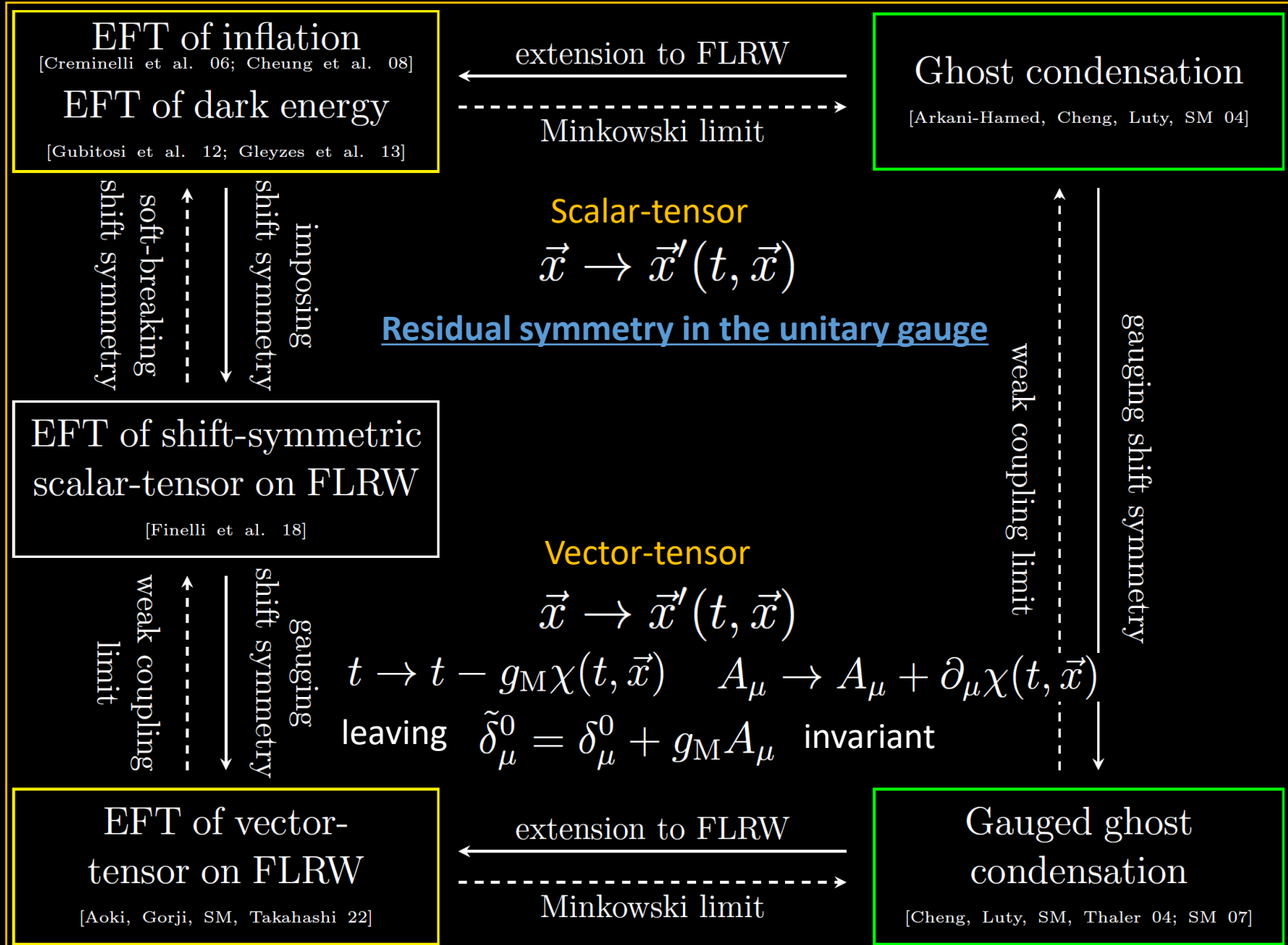
$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \dots \right]$$

Consistency relations  S is invariant under spatial diffeo but the background breaks it.

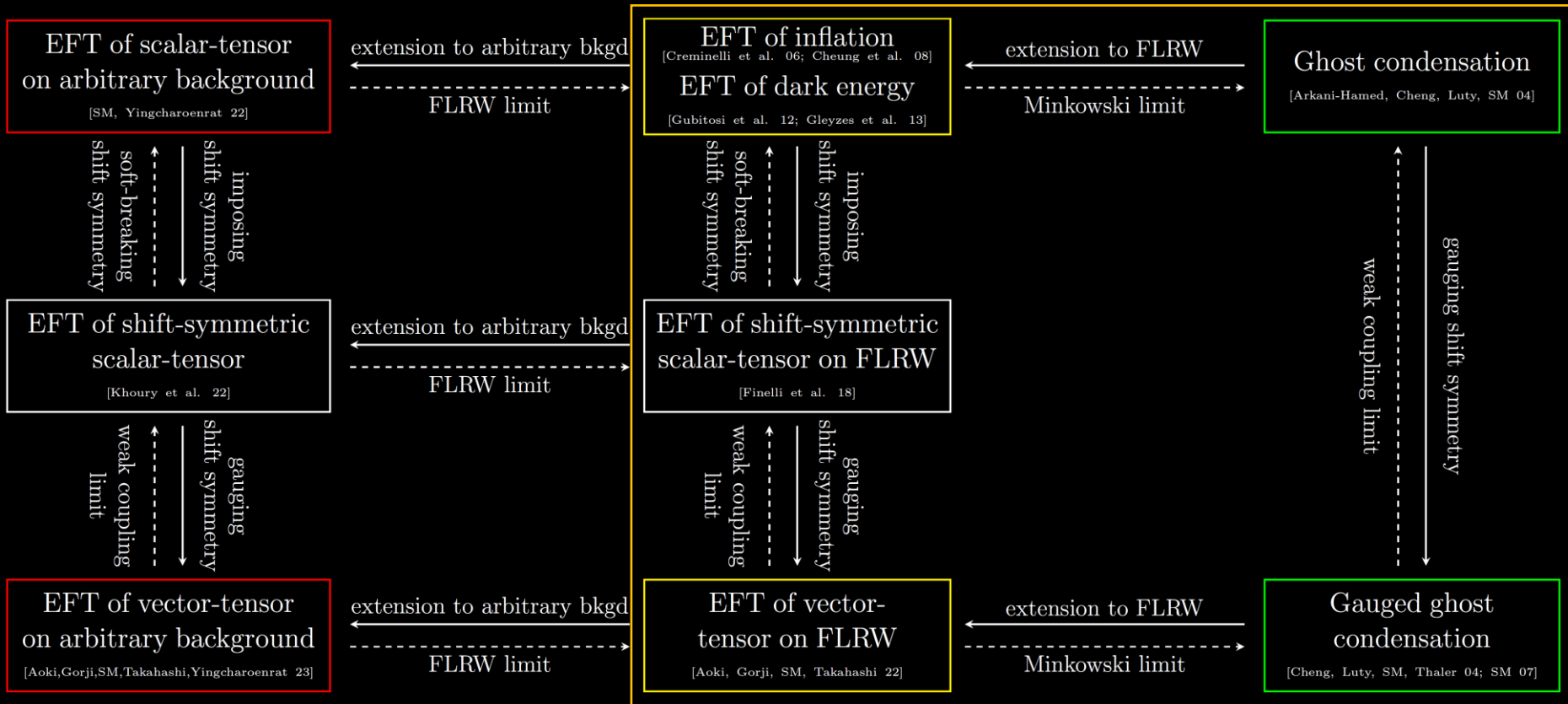
$$\frac{d}{dx^i} \bar{F} = \bar{F}_{g^{\tau\tau}} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \frac{\partial \bar{K}}{\partial x^i} + \dots$$

- Majorities of inflation/DE models are described by scalar-tensor gravity with timelike scalar profile.
- Ghost condensation universally describes all scalar-tensor theories of gravity with timelike scalar profile on Minkowski background respecting time translation / reflection symmetry (up to shift / reflection of the scalar).
- Extension of ghost condensation to FLRW backgrounds results in the EFT of inflation/DE.
- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.
- **EFT of scalar-tensor gravity with timelike scalar profile on arbitrary background** was developed. Consistency relations among EFT coefficients ensure the spatial diffeo invariance. **Applicable to BHs with scalar field DE.**
- **Any other applications? Let's discuss!**

Further extension of the web of EFTs



Further extension of the web of EFTs



Residual symmetry in the unitary gauge

Scalar-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

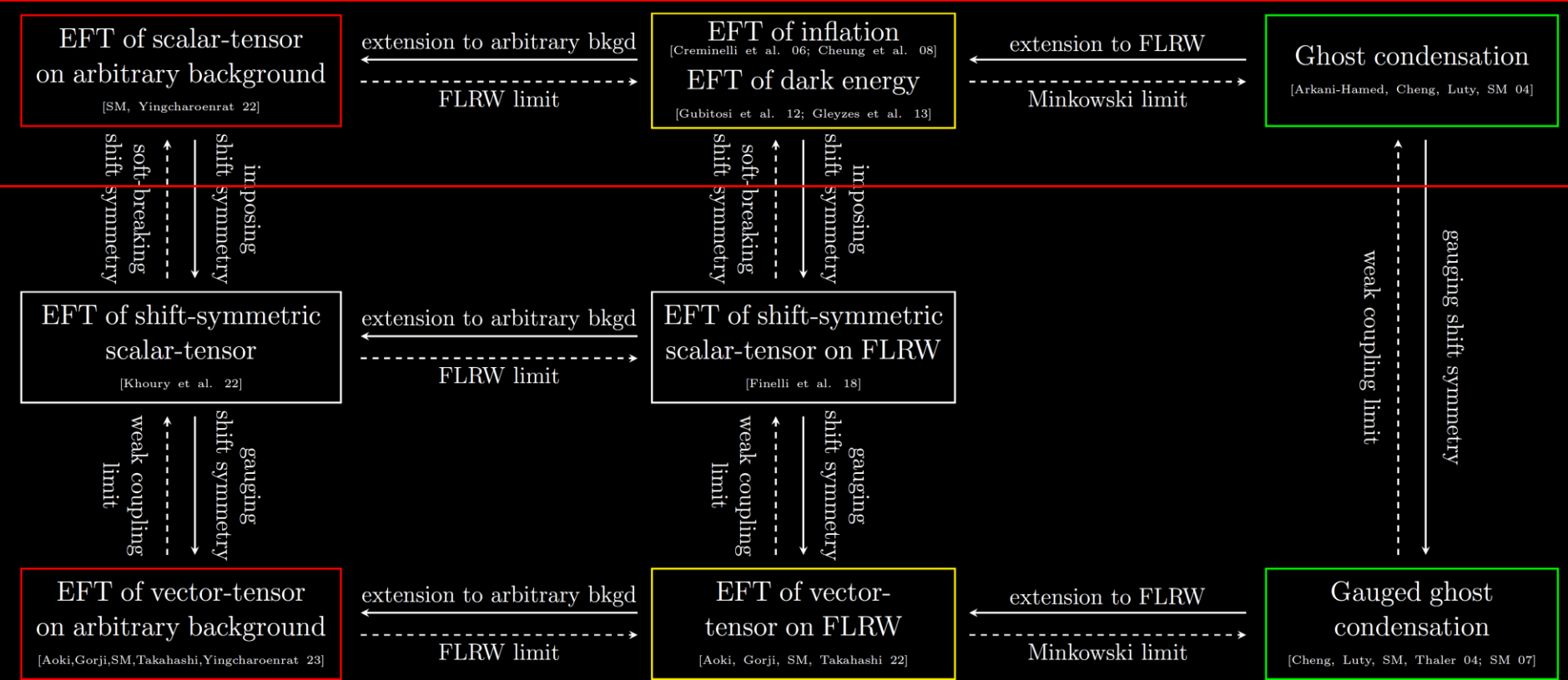
Vector-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

$$t \rightarrow t - g_M \chi(t, \vec{x}) \quad A_\mu \rightarrow A_\mu + \partial_\mu \chi(t, \vec{x})$$

$$\text{leaving } \tilde{\delta}_\mu^0 = \delta_\mu^0 + g_M A_\mu \text{ invariant}$$

Further extension of the web of EFTs



Residual symmetry in the unitary gauge

Scalar-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

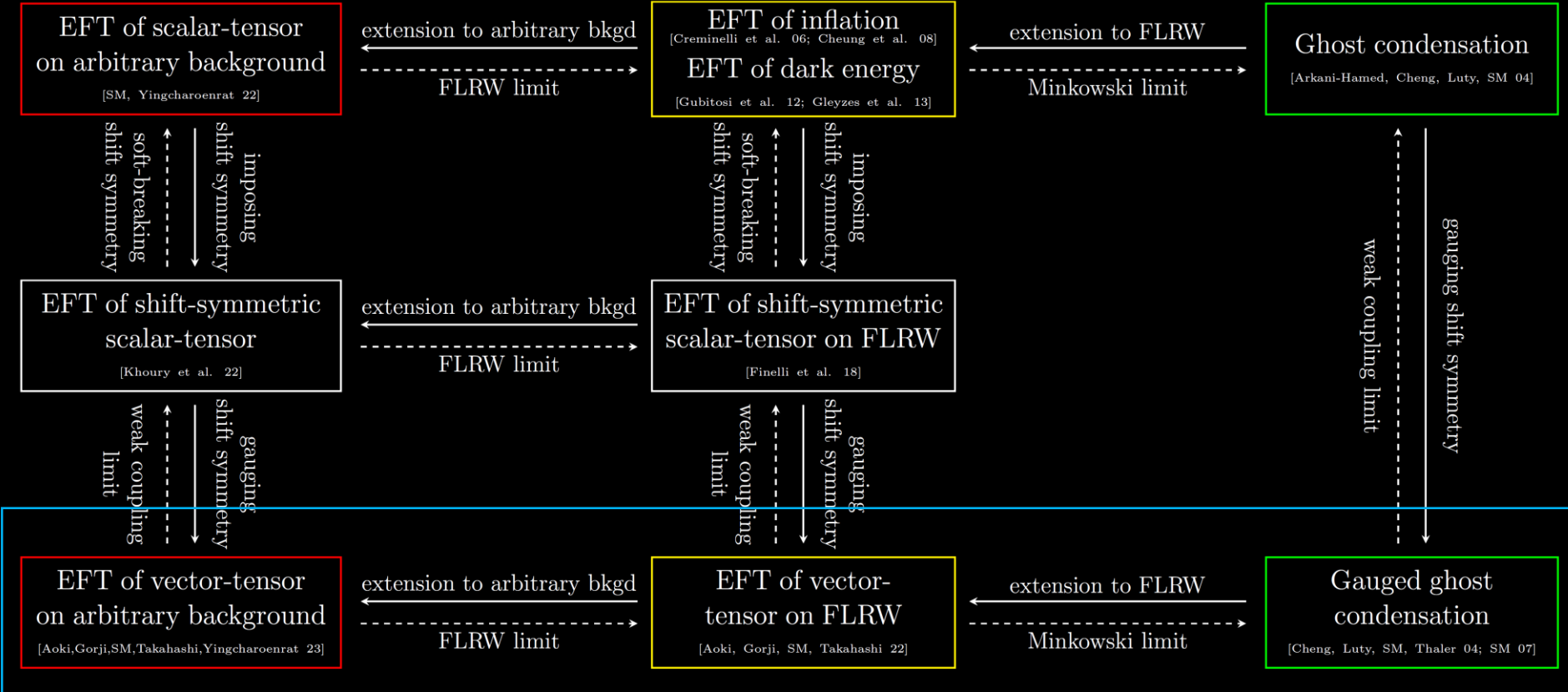
Vector-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

$$t \rightarrow t - g_M \chi(t, \vec{x}) \quad A_\mu \rightarrow A_\mu + \partial_\mu \chi(t, \vec{x})$$

$$\text{leaving } \tilde{\delta}_\mu^0 = \delta_\mu^0 + g_M A_\mu \text{ invariant}$$

Further extension of the web of EFTs



Residual symmetry in the unitary gauge

Scalar-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

See also "CMB spectrum in unified EFT of dark energy: scalar-tensor and vector-tensor theories", arXiv: 2405.04265

Vector-tensor

$$\vec{x} \rightarrow \vec{x}'(t, \vec{x})$$

$$t \rightarrow t - g_M \chi(t, \vec{x}) \quad A_\mu \rightarrow A_\mu + \partial_\mu \chi(t, \vec{x})$$

leaving $\tilde{\delta}_\mu^0 = \delta_\mu^0 + g_M A_\mu$ invariant

Thank you!



