MAX PLANCK INSTITUTE
FOR PHYSICS

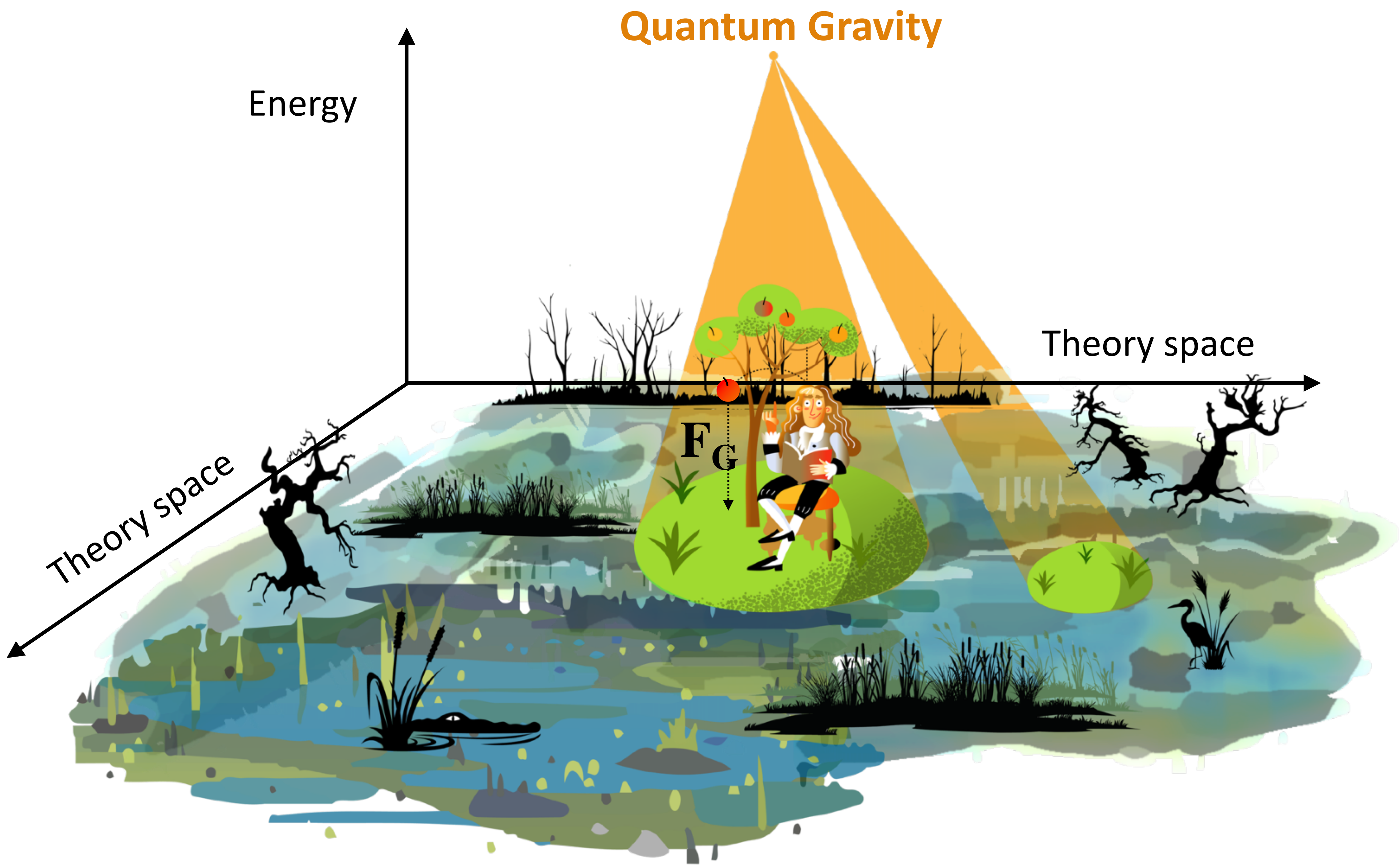
Infinite distances, the scalar potential and Ricci flow

[Saskia Demulder, Dieter Lüst, **TR** 2312.07674 & ongoing]

24TH HELLENIC SCHOOL AND WORKSHOPS ON ELEMENTARY PARTICLE PHYSICS AND GRAVITY
Quantum Gravity, Strings and the Swampland
Corfu, 5.09.2024

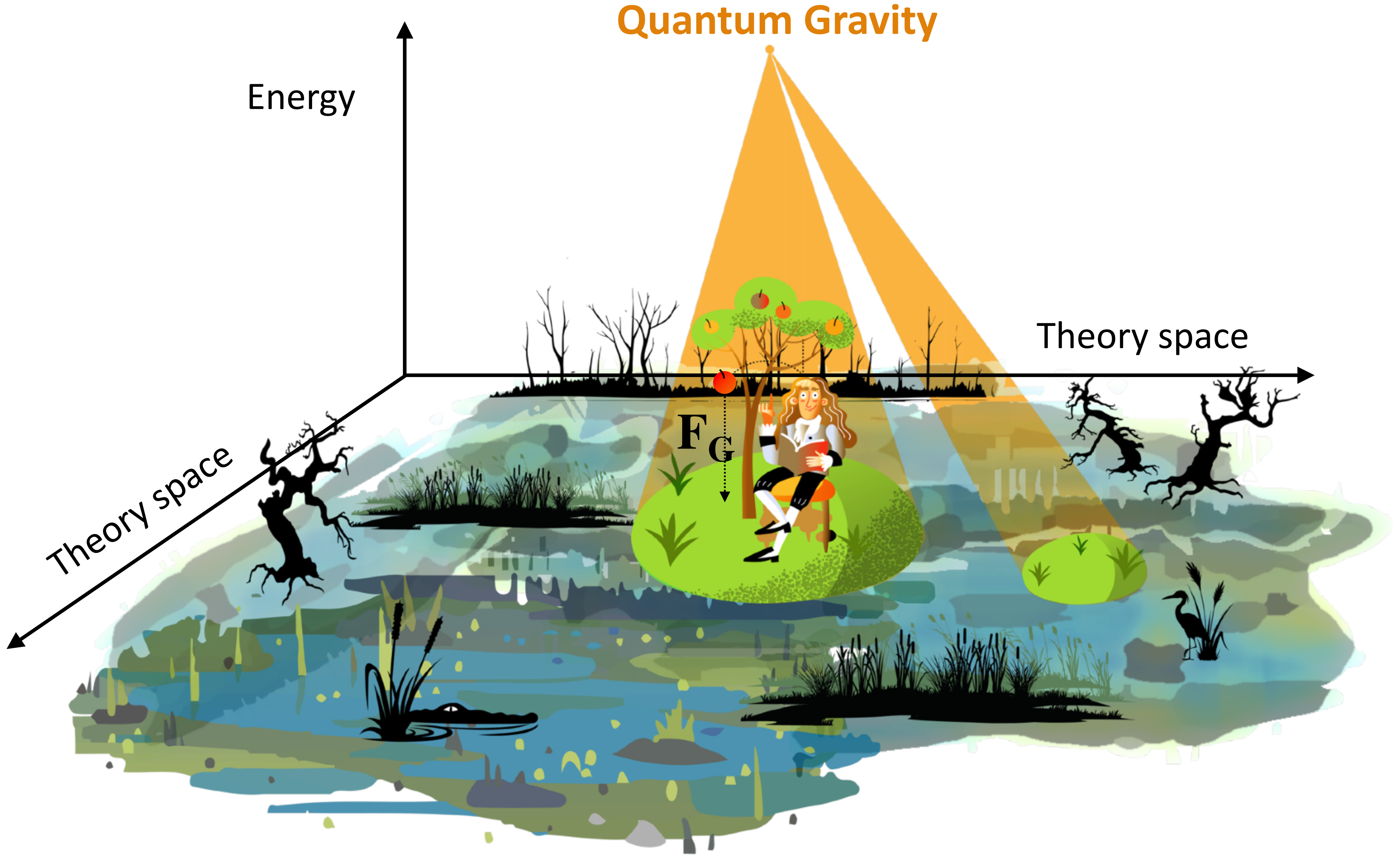
Thomas Raml

Quantum Gravity and the Swampland

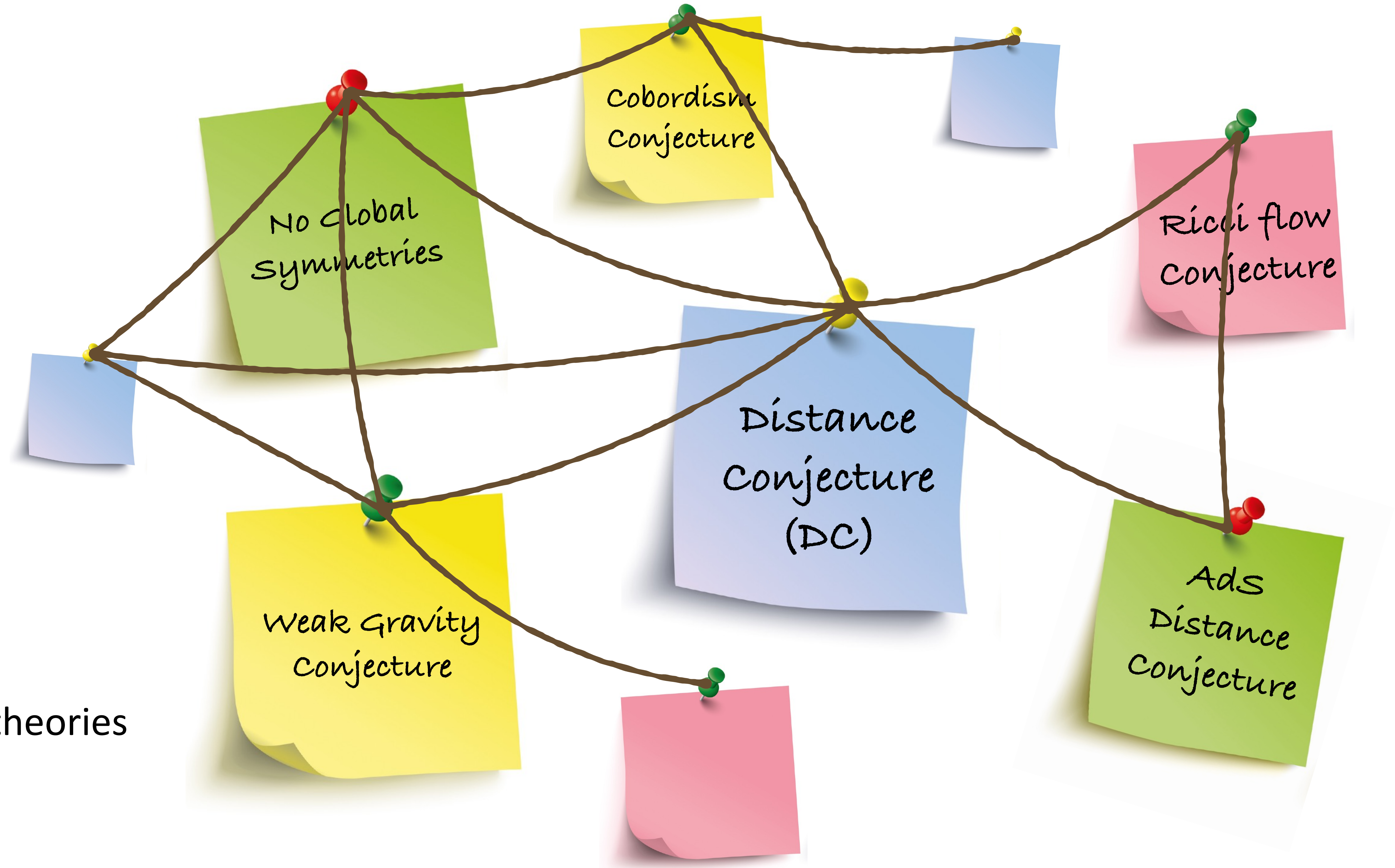


Quantum field theories that emerge in the low energy limit of a quantum gravity theory are very special

