What’s inside a black hole ?

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אוניברסיטת בן-גוריון

RB, Medved, Yagi, Sherf, Zigdon
Avitan, Shindelman, Simhon
2015 → --

- Q:What’s inside a large astrophysical BH?

  Growing interest motivated by GW experiments

- A1: We’ll never know – it’s inside the horizon
- A2: An ultra-compact BH-like object – “BH mimicker”

- Q: How can we tell?
- A: Emitted GW when they collide (not today)

Is GR right ?

- Yes, but …
- All tests to date are consistent with the GR predictions
- But, … Tests probe a region a few R_S away from the horizon

Deviations from GR ?

- Two perspectives:
  - Quantify how well GR does, “BSM”
  - Discover deviations from GR, may indicate a need for a new framework for describing BHs – My preference: string theory

➔Need consistent models which can describe a

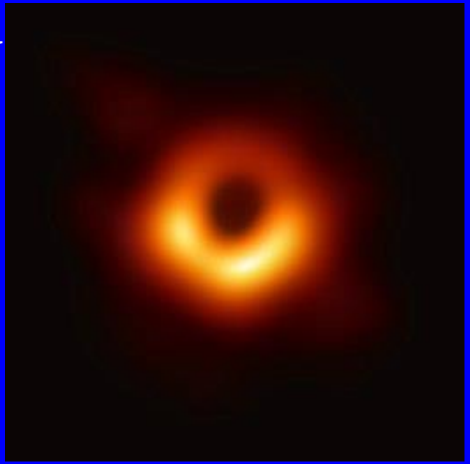
“black hole mimicker” – an object whose exterior geometry resembles that of a GR BH but differs in its interior geometry and matter composition

Quantum mechanics resolves BH singularity?

- **Common expectations:**
- Quantum effects relevant for curvature @ Planck scale → classical description changes only near singularity
- Horizon curvature extremely small →

  no quantum effects @ horizon scales

  Scale separation → horizon, singularity decoupled



This talk: indications to the contrary

Idea: resolving BH singularities requires

- **Horizon-scale** internal quantum structure

  Can be “mimicked” by

- **Horizon-scale** modifications to geometry

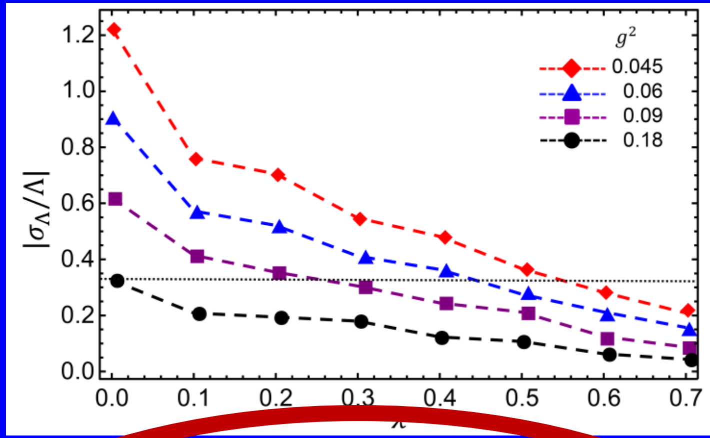
The case for horizon-scale corrections :

- **Assumption 1:**
- BHs are nonsingular states, do not collapse under their own gravity
- **Assumption 2:**
- Strong quantum effects “smear” the would-be singularity over horizon-size length scales
- **Consequence 1:** *strings keep popping up*
- Characteristic scale of excitations is the horizon (not Planck)
- **Consequence 2:** Self-consistency → significant departure from semiclassical gravity + exotic matter beyond the standard model

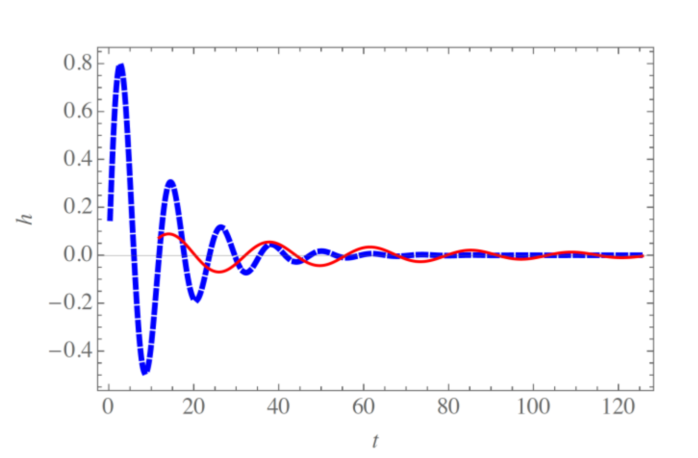
- What can be observed when two BH’s collide ?

RB, Medved, Yagi ’17
RB, Sherf ’21
RB, Medved, Shindelman, 2304.04984
Avitan, RB, Sherf, 2306.00173

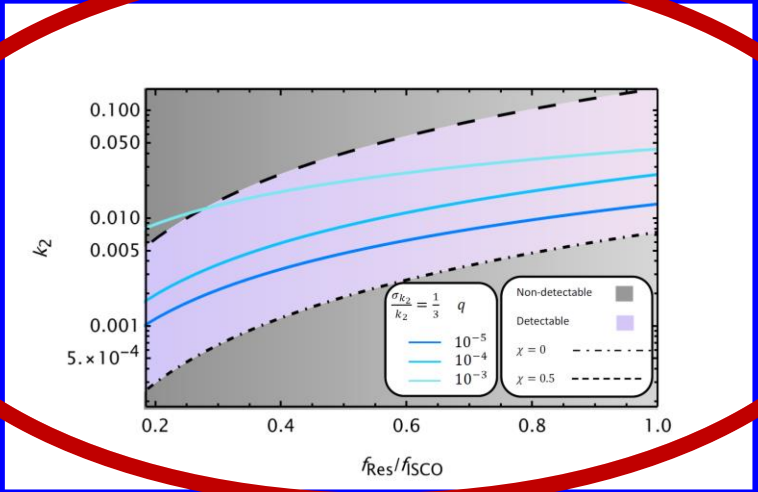
“Large” Love #



Additional GWs, longer decay time & lower amplitude than the leading signal.