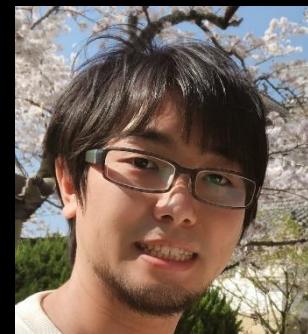
V. Yingcharoenrat



K. Takahashi



K. Tomikawa



K. Aoki



E. Serraille



M. A. Gorji



C. G. A. Barura



H. Kobayashi



N. Oshita

- Ref.
- arXiv: 2204.00228 w/ V. Yingcharoenrat
 - arXiv: 2208.02943 w/ K. Takahashi, V. Yingcharoenrat
 - arXiv: 2304.14304 w/ K. Takahashi, K. Tomikawa, V. Yingcharoenrat
 - arXiv: 2405.10813 w/ C. G. A. Barura, H. Kobayashi, N. Oshita, K. Takahashi, V. Yingcharoenrat
 - arXiv: 2406.04525 w/ N. Oshita and K. Takahashi
 - arXiv: 2407.xxxxx w/ E. Serraille, K. Takahashi, V. Yingcharoenrat
 - arXiv: 2111.08119 w/ K. Aoki, M. A. Gorji, K. Takahashi
 - arXiv: 2311.06767 w/ K. Aoki, M. A. Gorji, K. Takahashi, V. Yingcharoenrat

scalar-tensor

vector-tensor

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Backup slides

Stealth solutions in k-essence

Mukohyama 2005

- Action in Einstein frame

$$I = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X) \right] \quad X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

- EOMs $\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} P'(X) g^{\mu\nu} \partial_\nu \phi) = 0$

$$M_{\text{Pl}}^2 G_{\mu\nu} = 2P'(X) \partial_\mu \phi \partial_\nu \phi + P(X) g_{\mu\nu}$$

- **Stealth sol with $X = X_0$, where $P'(X_0) = 0$**

$$G_{\mu\nu} = \Lambda_{\text{eff}} g_{\mu\nu} \quad \Lambda_{\text{eff}} = P(X_0) / M_{\text{Pl}}^2$$

- $X = X_0 (\neq 0)$

↔ $u^\mu = g^{\mu\nu} \partial_\nu \phi$ defines geodesic congruence
($u^\nu \nabla_\nu u^\mu = -\nabla^\mu X / 2 = 0$)

↔ $\phi / \sqrt{|X_0|}$ defines Gaussian normal coord.

Stealth solutions in k-essence

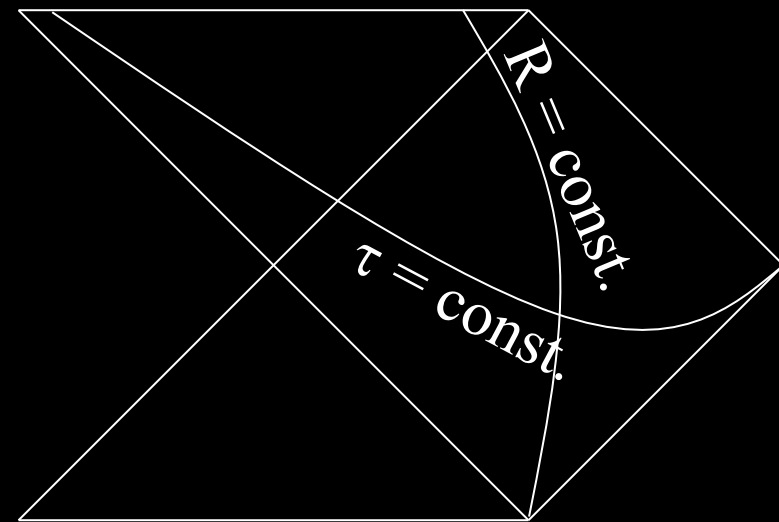
Mukohyama 2005

- Any metric locally admits Gaussian normal coord.
- If $P'(X)$ has a real root X_0 then any vacuum GR sol with $\Lambda_{\text{eff}} = P(X_0)/M_{\text{Pl}}^2$ locally leads to a stealth sol.
- **Schwarzschild metric admits a “globally” well-behaved Gaussian normal coord.** (Lemaitre reference frame)

$$g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau, R)} + r^2(\tau, R) d\Omega^2$$

$$r(\tau, R) = \left[\frac{3}{2} \sqrt{r_g} (R - \tau) \right]^{2/3}$$

- **Stealth Schwarzschild** solution with $\phi = \sqrt{X_0} \tau$, if $P'(X)$ has a positive root X_0 and if Λ_{eff} is canceled by Λ_{bare}



Stealth solutions with $\phi = qt + \psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) **Schwarzschild-dS in DHOST** (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- **Kerr-dS in DHOST** (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, **perturbations around most of those stealth solutions are infinitely strongly coupled** (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- **Approximately stealth solution in ghost condensate does not suffer from strong coupling** (Mukohyama 2005).
Why?

Origin of strong coupling

- EFT around stealth Minkowski sol. (= ghost condensate) \rightarrow universal dispersion relation without the usual k^2 term

$$\omega^2 = \alpha k^4 / M^2$$

- For $\alpha = O(1)$ (>0), EFT is weakly coupled all the way up to $\sim M$. [$E_{\text{cubic}} \simeq |\alpha|^{7/2} M$]
- If eom's for perturbations are strictly 2nd order (as in DHOST) then $\alpha = 0$ and the dispersion relation loses dependence on k
 \rightarrow strong coupling
- [For $\omega^2 = c_s^2 k^2$, strong coupling @ $E \sim c_s^{7/4} M$]

Strong coupling scales

- EFT of inflation in decoupling limit

$$S_\pi = M_{\text{Pl}}^2 \int dt d^3 \vec{x} a^3 \left[-\frac{\dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \dot{H} \left(\frac{1}{c_s^2} - 1 \right) \left(\frac{c_3}{c_s^2} \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) + \mathcal{O}(\pi^4, \tilde{c}^2) + \mathcal{L}_{\tilde{\delta}K, \tilde{\delta}R}^{(2)} \right]$$

$$\frac{1}{c_s^2} = 1 + \frac{4\lambda_1}{-\dot{H}}, \quad c_3 = c_s^2 - \frac{8c_s^2 \lambda_2}{-\dot{H}} \left(\frac{1}{c_s^2} - 1 \right)^{-1}$$

- If $c_s^2 \simeq \text{const}$ is not too small, $\mathcal{L}_{\tilde{\delta}K, \tilde{\delta}R}^{(2)}$ can be ignored. We further assume $0 < c_s < 1$.

$$S_\pi = \int dt d^3 \vec{x} a^3 (c_s \epsilon M_{\text{Pl}}^2 H^2) \left[\dot{\pi}^2 - \frac{(\tilde{\partial}_i \pi)^2}{a^2} + \left(\frac{1}{c_s^2} - 1 \right) \dot{\pi} \left(c_3 \dot{\pi}^2 - \frac{(\tilde{\partial}_i \pi)^2}{a^2} \right) + \dots \right]$$

$$\vec{x} = c_s \vec{\tilde{x}}$$

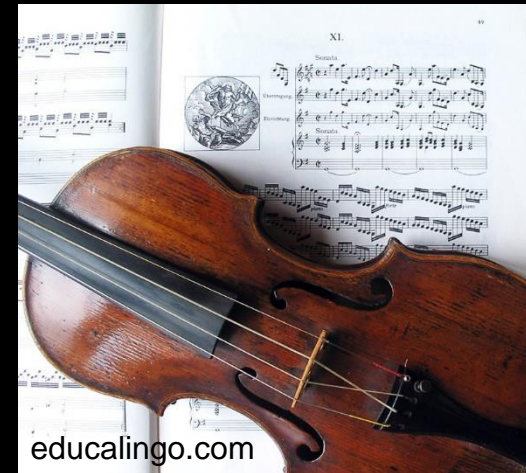
$$\dot{\pi}^2 \sim \frac{(\tilde{\partial}_i \pi)^2}{a^2} \sim \frac{E^4}{c_s \epsilon M_{\text{Pl}}^2 H^2} \left(\frac{1}{c_s^2} - 1 \right) |\dot{\pi}| \Big|_{E=E_{\text{cubic}}} \sim \frac{1}{\max[|c_3|, 1]}$$

$$\rightarrow E_{\text{cubic}} \lesssim \frac{(c_s^5 \epsilon M_{\text{Pl}}^2 H^2)^{1/4}}{\sqrt{1 - c_s^2}} \rightarrow 0 \quad (c_s^5 \epsilon / (1 - c_s^2)^2 \rightarrow 0)$$

A solution: scordatura

Motohashi & Mukohyama 2019

- Detuning of degeneracy condition recovers $\omega^2 = \alpha k^4 / M^2$ and uplifts the strong coupling scale to $\sim |\alpha|^{7/2} M$. If the amount of detuning is at most of $O(1)$ then an apparent ghost is heavy enough to be integrated out.
- Scordatura = weak and controlled detuning of degeneracy condition
- Scordatura DHOST realizes ghost condensation near stealth solutions while it behaves as DHOST away from them.



Strong coupling scales

- De Sitter limit = small c_s^2 limit

$$S_\pi = M_{\text{Pl}}^2 \int dt d^3 \vec{x} a^3 \left[4\lambda_1 \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) + 4(\lambda_1 - 2\lambda_2) \dot{\pi}^3 \right. \\ \left. + \lambda_3 \left(H - \frac{\partial_j^2 \pi}{a^2} \right) \frac{(\partial_i \pi)^2}{a^2} + (\lambda_4 + \lambda_5) \frac{(\partial_i^2 \pi)^2}{a^4} + \dots \right]$$

$$\lambda_1 = \frac{M^4}{8M_{\text{Pl}}^2}, \quad \lambda_3 = \frac{M^3 \beta}{2M_{\text{Pl}}^2}, \quad \lambda_4 = -\frac{M^2(\alpha + \gamma)}{2M_{\text{Pl}}^2}, \quad \lambda_5 = \frac{M^2 \gamma}{2M_{\text{Pl}}^2}$$

$$S_\pi = \frac{M^4}{2} \int dt d^3 \vec{x} a^3 \left[\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} - \frac{\alpha}{M^2} \frac{(\partial_i^2 \pi)^2}{a^4} + \frac{\beta}{M} \left(H - \frac{\partial_j^2 \pi}{a^2} \right) \frac{(\partial_i \pi)^2}{a^2} + \dots \right]$$

$$E^{-1} p^{-3} M^4 (E\pi)^2 \sim 1 \quad \Rightarrow \quad \pi \sim \frac{E^{3/2}}{p^{1/2} M^2}$$

$$\left. \frac{E\pi p^2}{E^2} \right|_{E=E_{\text{cubic}}} \sim 1 \quad \Rightarrow \quad \left(\frac{p}{E} \right)^{7/4} \frac{E}{M} \Big|_{E=E_{\text{cubic}}} \sim 1$$

$$\frac{\omega^2}{M^2} = \alpha \frac{k^4}{M^4 a^4} \quad \text{for} \quad \max \left[c_s^2, \left| \beta \right| \frac{H}{M} \right] \ll \left| \alpha \right| \frac{k^2}{M^2 a^2} \ll 1$$

$$\Rightarrow \quad E_{\text{cubic}} \simeq |\alpha|^{7/2} M$$

Stealth solutions with $\phi = qt + \psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzschild-dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019). This means the solutions cannot be trusted.
- Fortunately, Scordatura (= detuning of degeneracy condition) solves the strong coupling problem (Motohashi & Mukohyama 2019), if and only if the scalar profile is timelike.
- EFT of ghost condensation already includes scordatura (Arkani-Hamed & Cheng & Luty & Mukohyama 2004)
- Approximate Schwarzschild in ghost condensation (Mukohyama 2005). Also in quadratic HHOST (DeFelice & Mukohyama & Takahashi, JCAP 03 (2023) 050).

Approximately stealth BH in ghost condensate

Mukohyama 2005

- Two time scales: $t_{\text{BH}} \ll t_{\text{GC}} \sim M_{\text{Pl}}^2/M^3$
- For $t_{\text{BH}} \ll t \ll t_{\text{GC}}$, a usual BH sol is a good approximation \rightarrow approximately stealth

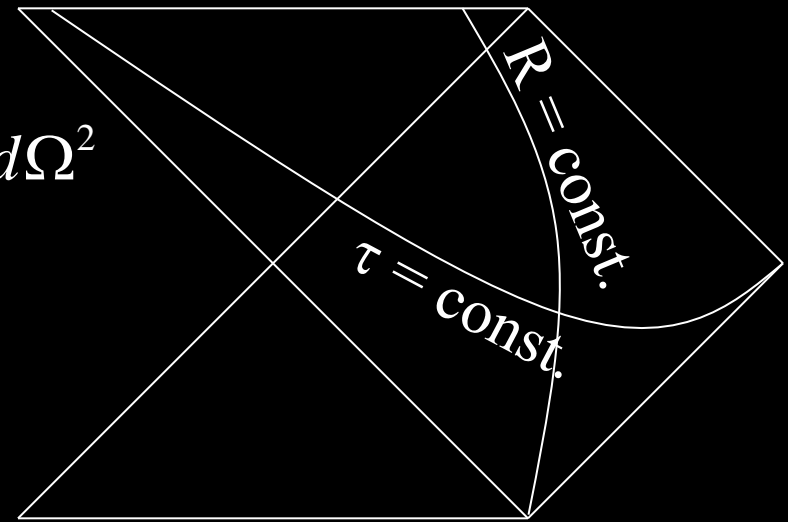
Schwarzschild metric:

$$g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau, R)} + r^2(\tau, R) d\Omega^2$$

$$r(\tau, R) = \left[\frac{3}{2} \sqrt{r_g} (R - \tau) \right]^{2/3}$$

$$E = -\xi^\mu p_\mu \quad \xi^\mu = \partial_\tau + \partial_R$$

$\phi = M^2 \tau \rightarrow$ Exact sol in the absence of higher derivative terms



Approximately stealth BH in ghost condensate

Mukohyama 2005; Cheng, Luty, Mukohyama and Thaler 2006

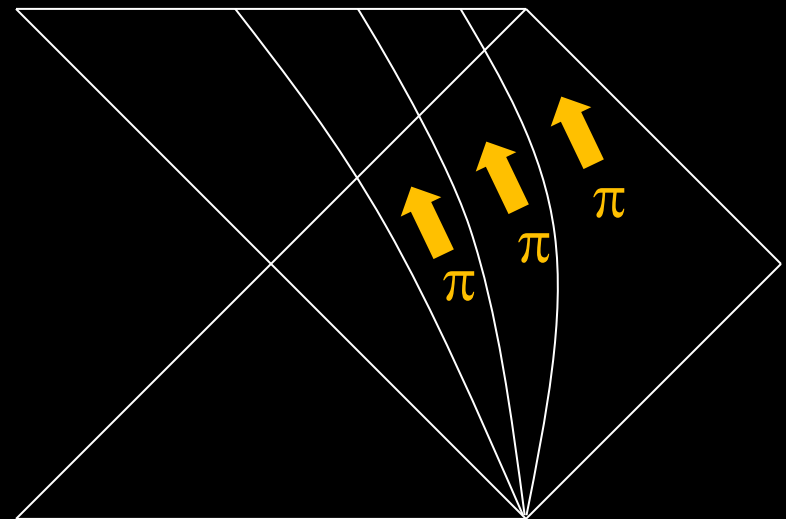
- A tiny tadpole due to higher derivative terms is canceled by extremely slow time-dependence.
- As a result, $\pi = \delta\phi$ starts accreting gradually.
- XTE J1118+480 ($M_{bh} \sim 7M_{sun}$, $r \sim 3R_{sun}$, $t \sim 240\text{Myr}$ or 7 Gyr) $\longrightarrow M < 10^{12}\text{GeV}$ much weaker than $M < 100\text{GeV}$

$$M_{bh} = M_{bh0} \times \left[1 + \frac{9\alpha M^2}{4M_{Pl}^2} \left(\frac{3M_{Pl}^2 v}{4M_{bh0}} \right)^{2/3} \right]$$

v : advanced null coordinate

α : coefficient of h.d. term

See DeFelice, Mukohyama, Takahashi, arXiv: 2212.13031 for a similar formula in more general HOST.



