UV/IR mixing and towards new paradigms for v. high energy physics

Steve Abel *(IPPP)*

Based on the following set of papers on NON-SUPERSYMMETRIC strings …

- **w/ Dienes and Nutricati — arXiv:2407.11160**
- **w/ Dienes and Nutricati — *Phys.Rev.D* 107 (2023) 12, 126019 ; arXiv:2303.08534**
- **w/ Keith Dienes — *Phys.Rev.D* 104 (2021) 12, 126032; arXiv:2106.04622**
- w/ Dienes+Mavroudi *Phys.Rev*.D 97 (2018) 12, 126017, arXiv: 1712.06894
- w/ Stewart, *Phys.Rev.D* 96 (2017) 10, 106013 arXiv:1701.06629
- Aaronson, SAA, Mavroudi, *Phys.Rev.D* 95, (2016) 106001, arXiv:1612.05742
- SAA JHEP 1611 (2016) 085, arXiv:1609.01311
- w/ Dienes+Mavroudi *Phys.Rev. D91*, (2015) 126014, arXiv:1502.03087

Themes of this talk …

There is a whole raft of SUSY-like supertrace identities associated UV/IR mixing that have not been noticed before

In this talk I will demonstrate this by showing how they appear in any closed string theory

These identities seem to have profound implications: e.g. they forbid power law running (Non-SUSY non-renormalisation theorems)

e.g. they imply scale invariance at the string scale

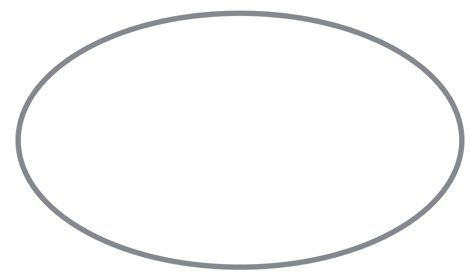
Outline

- How UV/IR mixing constrains theories
- Higher dimensions
- Theories with higher dimensional limits
- Surprising behaviour!

How UV/IR mixing constrains theories: string theory example

Understanding UV/IR mixing: the one-loop cosmological constant done in a stringy way

As a useful laboratory let's derive Λ the one-loop cosmological constant: we can do this as an integral over all distinct loops of massive propagators of mass M as follows:



For our discussion this can be written in a "stringy way" using a Schwinger worldline parameter, t :

$$\Lambda = -\frac{1}{2}\sum_{\text{states}}\int \frac{d^4k}{(2\pi)^4}(-1)^F \log\left(k^2+M_{\text{state}}^2\right) \;=\; -\frac{1}{2}\sum_{\text{states}}\int \frac{d^4k}{(2\pi)^4}\int_0^\infty \frac{dt}{t}(-1)^F e^{-t(k^2+M_{\text{state}}^2)}$$

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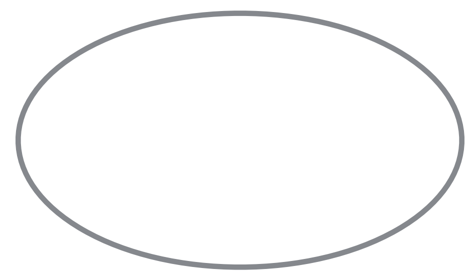


For our discussion this can be written in a "stringy way" using a Schwinger worldline parameter, t :

$$\Lambda \;=\; -\frac{1}{32\pi^2}\int_{M_{UV}^{-2}}^{\infty} \frac{dt}{t^2}\, g(t)$$

where we identify a "particle partition function" which is a graded sum over the spectral density: THIS WILL BE THE HERO IN OUR DISCUSSION

$$g(t) = \sum_{\text{states}} \frac{1}{t}(-1)^F e^{-tM^2_{\text{state}}}$$

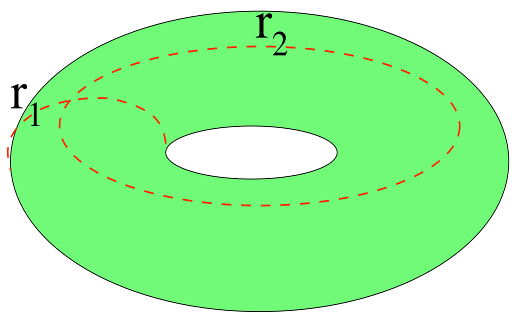
To orient you: if I perform this with cut-off it gives the precursor to the Coleman-Weinberg potential:

$$\Lambda = -\frac{M_{UV}^4}{64\pi^2}\mathrm{Str}_{EFT}\mathbf{1} + \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}_{EFT}M^2 - \mathrm{Str}_{EFT}\left[\frac{M^4}{64\pi^2}\log c\frac{M^2}{M_{UV}^2}\right]$$

where here $\mathrm{Str}_{EFT} \equiv \sum_{\text{states in EFT}} (-1)^F$ is the graded sum over states in the theory

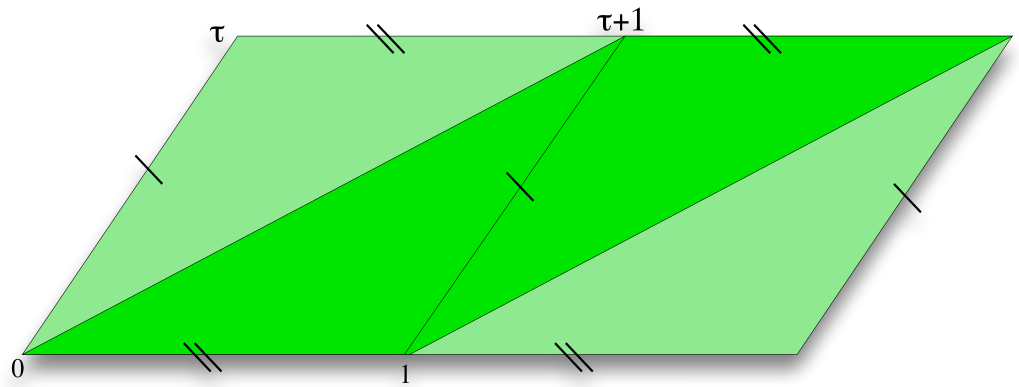
How does string theory get to be UV-complete and so avoid the need for the cut-off M_{UV}? Importantly I want to think about the theory generically TODAY, when SUSY (if it was ever there) is absent: I am not interested in model specific things.

Instead of a circle, closed string theory instead maps out a torus:



Instead of a circle, closed string theory instead maps out a torus:

But Modular Invariance implies torus can be mapped to parallelogram in complex plane, defined by single parameter τ,



$\tau \to \tau + 1$ redefines torus :

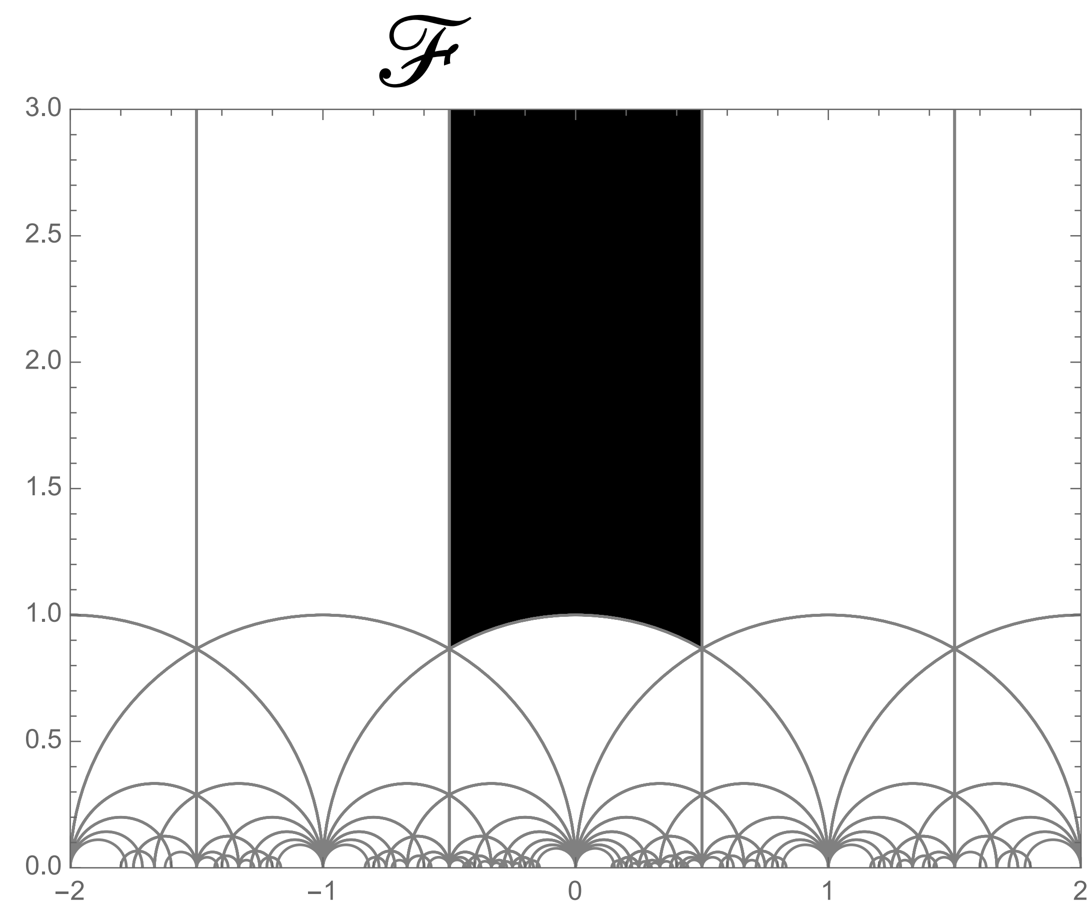
$\tau \to -1/\tau$ swops σ_1 and σ_2 and just reorients torus