Resonant excitation of internal structure



Plan

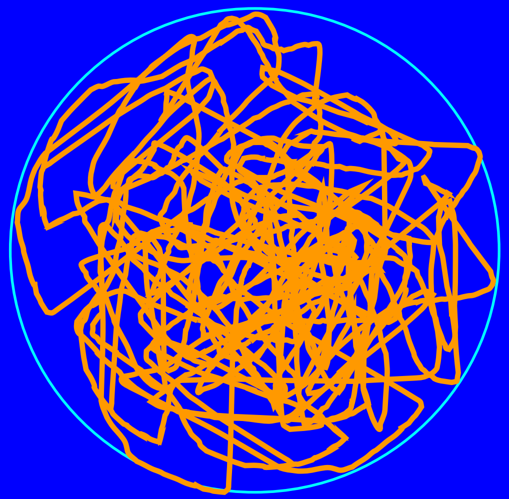
Singularity resolution by:

- **Maximal entropy on horizon scale**
- Maximally negative pressure on horizon scale
  - Static frozen star sourced by a “string fluid”
  - Rotating frozen star sourced by a “string fluid” (time allowing)
- Stability and internal structure (not today)

BH as a bound state of highly excited strings

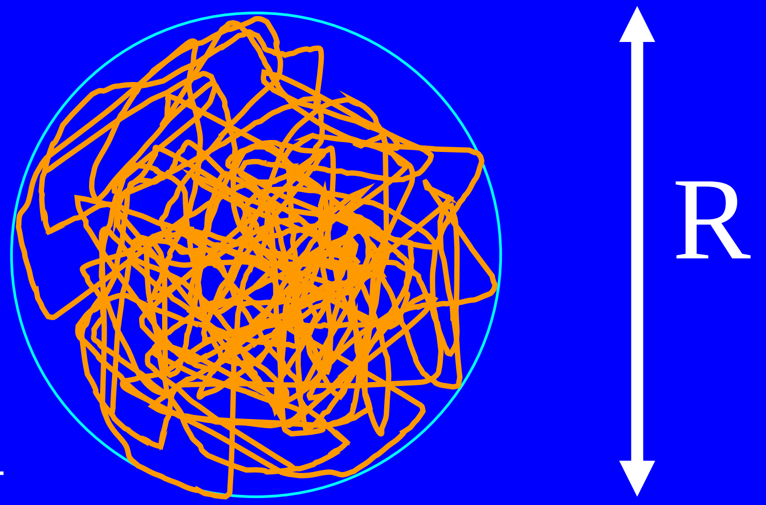
RB+Medved '16

"neutral fuzzball", "string star"



Maximal entropy prevents collapse to a singularity

RB+Medved, 1505.07131

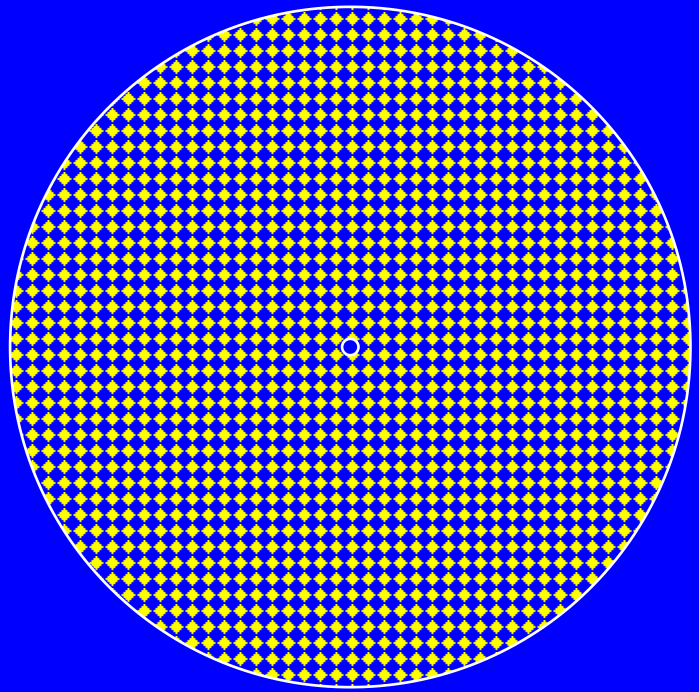


- Entropy ~ L/ℓ_s
- For their size to decrease strings must cut themselves ➔ entropy decreases exponentially

- Each sphere (almost) a horizon
- Gravity strongly coupled
- No classical geometry

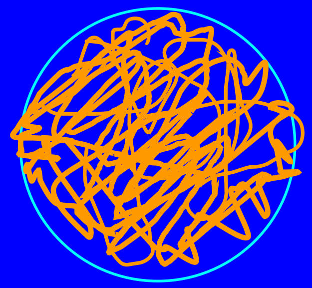
New "quantum pressure" prevents collapse

neutron star



Quantum pressure – Fermi pressure

"string star"
"neutral fuzzball" } **BH**



Quantum pressure – string entropy

Scale invariance ➔ Scale determined by M, not l_P

Plan

Singularity resolution by:

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- **Maximally negative pressure on horizon scale**
  - **Static frozen star sourced by a “string fluid”**
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- Stability and internal structure (not today)

Frozen Star model:

RB, Medved '19
RB, Medved, Simhon '22
RB, Medved, Shindelman, Simhon '23
RB, Medved, Shindelman '23
RB, Medved, '23
RB, Medved, Simhon, '23
RB, Medved, Simhon, '24
RB, Medved, '24
RB, Medved, Simhon, to appear
RB, Medved, Shindelman, to appear

Horizon-scale modifications to geometry

$$p_r = -\rho = -\frac{1-\varepsilon^2}{r^2 l_P^2}$$

$$2m(r) = r\left(1-\varepsilon^2\right)$$

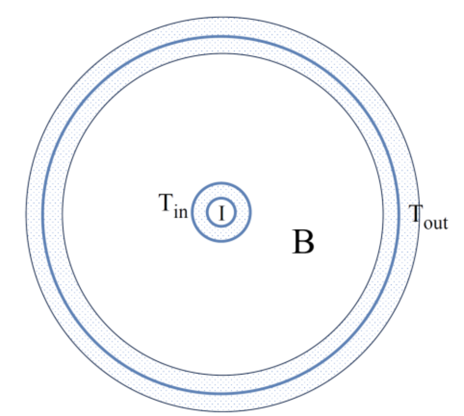
$$ds^2 = -\varepsilon^2 dt^2 + \frac{1}{\varepsilon^2} dr^2 + r^2 d\Omega_2^2 \ .$$

$$R \approx 2M(1+\varepsilon^2)$$

- Each sphere (almost) a horizon
- Gravity weakly coupled
- Regular classical geometry