Quantum Gravity and the Swampland

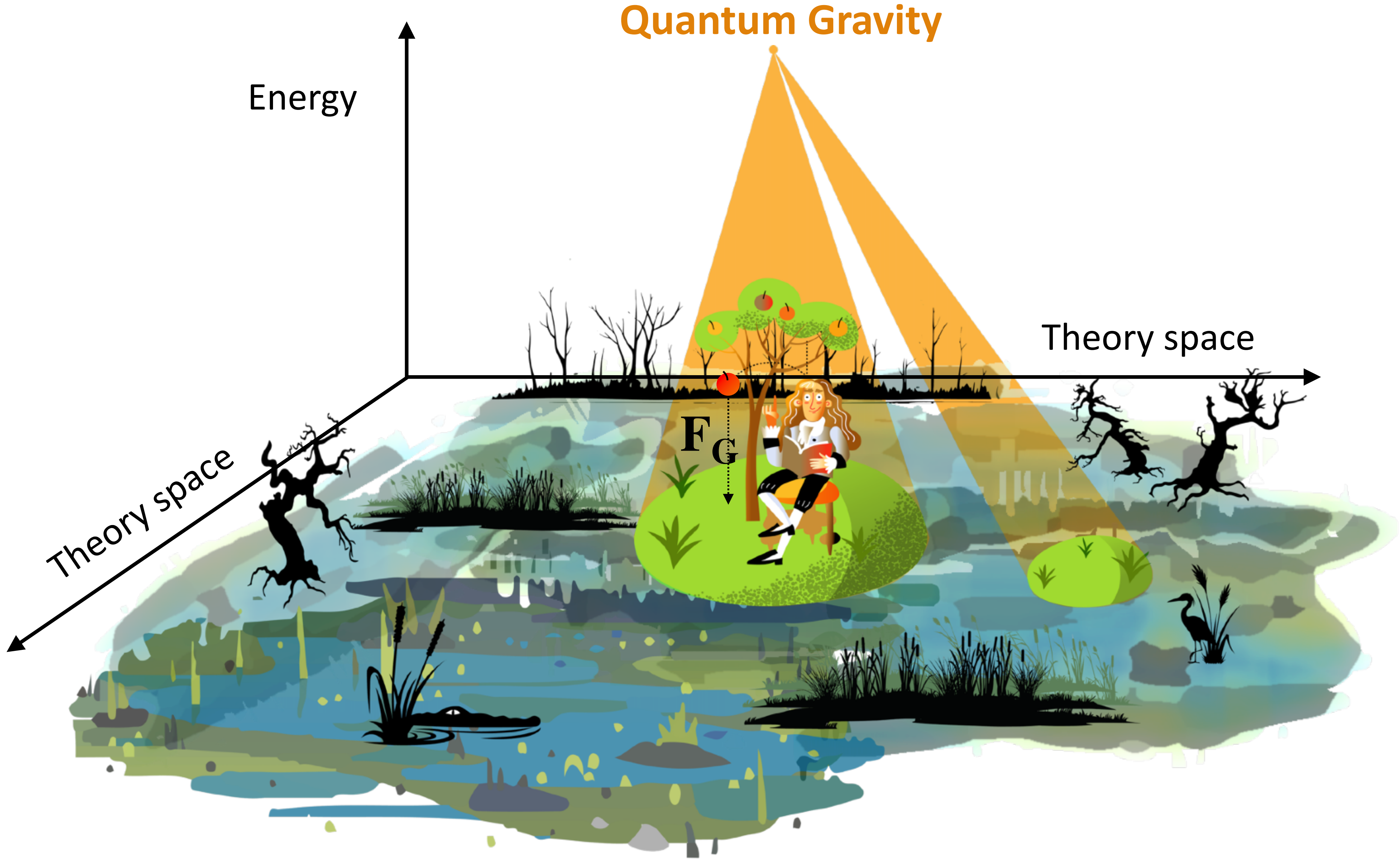


Quantum field theories that emerge **in the low energy limit** of a **quantum gravity theory** are **very special**

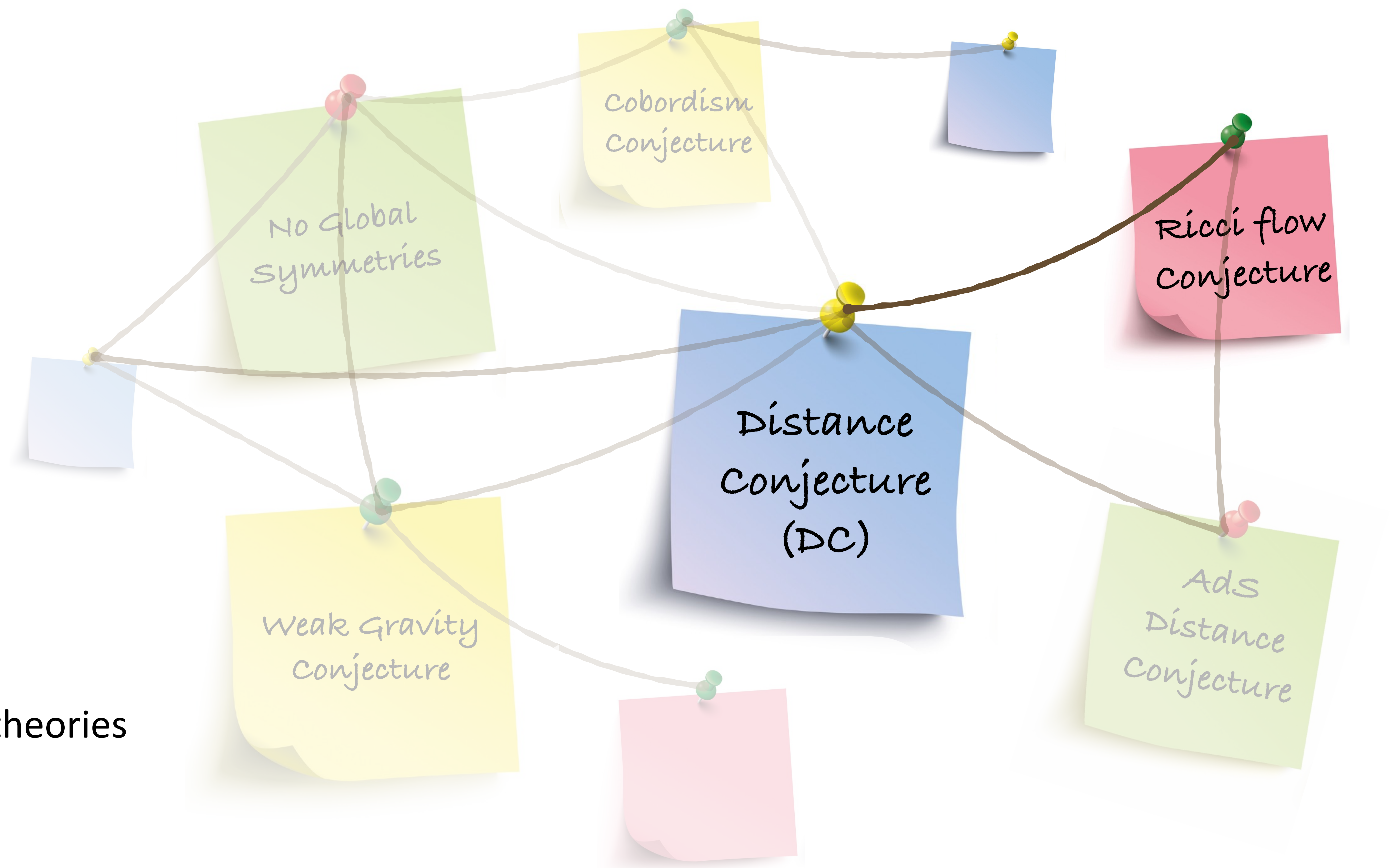


Swampland program:
formulate criteria that identify consistent quantum gravity theories by defining a set of constraining **conjectures**

Quantum Gravity and the Swampland



Quantum field theories that emerge **in the low energy limit** of a **quantum gravity theory** are **very special**



Swampland program:
formulate criteria that identify consistent quantum gravity theories by defining a set of constraining **conjectures**

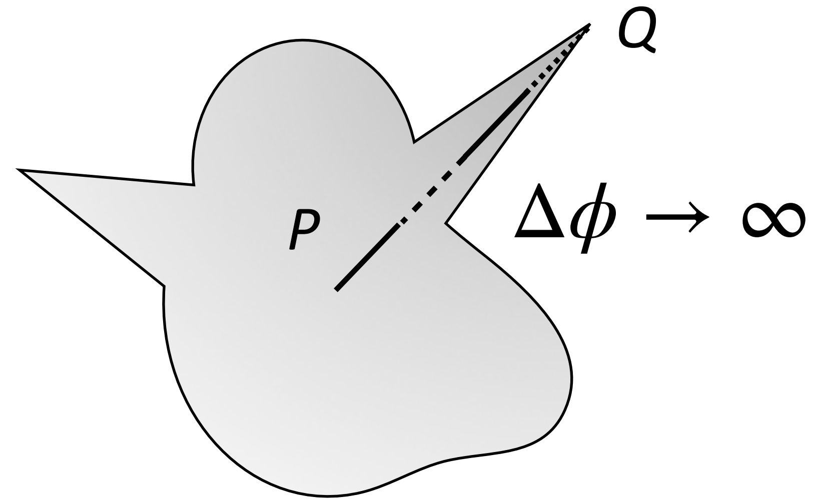
Recap: Distance Conjecture & S^1

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

when going to **large distances in its moduli space**,
we encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \qquad \text{when} \qquad \Delta\phi \to \infty\,, \qquad \Delta\phi \equiv d(P,Q)$$

Moduli space



It describes the parameters
of the internal space

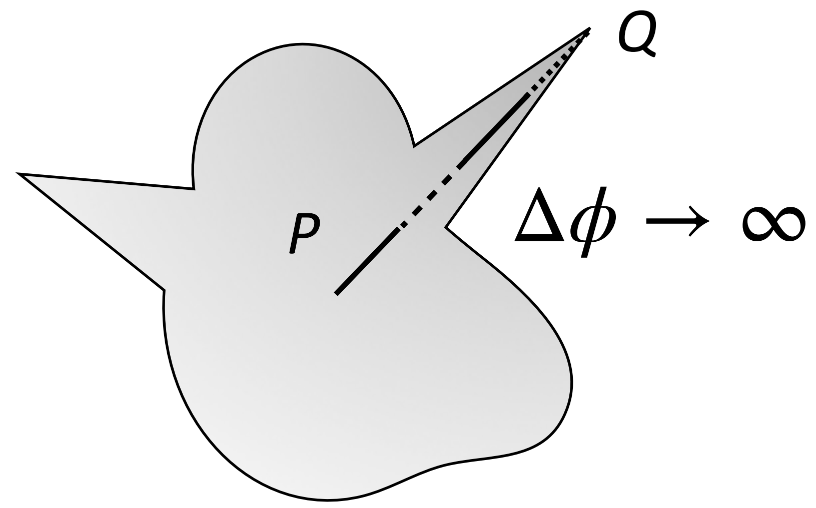
Recap: Distance Conjecture & S^1

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

when going to **large distances in its moduli space**,
we encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P) e^{-\lambda \Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty\,, \quad \Delta\phi \equiv d(P,Q)$$

Moduli space



It describes the parameters of the internal space

Example: Circle compactification

$$S_{\text{EH}} \sim \int \mathrm{d}^{D-1}x \sqrt{-g} \left(\mathscr{R}(g) - \frac{c}{R^2}(\partial R)^2 \right)$$

For $R \to \infty$
Infinite tower of
massless **KK**-modes

$$m_{KK}^2 \sim \frac{1}{R^2}$$

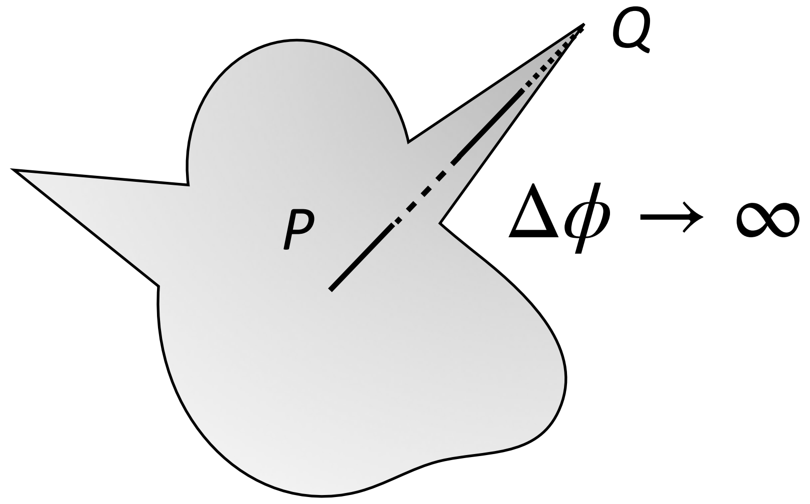
&

For $R \to 0$
Infinite tower of
massless **winding**-modes

$$m_w^2 \sim R^2$$

Recap: Distance Conjecture & S^1

Moduli space



It describes the parameters
of the internal space

In any consistent theory of quantum gravity: [Ooguri, Vafa '06]

when going to **large distances in its moduli space**,
we encounter an **infinite tower of states** which **become light** exponentially

$$M(Q) \sim M(P) e^{-\lambda \Delta\phi} \qquad \text{when} \qquad \Delta\phi \to \infty\,, \qquad \Delta\phi \equiv d(P,Q)$$