Summary of stealth BH with timelike scalar profile

- Stealth solutions = backgrounds with GR metric and non-trivial scalar profile → examples of BH solutions with timelike scalar profile
- They suffer from strong coupling problem, which is solved by scordatura (= controlled detuning of degeneracy condition)
- DHOST/Horndeski do not include scordatura but U-DHOST does (DeFelice, Mukohyama, Takahashi 2022) .
- EFT of ghost condensation already included scordatura.
- Approximately stealth solutions in ghost condensation (Mukohyama 2005) and in more general HOST with scordatura (DeFelice & Mukohyama & Takahashi, arXiv: 2212.13031) are stealth at astrophysical scales (no need for screening?, c.f. arXiv:1402.4737 by Davis, Gregory, Jha & Muir) and are free from the strong coupling problem.

- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to learn something about the EFT of DE by BHs.
- This would require **the scalar field profile to be timelike near BH**. Otherwise, the two EFTs, one for DE and the other for BH, can be unrelated to each other (unless a UV completion is specified).

EFT of scalar-tensor gravity on arbitrary background with timelike scalar profile

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
[arXiv: 2204.00228 w/ V.Yingcharoenrat]

Background analysis

- Spherically symmetric, static background

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

- Lemaitre coordinates

$$ds^2 = -d\tau^2 + [1 - A(r)]d\rho^2 + r^2 d\Omega^2$$

- Shift and Z_2 symmetries

$$\Phi \rightarrow \Phi + \text{const.}$$

$$\Phi \rightarrow -\Phi$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} R - \Lambda(r) - c(r)g^{\tau\tau} - \tilde{\beta}(r)K - \alpha(r)\bar{K}_\nu^\mu K_\mu^\nu - \zeta(r)n^\mu \partial_\mu g^{\tau\tau} \right. \\ \left. + \frac{1}{2}m_2^4(r)(\delta g^{\tau\tau})^2 + \frac{1}{2}\tilde{M}_1^3(r)\delta g^{\tau\tau}\delta K + \frac{1}{2}M_2^2(r)\delta K^2 + \frac{1}{2}M_3^2(r)\delta K_\nu^\mu \delta K_\mu^\nu \right. \\ \left. + \frac{1}{2}\mu_1^2(r)\delta g^{\tau\tau}\delta^{(3)}R + \frac{1}{2}\lambda_1(r)_\nu^\mu \delta g^{\tau\tau}\delta K_\mu^\nu + \frac{1}{2}\mathcal{M}_1^2(r)(\bar{n}^\mu \partial_\mu \delta g^{\tau\tau})^2 \right. \\ \left. + \frac{1}{2}\mathcal{M}_2^2(r)\delta K(\bar{n}^\mu \partial_\mu \delta g^{\tau\tau}) + \frac{1}{2}\mathcal{M}_3^2(r)\bar{h}^{\mu\nu}\partial_\mu \delta g^{\tau\tau}\partial_\nu \delta g^{\tau\tau} \right]$$

- Tadpole cancellation condition

$$\Lambda - c = M_{\star}^2 (\bar{G}^{\tau}_{\rho} - \bar{G}^{\rho}_{\rho}),$$

$$\Lambda + c + \frac{2}{r^2} \sqrt{\frac{B}{A}} \left(r^2 \sqrt{1-A} \zeta \right)' = -M_{\star}^2 \bar{G}^{\tau}_{\tau},$$

$$\left[\partial_{\rho} \bar{K} + \frac{1-A}{r} \left(\frac{B}{A} \right)' \right] \alpha + \frac{A'B}{2A} \alpha' + \sqrt{\frac{B(1-A)}{A}} \tilde{\beta}' = -M_{\star}^2 \bar{G}^{\tau}_{\rho},$$

$$\frac{1}{2r^2} \sqrt{\frac{B}{A}} \left[r^4 \sqrt{\frac{B}{A}} \left(\frac{1-A}{r^2} \right)' \alpha \right]' = M_{\star}^2 (\bar{G}^{\rho}_{\rho} - \bar{G}^{\theta}_{\theta}),$$

$$\bar{G}^{\tau}_{\tau} = -\frac{[r(1-B)]'}{r^2} + \frac{1-A}{r} \left(\frac{B}{A} \right)', \quad \bar{G}^{\rho}_{\rho} = -\frac{[r(1-B)]'}{r^2} - \frac{1}{r} \left(\frac{B}{A} \right)',$$

$$\bar{G}^{\tau}_{\rho} = -\frac{1-A}{r} \left(\frac{B}{A} \right)', \quad \bar{G}^{\theta}_{\theta} = \frac{B(r^2 A)'}{2r^2 A} + \frac{(r^2 A)'}{4r^2} \left(\frac{B}{A} \right)'.$$

Applications to BHs with timelike scalar profile

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[arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
[see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]

Odd-parity perturbations

- General odd-parity perturbations

$$\delta g_{\tau\tau} = \delta g_{\tau\rho} = \delta g_{\rho\rho} = 0 ,$$

$$\delta g_{\tau a} = \sum_{\ell,m} r^2 h_{0,\ell m}(\tau, \rho) E_a{}^b \bar{\nabla}_b Y_{\ell m}(\theta, \phi) ,$$

$$\delta g_{\rho a} = \sum_{\ell,m} r^2 h_{1,\ell m}(\tau, \rho) E_a{}^b \bar{\nabla}_b Y_{\ell m}(\theta, \phi) ,$$

$$\delta g_{ab} = \sum_{\ell,m} r^2 h_{2,\ell m}(\tau, \rho) E_{(a|}{}^c \bar{\nabla}_c \bar{\nabla}_{|b)} Y_{\ell m}(\theta, \phi) ,$$

- Gauge fixing ($\ell \geq 2$) $h_2 \rightarrow 0$
- Master variable

$$\chi = \dot{h}_1 - \partial_\rho h_0 - p_4 h_1$$

• Quadratic action $S_2 = \int d\tau d\rho \mathcal{L}_2$

$$\frac{(j^2 - 2)(2\ell + 1)}{2\pi j^2} \mathcal{L}_2 = s_1 \dot{\chi}^2 - s_2 (\partial_\rho \chi)^2 - s_3 \chi^2$$

$$s_1 = \frac{j^2 - 2}{2\sqrt{1 - A}} \frac{(M_\star^2 + M_3^2)^2 r^6}{(j^2 - 2)M_\star^2 + (M_\star^2 + M_3^2)r^2 p_4^2}$$

$$s_2 = \frac{(M_\star^2 + M_3^2)r^6}{2(1 - A)^{3/2}} \quad j^2 \equiv \ell(\ell + 1)$$

$$s_3 = j^2 \frac{(M_\star^2 + M_3^2)r^4}{2\sqrt{1 - A}} + \mathcal{O}(j^0)$$

$$p_4 \equiv \sqrt{\frac{B}{A(1 - A)}} \left(\frac{A'}{2} + \frac{1 - A}{r} \right) \frac{\alpha + M_3^2}{M_\star^2 + M_3^2}$$

- Sound speeds

$$c_\rho^2 = \frac{\bar{g}_{\rho\rho}}{|\bar{g}_{\tau\tau}|} \frac{s_2}{s_1} = \frac{M_\star^2}{M_\star^2 + M_3^2} + \frac{r^2 p_4^2}{j^2 - 2}$$

$$c_\theta^2 = \lim_{\ell \rightarrow \infty} \frac{r^2}{|\bar{g}_{\tau\tau}|} \frac{s_3}{j^2 s_1} = \frac{M_\star^2}{M_\star^2 + M_3^2}$$

- For $p_4=0$, i.e. $\alpha + M_3^2 = 0$

$$c_\rho^2 = c_\theta^2 = \frac{M_\star^2}{M_\star^2 + M_3^2} \equiv c_T^2$$

- Stability $s_1 > 0$, $c_\rho^2 > 0$, $c_\theta^2 > 0$

$$M_\star^2 + M_3^2 > 0, \quad M_\star^2 > 0$$

- Going back to Schwarzschild coordinates

$$\frac{(j^2 - 2)(2\ell + 1)}{2\pi j^2} \mathcal{L}_2 = a_1 (\partial_t \chi)^2 - a_2 (\partial_r \chi)^2 + 2a_3 (\partial_t \chi)(\partial_r \chi) - a_4 \chi^2$$

$$a_1 = \frac{s_1 - (1 - A)^2 s_2}{\sqrt{A^3 B(1 - A)}}, \quad a_2 = \sqrt{\frac{B(1 - A)}{A}} (s_2 - s_1),$$

$$a_3 = \frac{(1 - A)s_2 - s_1}{A}, \quad a_4 = \sqrt{\frac{A}{B(1 - A)}} s_3.$$

- Generalized Regge-Wheeler equation

$$\frac{\partial^2 \Psi}{\partial \tilde{t}^2} - c_{r_*}^2 \frac{\partial^2 \Psi}{\partial r_*^2} + V_{\text{eff}} \Psi = 0 \quad \Psi = \sqrt{\Gamma} \chi$$

$$V_{\text{eff}} \equiv \frac{a_4}{\tilde{a}_1} + \frac{1}{2\sqrt{AB} \tilde{a}_1} \frac{d^2 \Gamma}{dr_*^2} - \frac{1}{4\tilde{a}_1 a_2} \left(\frac{d\Gamma}{dr_*} \right)^2 \quad \Gamma \equiv \frac{a_2}{\sqrt{AB}}$$

$$\tilde{t} = t + \int \frac{a_3}{a_2} dr \quad r_* = \int \frac{1}{\sqrt{AB}} dr \quad \tilde{a}_1 = a_1 + \frac{a_3^2}{a_2}$$

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→ Quasi-normal mode
[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

QNM of stealth Schwarzschild BH

[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

- Background with $2m=1$

$$A(r) = B(r) = 1 - 1/r \quad ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

- Set $p_4 = 0$ to make c_T^2 finite @ $r \rightarrow \infty$
- Generalized Regge-Wheeler potential

$$V_{\text{eff}}(r) = (1 + \alpha_T) f(r) \left[\frac{\ell(\ell + 1)}{r^2} - \frac{3r_g}{r^3} \right] \quad f(r) = 1 - r_g/r$$

$$\alpha_T \equiv c_T^2 - 1 = \alpha / (M_\star^2 - \alpha) \quad r_g \equiv r_H / (1 + \alpha_T)$$

- QNM frequency

$$\omega = \omega_{\text{GR}} (1 + \alpha_T)^{3/2}$$
$$\rightarrow \omega_{\text{GR}} \quad (c_T^2 \rightarrow 1)$$

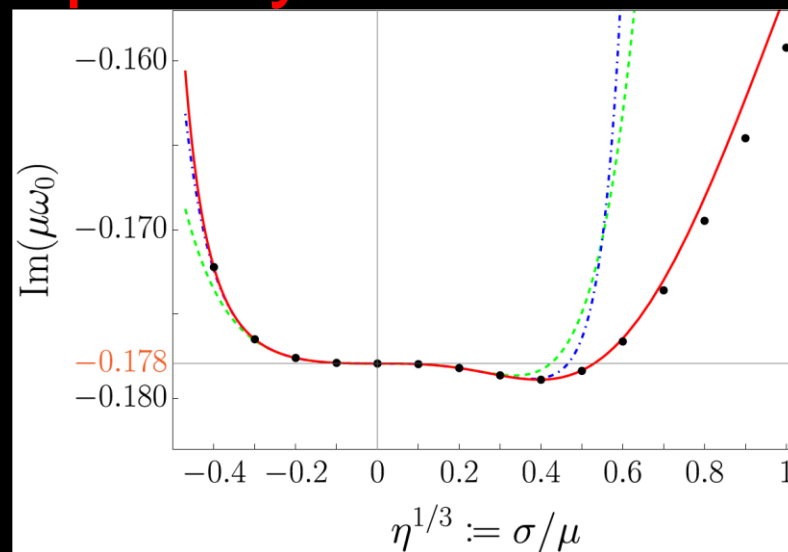
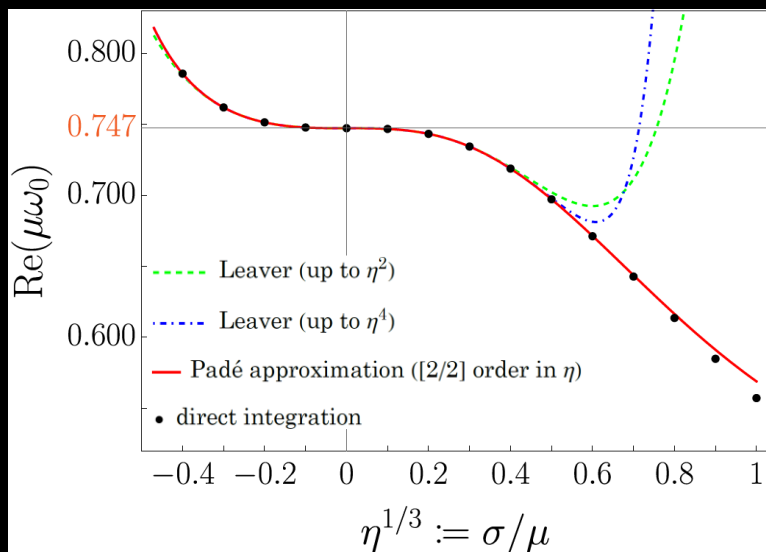
QNM of Hayward BH

[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

- Non-singular BH background

$$A = B = 1 - \frac{\mu r^2}{r^3 + \sigma^3}$$

- Set $p_4 = M_3^2 = 0$ to ensure $c_T^2 = 1 @ r \rightarrow \infty$
- Fundamental QNM frequency



- Overtones show more prominent deviations [Konoplya, arxiv: 2310.19205]

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 - Quasi-normal modes deviate from GR
[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
 - Static Tidal Love number
[arXiv: 2405.10813 w/C.G.A.Barura, H.Kobayashi, N.Oshita, K.Takahashi, V.Yingcharoenrat]

Tidal Love number of Hayward BH

[arXiv: 2405.10813 w/C.G.A.Barura, H.Kobayashi, N.Oshita, K.Takahashi, V.Yingcharoenrat]

- TLNs \leftarrow regularity @ horizon $x \equiv r/r_g$

$$\tilde{\psi}(x) = x^{\ell+1} [1 + \mathcal{O}(x^{-1})] + K_\ell(\eta) x^{-\ell} [1 + \mathcal{O}(x^{-1})]$$

- Analytic continuation of multipole index ℓ
 \rightarrow Separation of growing & decaying sols.