Exactly Schwarzschild for r > R

Needs smoothing @ surface and core ✓



Maximally negative pressure prevents collapse

Horizon-scale modifications to geometry

$$-p_{radial} = \rho = \frac{1-\varepsilon^2}{r^2 l_P^2}$$

$$p_\perp = 0$$

General Relativistic Fluid Spheres

H. A. Buchdahl*
Institute for Advanced Study, Princeton, New Jersey
(Received June 16, 1959)

$$\Delta = 1 - 2M/R \geq 1/9 \qquad (1.1)$$

$$R \approx 2M(1+\varepsilon^2)$$

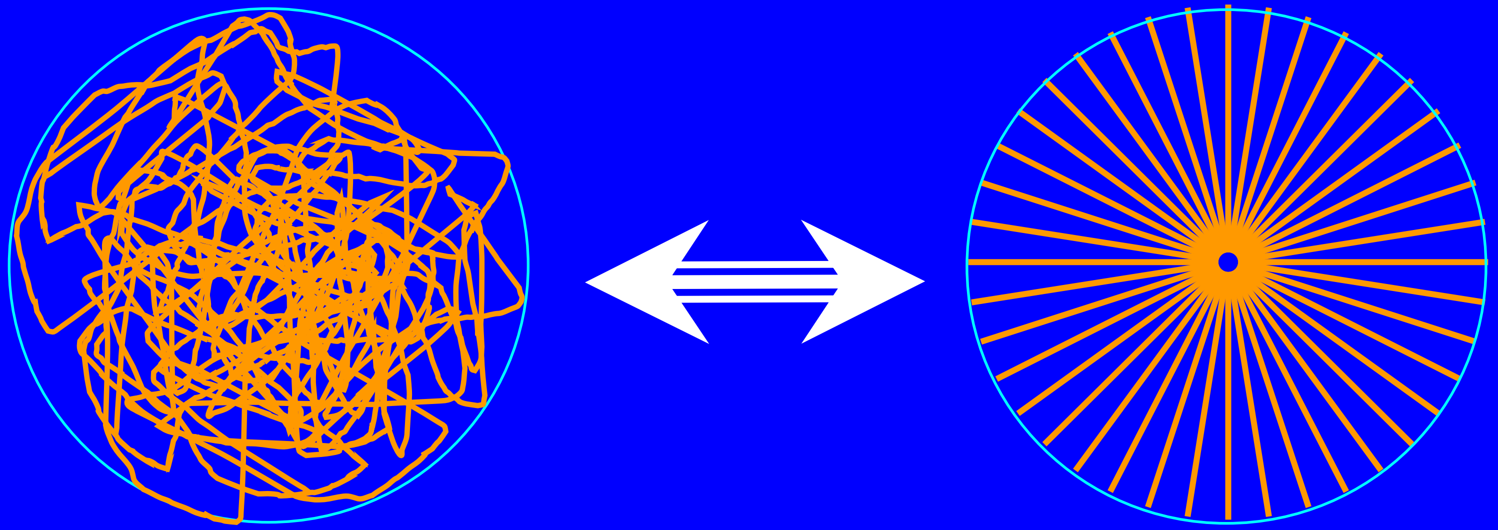
$$\varepsilon^2 \ll 1$$

$$R \geq 2M \times \frac{9}{8}$$

Looks EXACTLY like a BH from the outside

Frozen star model

Radial rigid strings



Each sphere (almost) a horizon

Totally rigid classical geometry

Sourced by a “string fluid”

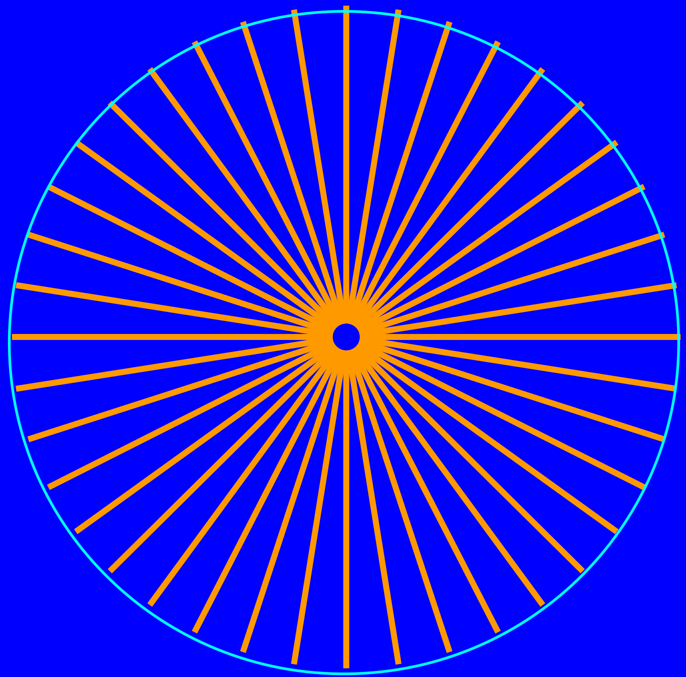
Frozen star : BH sourced by rigid strings

Letelier '79 (in some limit)

(in a Cosmological context)
Letelier '83
Rabinowitz, Guendelman '93
…

- Spherically symmetric collection of rigid strings with tension $1/\alpha'$
- The total mass enclosed in a sphere of radius r,

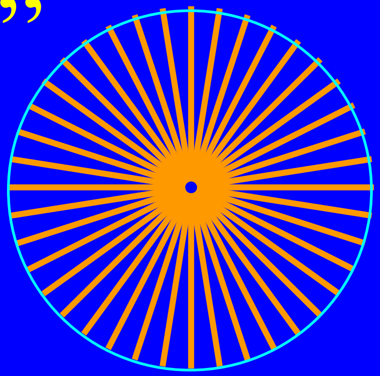
$$m(r) = 4\pi \int_0^r 1/\alpha' = 4\pi/\alpha' \; r$$



- Comparing with $2Gm(r) \cong r$

$$\Longrightarrow \quad \alpha' = 8\pi G \qquad g_s^2 = \frac{1}{8\pi}$$

Frozen star model: Sourced by a "string fluid"



Tachyon effective action

Born-Infeld Lagrangian

End point of tachyon condensation V→0

$$\mathcal{L} \;=\; -V(T)\sqrt{-Det\,(g+2\pi\alpha'\mathcal{F})} \;+\; \sqrt{-g}A^a J_a$$

L= 0 for V → 0 ➔ Hamiltonian

$$D^i \;=\; \frac{\delta\mathcal{L}}{\delta(\partial_0 A_i)}$$

$$\mathcal{H} \;=\; \frac{\delta\mathcal{L}}{\delta(\partial_0 A_i)} - \mathcal{L} \;=\; \frac{1}{2\pi\alpha'}\sqrt{D^i D_i + \mathcal{P}^i\mathcal{P}_i}$$

$$\nabla_i D^i \;=\; J_0$$

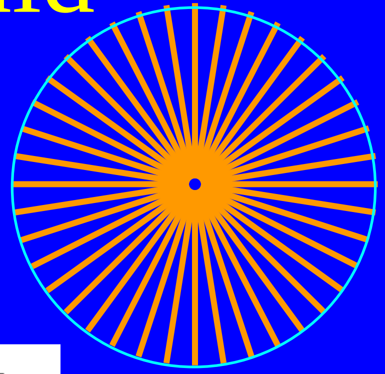
Gibbons, Hori, Yi 0009061
Gibbons 0106059
Yee, Yi 0402027
Sen (Review) 0410103

$$B^i \;=\; \frac{1}{2}\epsilon^{ijk}\mathcal{F}_{jk}$$

$$\mathcal{H} = \sqrt{\mathbf{D}^2 + (\mathbf{D}\times\mathbf{B})^2}$$

$$\mathcal{P}_i \;=\; -\mathcal{F}_{ij}D^j \;=\; \left(\vec{D}\times\vec{B}\right)_i$$