Thus the integral over all diagrams does not cover the whole τ plane but takes the form $(\mathcal{M} = M_s/2\pi)$…

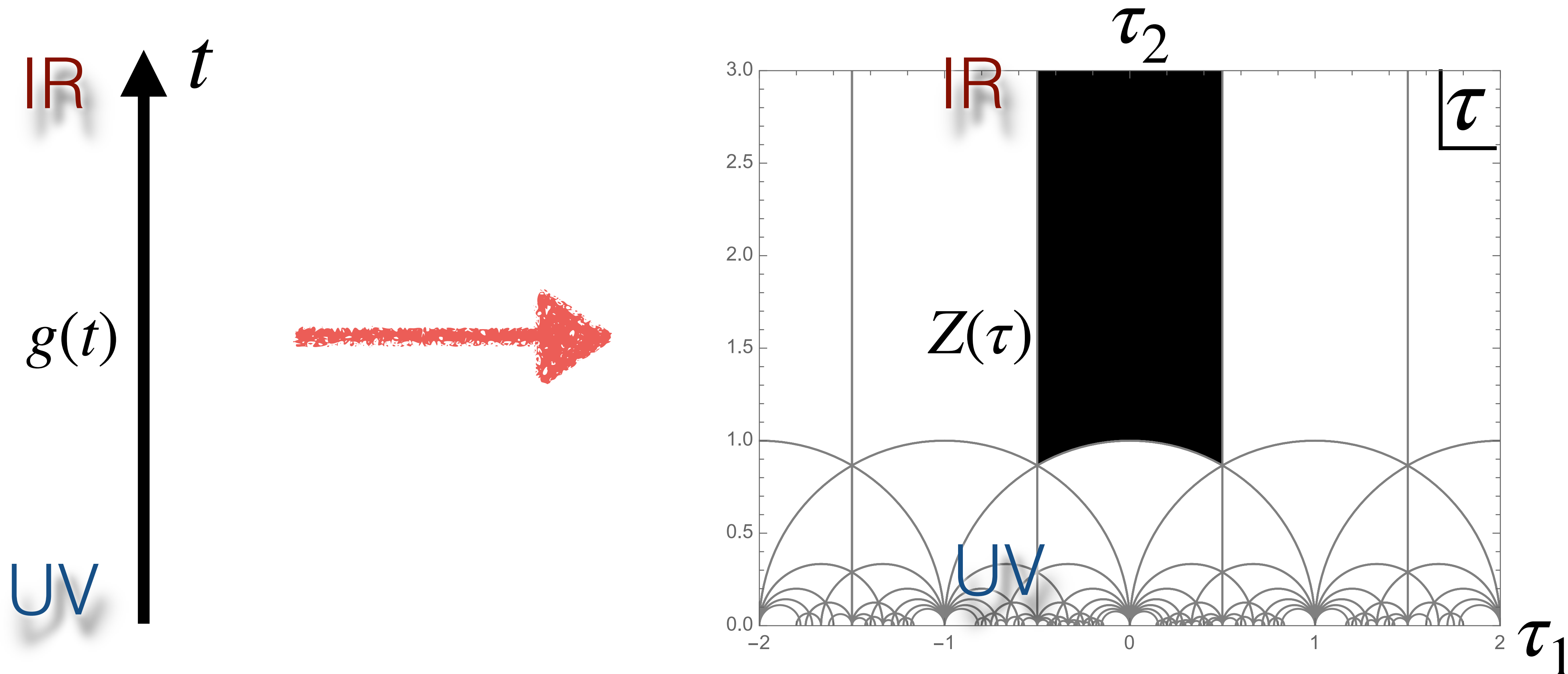
$$\Lambda \;=\; -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, Z(\tau,\overline{\tau})$$

where $Z(\tau) = Z(\tau')$ when $\tau' = \dfrac{a\tau + b}{c\tau + d}$

$Z(\tau)$ is the string version of the particle $g(t)$ and holds all the information about the spectrum. *All amplitudes look similar to this*.



Usual cartoon …



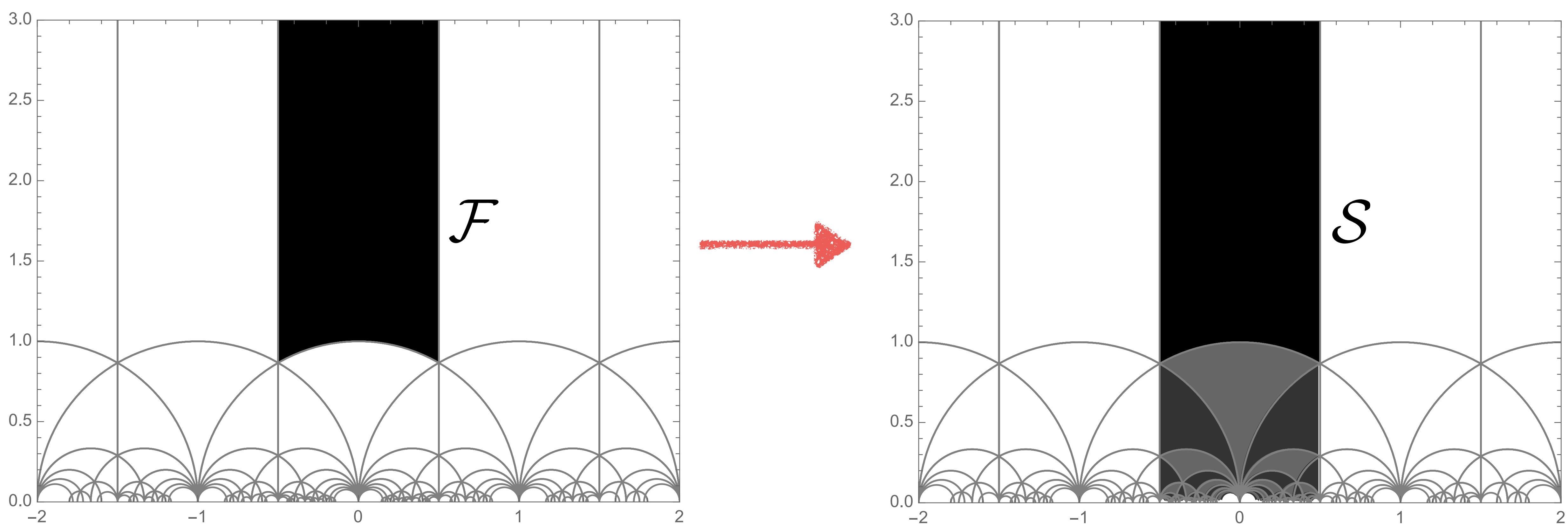
Particle

Strings: UV is "missing"

This is the textbook explanation of stringy finiteness. ***However:*** a method due to Rankin and Selberg (1939/40) expresses the integral in terms of the completely particle theory expression $g(\tau_2)$ of **physical (level-matched) states** —

$$
\begin{aligned}
g(\tau_2) &= -\frac{\mathcal{M}^4}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \; Z(\tau) \\
&= -\frac{\mathcal{M}^4}{2}\tau_2^{-1} \sum_{\text{states}} (-1)^F e^{-\pi\tau_2\alpha' M^2_{\text{state}}}
\end{aligned}
$$