- Expansion w.r.t. η
 $\eta \equiv \sigma^3 / r_g^3$

$$K_\ell(\eta) = \sum_{k \geq 0} \eta^k K_\ell^{(k)}$$

- Static tidal Love numbers are non-vanishing

$$K_{\ell=2} = \frac{7}{20} \eta^2 - \frac{11}{20} \eta^3 + \frac{2}{5} \eta^4 + \dots$$

$$K_{\ell=3} = \frac{5}{42} \eta + \frac{1417}{504} \eta^2 - \frac{1285}{1008} \eta^3 + \frac{3713}{4032} \eta^4 + \dots$$

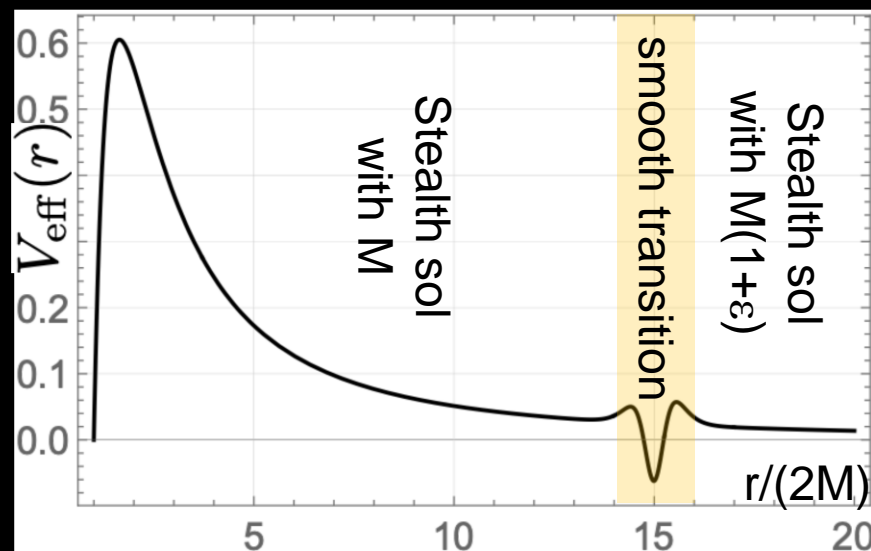
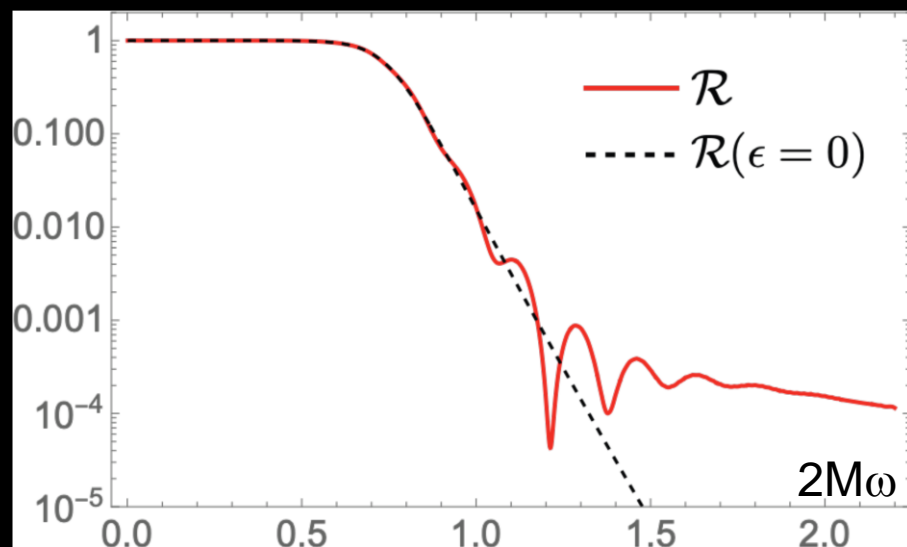
$$K_{\ell=4} = \frac{23}{840} \eta + \left(\frac{110051}{50400} - \frac{24}{25} \log x \right) \eta^2 + \dots \quad \text{logarithmic running}$$

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[arXiv: 2406.04525 w/N.Oshita and K.Takahashi]

(In)stability of greybody factor

[arXiv: 2406.04525 w/N.Oshita and K.Takahashi]



$$\mathcal{R}(\omega) := \left| \frac{A_{\text{out}}}{A_{\text{in}}} \right|^2 = 1 - \Gamma(\omega)$$

Reflectivity Greybody factor

$$\psi_{\text{in}} = \begin{cases} e^{-i\omega r_*} & \text{for } r_* \rightarrow -\infty, \\ A_{\text{out}}(\omega)e^{i\omega r_*} + A_{\text{in}}(\omega)e^{-i\omega r_*} & \text{for } r_* \rightarrow \infty, \end{cases}$$

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- Even-parity perturbation around spherical BH
[work in progress w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
- Rotating BH
[work in progress w/ N.Oshita & K.Takahashi & Z.Wang & V.Yingcharoenrat]

Extension of EFT of inflation to arbitrary background = EFT of BH perturbations

Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

- We call it EFT of BH perturbations simply because we applied it to BH in the presence of DE.
- Can be applied to any background as far as the scalar profile is timelike.

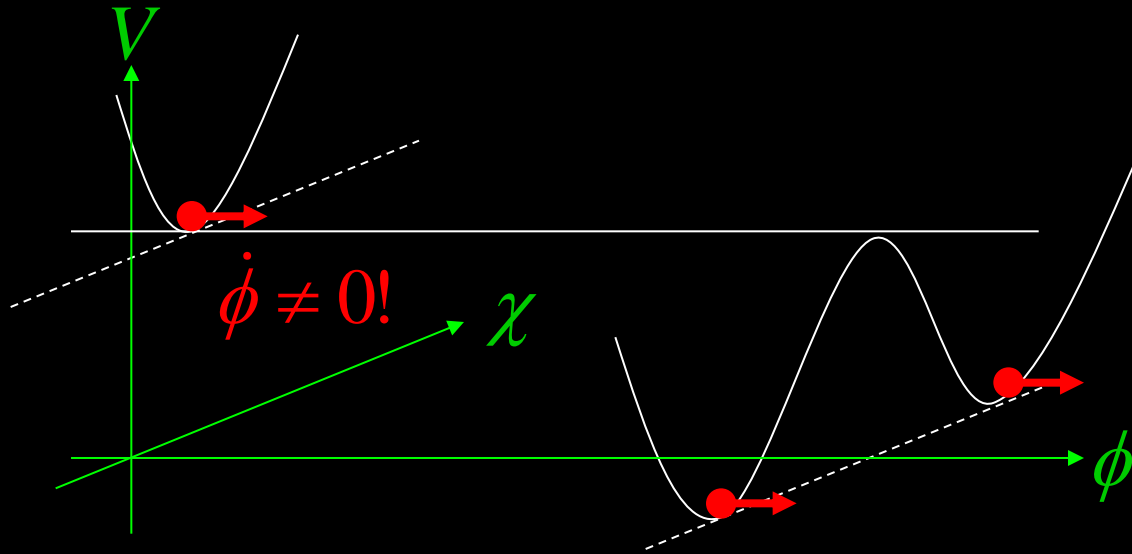
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- Can be applied to e.g. astrophysics after inflation with ever rolling inflaton, such as ghost inflation.

Ghost inflation

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga 2004



Similar to hybrid inflation but **NOT SLOW ROLL**

Scale-invariant perturbations

cf. tilted ghost inflation, Senatore (2004)

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\pi}{\dot{\phi}} \sim \left(\frac{H}{M}\right)^{5/4}$$

$$\delta\pi \sim M \cdot (H/M)^{1/4} \quad \dot{\phi} \sim M^2$$

[compare $\frac{H}{M_{Pl}\sqrt{\epsilon}}$]

scaling dim of π



Prediction of Large non-Gauss.

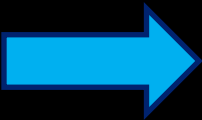
Leading non-linear interaction $\beta \frac{\dot{\pi}(\nabla \pi)^2}{M^2}$

non-G of $\sim \beta \left(\frac{H}{M}\right)^{1/4}$ ← scaling dim of op.
 $\sim \beta \left(\frac{\delta\rho}{\rho}\right)^{1/5}$

$$\int dt d^3x \left[\frac{1}{2} \dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2 \pi)^2}{M^2} + \dots \right]$$

[Really “0.1” $\times (\delta\rho/\rho)^{1/5} \sim 10^{-2}$. **VISIBLE.**

In usual inflation, non-G $\sim (\delta\rho/\rho) \sim 10^{-5}$ too small.]

 $f_{\text{NL}} \sim 82 \beta \alpha^{-4/5}$, equilateral type

Planck 2018 constraint (equilateral type)

$$f_{\text{NL}} = -26 \pm 47 \text{ (68\% CL statistical)} \Rightarrow -0.89 \leq \beta \alpha^{-4/5} \leq 0.26$$

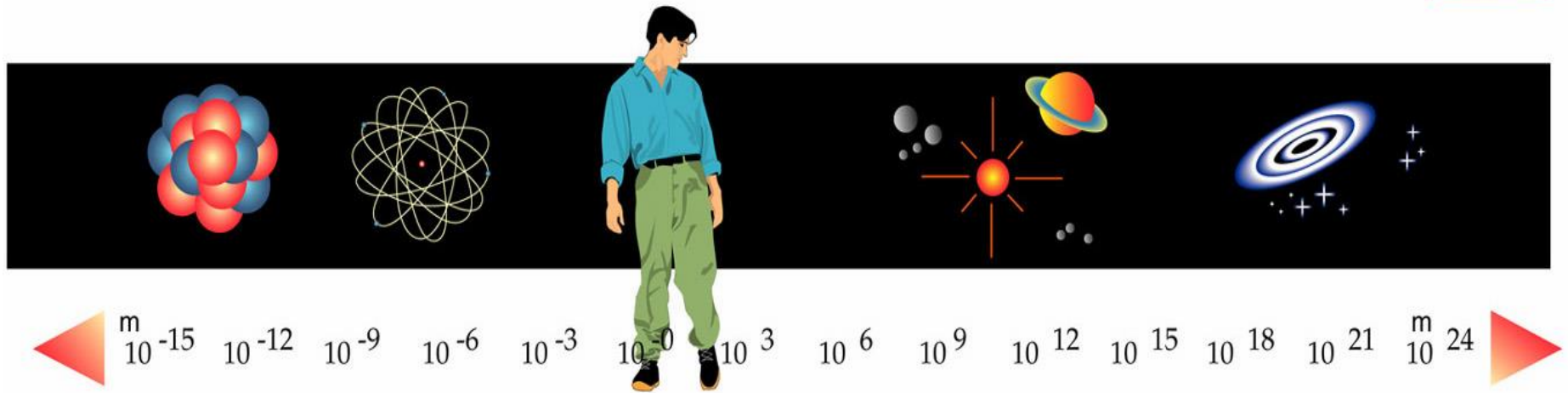
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Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

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- Can be applied to any background as far as the scalar profile is timelike.
- Can be applied to e.g. astrophysics after inflation with ever rolling inflaton, such as ghost inflation.
- Any other applications? Depending on them, we may have to change the name... Let's discuss!

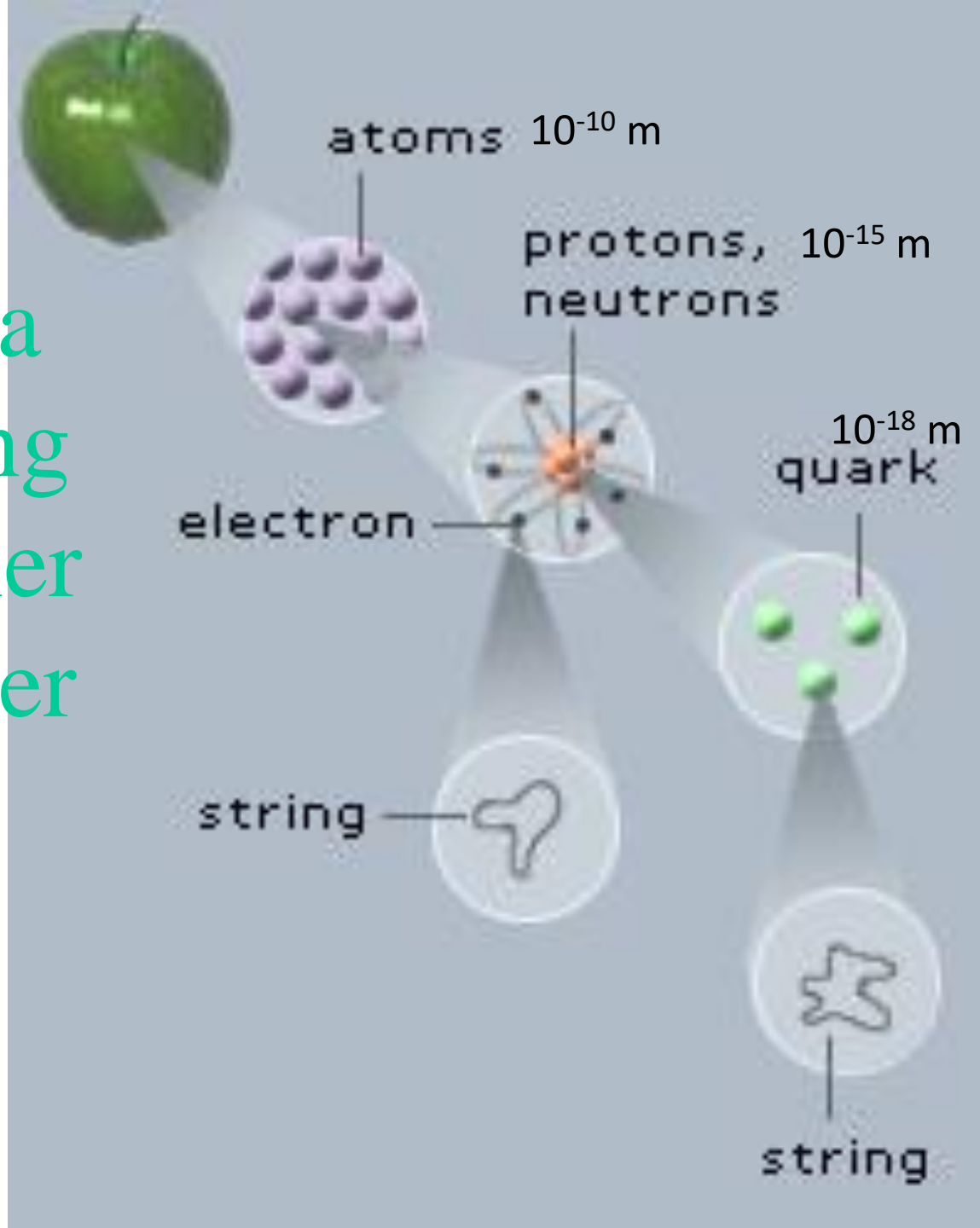
More backup slides

There are Frontiers in Physics:



at Short and Long Scales

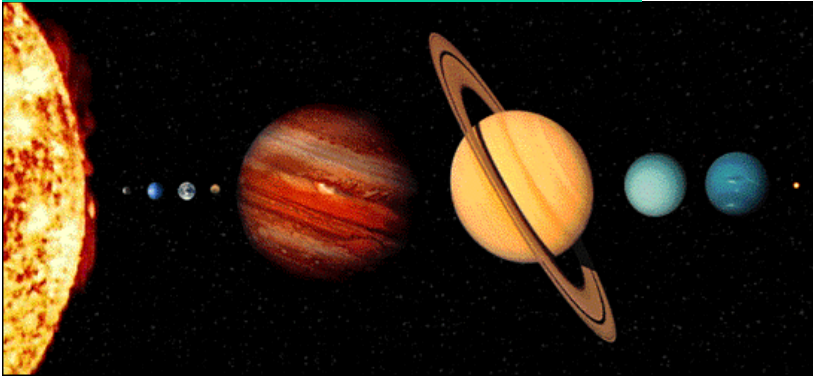
There is a story going into smaller and smaller scales.