Frozen star model: Sourced by a "string fluid"



End point of tachyon condensation

$$\mathcal{H} = \sqrt{\mathbf{D}^2 + (\mathbf{D} \times \mathbf{B})^2}$$

$$\mathcal{L}' = \mathcal{H} - \frac{1}{2} F_{ij} K^{ij}$$

antisymmetric tensor $K_{\mu\nu}$

(Hodge) Dual Lagrangian

$$\mathcal{L}' = \frac{1}{2\pi\alpha'} \sqrt{-\frac{1}{2} \mathcal{K}^{ab} \mathcal{K}_{ab}}$$

Nielsen, Olesen '73

"Gravitational confinement"

$$K \equiv D_i \, dt \wedge dx^i + \frac{1}{2} K_{ij} dx^i \wedge dx^j$$

$$D_i \equiv \mathcal{K}_{ti}$$

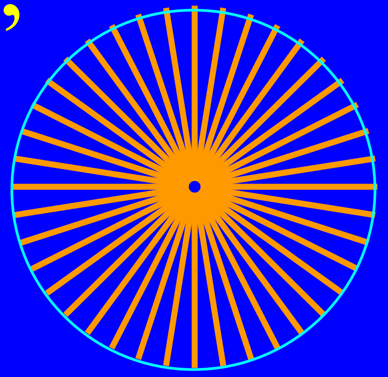
$$K_{\mu[\nu} K_{\kappa\lambda]} = 0$$

$$\nabla^\mu K_{\mu[\nu} K_{\kappa\lambda]} = 0$$

$$\nabla^\mu \mathcal{K}_{\mu\nu} = 0$$

$\mathcal{K}$ ~ 2D area element

Frozen star model: Sourced by “string fluid”



$$\mathcal{L}' = \frac{1}{2\pi\alpha'}\sqrt{-\frac{1}{2}\mathcal{K}^{ab}\mathcal{K}_{ab}}$$

$$T_{\mu\nu} = K_\mu{}^\lambda K_{\nu\lambda}/\sqrt{-\tfrac{1}{2}K^2}$$

$$K \equiv D_i\, dt \wedge dx^i + \frac{1}{2} K_{ij} dx^i \wedge dx^j$$

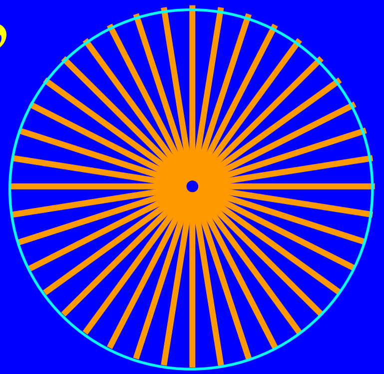
$$\nabla_i D^i = J_0$$

$$T_{00} = \frac{1}{2\pi\alpha'}\frac{D^i D_i}{\sqrt{D^i D_i - \frac{1}{2}K^{ij}K_{ij}}} = \mathcal{H}\,,$$

$$T_{0i} = \frac{1}{2\pi\alpha'}\frac{D^j K_{ij}}{\sqrt{D^i D_i - \frac{1}{2}K^{ij}K_{ij}}} = -\mathcal{P}_i\,,$$

$$T_{ij} = \frac{1}{2\pi\alpha'}\frac{-D^i D_j + K_i{}^k K_{jk}}{\sqrt{\pi^i\pi_i - \frac{1}{2}K^{ij}K_{ij}}} = \frac{1}{2\pi\alpha'}\frac{-D_i D_j + \mathcal{P}_i\mathcal{P}_j}{\mathcal{H}}$$

Frozen star model: Sourced by "string fluid"



$$T_{00} = \frac{1}{2\pi\alpha'} \frac{D^i D_i}{\sqrt{D^i D_i - \frac{1}{2} K^{ij} K_{ij}}} = \mathcal{H} \,,$$

$$T_{0i} = \frac{1}{2\pi\alpha'} \frac{D^j K_{ij}}{\sqrt{D^i D_i - \frac{1}{2} K^{ij} K_{ij}}} = -\mathcal{P}_i \,,$$

$$T_{ij} = \frac{1}{2\pi\alpha'} \frac{-D^i D_j + K_i{}^k K_{jk}}{\sqrt{\pi^i \pi_i - \frac{1}{2} K^{ij} K_{ij}}} = \frac{1}{2\pi\alpha'} \frac{-D_i D_j + \mathcal{P}_i \mathcal{P}_j}{\mathcal{H}}$$

$$p_r = g^{rr} g^{tt} K_{tr}^2 / |K_{tr}| = -\rho$$

$$p_\perp = 0$$

Simplest solution: only electric flux in the radial direction – electric monopole

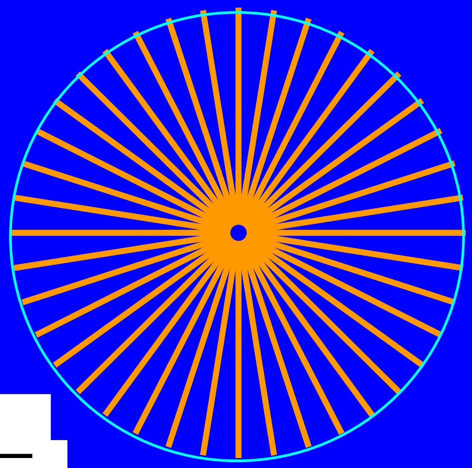
$$D_r \equiv K_{tr} \neq 0 \,, P_i = 0, K_{ij} = 0$$

The **frozen star** model

$$K_{tr} = (1 - \varepsilon^2)/(8\pi\, r^2)$$

Needs smoothing @ surface and core

Frozen star model:
BH sourced by “string fluid”



$$\sqrt{D^r D_r} = \frac{\alpha'}{4G}\frac{1}{r^2}$$

$$\rho_{FS} = \frac{1}{8\pi G}\frac{1-\varepsilon^2}{r^2} = \frac{1}{2\pi\alpha'}\sqrt{D^r D_r}$$