Example: Circle compactification

$$S_{\text{EH}} \sim \int \mathrm{d}^{D-1}x \sqrt{-g} \left(\mathscr{R}(g) - \frac{c}{R^2} (\partial R)^2 \right)$$

Moduli space:

$R = 0$
infinite distance point

$R \to \infty$
infinite distance point

For $R \to \infty$
Infinite tower of
massless **KK**-modes

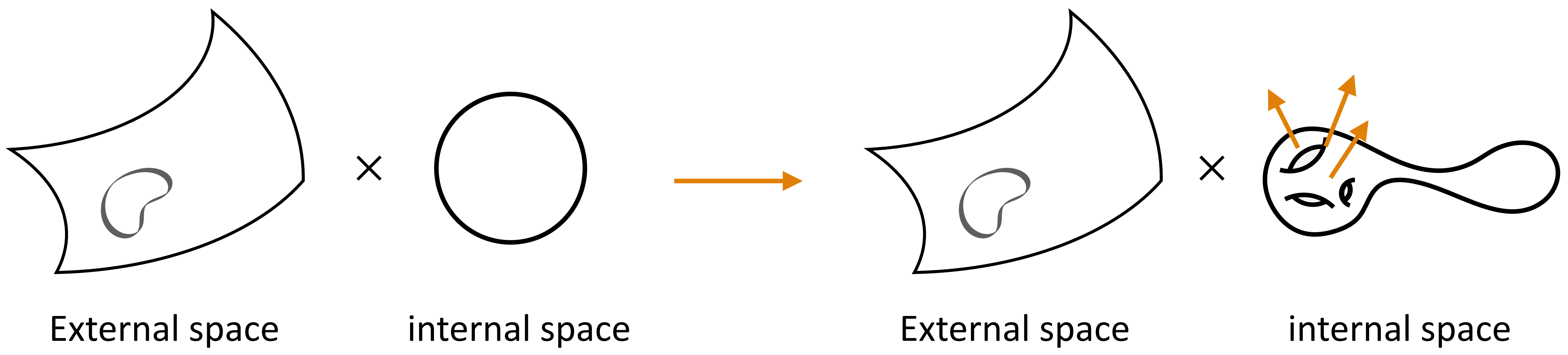
$$m_{KK}^2 \sim \frac{1}{R^2}$$

&

For $R \to 0$
Infinite tower of
massless **winding**-modes

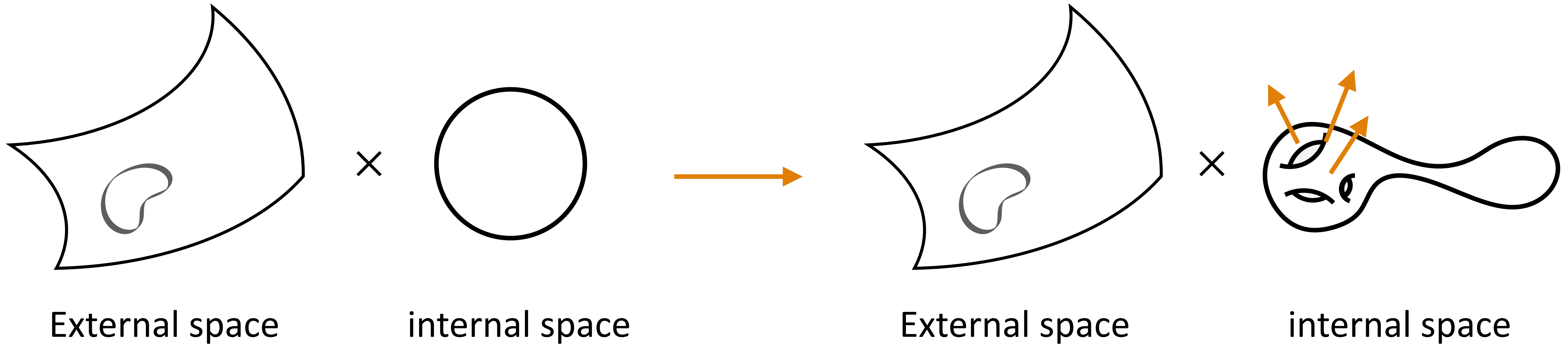
$$m_w^2 \sim R^2$$

What about more complicated compact geometries?



A much more challenging question…

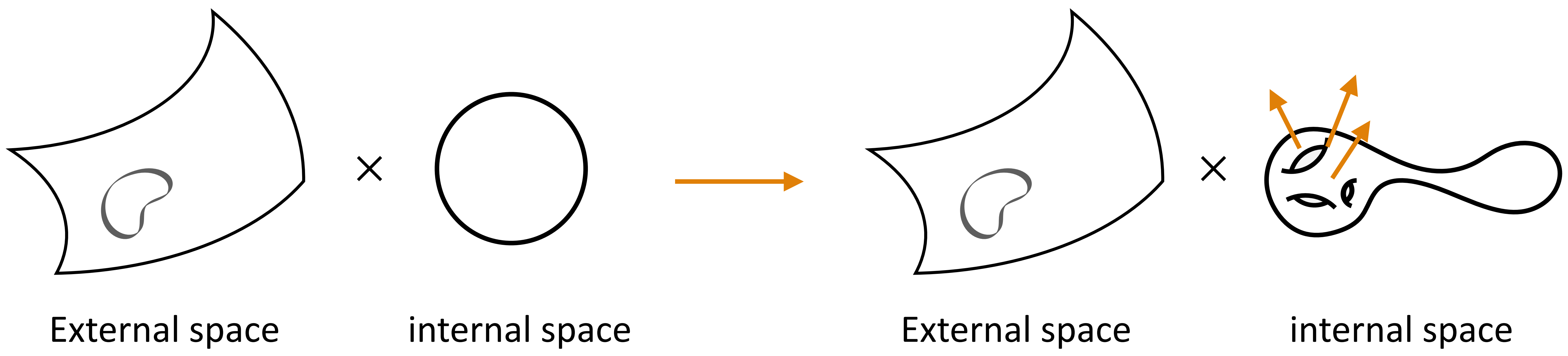
What about more complicated compact geometries?



A much more challenging question…

▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential** $V(\varphi) \supset \mathscr{R}, H, \ldots$

What about more complicated compact geometries?

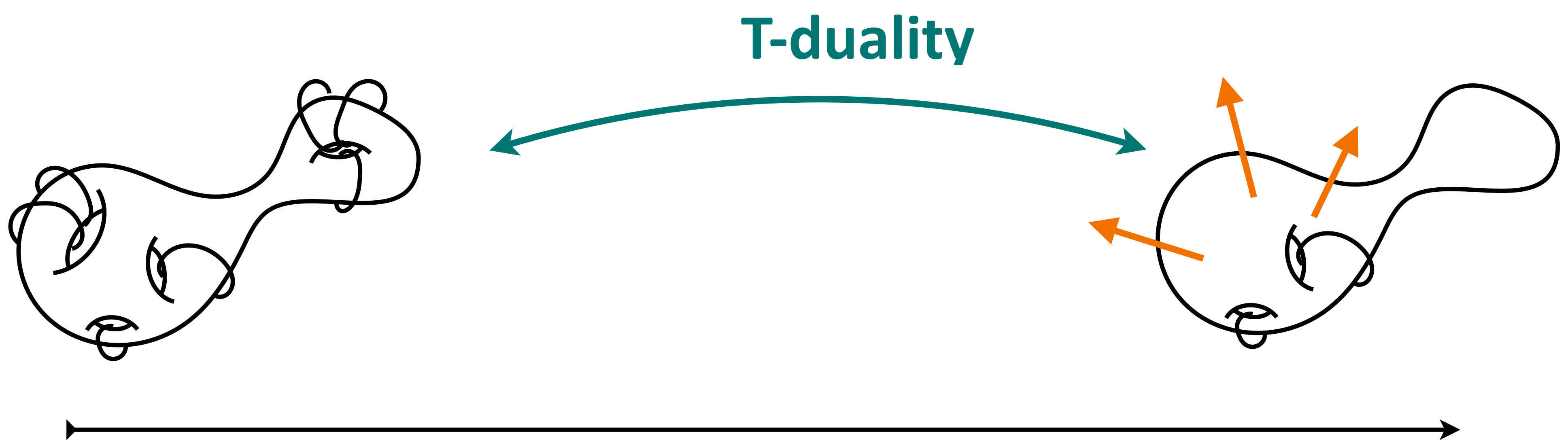


A much more challenging question…

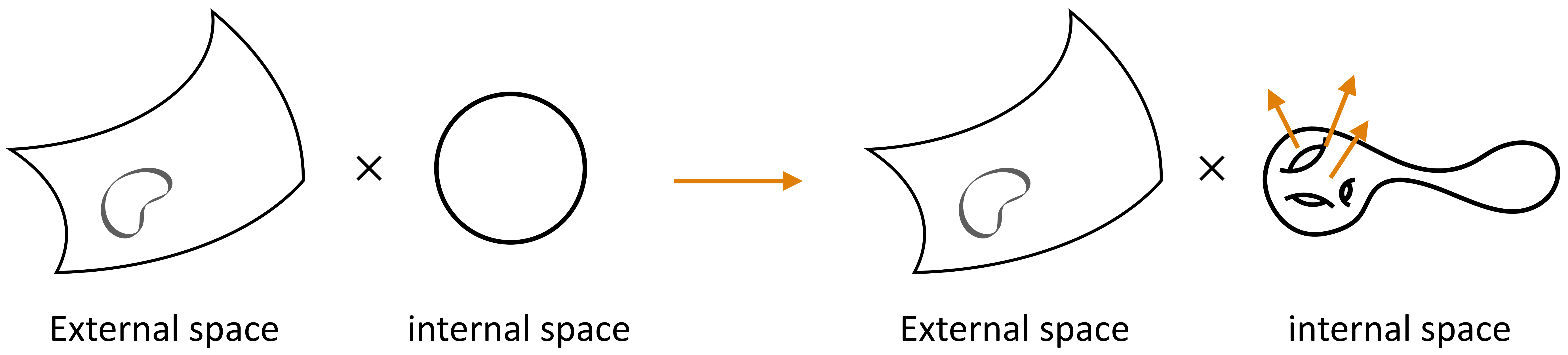
▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**

$$V(\varphi) \supset \mathscr{R}, H, \ldots$$

▷ Under **T-duality** the backgrounds may display **changes in topology**



What about more complicated compact geometries?