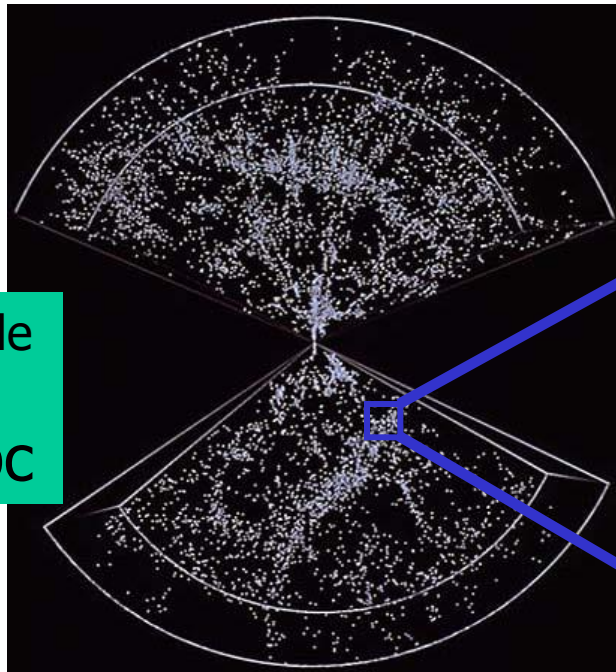
Also at Large scales

(pc = 3.3 light year = 3.1×10^{18} cm)

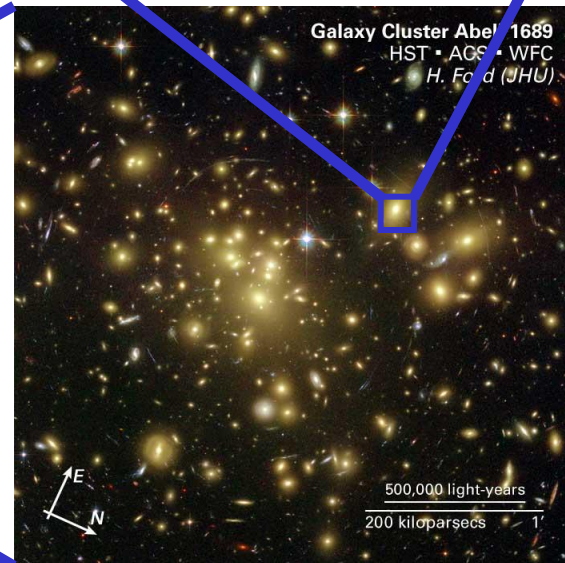
Solar system 10^{15} cm



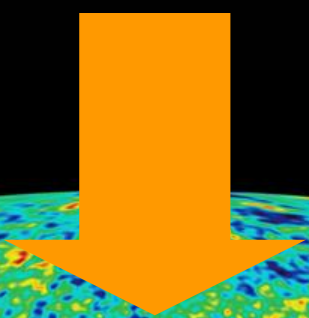
Galaxy
10 kpc



Large scale
structure
100 Mpc



Cluster
of
galaxies
Mpc



Physics @ largest scale

COSMOLOGY

History of Our Universe

Dark Energy
Accelerated Expansion

Cosmic microwave
background

Development of
Galaxies, Planets, etc.

Inflation

Dark energy

WMAP

Quantum
Fluctuations

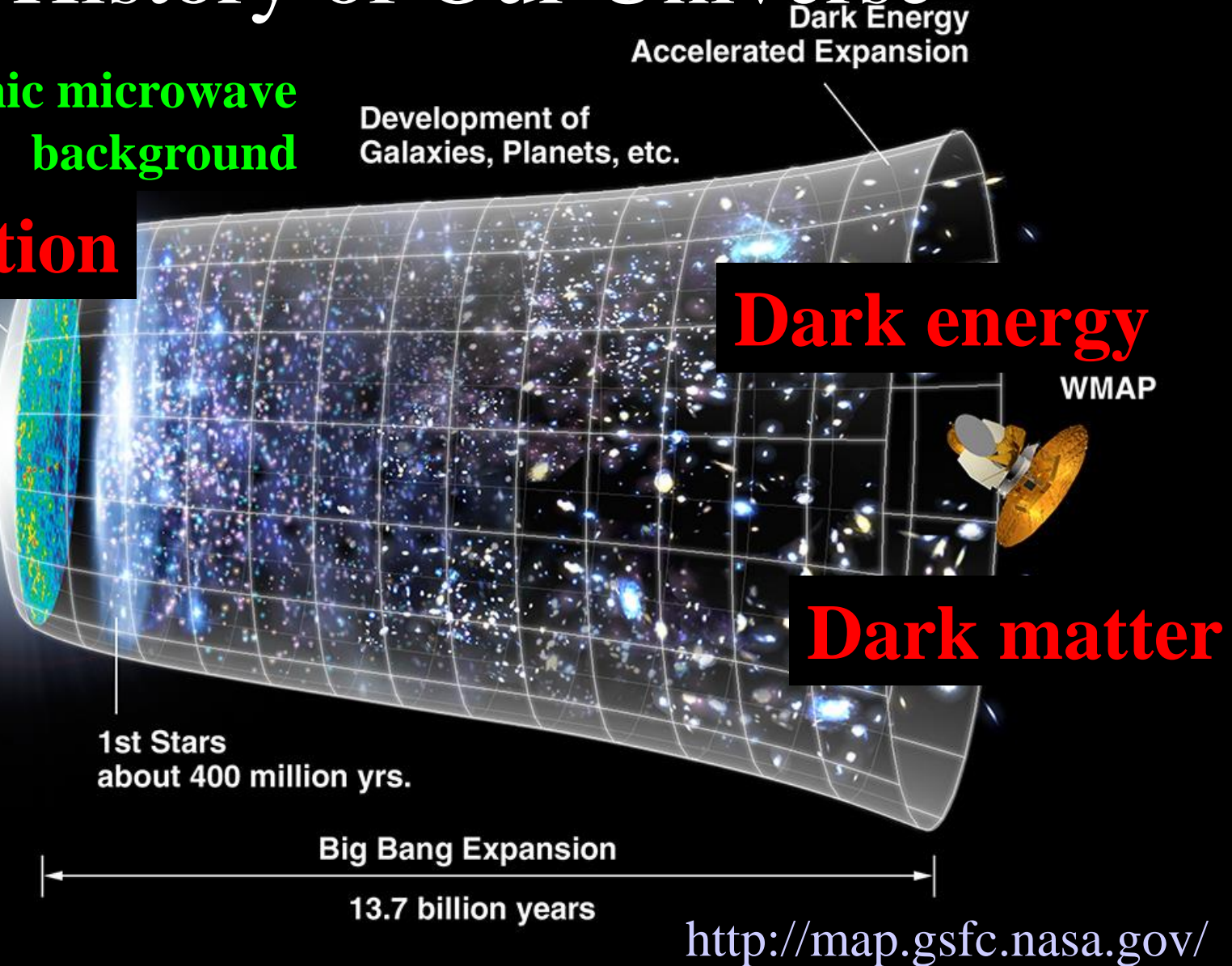
Dark matter

1st Stars
about 400 million yrs.

Big Bang Expansion

13.7 billion years

<http://map.gsfc.nasa.gov/>



History of Our Universe

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Accelerated Expansion

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**3 major mysteries
in modern
cosmology**

Dark matter

Quantum
Fluctuations

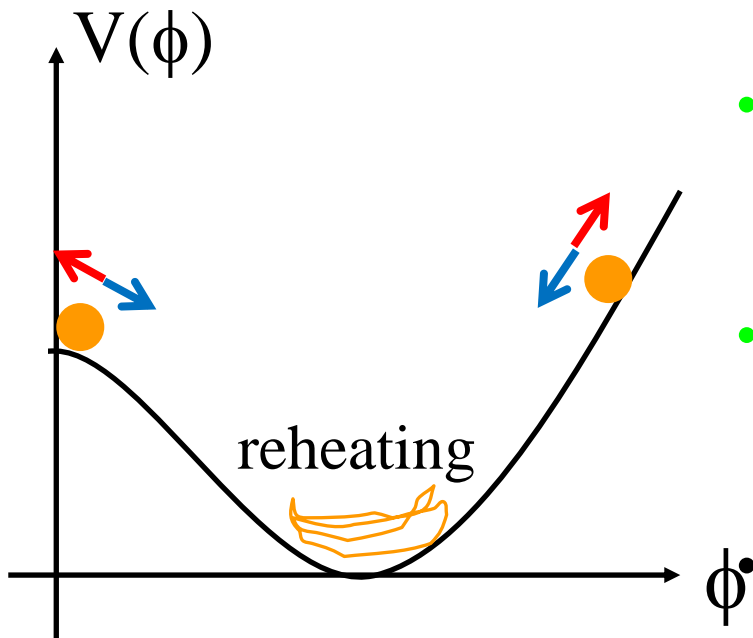
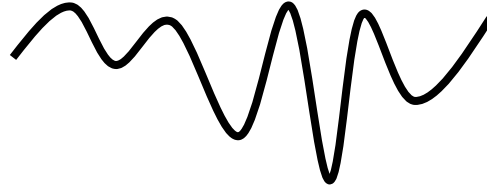
We (almost) know they are/were there...
But, we don't know what they are.

<http://map.gsfc.nasa.gov/>

Two phases of the accelerated expansion of the universe

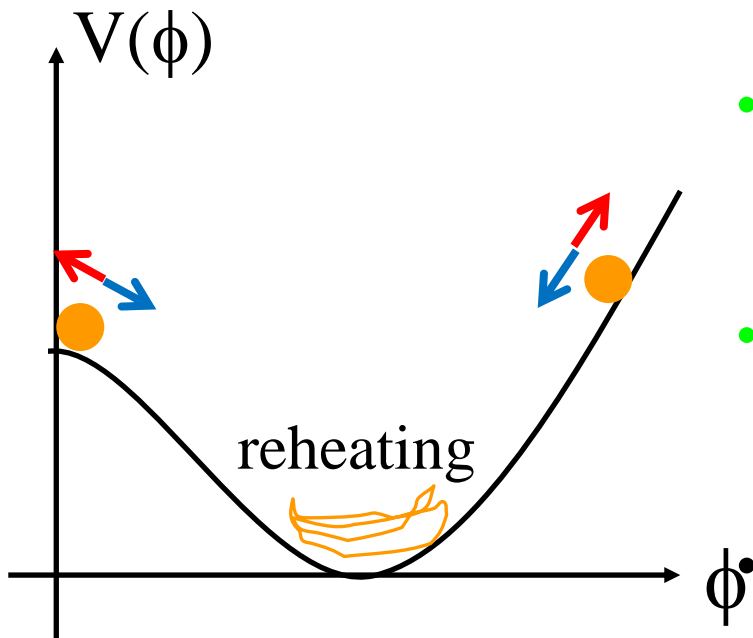
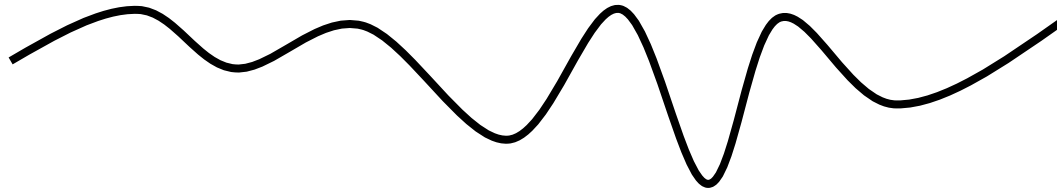
- **Inflation** in the **early universe**
- Accelerated expansion of the **late-time universe** driven by **dark energy**

Inflation generates tiny inhomogeneities



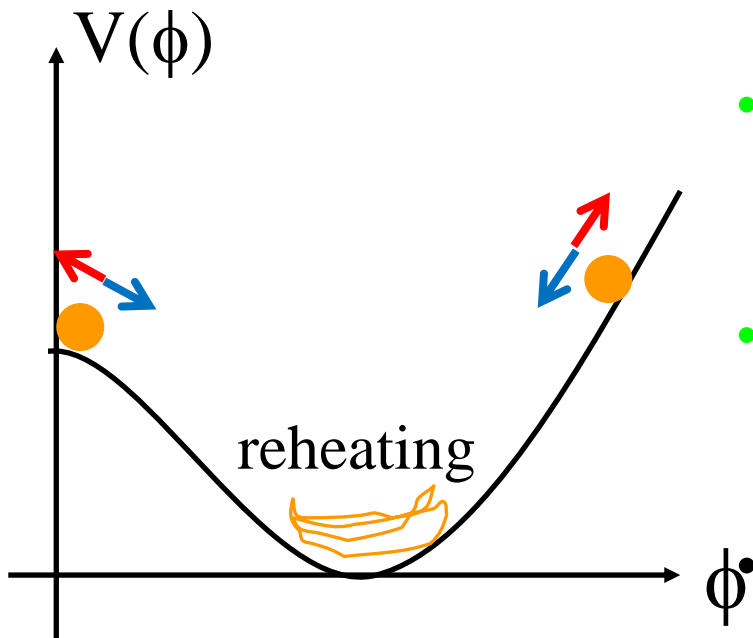
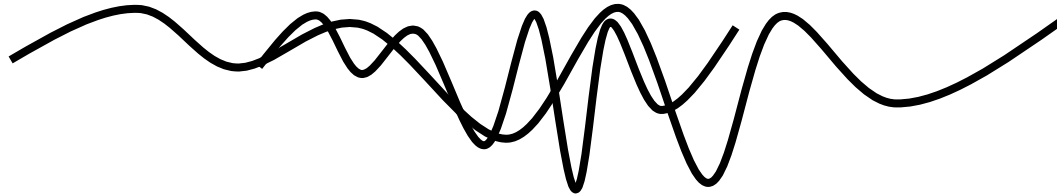
- Quantum effects become important in the early universe
 - **Quantum mechanically**, the inflaton ϕ (alarm clock) moves **forward** or **backward** slightly due to fluctuations
 - **Exponential expansion** stretches microscopic fluctuations to macroscopic lengths
- If inflation ends a little **earlier** (or **later**) than the surrounding area, the energy density will be **lower** (**higher**) than the surrounding area.