$$\nabla_r D^r = q_{core}\delta^{(3)}(\vec{r}\,) + \sigma_{surface}\delta(r-R)$$

$$q_{core} = \pi\frac{\alpha'}{G}$$

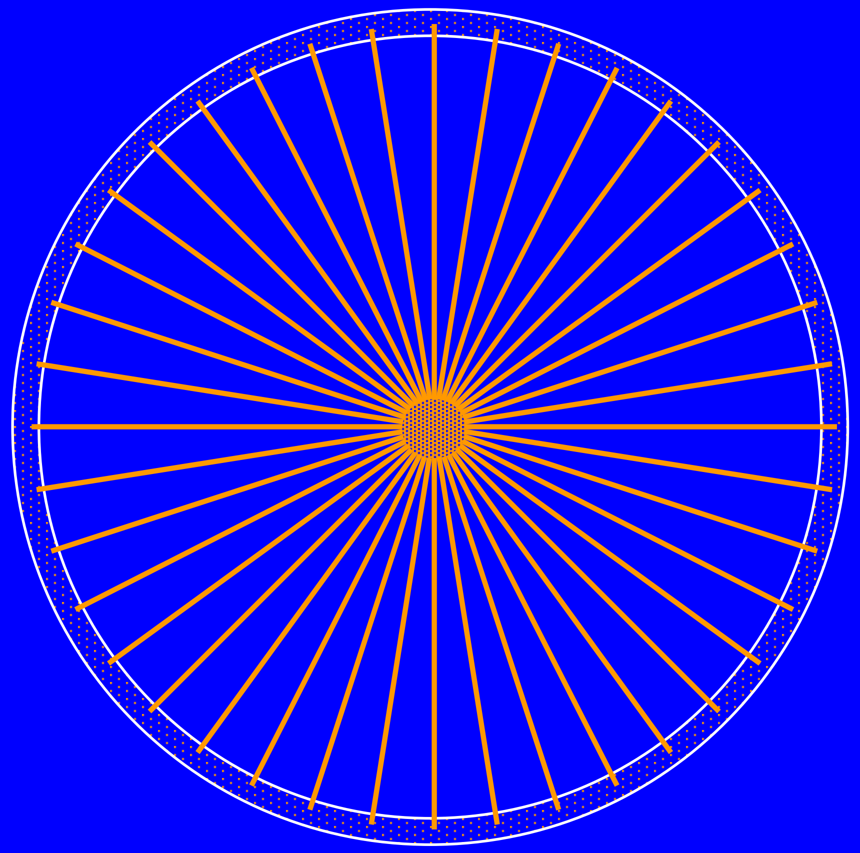
$$q_{surface} = -\pi\frac{\alpha'}{G}$$

Stability is ensured by fixing the mass

Frozen star :
a regular BH mimicker

- Einstein equations ✓
- Completely regular ✓
- Arbitrary compactness ✓
- Black ✓
- Null energy condition ✓
- No pathologies ✓
- Ultrastable (not today) ✓
- Temperature, Entropy ✓

AFAIK – first example



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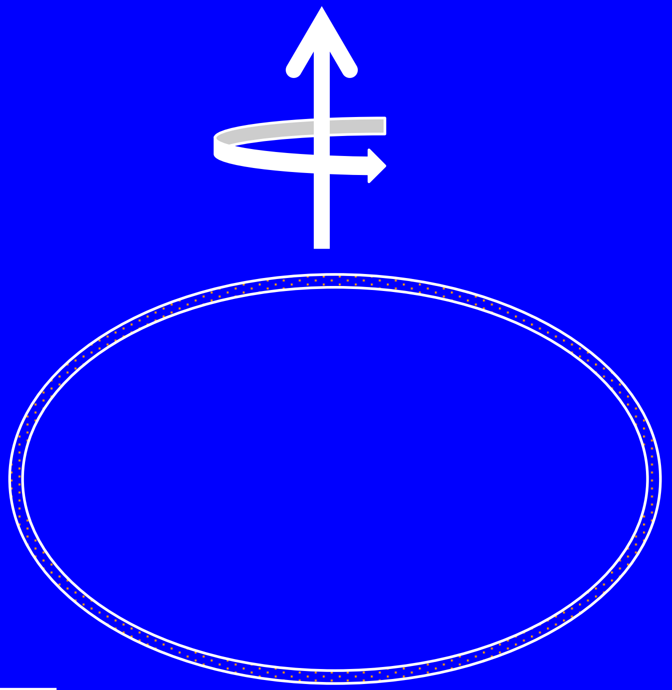
Reminder: Kerr BH



$$ds^2\Big|_{BL} = -\frac{\Delta - a^2\sin^2\theta}{\Sigma}dt^2 + \frac{\Delta}{\Sigma}dr^2 - \frac{4Mar\sin^2\theta}{\Sigma}dtd\phi + \Sigma d\theta^2 + \left[r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right]\sin^2\theta d\phi^2 \,,$$

$$\Sigma = g_{\theta\theta} = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 + a^2 - 2Mr$$

$$\Omega_H = \frac{a}{r_+^2 + a^2} = \frac{a}{2Mr_+}$$

$$R^2 = r^2 + a^2$$

$$A = 4\pi R_+^2$$

ZAMO – co-rotating observer

$$\chi^a = \xi^a + \Omega_H \psi^a = (1, 0, 0, \Omega_H)$$

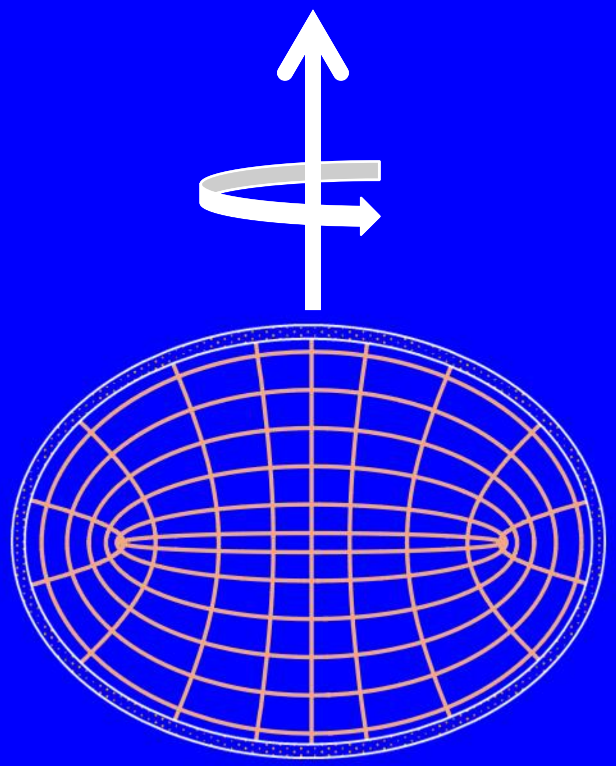
Near horizon geometry

$$ds^2 \to -\Delta/\Sigma\, d\chi^2 + \frac{dr^2}{\Delta/\Sigma} + \Sigma\, d\theta^2 + \frac{(r^2+a^2)^2}{\Sigma}\sin^2\theta\ d\phi^2$$