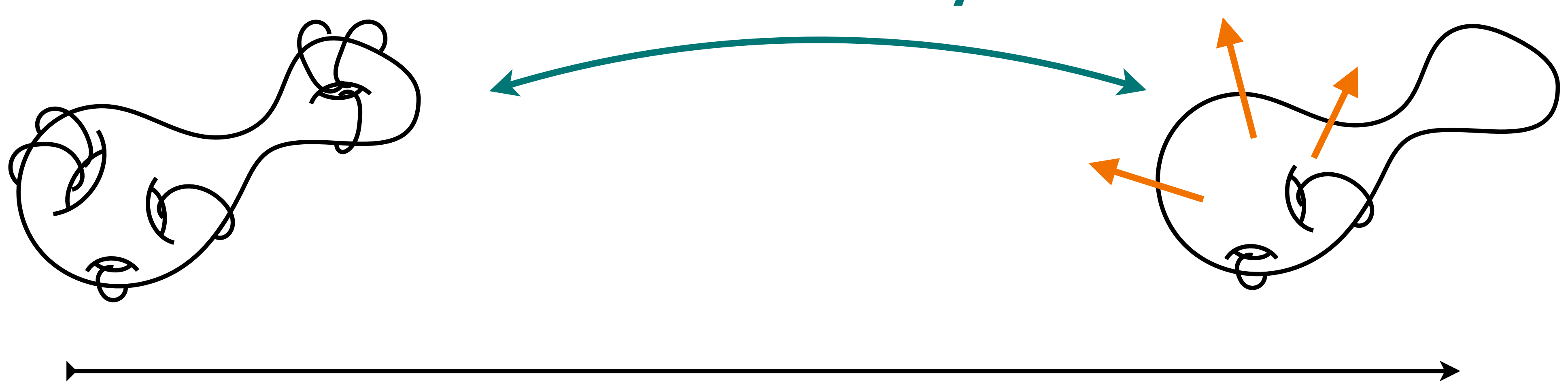


A much more challenging question…

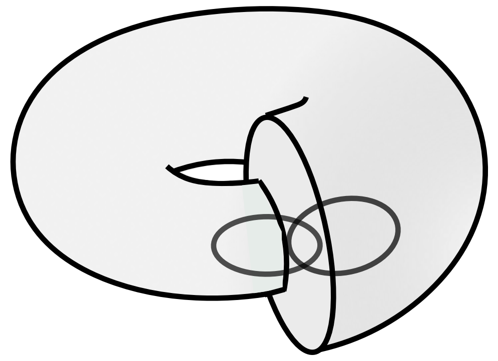
▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential**

$$V(\varphi) \supset \mathscr{R}, H, \dots$$

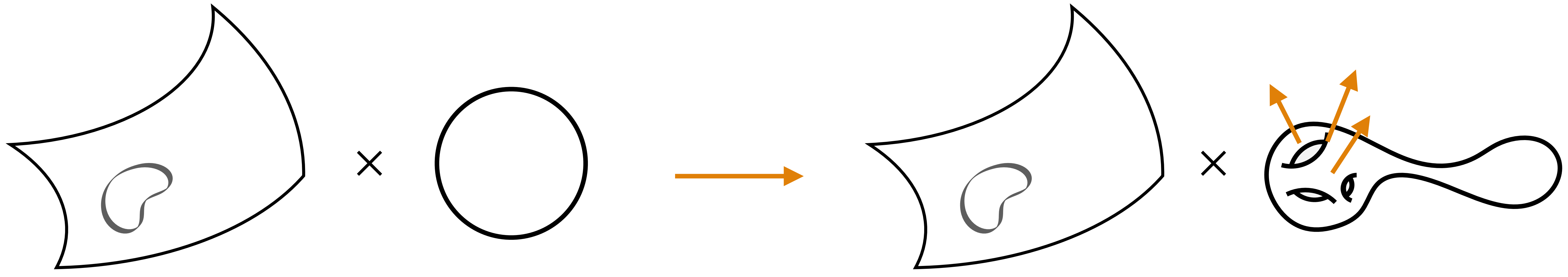
▷ Under **T-duality** the backgrounds may display **changes in topology**



▷ **Non-geometric backgrounds**

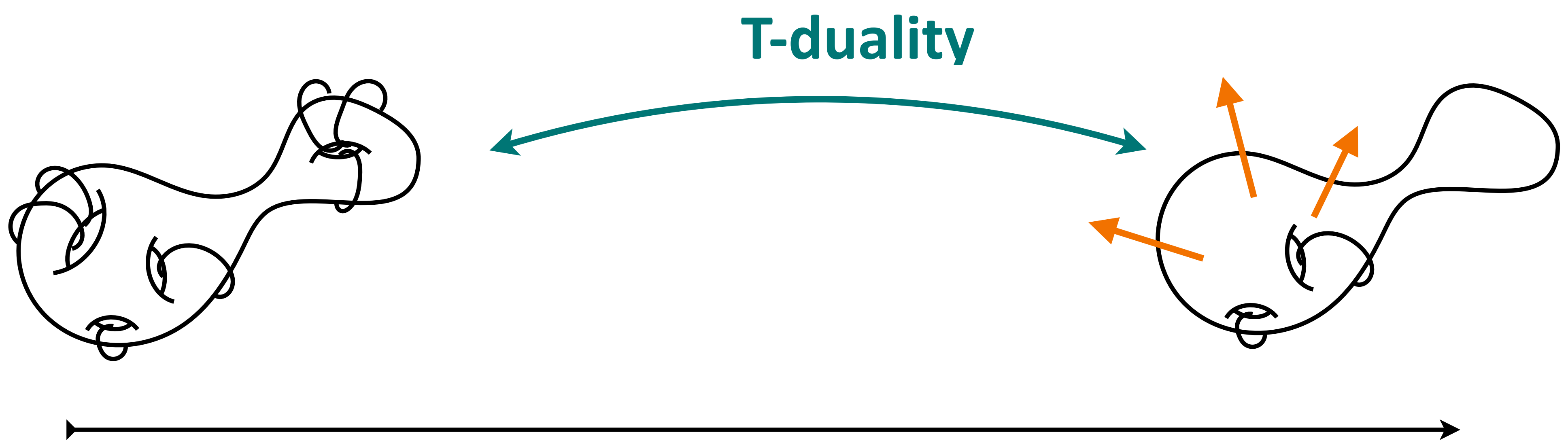


What about more complicated compact geometries?

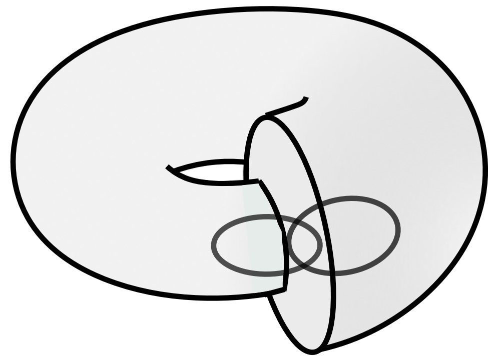


A much more challenging question…

- ▷ Backgrounds display **curvature and/or fluxes**: sources a **scalar potential** $V(\varphi) \supset \mathscr{R}, H, \ldots$
- ▷ Under **T-duality** the backgrounds may display **changes in topology**



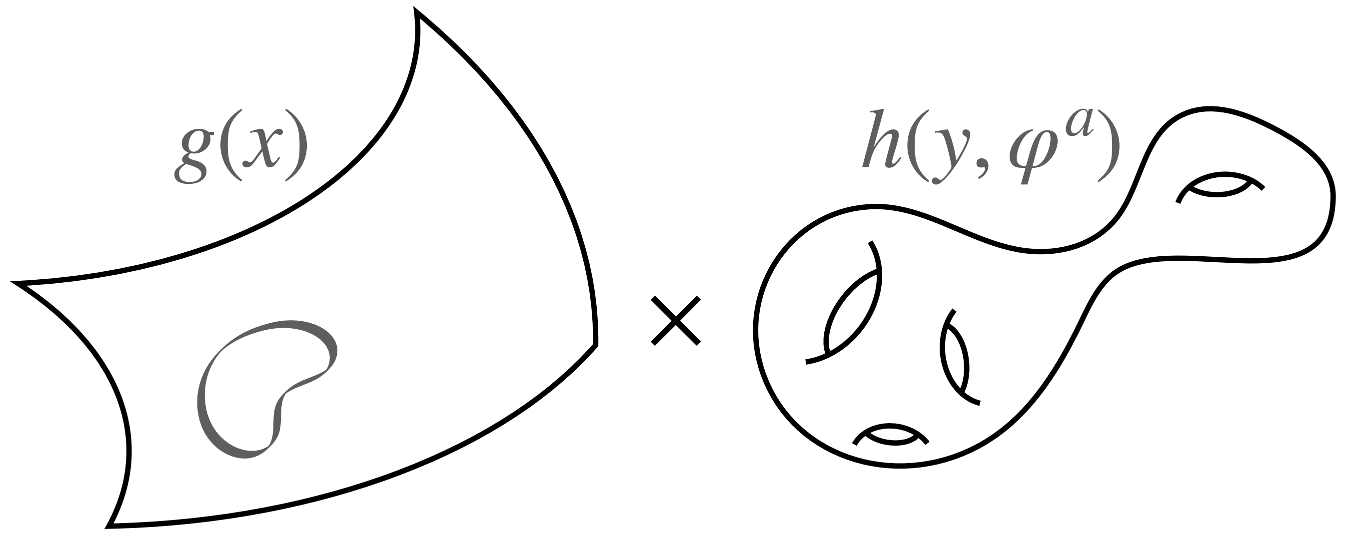
- ▷ **Non-geometric backgrounds**



Do these properties modify the Swampland Distance Conjecture?

Generic setup & reduction

$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I \Phi \partial^I \Phi\right)$$



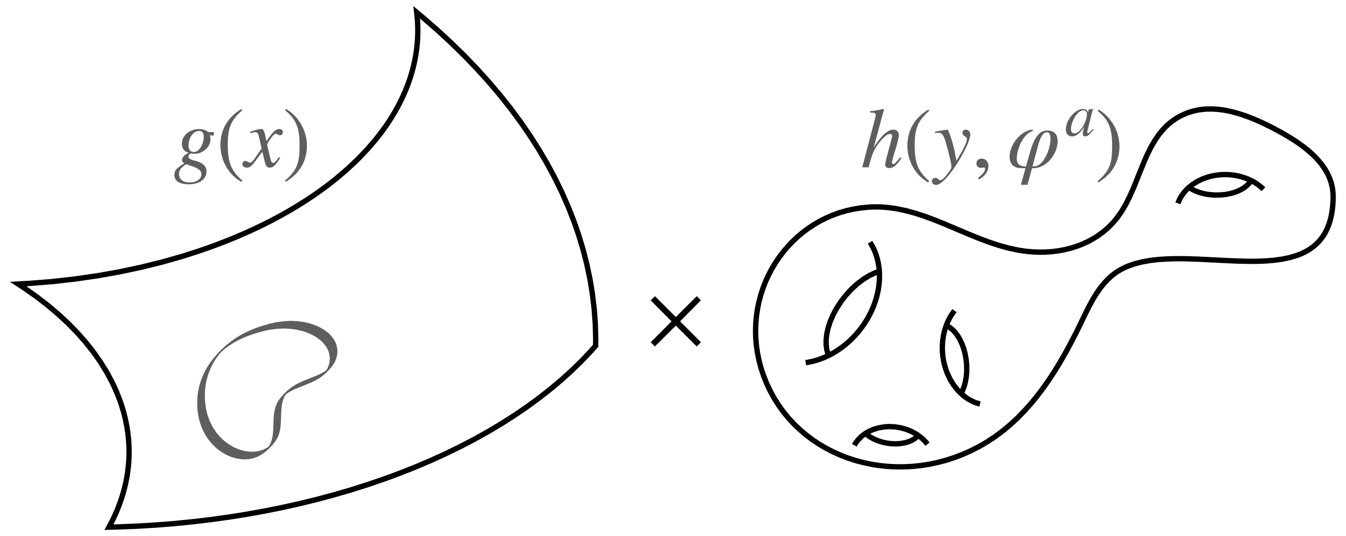
External space

internal space

$dim = n$

Generic setup & reduction

$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I\Phi\partial^I\Phi\right)$$