Inflation generates tiny inhomogeneities



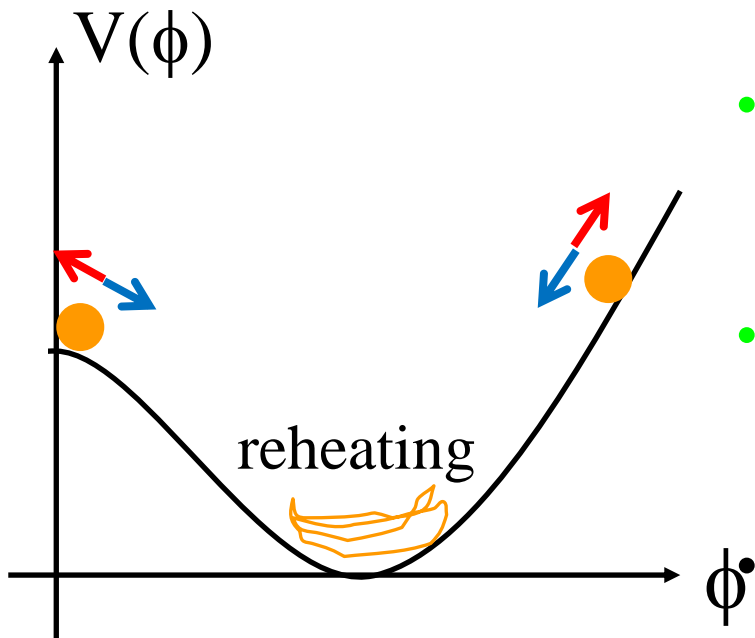
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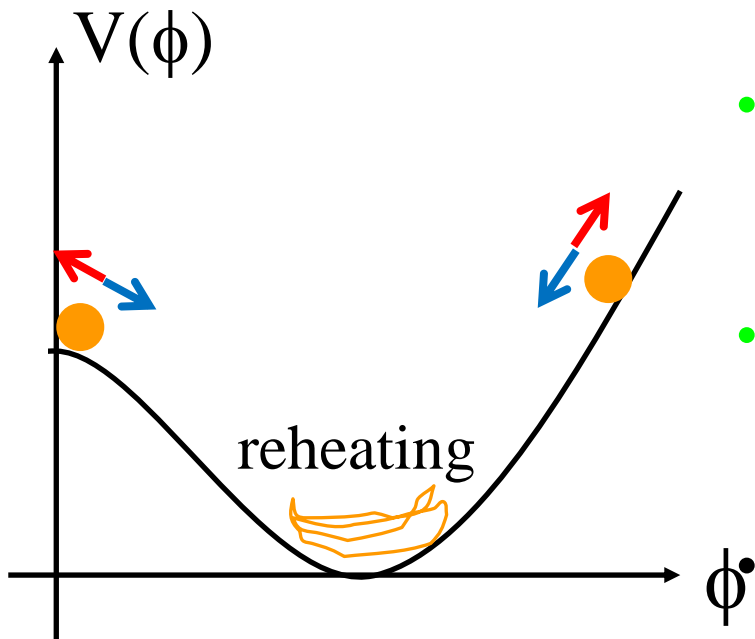
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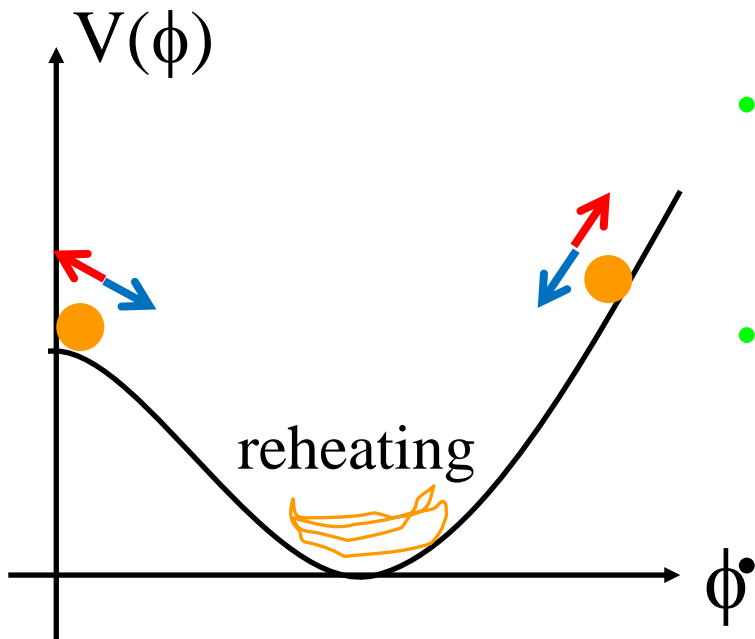
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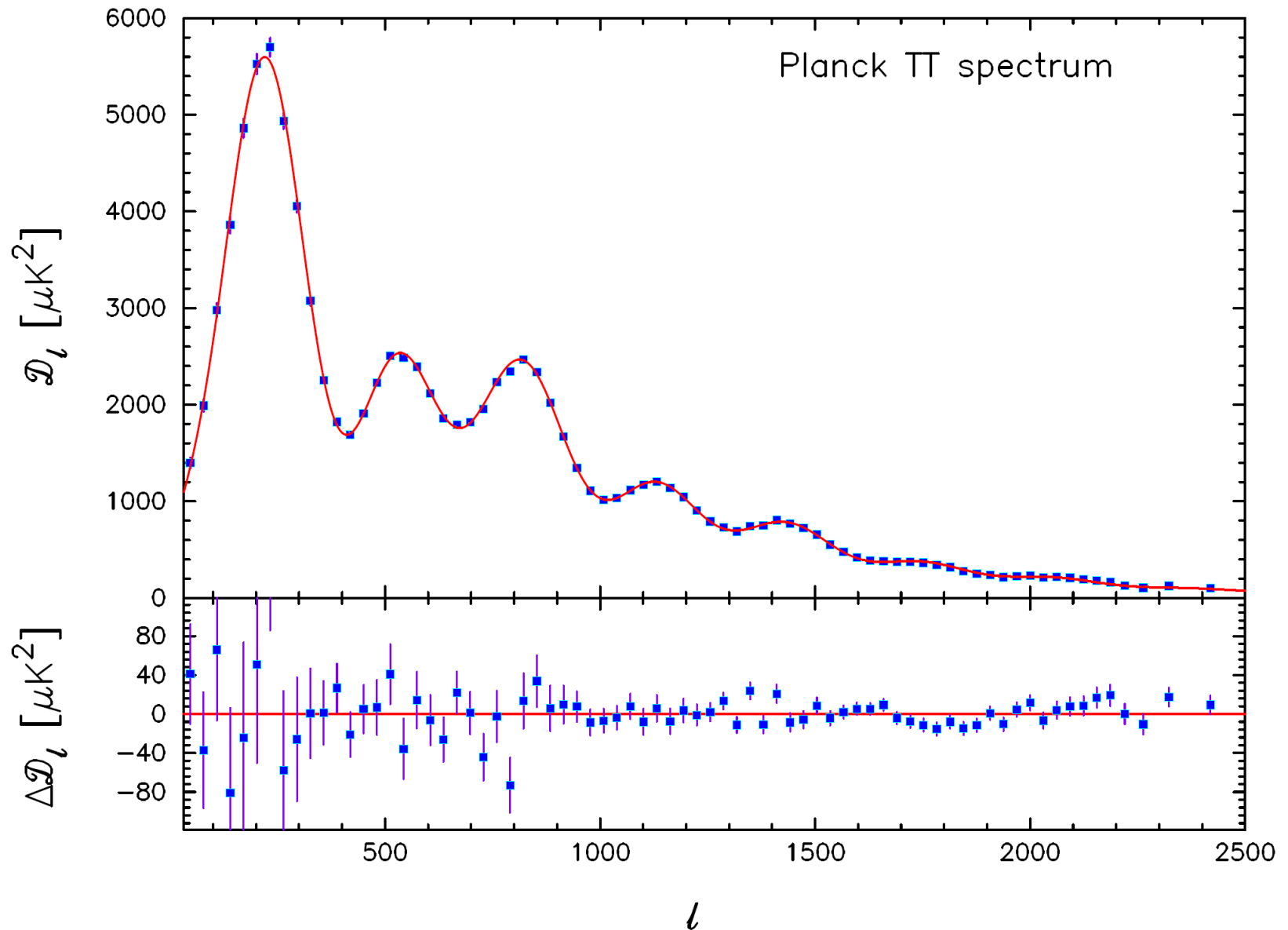
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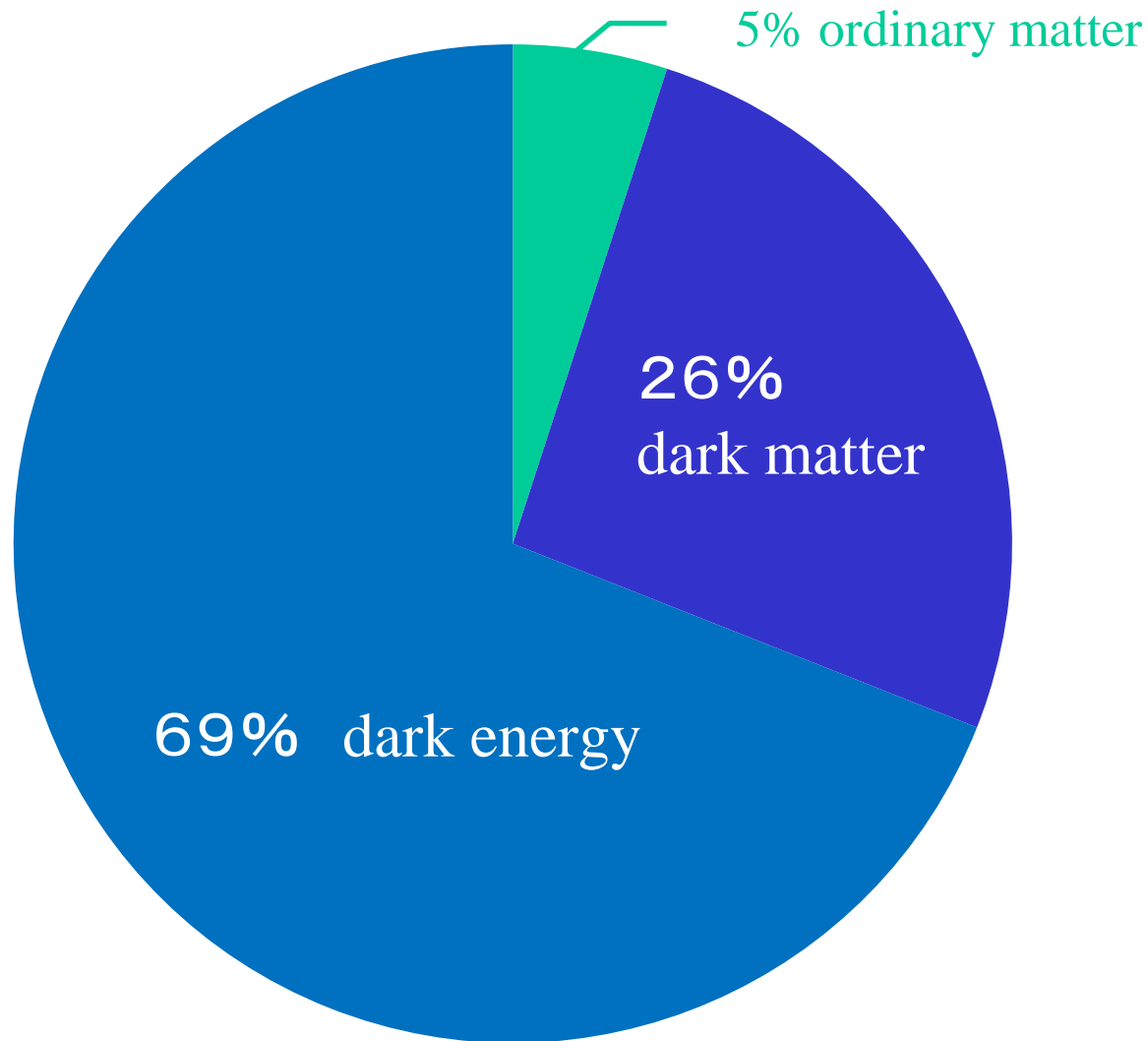
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- If inflation ends a little **earlier** (or **later**) than the surrounding area, the energy density will be **lower** (**higher**) than the surrounding area.

Perfect match with observation

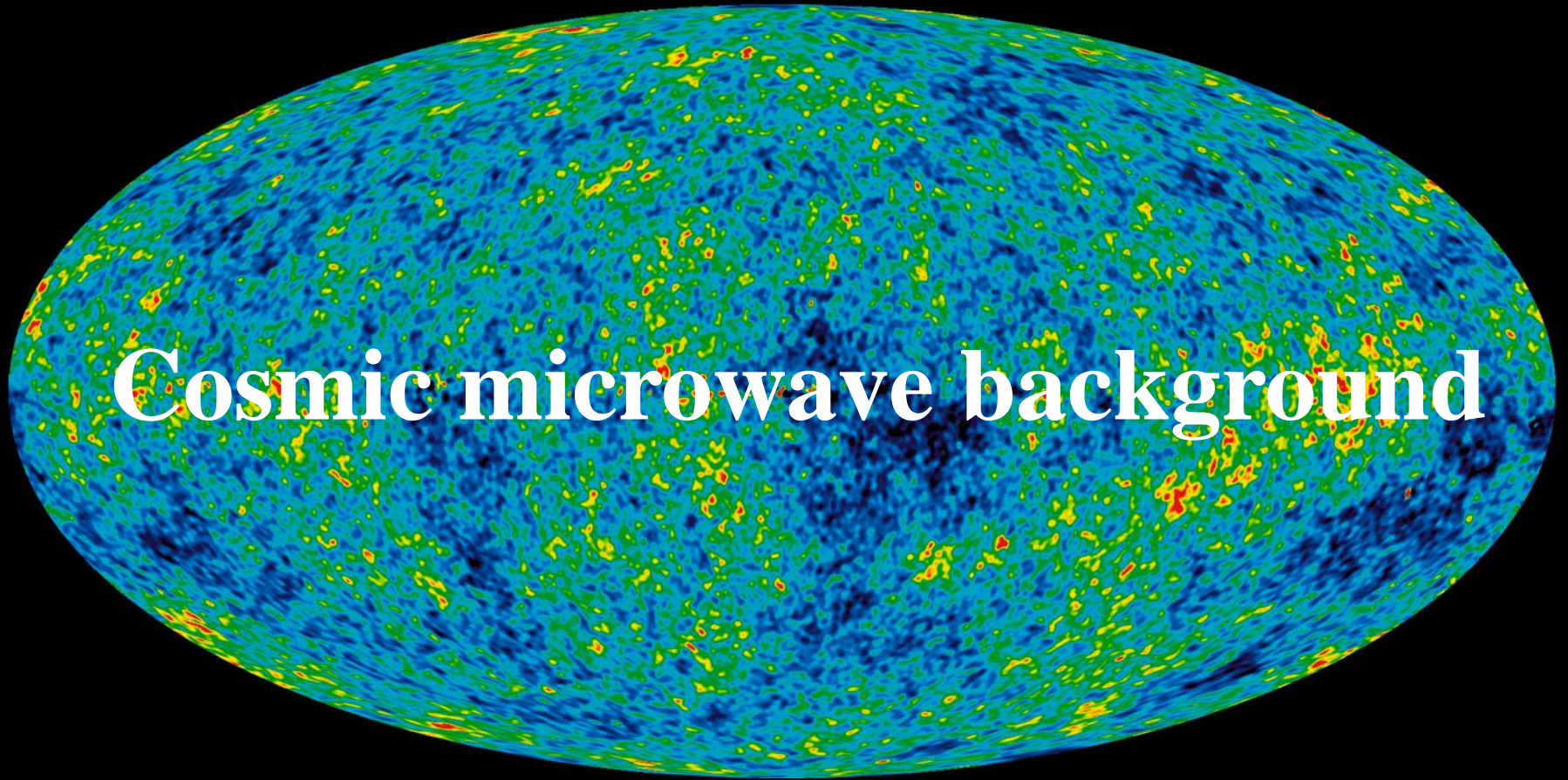


The composition of the universe:

95% unknown!