RB, Medved, 2310.16467

RB, Medved, Simhon, to appear

Rotating frozen star



Near horizon geometry

$$ds^2 \rightarrow -\Delta/\Sigma\, d\chi^2 + \frac{dr^2}{\Delta/\Sigma} + \Sigma\, d\theta^2 + \frac{(r^2+a^2)^2}{\Sigma}\sin^2\theta\, d\phi^2$$

$$ds^2 = -\varepsilon^2 dt^2 + \frac{1}{\varepsilon^2} dr^2 + r^2 d\Omega_2^2\ .$$

$$\Delta/\Sigma \rightarrow \varepsilon^2$$

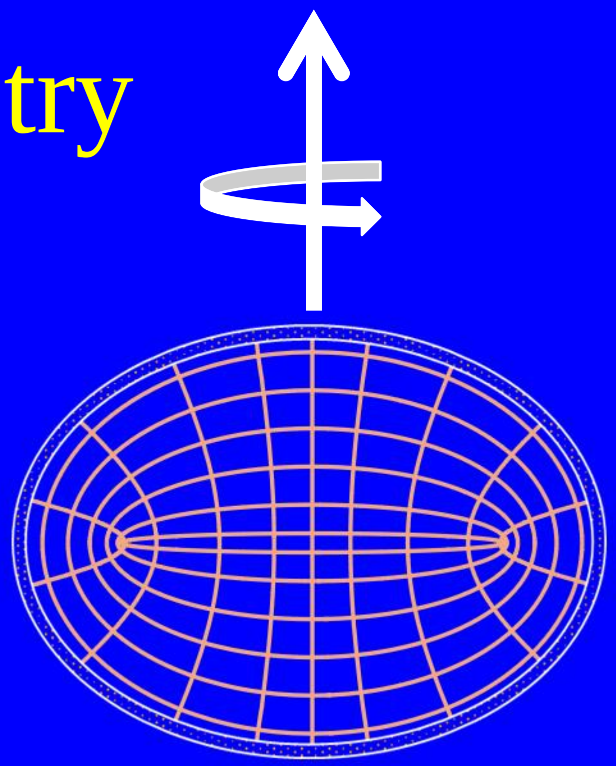
Near horizon geometry extended throughout

$$ds^2 \rightarrow -\varepsilon^2 d\chi^2 + \frac{dr^2}{\varepsilon^2} + \Sigma d\theta^2 + \frac{(r^2+a^2)^2}{\Sigma}\sin^2\theta d\phi^2$$

Everywhere in the interior

Rotating frozen star: symmetry

$$ds^2 \;\to\; -\varepsilon^2 d\chi^2 + \frac{dr^2}{\varepsilon^2} + \Sigma d\theta^2 + \frac{(r^2+a^2)^2}{\Sigma}\sin^2\theta d\phi^2$$



Only radial trajectories

$$\mathcal{E}^2 - (u^r)^2 \;=\; \varepsilon^2\left[g_{\theta\theta}(u^\theta)^2 + g_{\phi\phi}(u^{\phi\phi})^2 + k\right]$$

$$\mathcal{E}^2 - (u^r)^2 \;=\; \mathcal{O}[\varepsilon^2]$$

Carter constant vanishes identically

$$K^{ab} \;=\; \Sigma\, l^{(a} n^{b)} + r^2 g^{ab}$$

$$l^a \;=\; \left(\tfrac{1}{\varepsilon^2}, 1, 0, 0\right)\;,\quad n^a \;=\; (1, -\varepsilon^2, 0, 0)$$

$$C \;=\; K_{ab}u^a u^b \;=\; 0$$

$$\begin{aligned}\varepsilon^2 p_r^2 + \frac{r^2}{(a^2+r^2)^2}L^2 - \frac{1}{\varepsilon^2}\mathcal{E}^2 + k \;&=\; -\widetilde{C}\\ \frac{1}{\Sigma}p_\theta^2 + \frac{a^2\cos^2\theta}{(a^2+r^2)^2}L^2 \;&=\; +\widetilde{C}\end{aligned}$$

$\tilde{C}$ is the Carter constant per square mass

Rotating frozen star: mass

$$G^t{}_t = G^r{}_r = -\frac{(r^2+a^2)(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^3}$$

$$p_r = -\rho$$

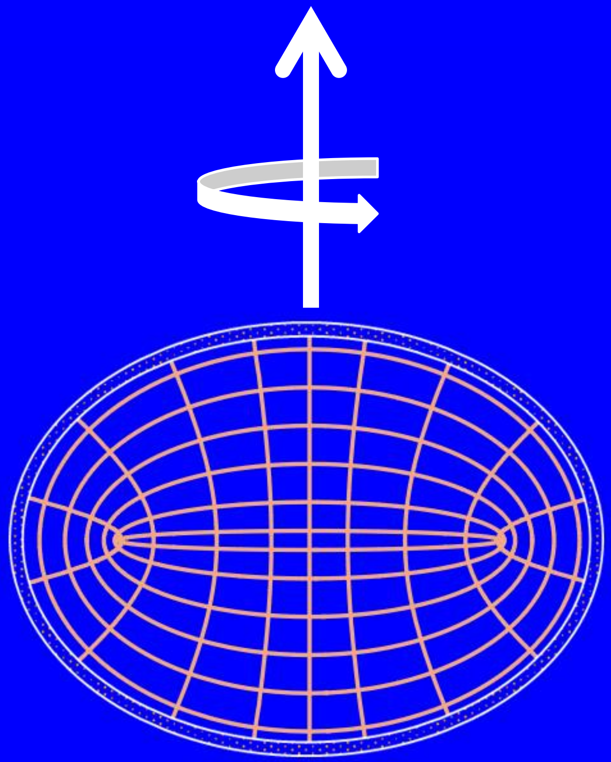
Units: $R_+^2 = r_+^2 + a^2 = 1$

Restored at the end

$$8\pi M_0 = -2\pi \int_0^1 dR \int_0^\pi d\theta \sqrt{g_{\theta\theta} g_{\phi\phi}}\, G^t{}_t$$

$$2M_0 = \frac{1}{2}\int_0^1 dR\, R^4 \int_0^\pi d\theta \sin\theta \frac{(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^3} = 1$$

$$M_0 = \frac{1}{2}R_+ = M_{irr}$$



"Rest mass" = Irreducible mass of the BH

Rotating frozen star: angular momentum

$$ds^2 = \left(\frac{a^2 R^4 \sin^2\theta}{\Sigma} - \varepsilon^2\right) dt^2 - 2a\sin^2\theta \frac{R^4}{\Sigma} dt d\phi + \frac{1}{\varepsilon^2} dr^2 + \Sigma d\theta^2 + \frac{R^4}{\Sigma}\sin^2\theta d\phi^2$$