RS use a transform to unfold $\mathscr{F}$ to the critical strip $\mathscr{S}$



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$$g(\tau_2) = -\frac{\mathcal{M}^4}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \; Z(\tau)$$

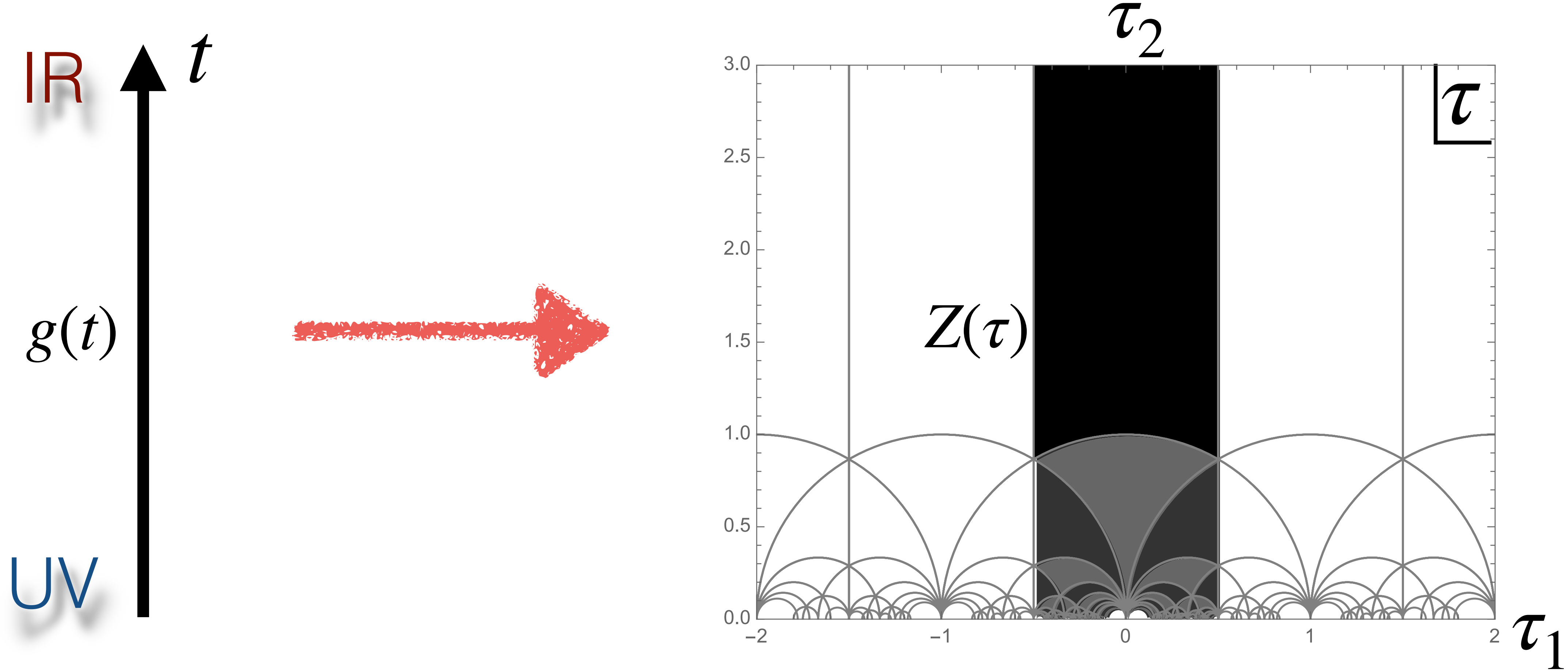
$$= -\frac{\mathcal{M}^4}{2} \tau_2^{-1} \sum_{\text{states}} (-1)^F e^{-\pi\tau_2 \alpha' M^2_{\text{state}}}$$

This gives the following answer …

$$-\frac{\mathcal{M}^4}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau, \overline{\tau}) = \frac{\pi}{3} \lim_{\tau_2 \to 0} g(\tau_2)$$

- Rankin, Selberg (1939/40)
- Zagier (1981)

In string theory: Kutasov, Seiberg; McClain, Roth, O'Brien, Tan; Dienes; Angelantonj, Florakis, Pioline, Rabinovici



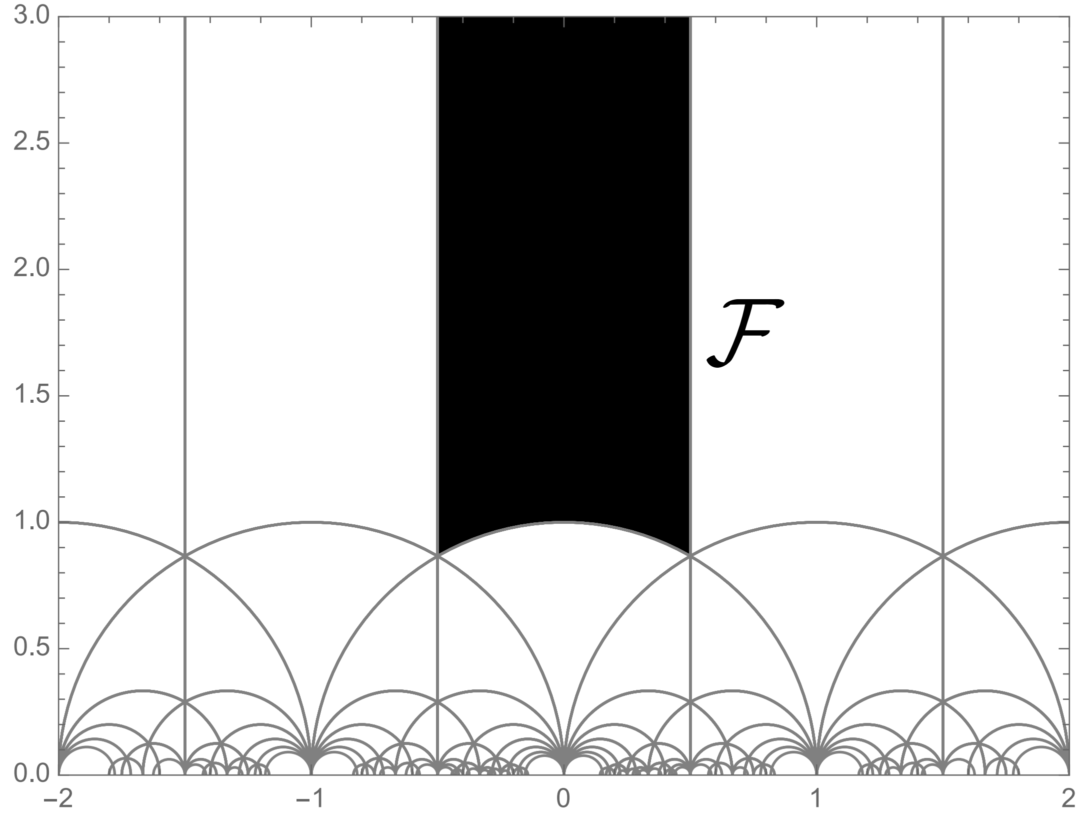
Particle

Strings according to RS: infinite sum over fundamental domains divided by infinite overcounting

Note the labels “UV” and an “IR” on the string integral no longer make sense.