Inflation, dark energy & dark matter
are (almost) confirmed by

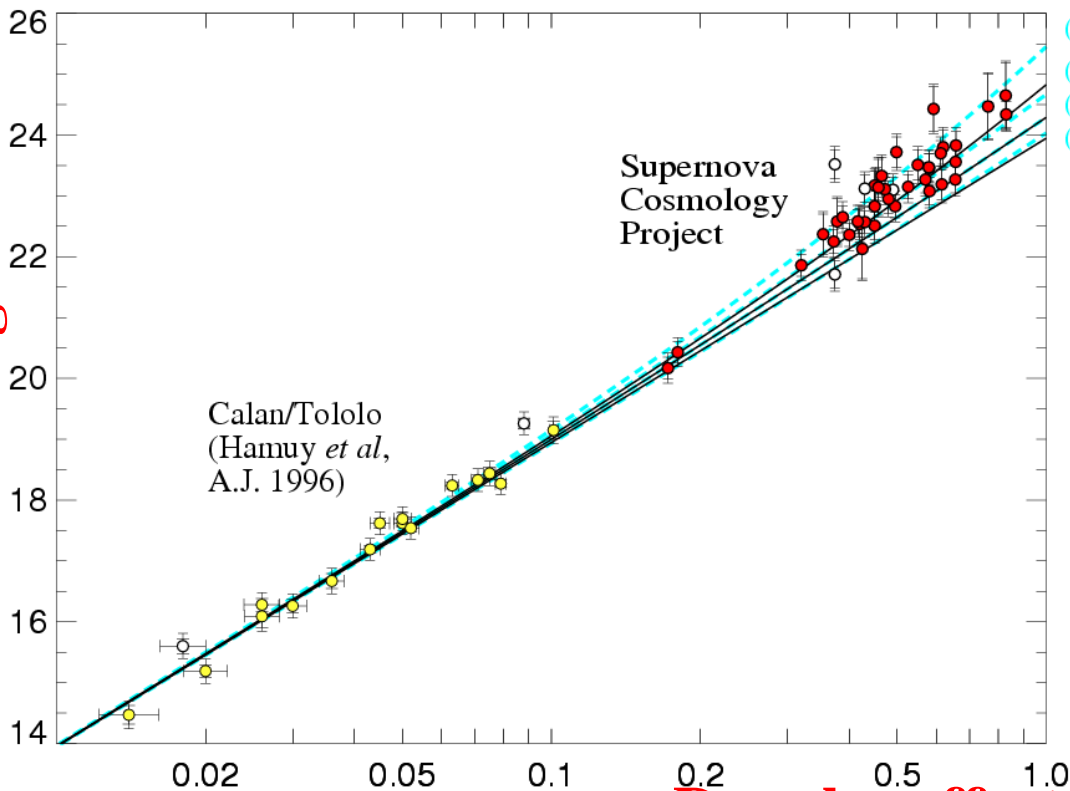


Cosmic microwave background

& Supernava observation

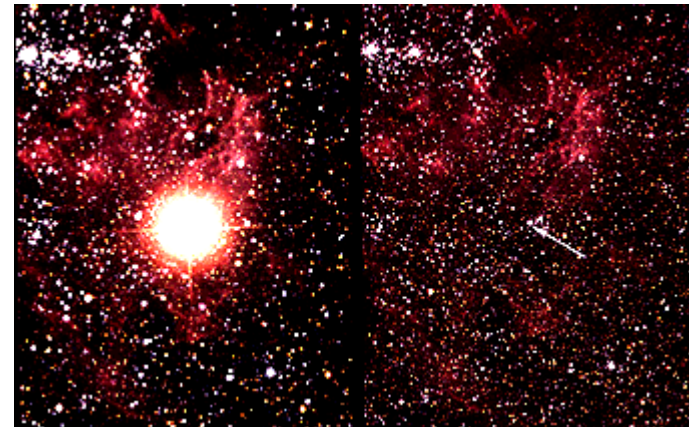
effective degree of darkness \uparrow distance
 (Farther)
 (Further back in time)

Perlmutter, *et al.* (1998)



$(\Omega_M, \Omega_\Lambda) =$
 (0, 1) $(0, 0)$
 (0.5, 0.5) $(1, 0)$
 (1, 0) $(1.5, -0.5)$ $(2, 0)$
 Flat $\Lambda = 0$

Explosion of a heavy star
 10 billion times brighter than the sun



Doppler effect ~ expansion of universe

MORE REDSHIFT \rightarrow
 (More total expansion of universe since the supernova explosion)

<http://supernova.lbl.gov/>

Two phases of the accelerated expansion of the universe

- **Inflation** in the **early universe**
- Accelerated expansion of the **late-time universe** driven by **dark energy**

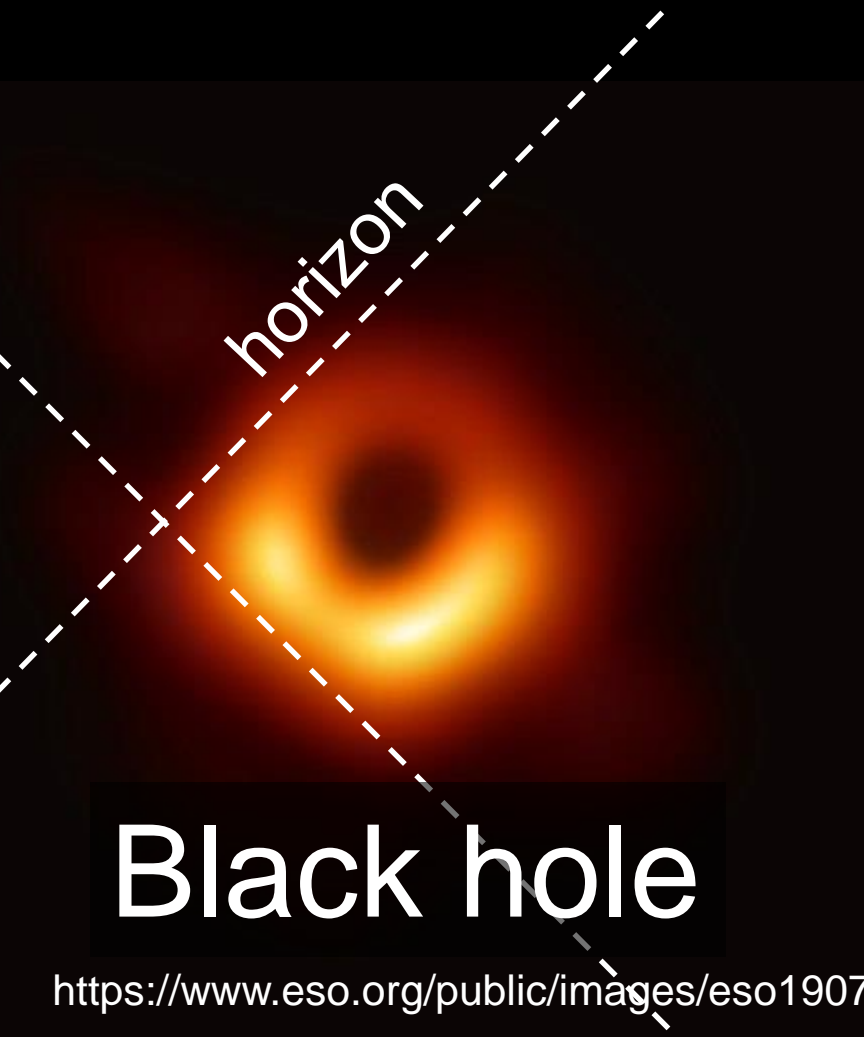
We (almost) know they (or something like them)
are/were there...

But, we don't know what they are.

- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.

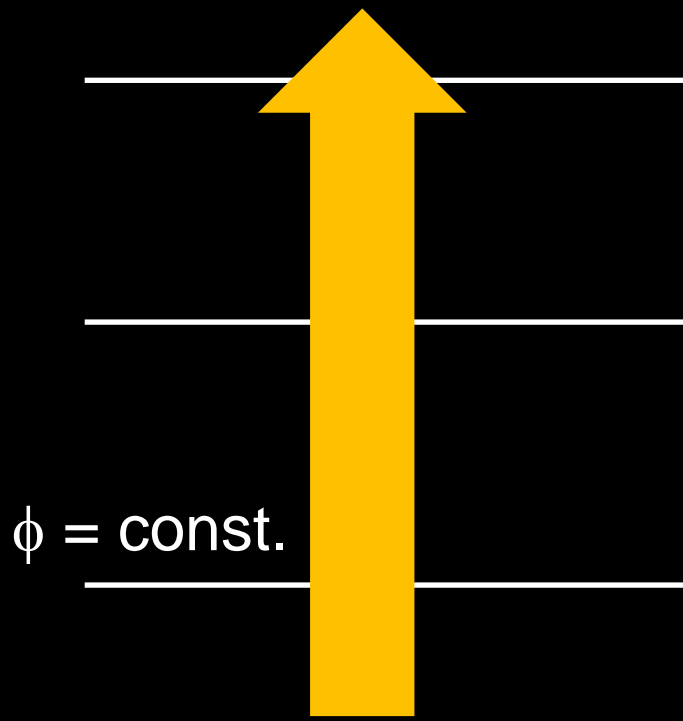
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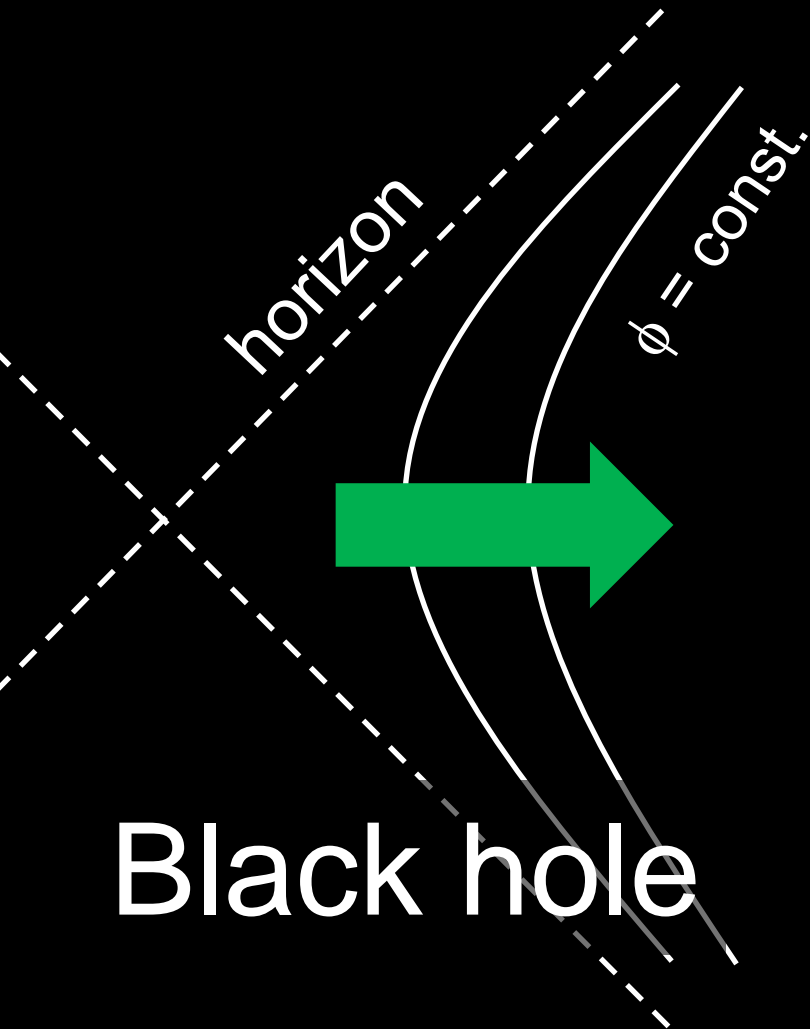
<https://www.eso.org/public/images/eso1907a/>