Not a ZAMO – spacetime rotates@ Ω_H

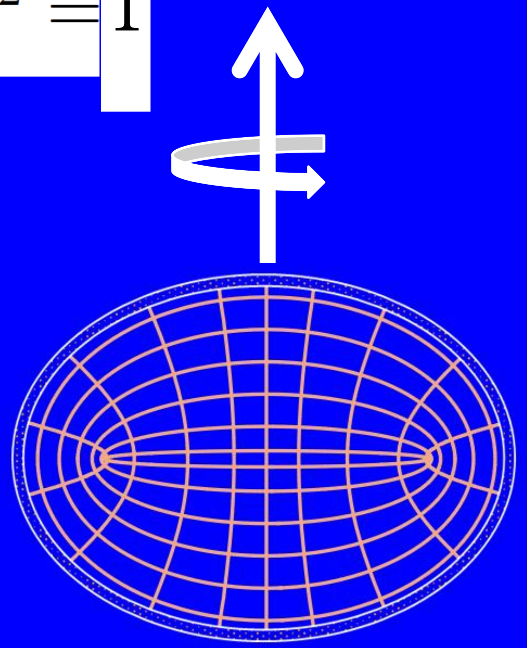
Units: $R_+^2 = r_+^2 + a^2 = 1$

Restored at the end

$$G^t{}_\phi = a\, G^t{}_t$$

$$8\pi J_0 = -2\pi \int_0^1 dR \int_0^\pi d\theta \sqrt{g_{\theta\theta} g_{\phi\phi}}\, G^t{}_\phi$$

$$J_0 = a\, M_{irr}$$



Rotating frozen star

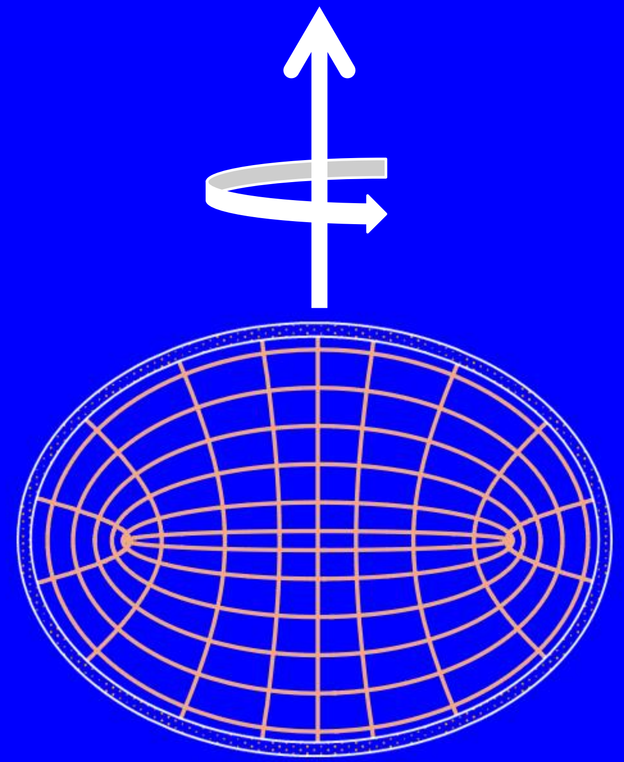
Adding rotational energy in the standard Kerr way using the 1st law

$$M_0 = \frac{1}{2} R_+ = M_{irr}$$

$$a = J_0/M_0$$

$$M^2 = M_{irr}^2 + \frac{1}{4}\frac{J^2}{M_{irr}^2}$$

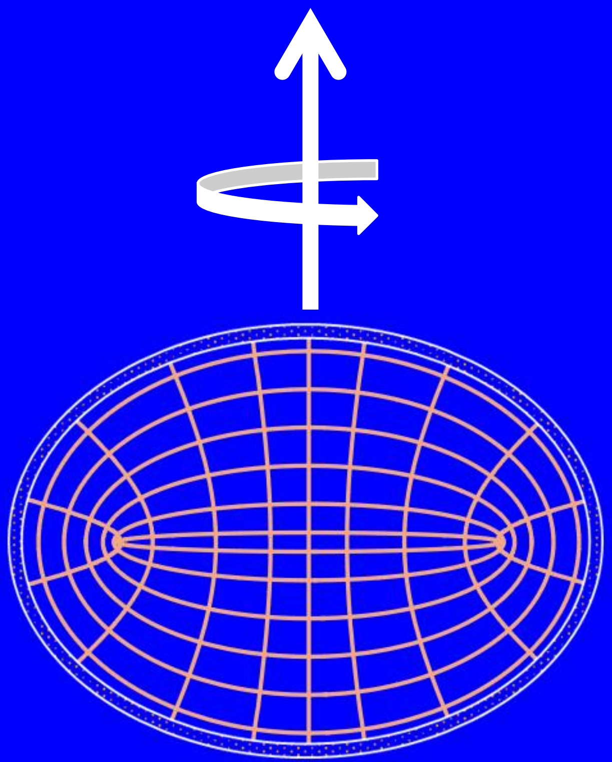
$$J = aM$$



Rotating frozen star: Sourced by “string fluid”

$$G^t{}_t = G^r{}_r = -\frac{(r^2+a^2)\,(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^3}$$

$$p_r = -\rho$$



$$\frac{1}{8\pi G}\frac{R^2\,(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^3} = \frac{1}{2\pi\alpha'}\sqrt{D^r D_r}$$

$$D_r = \frac{\alpha'}{4G}\frac{R^2\,(r^2-3a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^3}$$

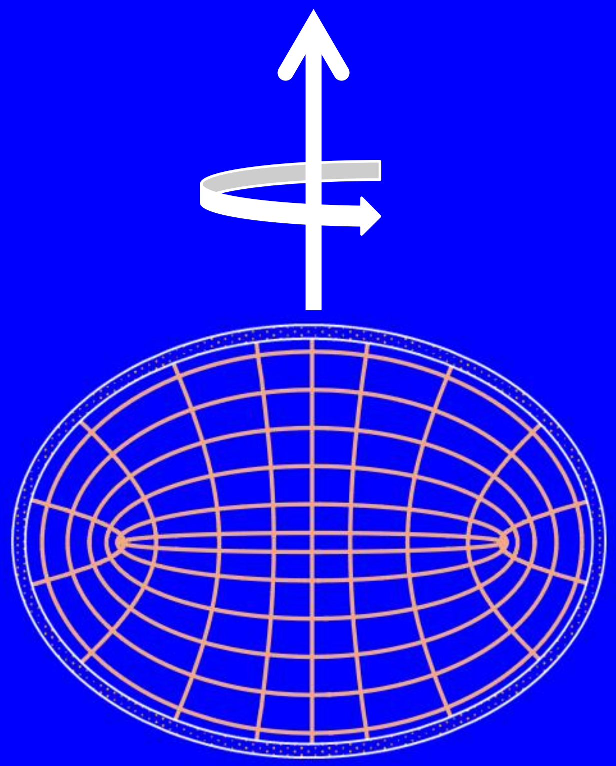
$$\begin{aligned}
T_{00} &= \frac{1}{2\pi\alpha'}\frac{D^i D_i}{\sqrt{D^i D_i - \frac{1}{2}K^{ij}K_{ij}}} = \mathcal{H}\,,\\
T_{0i} &= \frac{1}{2\pi\alpha'}\frac{D^j K_{ij}}{\sqrt{D^i D_i - \frac{1}{2}K^{ij}K_{ij}}} = -\mathcal{P}_i\,,\\
T_{ij} &= \frac{1}{2\pi\alpha'}\frac{-D^i D_j + K_i{}^k K_{jk}}{\sqrt{\pi^i\pi_i - \frac{1}{2}K^{ij}K_{ij}}} = \frac{1}{2\pi\alpha'}\frac{-D_i D_j + \mathcal{P}_i\mathcal{P}_j}{\mathcal{H}}
\end{aligned}$$