Let's pause for a minute to see (as physicists) why this is remarkable:

$\pi\alpha'\tau_2$ clearly plays the role of the Schwinger parameter t when $\tau_2 \geq 1$: by naively integrating over the fundamental domain, we physicists see a result that mimics EFT …

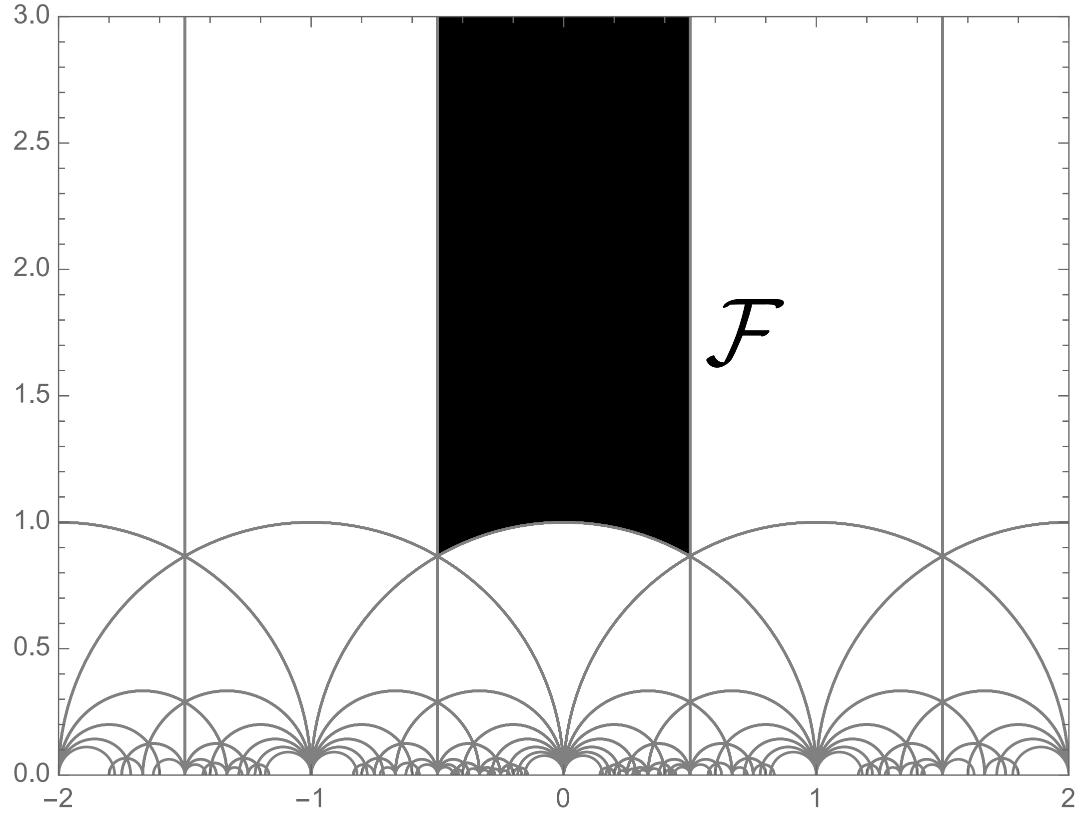


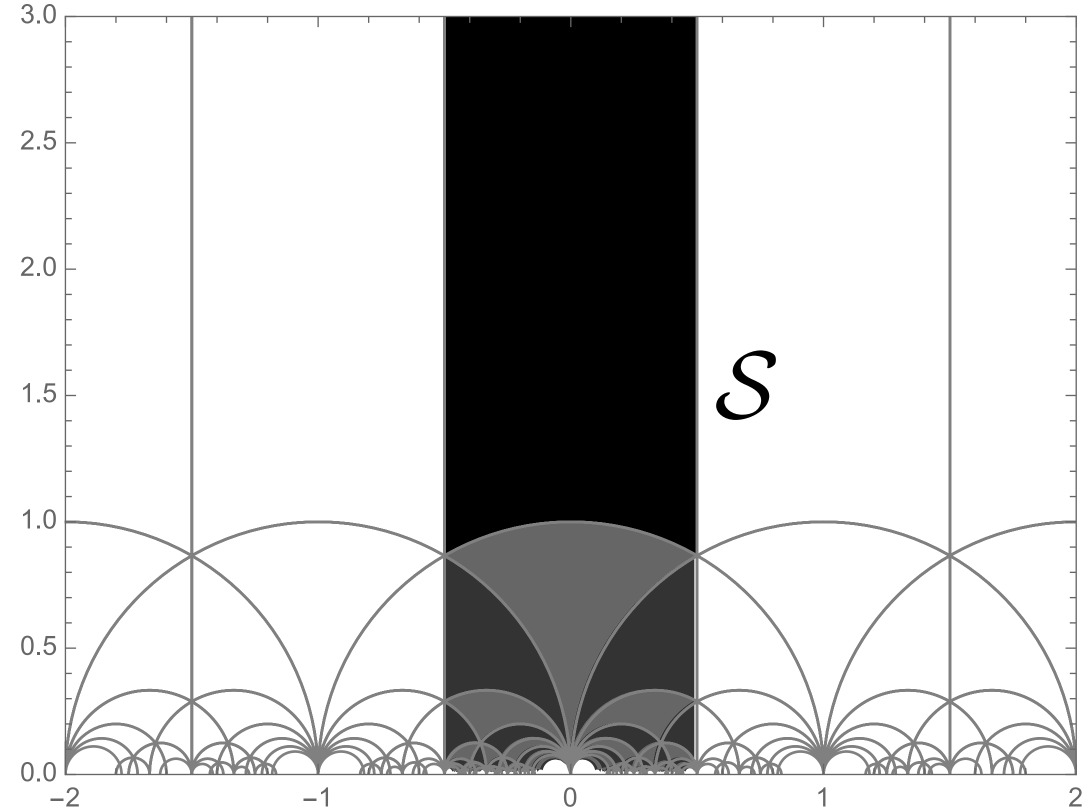
$$\Lambda \approx \int_1^\infty \frac{d\tau_2}{\tau_2^2} g(\tau_2)$$

$$\approx -\frac{\mathcal{M}^4}{2} \int_1^\infty \frac{d\tau_2}{\tau_2^3} \sum_{\text{states}} (-1)^F e^{-\pi\tau_2\alpha' M_{\text{state}}^2}$$

Let's pause for a minute to see (as physicists) why this is remarkable:

But this is equal to a ***very not EFT-like limit*** - it instead looks like a deep UV limit!!





$$\Lambda \approx \int_1^\infty \frac{d\tau_2}{\tau_2^2} g(\tau_2)$$

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So this is the ultimate UV/IR mixing. But it also implies something spectacular about the supertrace over the physical states …

To see this let's try and evaluate this RS limit:

$$\frac{\pi}{3} \lim_{\tau_2 \to 0} g(\tau_2) \;\; = -\frac{\mathcal{M}^4}{2} \lim_{\tau_2 \to 0} \sum_{\text{states}} (-1)^F \frac{1}{\tau_2} e^{-\pi \tau_2 \alpha' M^2_{\text{state}}}$$

It looks like it diverges because of the $1/\tau_2$ prefactor in $g(\tau_2)$!!!

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It looks like it diverges because of the $1/\tau_2$ prefactor in $Z(\tau_2)$!!! … *Unless* …

$$\lim_{\tau_2 \to 0} \sum_{\rm states} (-1)^F e^{-\pi \tau_2 \alpha' M^2_{\rm states}} = 0$$

Thus — if we define a stringy ***regulated supertrace*** appropriate for infinite towers of states for any operator $\mathcal{X}$,

$$\mathrm{Str}\,\mathcal{X} = \lim_{\tau_2\to 0} \sum_{\text{states}} (-1)^F \mathcal{X}_{\text{state}}\, e^{-\pi\tau_2\alpha' M^2_{\text{state}}}$$

then here (where $\mathcal{X} = const$ for the case of Λ) we see that any modular invariant 4D theory with a finite Λ obeys

$$\boxed{\mathrm{Str}\,\mathbf{1} = 0}$$

Any tachyon-free modular invariant theory in 4D has $\mathrm{Str}(\mathbf{1}) = 0$ ***even when no SUSY!***

- Dienes, Misaligned SUSY, 1994
- Dienes, Moshe, Myers 1995

Or to put it another way … if we expand $g(\tau_2)$ around $\tau_2 = 0$ in a generic particle theory it would go like

$$g(\tau_2) = \frac{1}{\tau_2} \times \left(C_0 + C_1\tau_2 + C_2\tau_2^2 + \ldots\right)$$

but in a modular invariant theory we have $C_0 = 0$ and it must instead go like

$$g(\tau_2) = \frac{1}{\tau_2} \times \left(C_1\tau_2 + C_2\tau_2^2 + \ldots\right)$$

Note we can express the integral as $\Lambda = \pi C_1/3$, where by expanding the exponential around τ_2 and picking off the first term C_1: we have

$$\Lambda = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

- Dienes, Misaligned SUSY, 1994
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- Dienes, Moshe, Myers 1995

This looks exactly like the leading piece in the Coleman Weinberg potential if the quartic M_{UV}^4 term magically vanishes. i.e. the condition $\mathrm{Str}1 = 0$ forces the quartic divergence term vanishing in any modular invariant theory. Only the first non-renormalisation theorem we will meet.