Timelike gradient

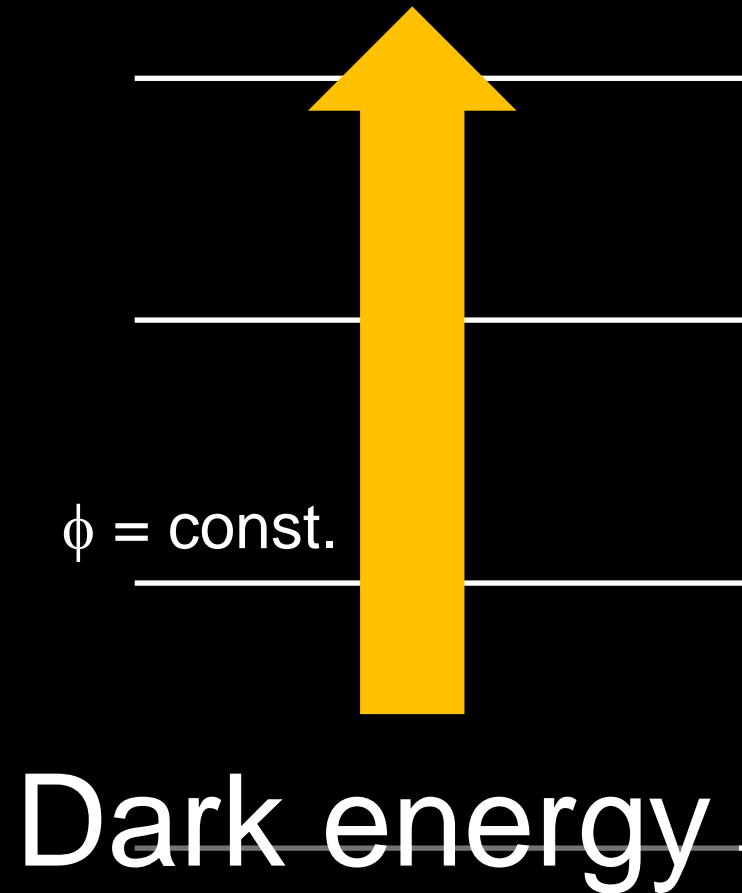


Dark energy

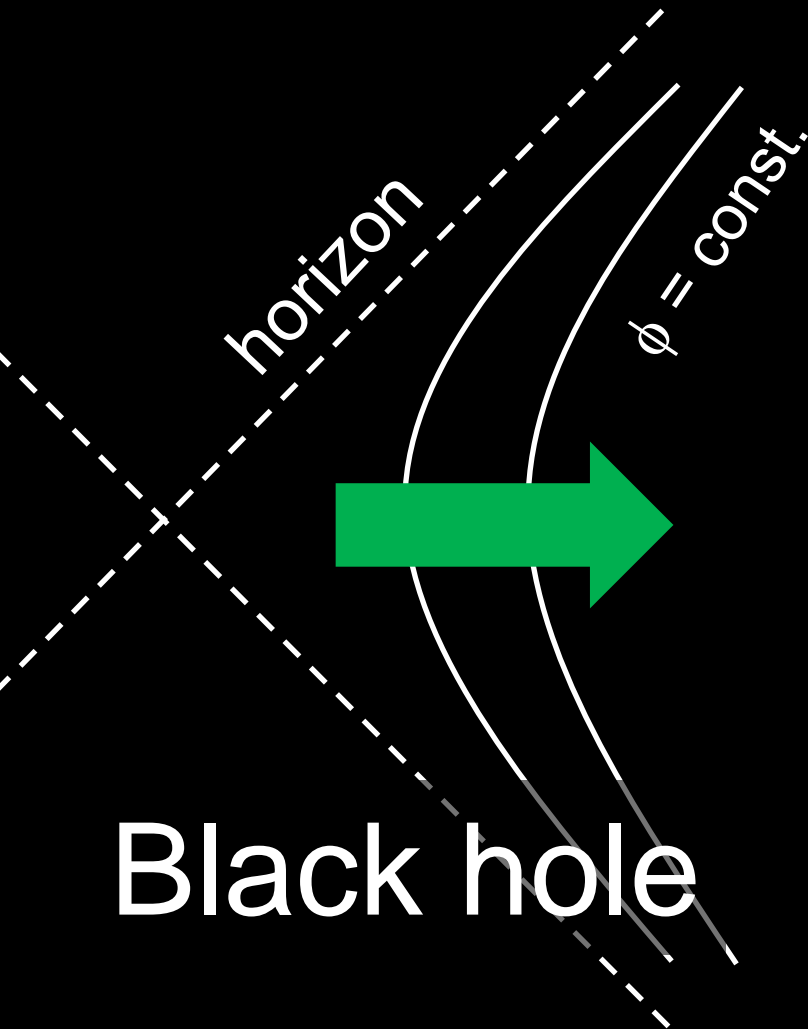
Unlucky case
Spacelike gradient



Timelike gradient

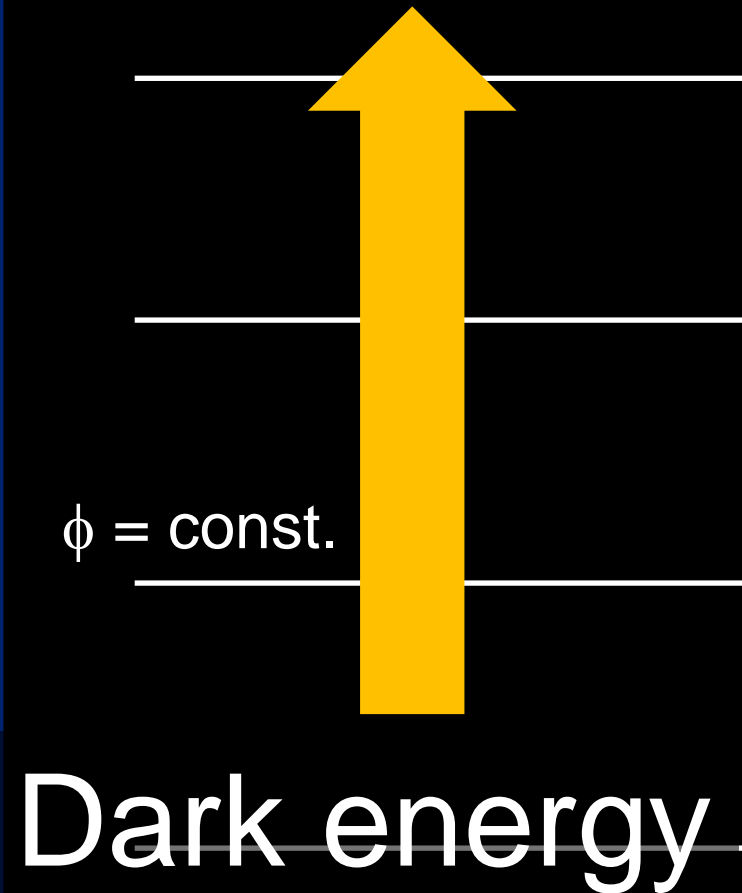


Unlucky case
Spacelike gradient



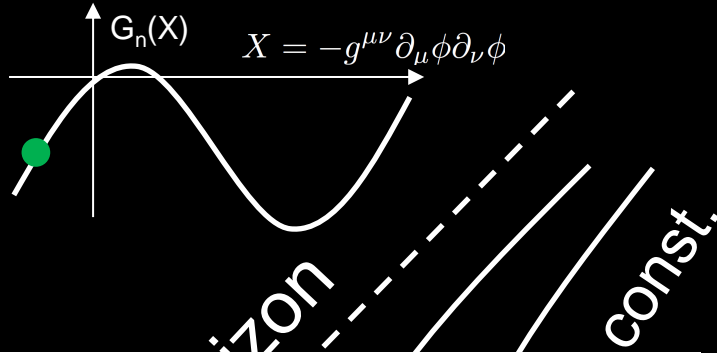
No smooth matching

Timelike gradient



Unlucky case

Spacelike gradient

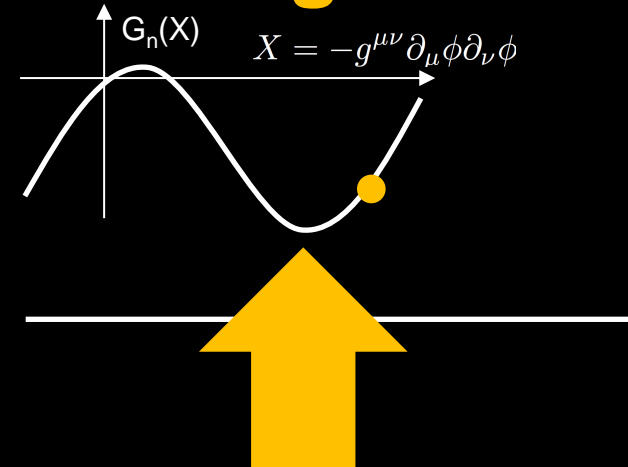


Taylor expansion
around $X = X_{\text{BH}} < 0$
($\beta_1, \beta_2, \beta_3, \dots$)

Black hole

No direct relation
between Taylor coefficients

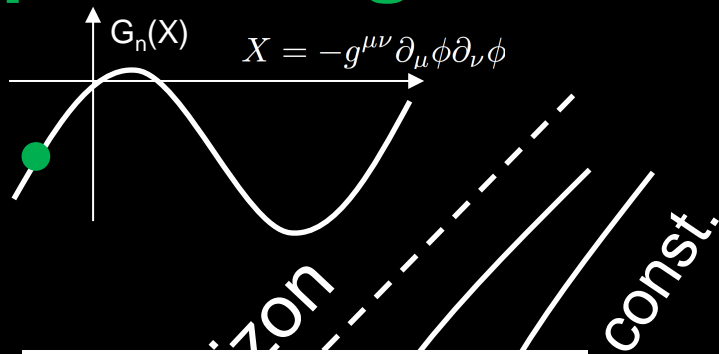
Timelike gradient



Taylor expansion
around $X = X_{\text{DE}} > 0$
($\alpha_1, \alpha_2, \alpha_3, \dots$)

Dark energy

Unlucky case
Spacelike gradient

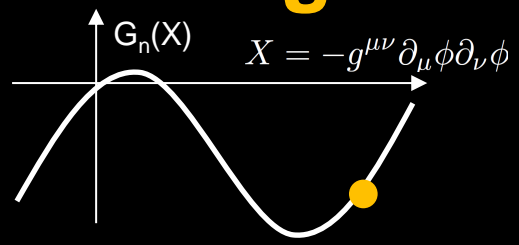


EFT2
 $(\beta_1, \beta_2, \beta_3, \dots)$

Black hole

No direct relation
between EFT1 & EFT2

Timelike gradient

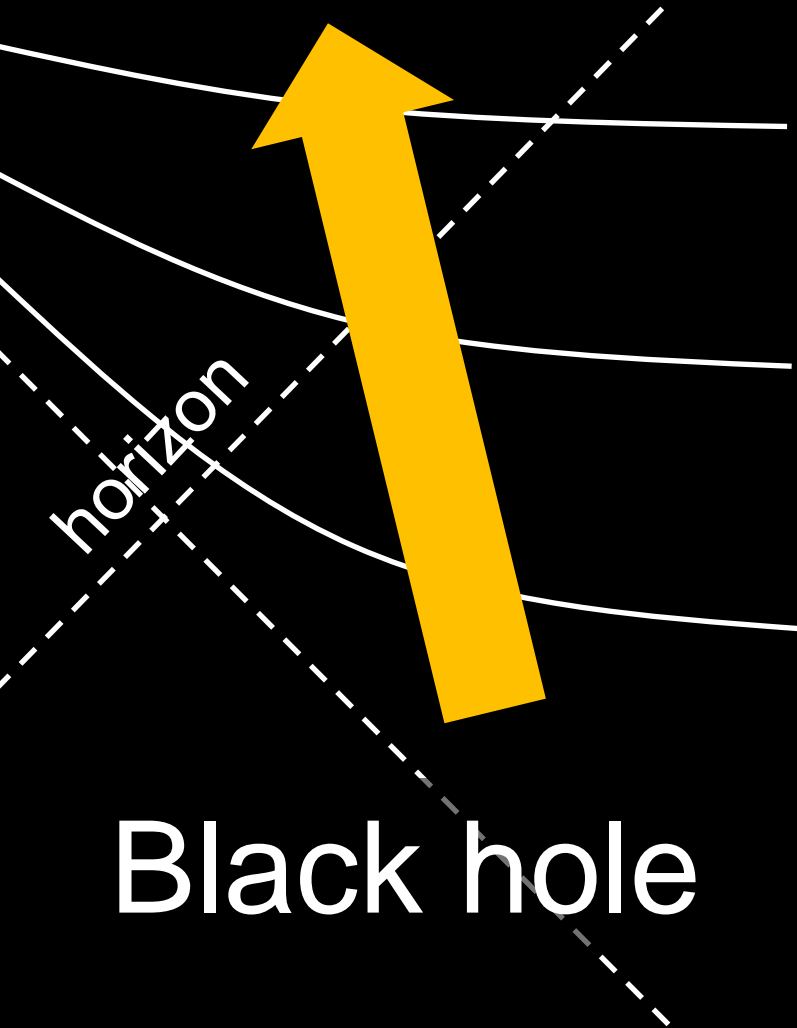


EFT1
 $\phi = (\alpha_1, \alpha_2, \alpha_3, \dots)$

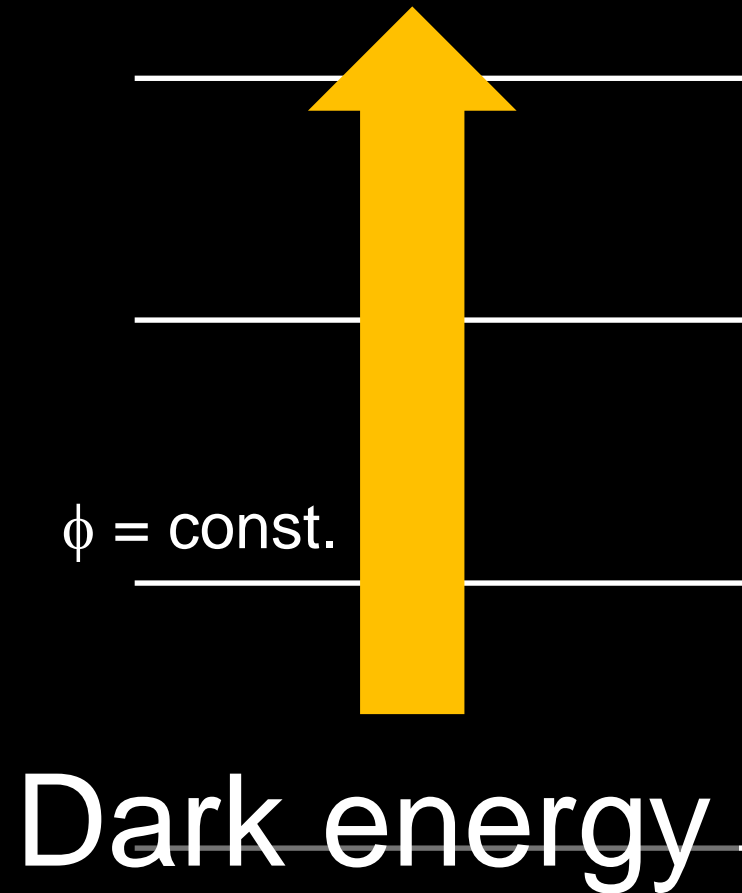
Dark energy

Lucky case

Timelike gradient

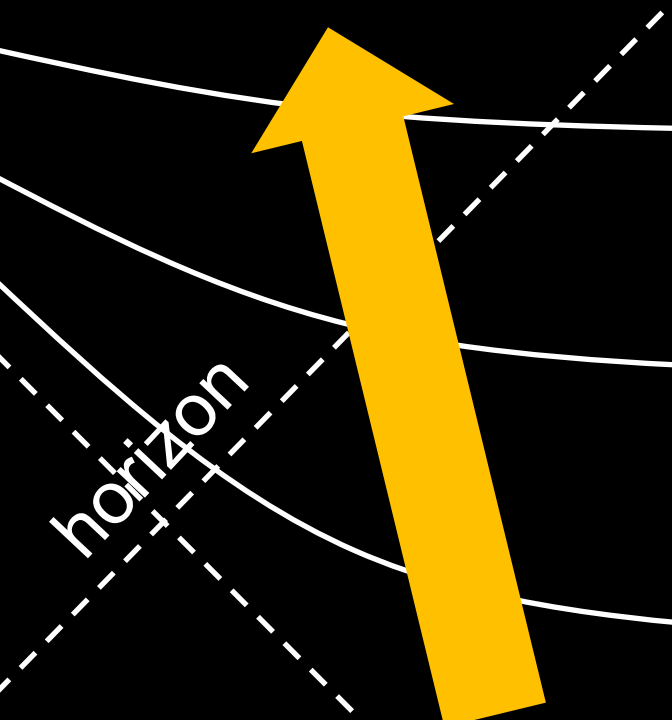


Timelike gradient



Lucky case

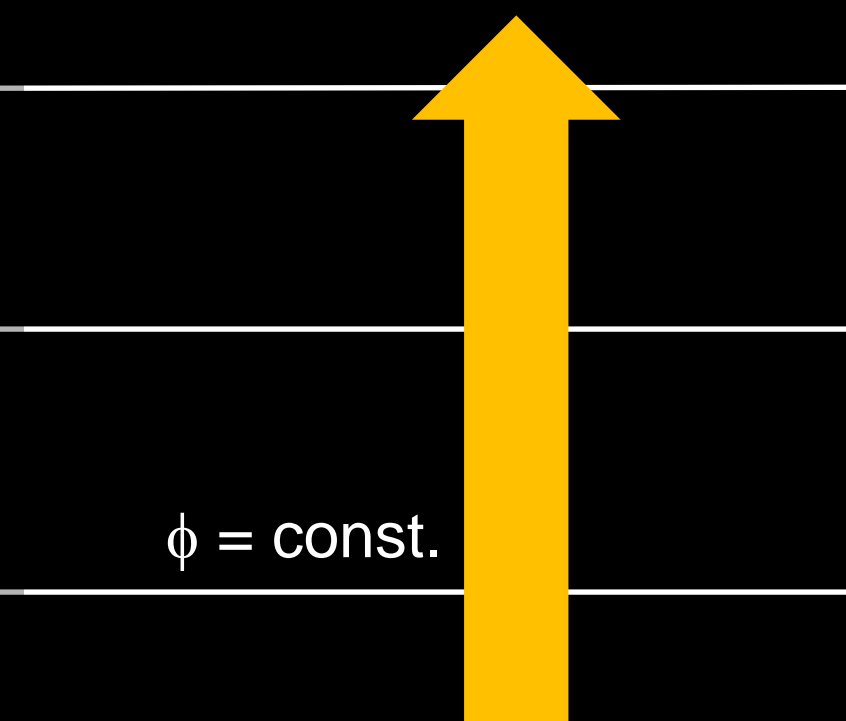
Timelike gradient



Black hole

Smooth matching!

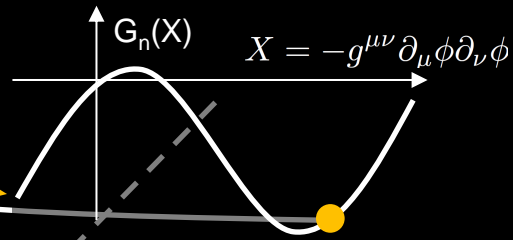
Timelike gradient



Dark energy

Lucky case

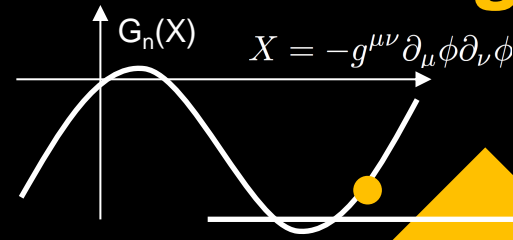
Timelike gradient



Taylor expansion
around $X = X_{BH} > 0$
($\alpha'_1, \alpha'_2, \alpha'_3, \dots$)

Black hole

Timelike gradient

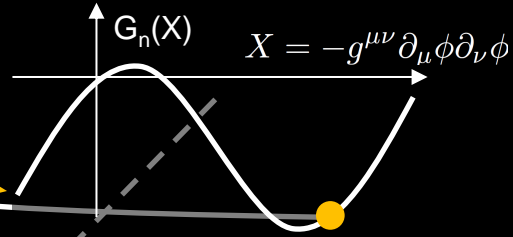


Taylor expansion
around $X = X_{DE} > 0$
($\alpha_1, \alpha_2, \alpha_3, \dots$)

Dark energy

Lucky case

Timelike gradient

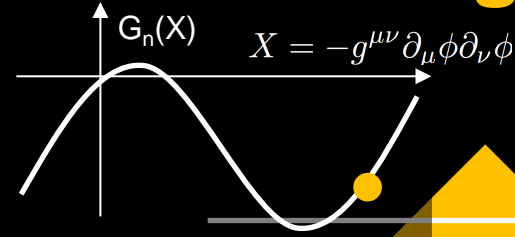


EFT

$(\alpha_1(t, \mathbf{x}^i), \alpha_2(t, \mathbf{x}^i), \alpha_3(t, \mathbf{x}^i), \dots)$

Black hole

Timelike gradient



EFT

$(\alpha_1(t, \mathbf{x}^i), \alpha_2(t, \mathbf{x}^i), \alpha_3(t, \mathbf{x}^i), \dots)$

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EFT of scalar-tensor gravity on arbitrary background with timelike scalar profile