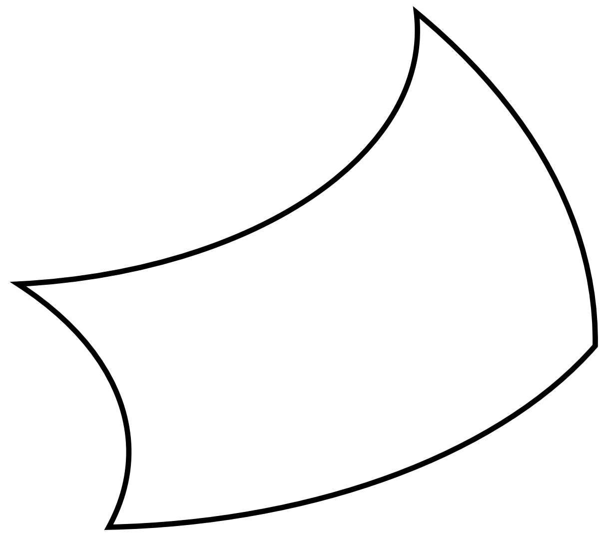
External space

internal space

$dim = n$

$$G(x,y) = g(x) \oplus h(y, \varphi^a(x))$$

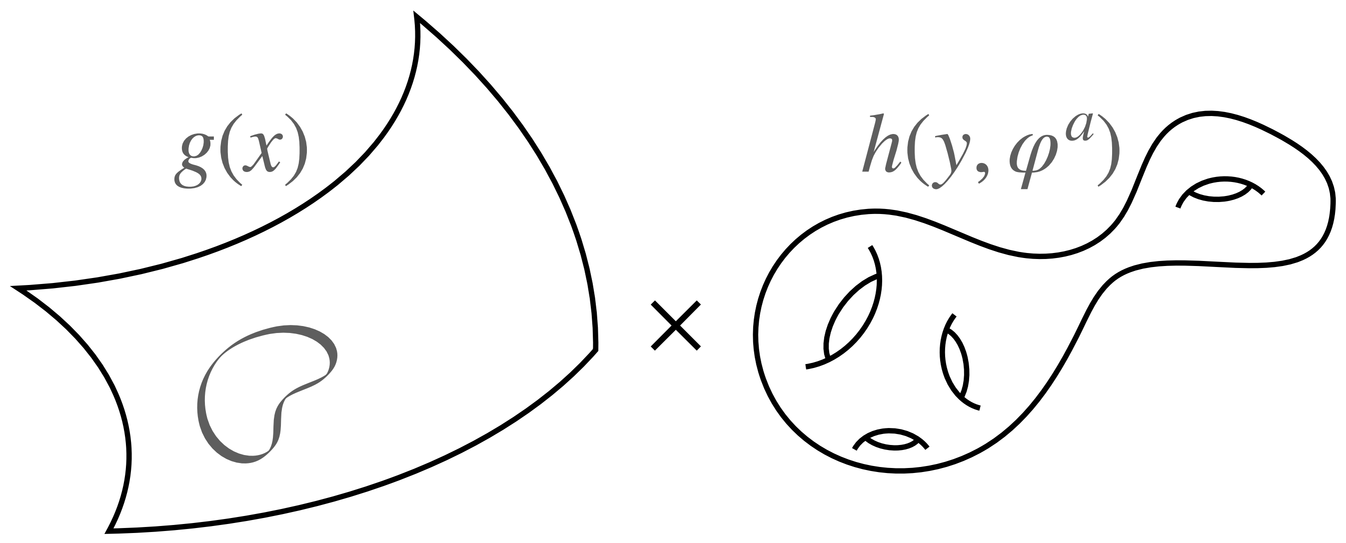
$$S \sim \int \mathrm{d}^{D-n}x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$



Low-energy EFT on “External manifold”

Generic setup & reduction

$$S = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D X \sqrt{-G}\, e^{-2\Phi}\left(\mathscr{R}(G) - \frac{1}{12}H_{IJK}H^{IJK} + 4\partial_I\Phi\partial^I\Phi\right)$$



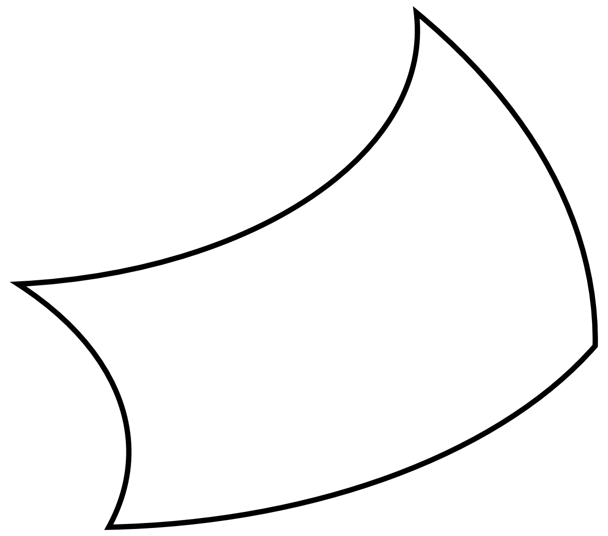
External space

internal space

$dim = n$

$$G(x,y) = g(x) \oplus h(y,\varphi^a(x))$$

$$S \sim \int \mathrm{d}^{D-n}x\sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi^a)\right)$$



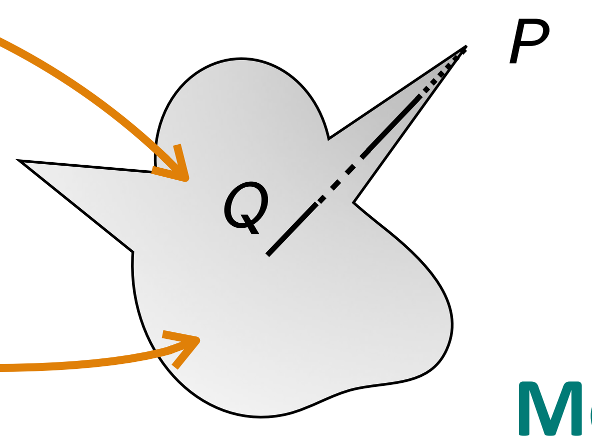
Low-energy EFT on “External manifold”

$$\gamma_{ab} \sim \int \mathrm{d}^n y\sqrt{h}\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a}h\ h^{-1}\partial_{\varphi_b}h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a}B\ h^{-1}\partial_{\varphi_b}B\right)\right)$$

metric

$$V(\varphi^i) \sim \int \mathrm{d}^n y\sqrt{h}\left(\mathscr{R}(h) - \frac{1}{12}H_{ijk}H^{ijk} + 4\partial_i\Phi\partial^i\Phi\right)$$

potential



Moduli space

Example: S^3 with H-flux

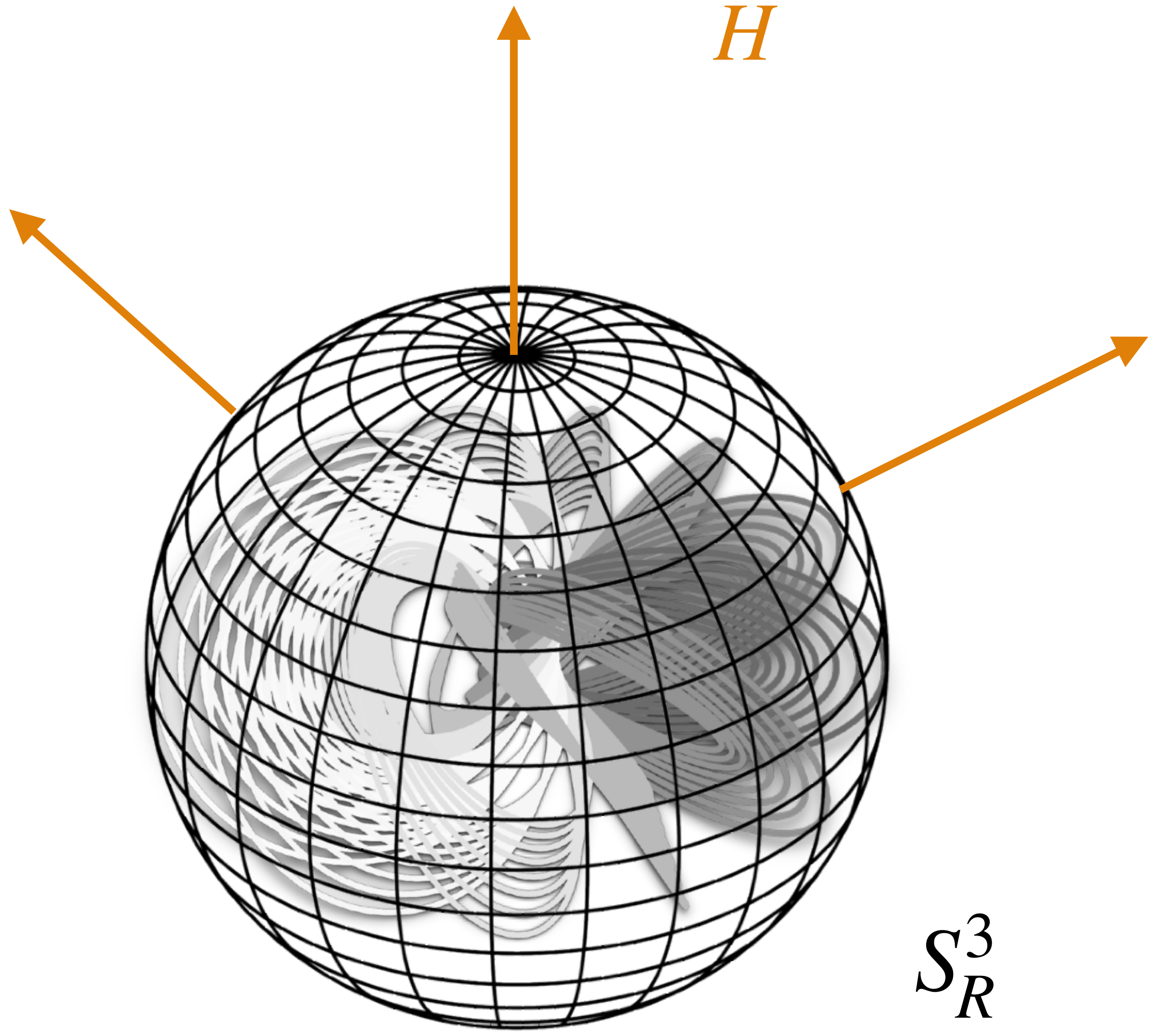
$$S_{\text{EH}} \sim \int \mathrm{d}^d x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a)\right)$$

$$ds^2 = R^2(\mathrm{d}\eta^2 + \mathrm{d}\xi_1^2 + \mathrm{d}\xi_2^2 + 2\cos(\eta)\mathrm{d}\xi_1^2\mathrm{d}\xi_2^2)$$

$$H = k\sin(\eta)\mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



~~winding~~

$\pi_1(S^3) = 0$

Example: S^3 with H-flux

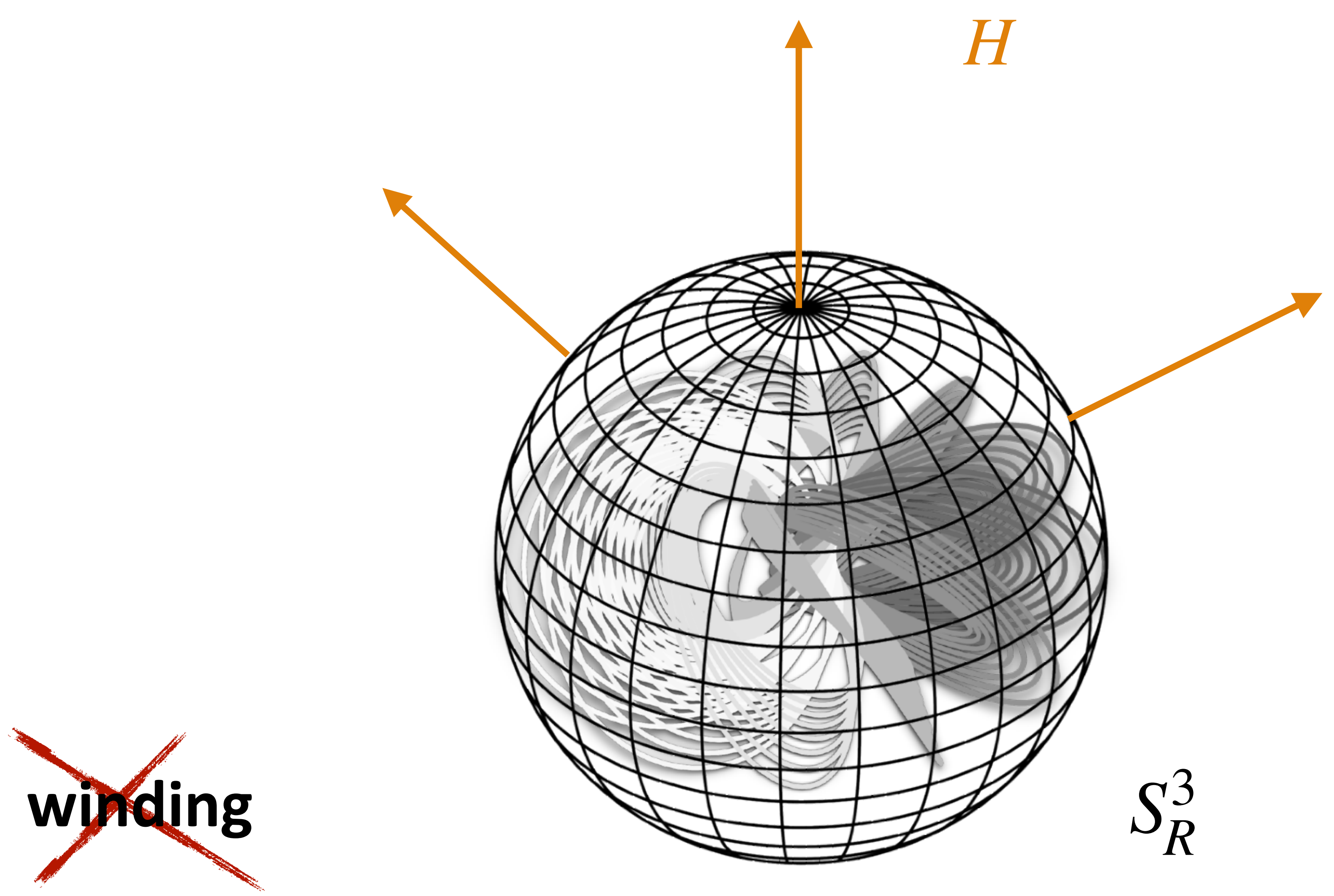
$$S_{\mathrm{EH}} \sim \int \mathrm{d}^d x \sqrt{-g}\left(\mathscr{R}(g) - \gamma_{ab}\partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi^a)\right)$$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

$$ds^2 = R^2(\mathrm{d}\eta^2 + \mathrm{d}\xi_1^2 + \mathrm{d}\xi_2^2 + 2\cos(\eta)\mathrm{d}\xi_1^2\mathrm{d}\xi_2^2)$$

$$H = k\sin(\eta)\mathrm{d}\eta \wedge \mathrm{d}\xi_1 \wedge \mathrm{d}\xi_2$$



$\pi_1(S^3) = 0$

How is **absence of winding modes** compatible with **T-duality**?