Higher dimensions

In theories with $D > 4$ space-time dimensions things get more constrained. The reason why is that $g(\tau_2)$ takes the form

$$g(\tau_2) = \frac{1}{\tau_2^{1+\delta/2}} \times \left(C_0' + C_1' \tau_2 + C_2' \tau_2^2 + \ldots \right)$$

But now applying Rankin-Selberg we see that in a theory with $D = 4 + \delta \ldots$

$\Longrightarrow$ we have $C_0', C_1', \ldots, C_{\delta/2}' = 0$

Thus in a theory with $D = 4 + \delta$ expanding the expression for $\Lambda^{(D)}$ we have

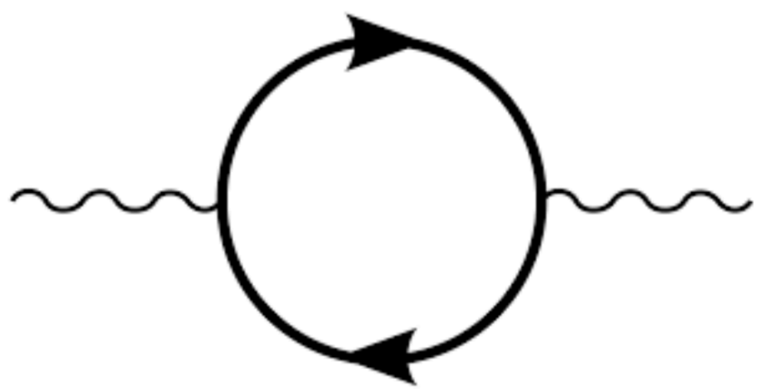
$$\mathrm{Str}' M^k = 0$$

for all $k < 2 + \delta$.

But in higher dimensions many more supertraces get constrained: let's now extend the discussion to more general amplitudes … $\langle \mathcal{X} \rangle$

Any amplitude one might want to calculate simply corresponds to the insertion of an operator $\mathcal{X}$ into the Λ integral.

For example vacuum polarisation amplitude to find one-loop gauge coupling correction $16\pi^2/g_G^2 = 16\pi^2/g_{\text{tree}}^2 + \Delta_G$:



$$\mathcal{X} = \mathbb{X}_0 + \tau_2 \mathbb{X}_1 + \tau_2^2 \mathbb{X}_2$$

$$\begin{cases} \mathbb{X}_0 = 0 \\ \mathbb{X}_1 \equiv \dfrac{\xi}{2\pi}\left(\overline{Q}_H^2 - \dfrac{\overline{E}_2}{12}\right) \\ \mathbb{X}_2 \equiv -2\left(\overline{Q}_H^2 - \dfrac{\overline{E}_2}{12}\right) Q_G^2 \end{cases}$$

Space-time helicity Gauge charges

For example in a 6 dimensional theory we find a *constraint* plus a one - loop contribution to $16\pi^2/g_G^2 = 16\pi^2/g_{\rm tree}^2 + \Delta_G$ of the form

$$\mathrm{Str}'\,\overline{Q}_H^2 - \tfrac{1}{12}\mathrm{Str}'_E\,\mathbf{1} \;=\; 0$$

and …

$$\Delta_G \;\approx\; \frac{\pi}{3} \times \left[-2\,\mathrm{Str}'\,(Q_G^2\overline{Q}_H^2) + \frac{1}{6}\mathrm{Str}'_E\,Q_G^2 - \frac{\xi}{2\pi}\mathrm{Str}'\left(\overline{Q}_H^2\widetilde{M}^2\right) + \frac{\xi}{24\pi}\mathrm{Str}'_E\,\widetilde{M}^2\right]$$

where $\widetilde{M}^2 \;\equiv\; \dfrac{M^2}{4\pi\mathcal{M}^2}$

Theories with higher dimensional limits

So the question is — what happens when a 4 dimensional theory has a decompactification limit to a higher dimensional theory?

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$$Z^{(4)} = \sum_{i=1}^{N} Z_i' \Theta_i$$

The i indicates a sum over different sectors … each with a "base" contribution Z_i' multiplying KK/winding factors Θ_i which turn into volumes in each large radius limit …

$$Z^{(4)} \to \tau_2^{-\delta/2} c_i Z_i' \, \mathcal{M}^{\delta} V_{\delta}$$

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$$Z^{(4)} \to {\tau_2}^{-\delta/2} c_i Z_i' \, \mathcal{M}^{\delta} V_{\delta}$$

i.e. at large radius the partition function is simply proportional to the higher dimensional theory

But at this point we notice a clash! … we know that the Z' have to satisfy many more *constraints* than the four dimensional theory

The only way to resolve this clash and for *physics to be smooth* at infinite radius is for all the constraints to *already* be satisfied in the 4D theory … it turns out this is independent of the compactification radius:

The 4D theory will inherit the precise stricter internal cancellations of any higher-dimensional theory to which can be decompactified.

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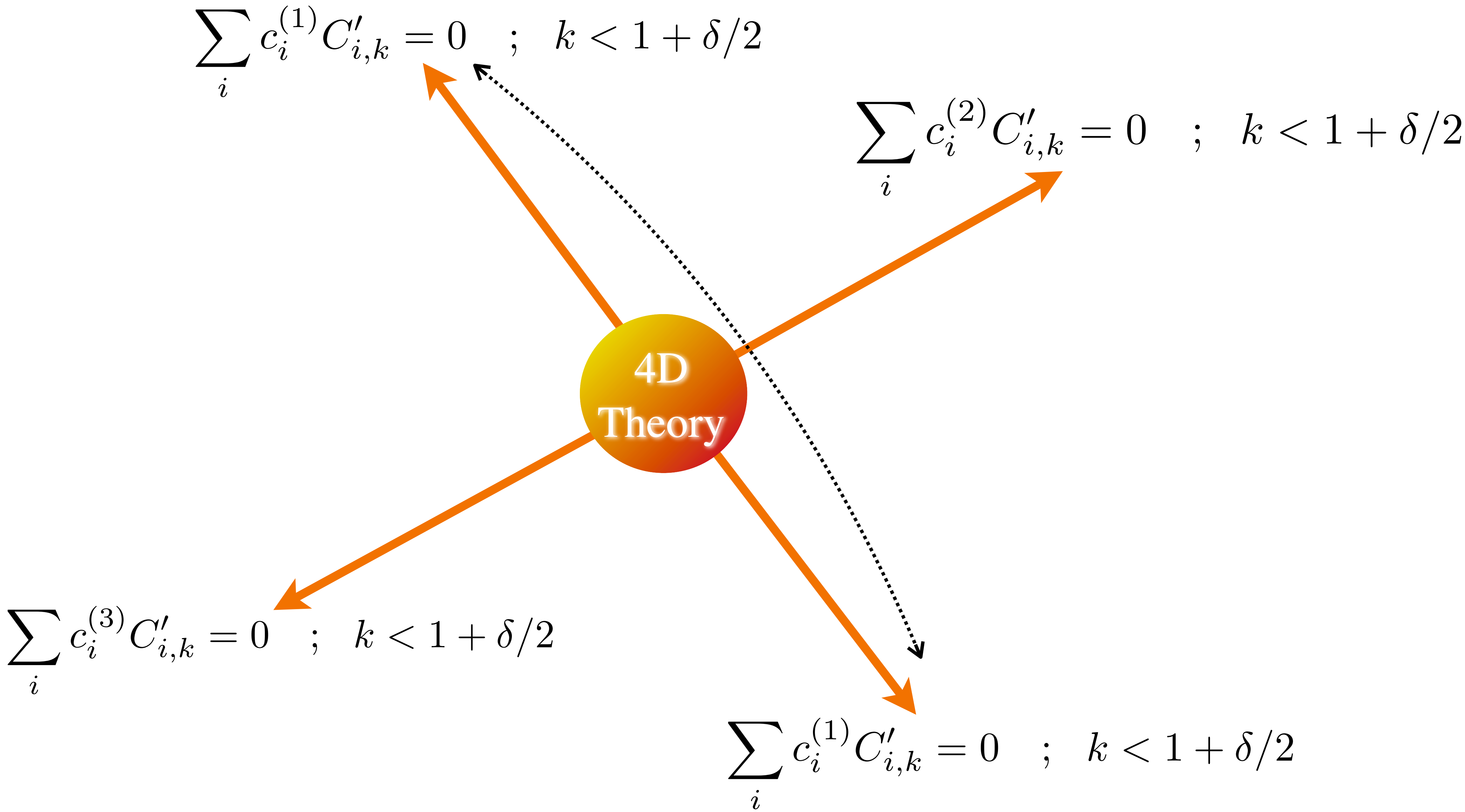
> *The 4D theory will inherit the precise stricter internal cancellations of any higher-dimensional theory to which can be decompactified.*

For example $16\pi^2 g_G^{-2} = 16\pi^2 g_{\text{tree}}^{-2} + \Delta_G$ in a theory with $\delta = 2$ decompactification:

$$\begin{aligned} \mathrm{Str}'\,\overline{Q}_H^2 - \tfrac{1}{12}\mathrm{Str}'_E\,\mathbf{1} &= 0 \\ \Delta_G &\approx \frac{\pi}{3}V_\delta\left[-2\,\mathrm{Str}'\,(Q_G^2\overline{Q}_H^2) + \frac{1}{6}\mathrm{Str}'_E\,Q_G^2 - \frac{\xi}{2\pi}\mathrm{Str}'\left(\overline{Q}_H^2\widetilde{M}^2\right) + \frac{\xi}{24\pi}\mathrm{Str}'_E\,\widetilde{M}^2\right] \end{aligned}$$

So the cartoon looks like this ...