The radial direction is a hyperbola

$$\frac{x^2 + y^2}{a^2 \sin^2 \theta} - \frac{z^2}{a^2 \cos^2 \theta} = 1$$

Cross sections of fixed $R = \sqrt{r^2 + a^2}$ are confocal ellipses

$$\frac{x^2 + y^2}{R^2} + \frac{z^2}{r^2} = 1$$



The surface at $r = 0$ is a degenerate ellipsoid whose height (range of the z coordinate) goes to zero with r but whose surface area, $A = 4\pi a^2$, nevertheless remains finite.

$$R(r = 0) = \sqrt{a^2}$$

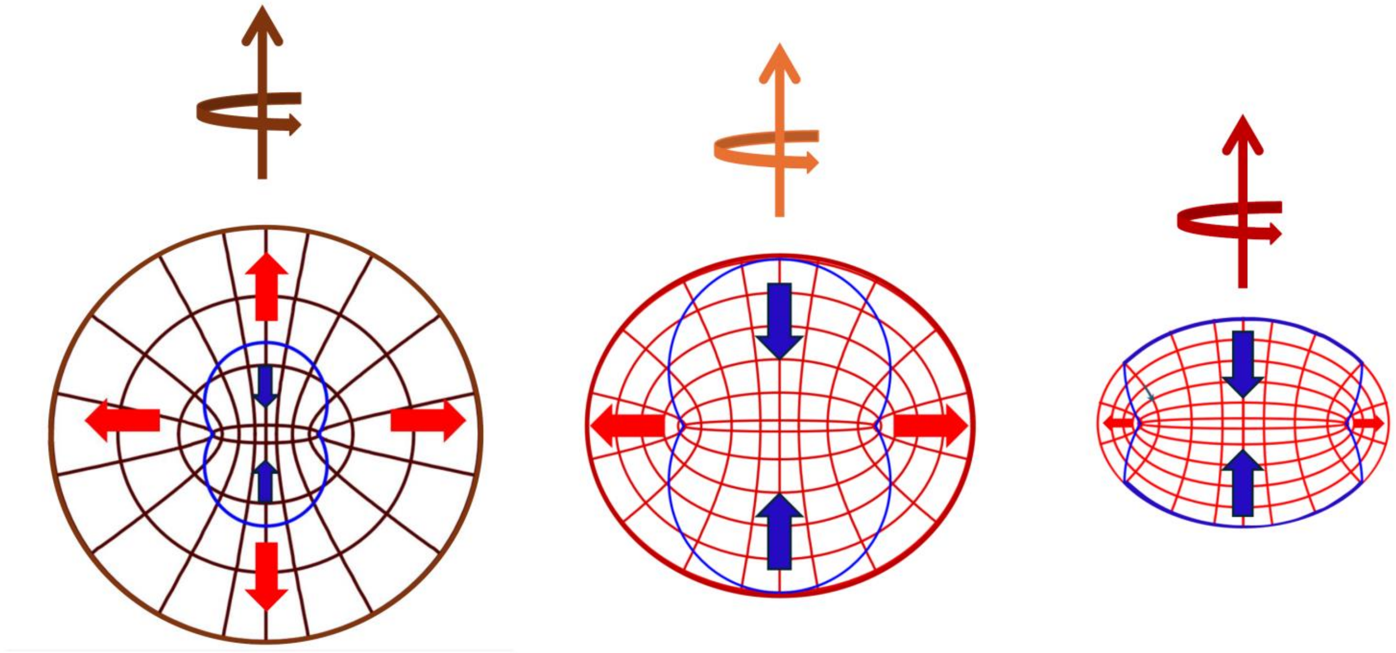
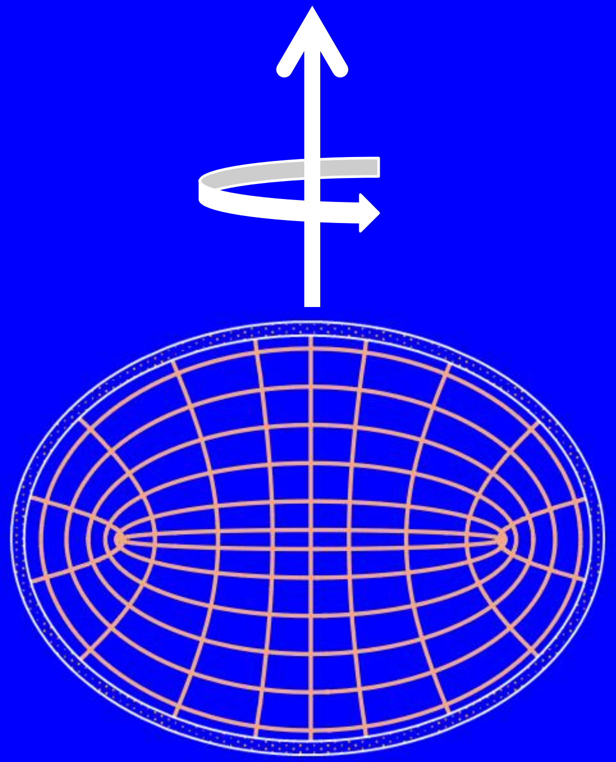


Figure 1: Shown are cross-sections of fixed ϕ of the frozen star for the cases $a = 0.3$ (left), $a = \sqrt{3}/2$ (middle) and $a = 1$ (right). The star is sourced by a fluid of electric-flux lines that either emanate from or end at an ellipsoidal charge distribution in its core or, alternatively, emanate from the blue curve and end either at the outer surface or the center of the star. The flux is directed along the constant-θ hyperbolae. The flux inside the regions bounded by the blue curves is negative, as indicated by the blue, inward-pointing arrows, while the flux outside this region is positive, as indicated by the red, outward-pointing arrows. The outer-surface charge distribution has the same angular distribution as that in the core, but is oppositely charged. The flux lines are rigid and, consequently, the star is ultrastable.

Frozen star :
a regular BH mimicker

- Einstein equations ✓
- Completely regular ✓
- Arbitrary compactness ✓
- Black ✓
- Null energy condition ✓
- No pathologies ✓
- Ultrastable (not today) ✓
- Temperature, Entropy (last year) ✓

AFAIK – first example



What’s inside a black hole ?

RB, Medved, Yagi, Sherf, Zigdon Avitan, Shindelman, Simhon 2015 → --

Ramy Brustein

אוניברסיטת בן-גוריון

- Q:What’s inside a large astrophysical BH?

  Growing interest motivated by GW experiments

- A1: We’ll never know – it’s inside the horizon
- A2: An ultra-compact BH-like object – “BH mimicker”

- Q: How can we tell?
- A: Emitted GW when they collide (if time allows)