What does this mean for the **Swampland Distance Conjecture**?

▸ T-duality:

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$

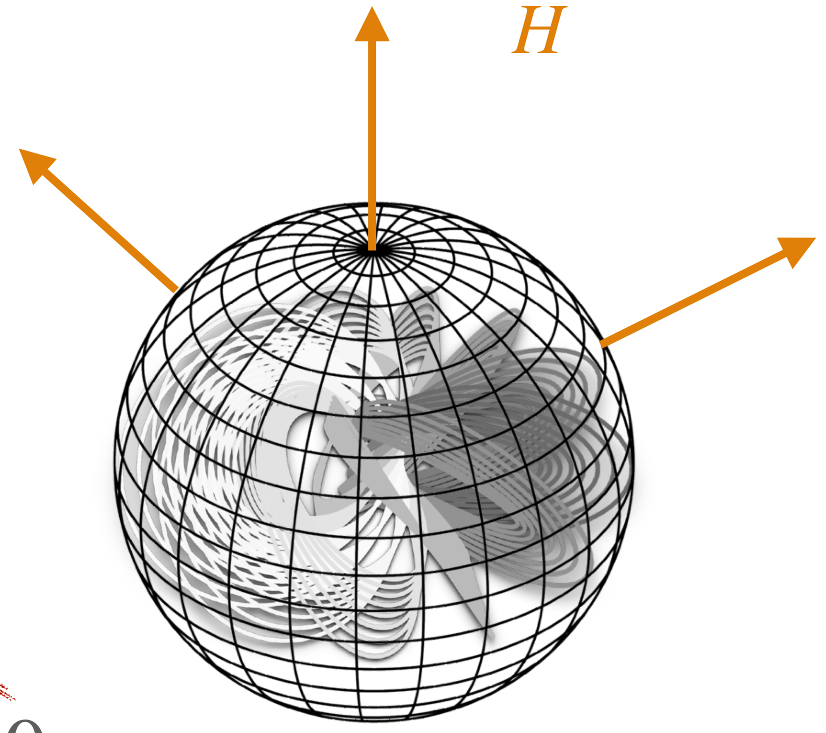
T-duality

along Hopf-fiber

$\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}}$ with $[\tilde{\mathbf{H}}] = \mathbf{1}$

$\downarrow$

$\mathbf{S^2_R}$



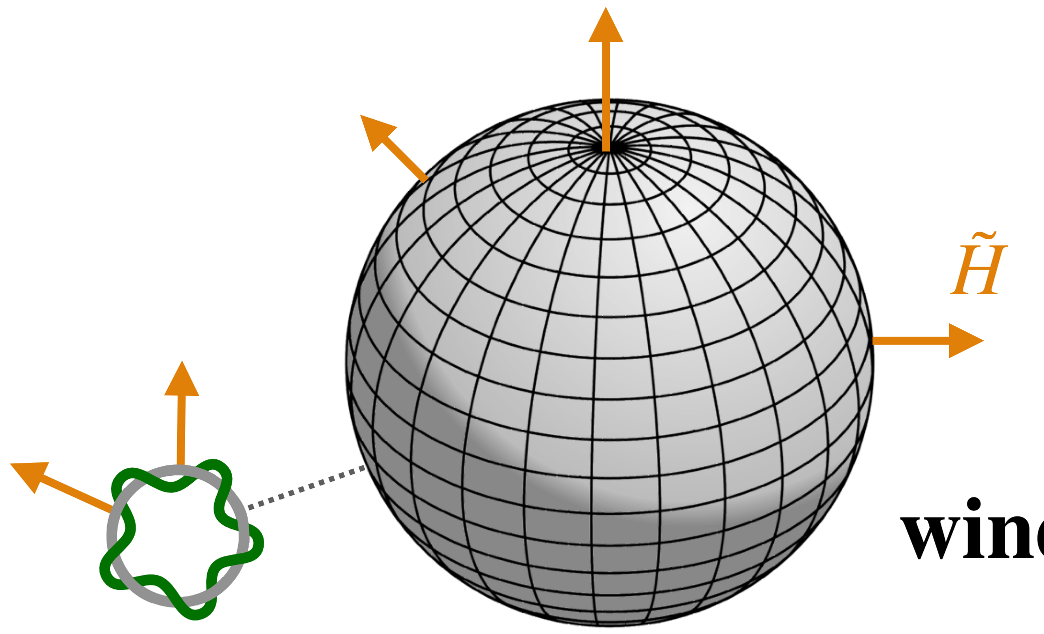
~~winding~~

$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



$S^1_{1/R}$ S^2_R

winding $\in \mathbb{Z}_k$

$\pi_1(\tilde{E}) = \mathbb{Z}_k$

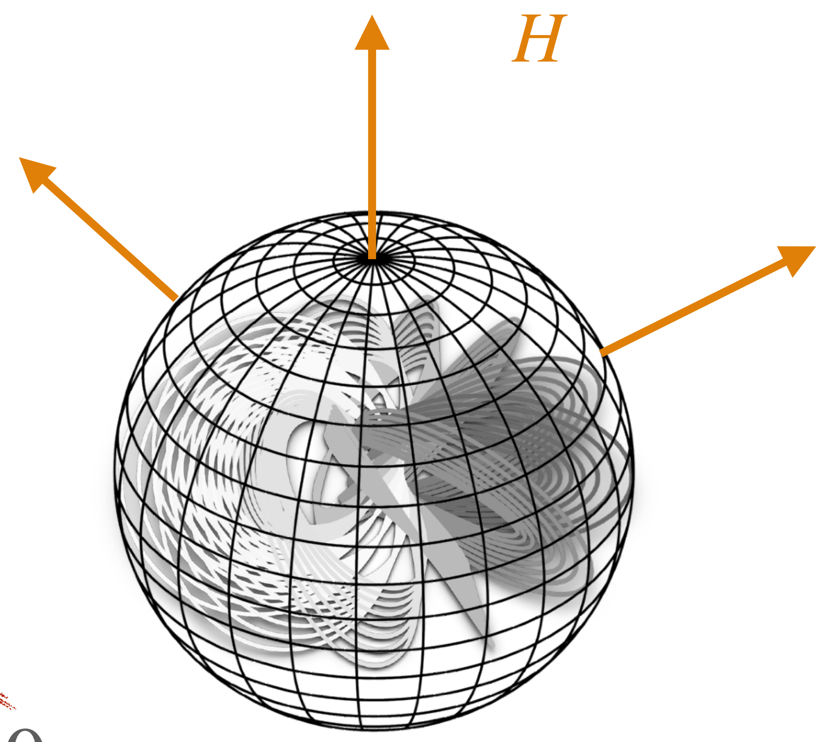
$c_1(\tilde{E}) = k$

▸ T-duality:

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$

T-duality ⟷ along Hopf-fiber

$$\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}} \to \mathbf{S^2_R} \quad \text{with } [\tilde{\mathbf{H}}] = \mathbf{1}$$



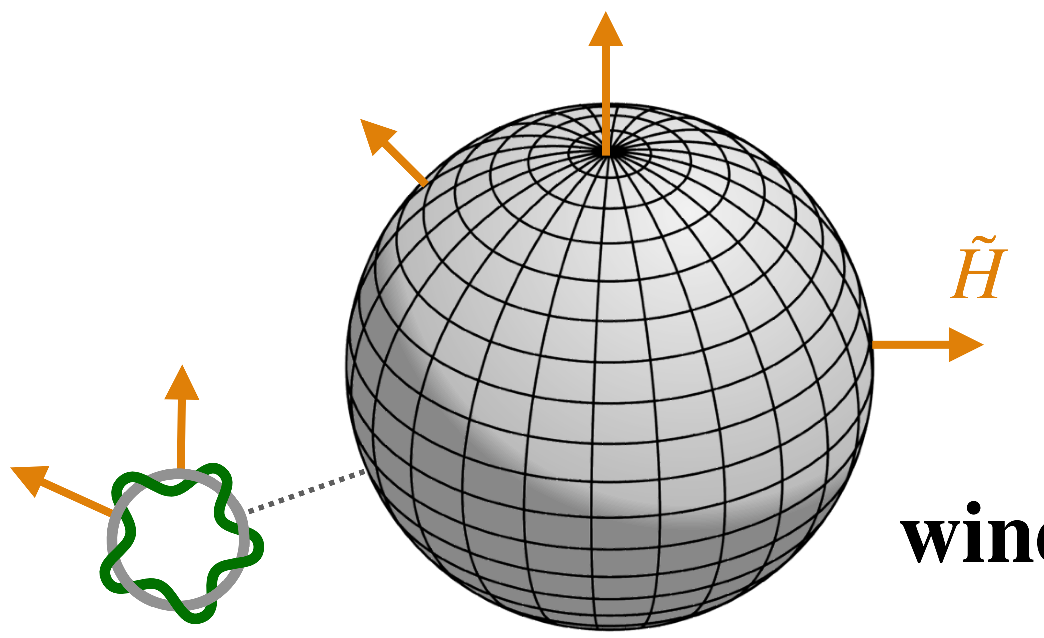
~~winding~~

$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



$S^1_{1/R}$ S^2_R

winding $\in \mathbb{Z}_k$

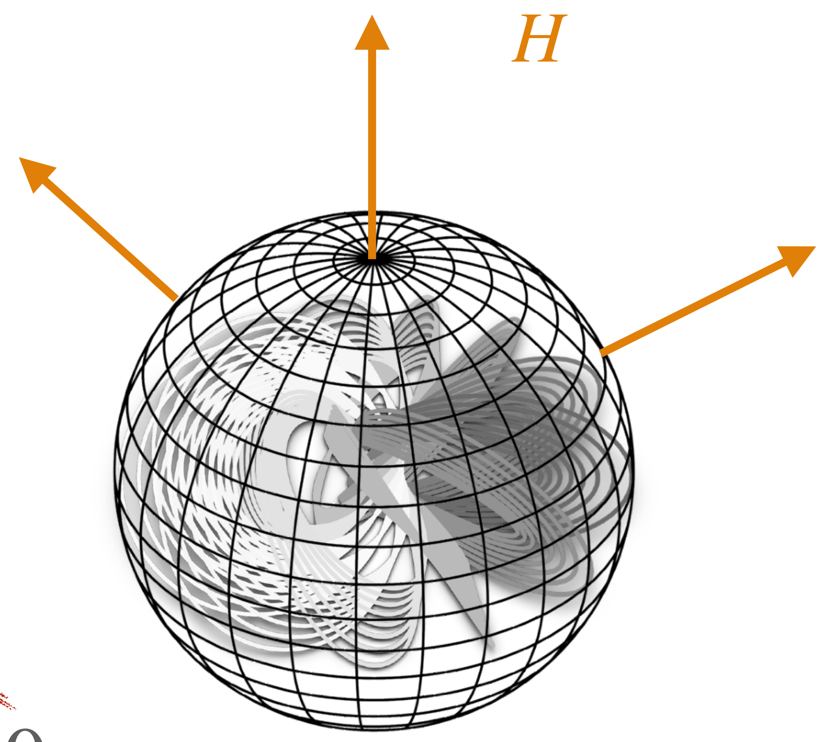
$\pi_1(\tilde{E}) = \mathbb{Z}_k$

$c_1(\tilde{E}) = k$

		$R = 0$	$R \to \infty$
S^3	winding	$\varnothing$	$\varnothing$
	momentum	heavy	light
$\tilde{E}$	winding	$\mathbb{Z}_k$ (heavy)	$\mathbb{Z}_k$ (light)
	momentum	heavy/ non-conserved	light
		no modes becoming light	tower of light states