Some of these endpoint theories related by duality transformations - but they all lead to a constraint that has to be satisfied in the 4D theory.

$\Longrightarrow$

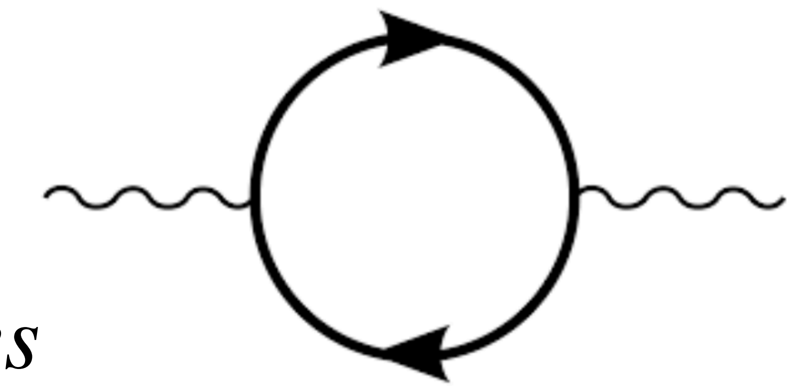
Surprising behaviour …

No power-law running …

Power law running is the expectation that contributions over towers of Kaluza-Klein modes resum to give a power-law scale dependence …

$$M_{KK} \sim k/R$$

$$\Delta_G = \sum_{KK\ states}$$



$$\sim C_2' \mu^\delta R^\delta = C_2' \mu^\delta V_\delta$$

which arises because a single δ-dimensional KK tower contribution to $g(t)$ goes like

$$g(t) \rightarrow \begin{cases} \frac{1}{t}(C_0' + C_1' t + C_2' t^2 + \ldots) & t \gg R^2 \\ \frac{R^\delta}{t^{1+\delta/2}}(C_0' + C_1' t + C_2' t^2 + \ldots) & t \ll R^2 \end{cases}$$

The crux of the matter: we saw that in modular invariant theories: $C_2' = 0$ if $\delta > 2$!

In other words there can be no $\delta > 2$ power law running, and moreover there is no contribution to *any* running (even logarithmic) from the states in the theory associated with $\delta > 2$ decompactification limits.

- The case of $\delta = 2$ is more subtle: these *can* give logarithmic running below the KK scale.
- However it is easy to see that however we define the energy scale *there can be no $\delta = 2$ power-law running if there is no $\delta > 2$ running (which as we just saw is unphysical)*.

Let’s see an example: running in a theory with a $\delta = 2$ decompactification limit

Modular invariant renormalisation:

- SAA, Dienes, 2021

To insert an energy scale μ we insert a cut-off function $\mathcal{G}(\mu, \tau)$ which removes log divergences from any massless states and which must itself be *modular invariant*

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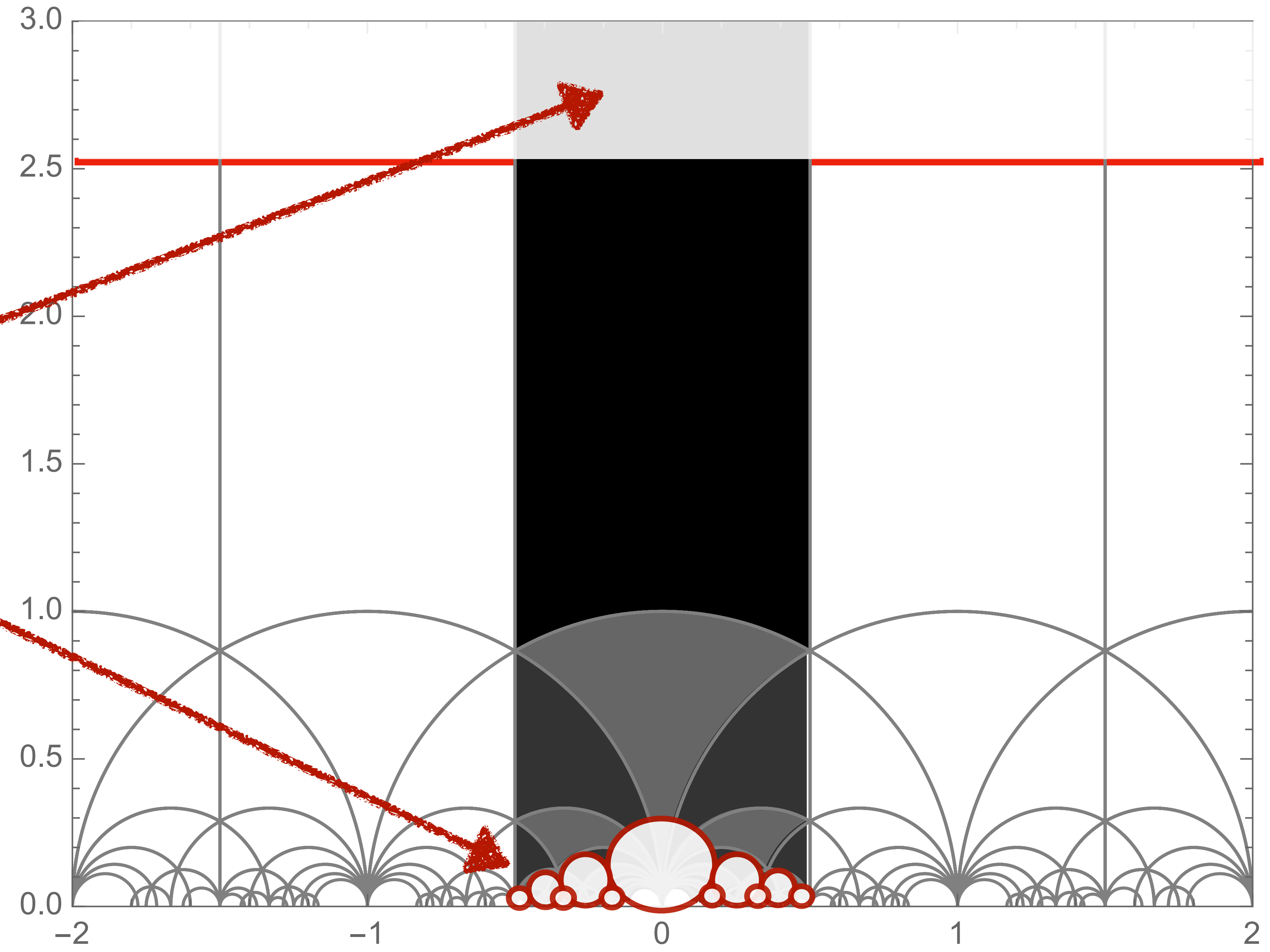
$$\tau_* \equiv M_s^2/\mu^2$$

$\mathscr{G}$ suppresses integral in a modular invariant way around all the cusps