▸ T-duality:

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$ ⟷ (T-duality along Hopf-fiber) ⟷ $\mathbf{S^1_{1/R}} \hookrightarrow \tilde{\mathbf{E}} \to \mathbf{S^2_R}$ with $[\tilde{\mathbf{H}}] = \mathbf{1}$

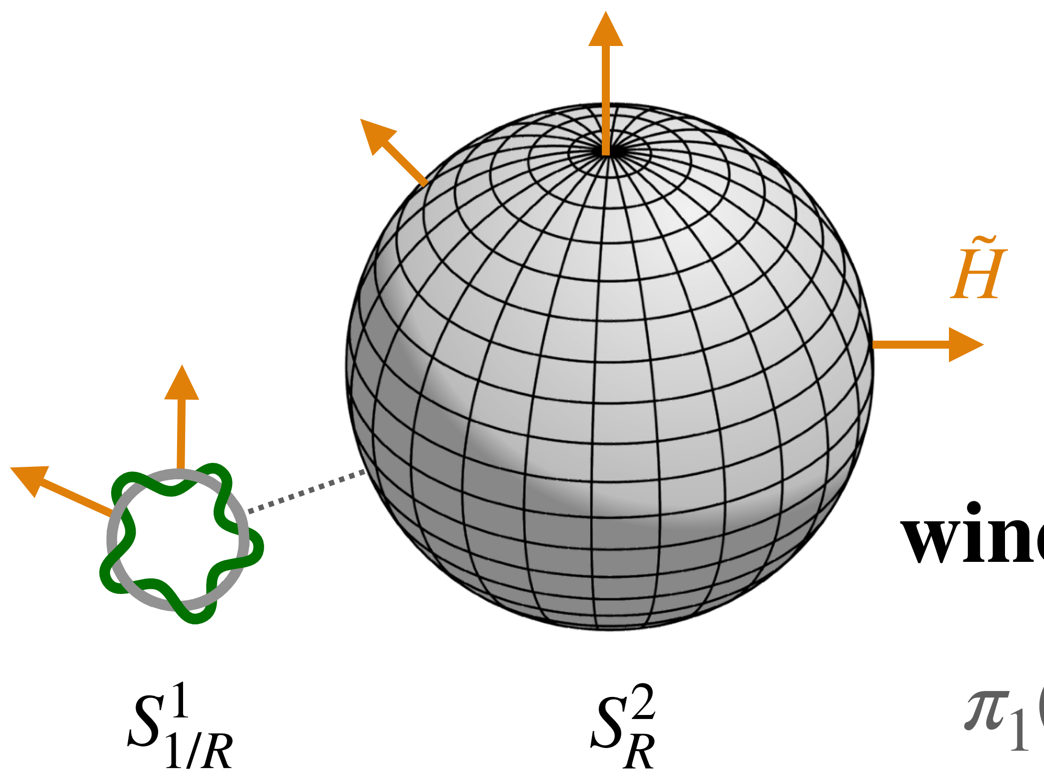


~~winding~~

$\pi_1(S^3) = 0$

$c_1(S^3) = 1$

$$\gamma_{RR} = \frac{3}{R^2}$$
$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$



winding $\in \mathbb{Z}_k$

$\pi_1(\tilde{E}) = \mathbb{Z}_k$

$c_1(\tilde{E}) = k$

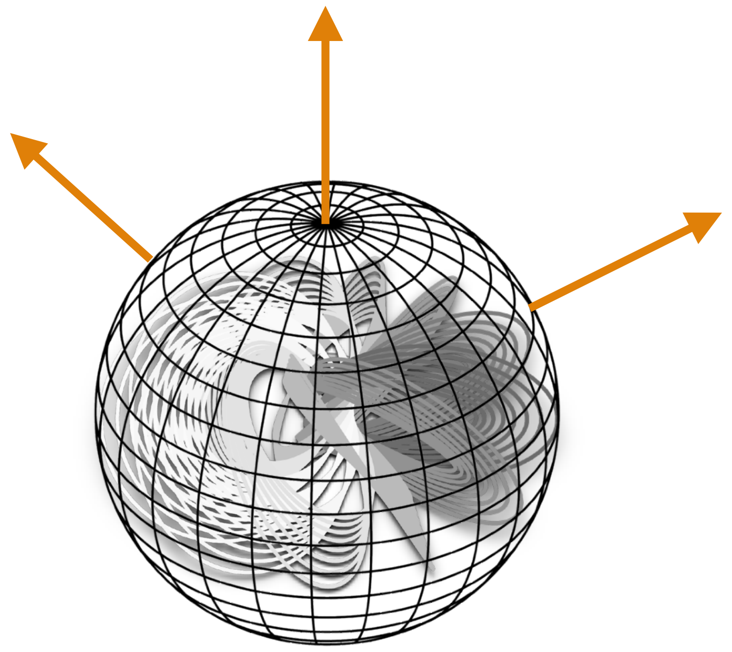
		$R = 0$	$R \to \infty$
S^3	winding	$\varnothing$	$\varnothing$
	momentum	heavy	light
$\tilde{E}$	winding	$\mathbb{Z}_k$ (heavy)	$\mathbb{Z}_k$ (light)
	momentum	heavy/ non-conserved	light
		no modes becoming light	tower of light states

▸ Distance Conjecture:

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

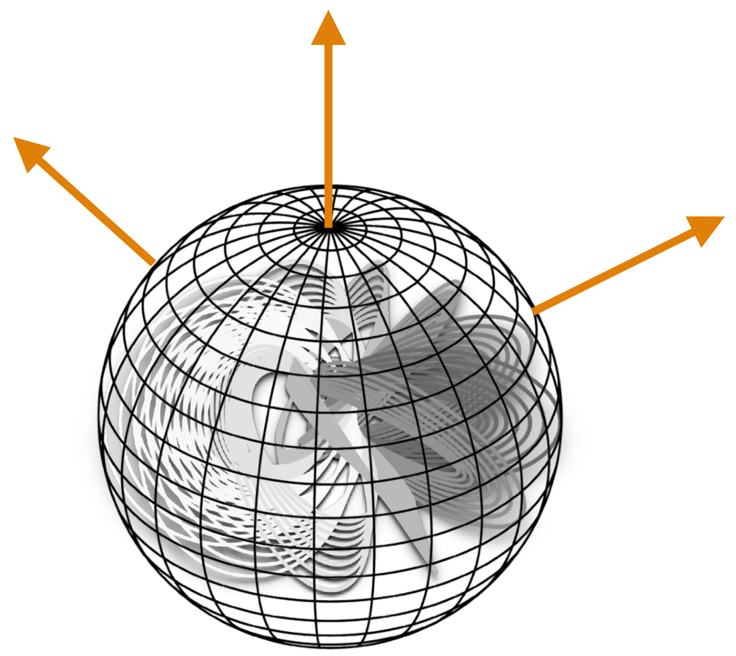
However there is **no tower of light states** for R → 0 which is an infinite distance limit

▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

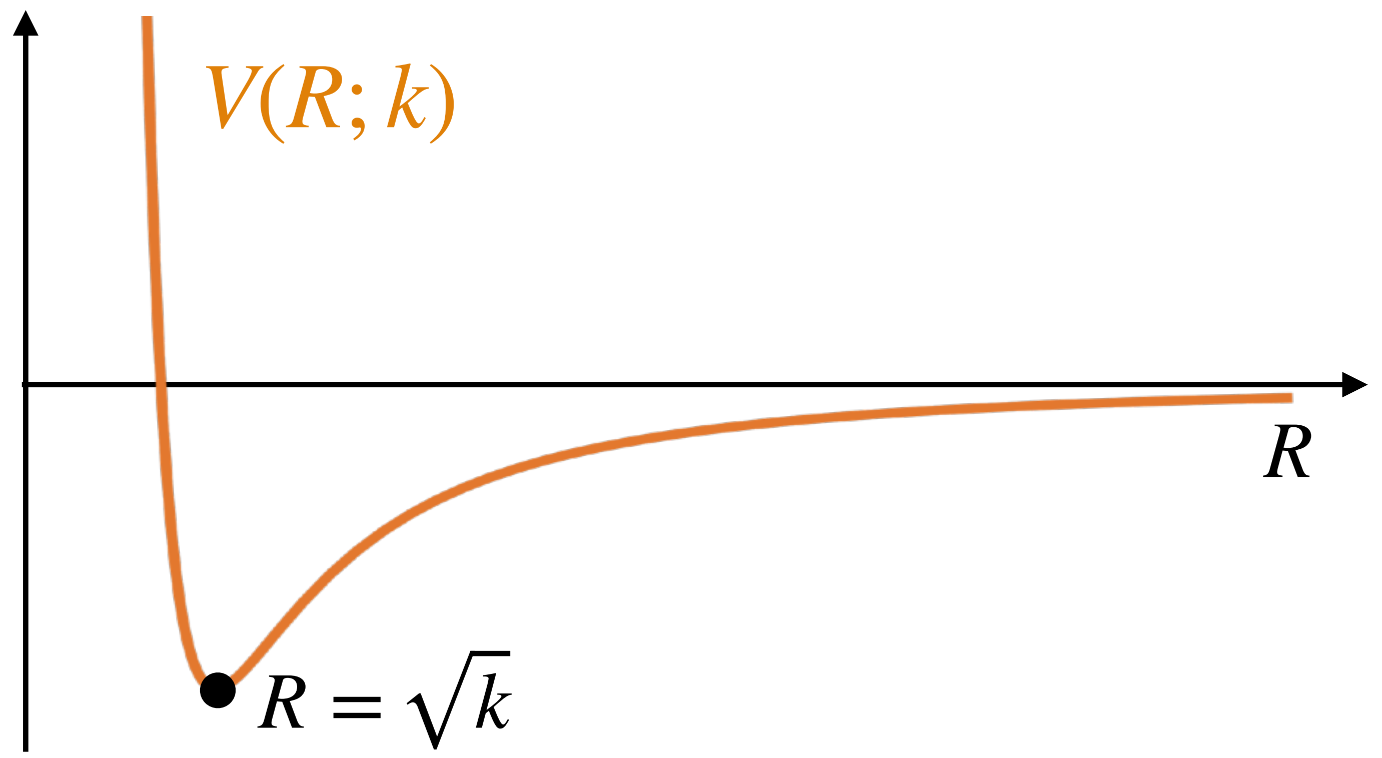
$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R → 0 which is an infinite distance limit