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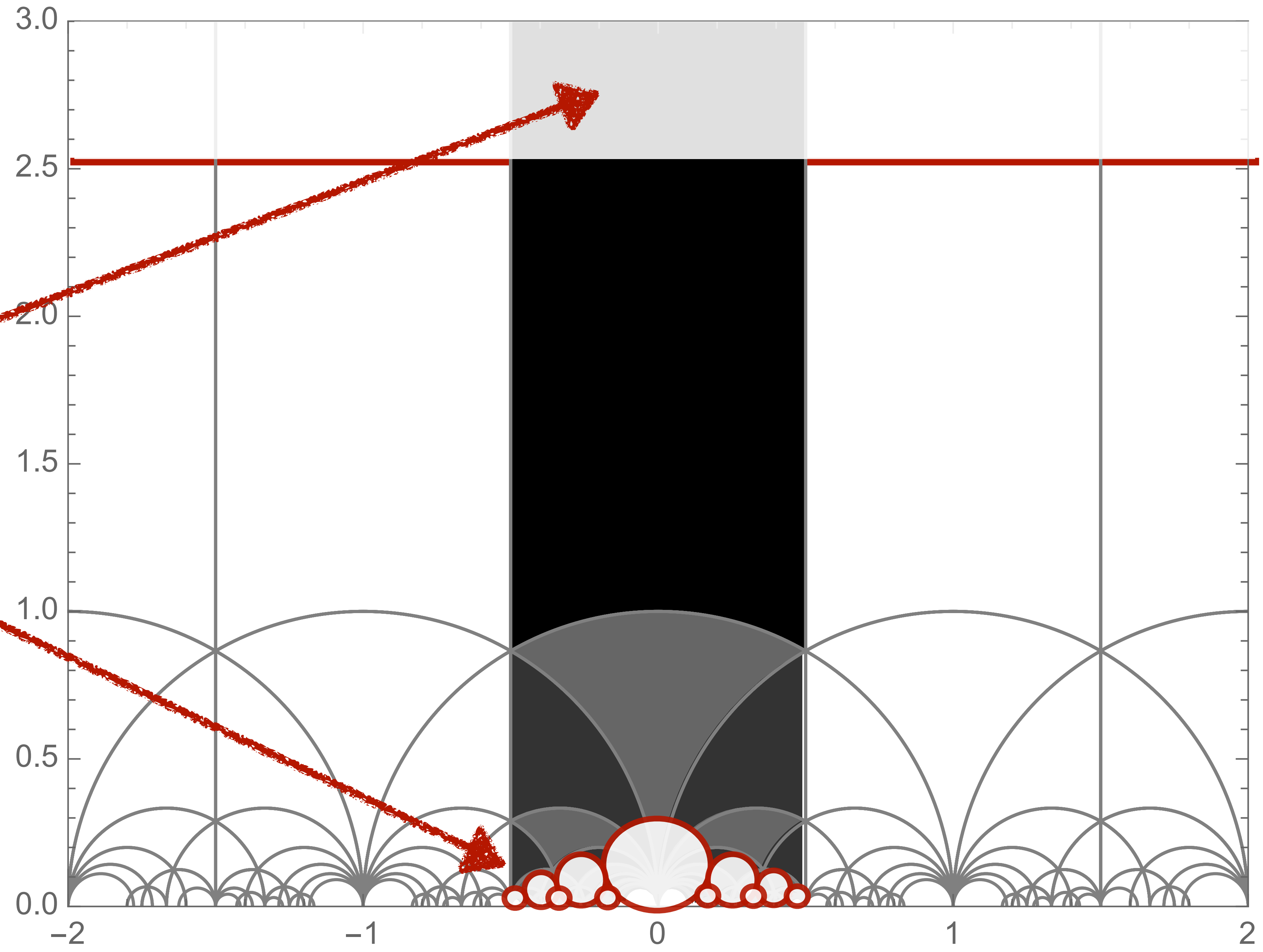
NOTE: Consistency requires $\mathcal{G}(\mu, \tau)$ invariant under

$$\mu \to M_s^2/\mu$$



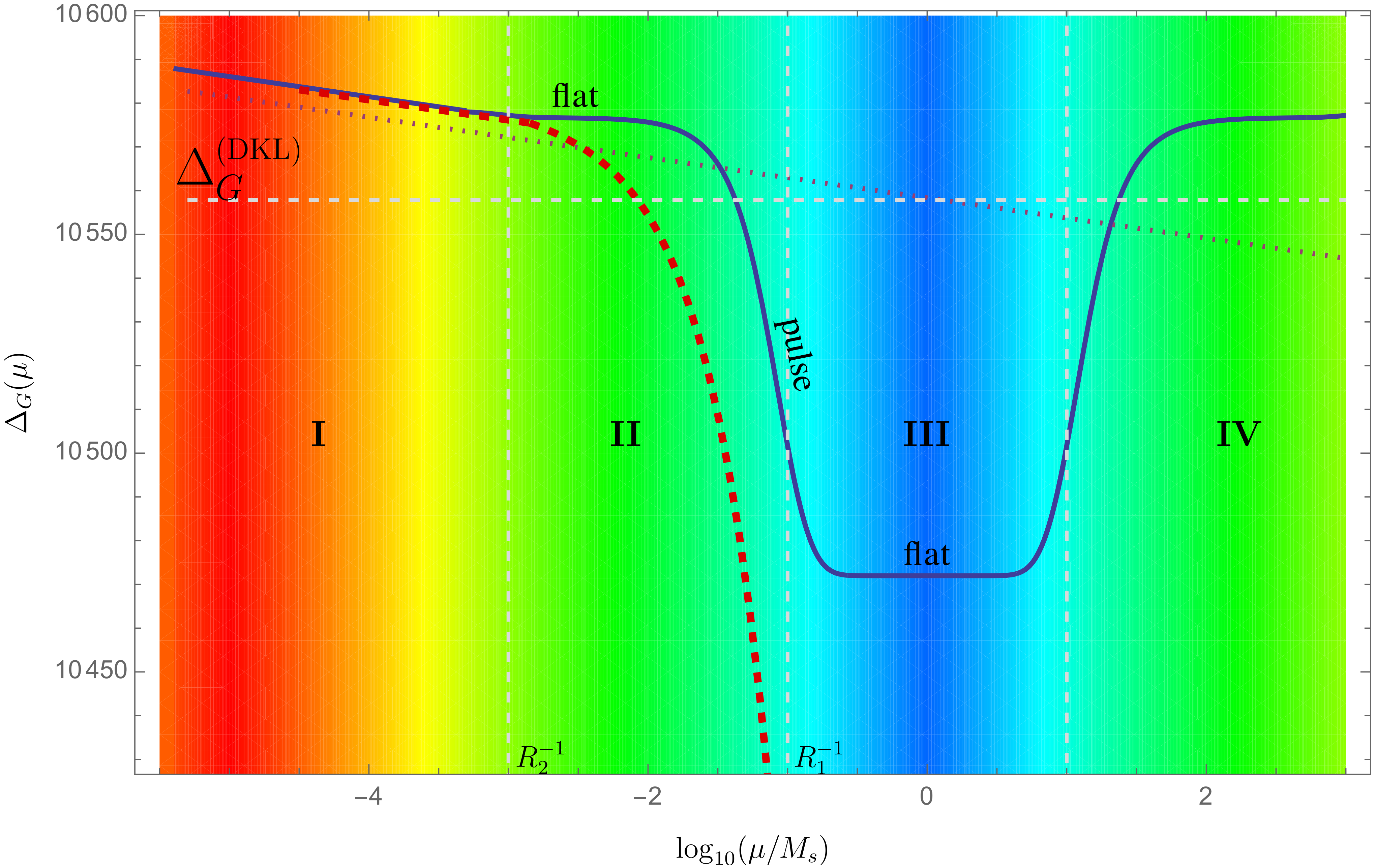
Using such a regulator cut-off function with a 2-torus volume factor we can compare $\Delta_G(\mu)$ with the famous result of Dixon, Kaplunovsky and Louis, but recovering energy dependence and the EFT …

SAA, Dienes, Nutricati

Rectangular torus:

$T_2 = R_2 R_1 = 10000$

$U_2 = R_2/R_1 = 100$



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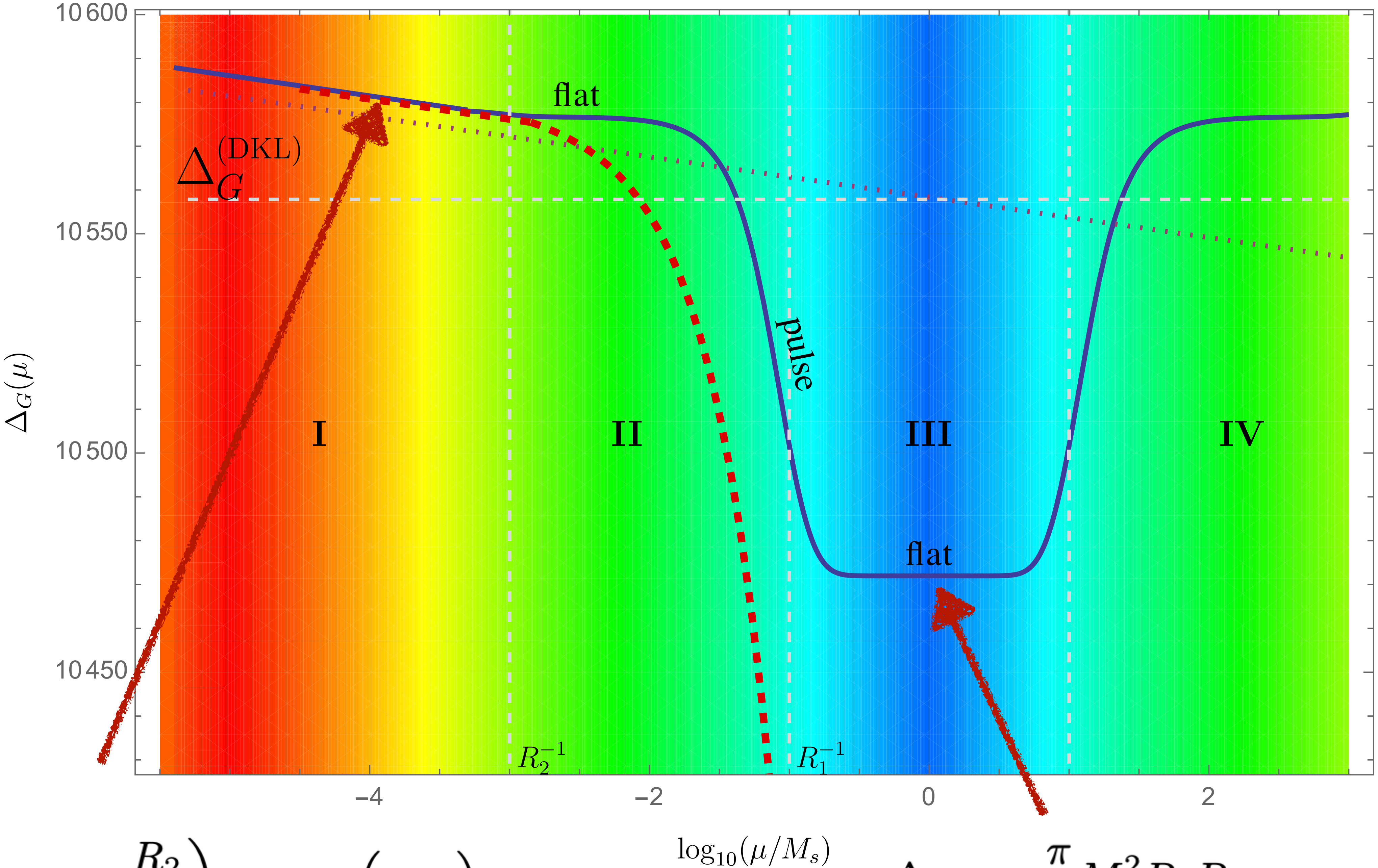
SAA, Dienes, Nutricati

Rectangular torus:

$T_2 = R_2 R_1 = 1000$

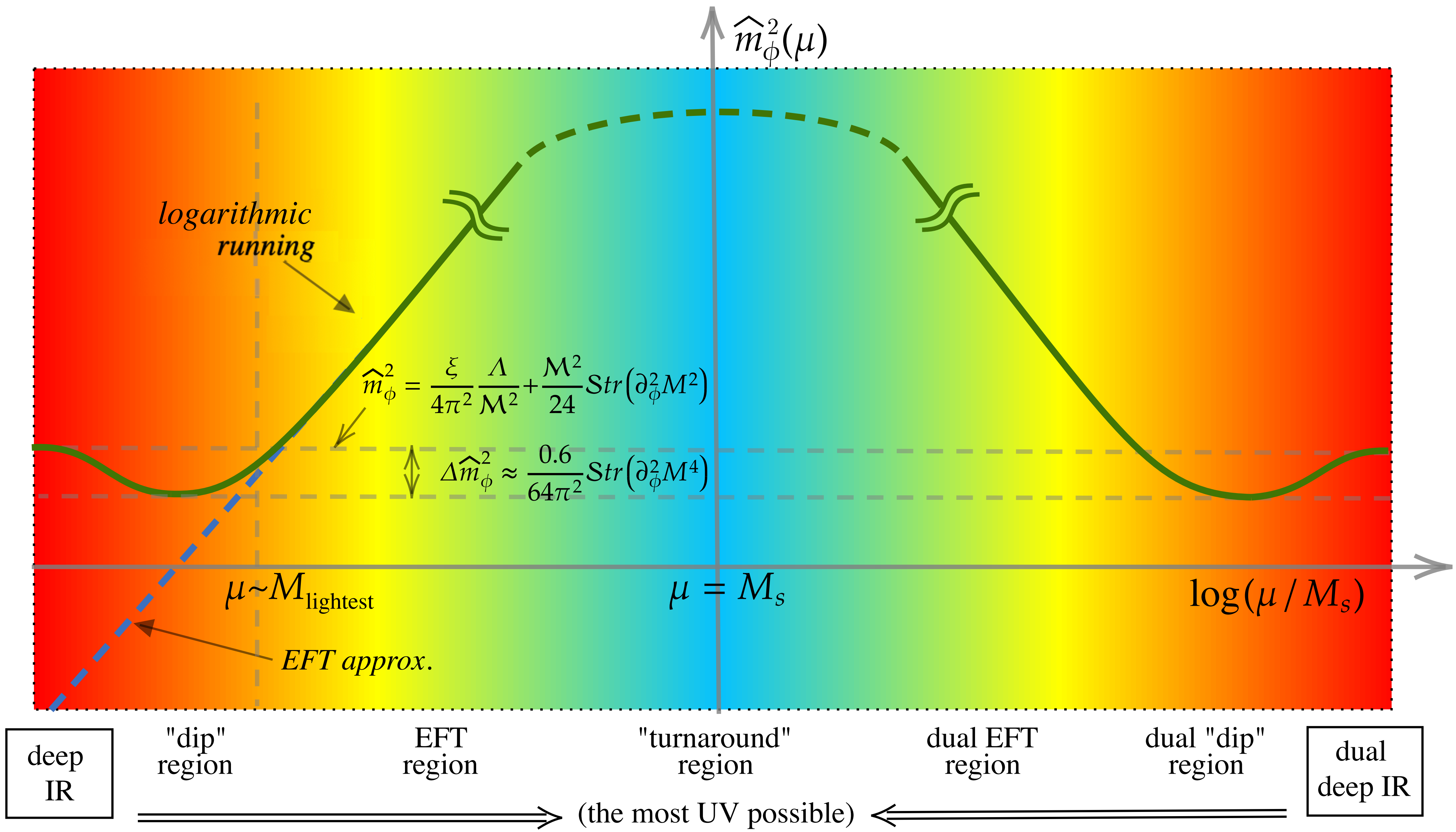
$U_2 = R_2/R_1 = 20$



$$\Delta_G \approx \frac{\pi}{3}\left(M_s^2 R_1 R_2 + \frac{R_2}{R_1}\right) - 2\log\left(\mu R_2\right)$$

$$\Delta_G \approx \frac{\pi}{3} M_s^2 R_1 R_2$$

Similarly we can get a scale dependent Λ … $\widehat{\Lambda}(\mu)$ and thus a stringy Coleman-Weinberg potential at $\mu \lesssim 1/R$ but it has complete $\mu \to M_s^2/\mu$ symmetry



$$\widehat{\Lambda}(\mu) \to \frac{1}{24}\mathcal{M}^2 \mathrm{Str} M^2 - \mathrm{Str}_{M \lesssim \mu}\left[\frac{M^4}{64\pi^2}\left(\log c\frac{M^2}{\mu^2} + c'\mu^4\right)\right]$$

Summary

- Using various novel techniques we learnt how an EFT emerges from a UV/IR mixed theory
- In a 4D theories this requires constraints which become more and more severe when there are decompactification limit
- Consistent theories already "know" they can decompactify
- A definition of energy scale consistent with UV/IR mixing implies scale invariance around $\mu = M_s$.
- This explains why for example we often found scale-invariant (e.g. $\mathcal{N} = 4$ SUSY sectors) when doing model building — but this is really to do with decompactification - it applies just the same in non-SUSY theories
- Potential implications for Dynamical Dark Matter and also "dark dimension" scenarios
- Phenomenological consequences - no power law running - Hagedorn behaviour and thermal duality?
- Removes "technical hierarchies": i.e. all the heavy modes yield a constant piece that may be large but which is always separated from the EFT modes.