…need to **take into account scalar potential**

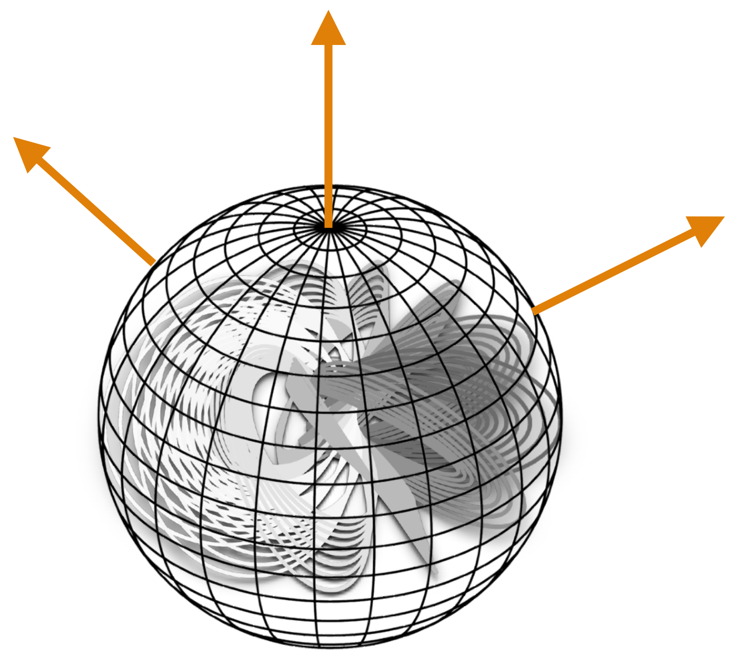


▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

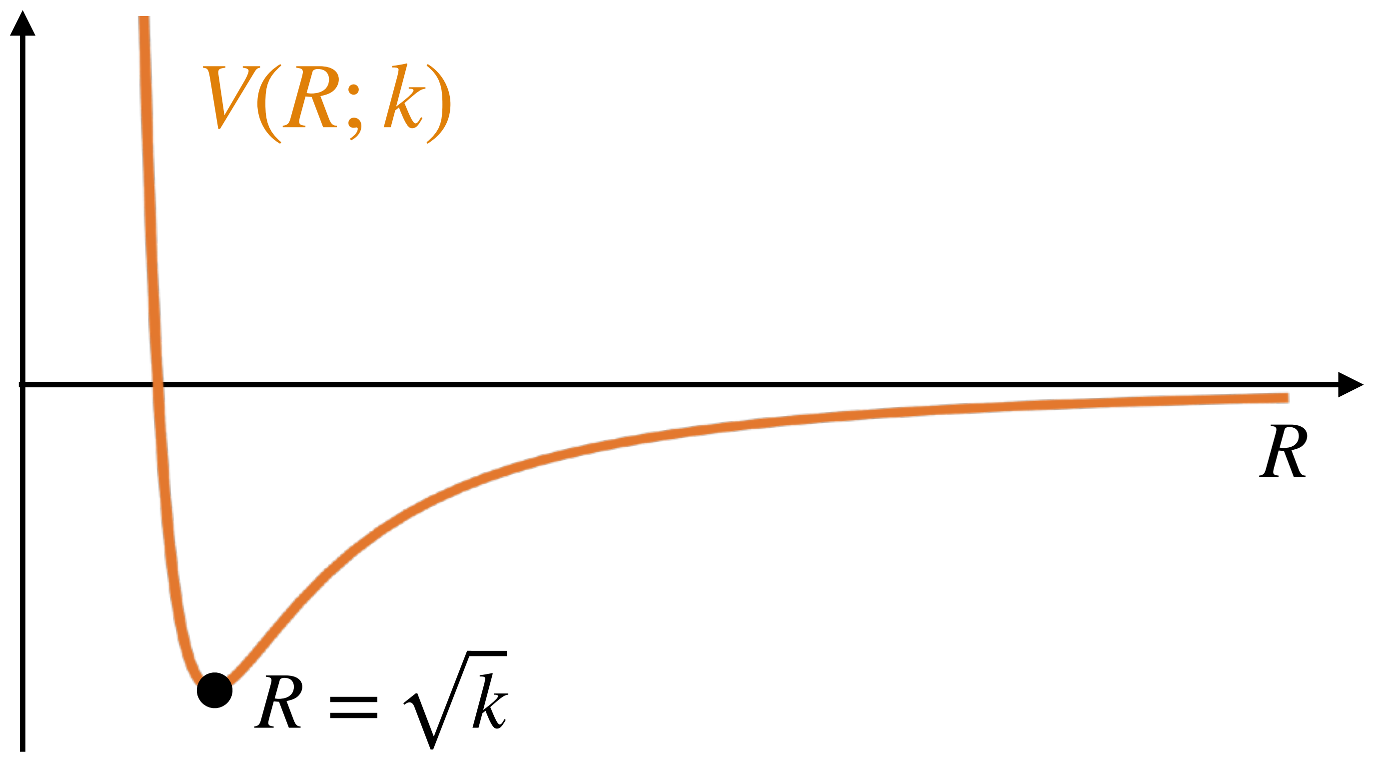
$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R → 0 which is an infinite distance limit

…need to **take into account scalar potential**



[Demulder, Lüst, TR '23]

In **effective field theories** that **can be lifted to a theory of quantum gravity** in the UV, a **divergence in the scalar potential** emerges when approaching an **infinite locus point** for which the target space geometry **cannot give rise to a light tower of states**.

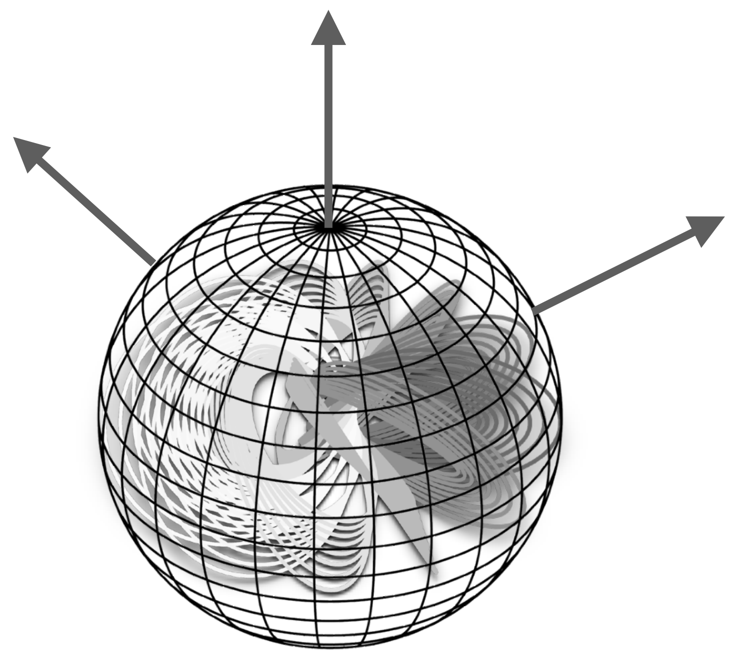
That is, the **potential signals pathological infinite distance loci** in the scalar field space.

▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

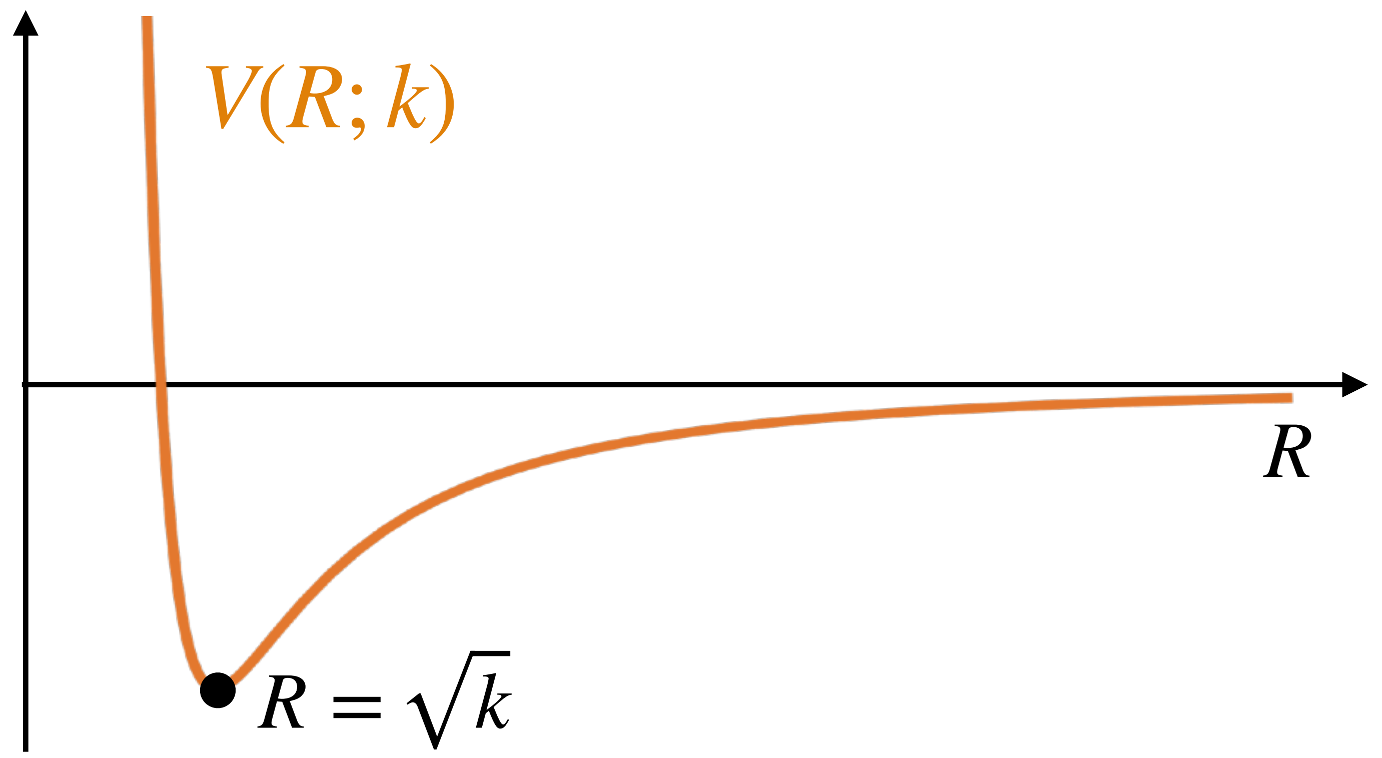
$\mathbf{S^3_R}$ with $\mathbf{[H] = k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R $\to$ 0 which is an infinite distance limit

…need to **take into account scalar potential**



[Demulder, Lüst, TR ’23]

In **effective field theories** that **can be lifted to a theory of quantum gravity** in the UV, a **divergence in the scalar potential** emerges when approaching an **infinite locus point** for which the target space geometry **cannot give rise to a light tower of states**.

That is, the **potential signals pathological infinite distance loci** in the scalar field space.

▸ Invariance of metric & flux variations:

Metric on moduli space given by

$$\gamma_{ab} \sim \int \mathrm{d}^n y \sqrt{h} \Big(\underbrace{\mathrm{tr}\big(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\big) - \mathrm{tr}\big(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\big)}_{=\frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]} \Big)$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[Demulder, Lüst, TR ’23]

▸ **Invariance of metric & flux variations:**

Metric on moduli space given by

$$\gamma_{ab} \sim \int d^n y \sqrt{h} \underbrace{\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right) \right)}_{= \frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]}$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[Demulder, Lüst, TR '23]

▸ **Example:** S^3 with k(x)=$R^2(x)$ $\xleftrightarrow{\text{T-duality}}$ $\tilde{E}$

$\tilde{\mathbf{E}}$ …modulus only in spacetime metric h

γ_{RR} obtained in standard way from "deWitt" metric

▸ **Invariance of metric & flux variations:**

Metric on moduli space given by

$$\gamma_{ab} \sim \int d^n y \sqrt{h} \underbrace{\left(\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} h \; h^{-1}\partial_{\varphi_b} h\right) - \mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right) \right)}_{= \frac{1}{2}\mathrm{tr}[(\mathscr{H}^{-1}\partial_{\varphi_a}\mathscr{H})^2]}$$

$$O(d,d) \ni \mathscr{H} = \begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix}$$

So by $O(d,d)$ invariance γ_{ab} **is invariant** under (abelian) **T-duality**.

[Demulder, Lüst, TR '23]

▸ **Example:** S^3 with k(x)=$R^2(x)$ $\xleftrightarrow{\text{T-duality}}$ $\tilde{E}$

$$H = R^2 \sin(\eta) d\eta \wedge d\xi_1 \wedge d\xi_2$$
$$B = -R^2 \cos(\eta) d\xi_1 \wedge d\xi_2$$

$\tilde{\mathbf{E}}$ ...modulus only in spacetime metric h

γ_{RR} obtained in standard way from "deWitt" metric

$\mathbf{S^3_R}$ with $[\mathbf{H}] = \mathbf{k} = \mathbf{R^2}$ ...modulus in h **and B**

also contribution $\mathrm{tr}\left(h^{-1}\partial_{\varphi_a} B \; h^{-1}\partial_{\varphi_b} B\right) \neq 0 \subset \gamma_{RR}$

$\tilde{\gamma}_{RR} = \gamma_{RR}$ only if **flux variation** are taken into account

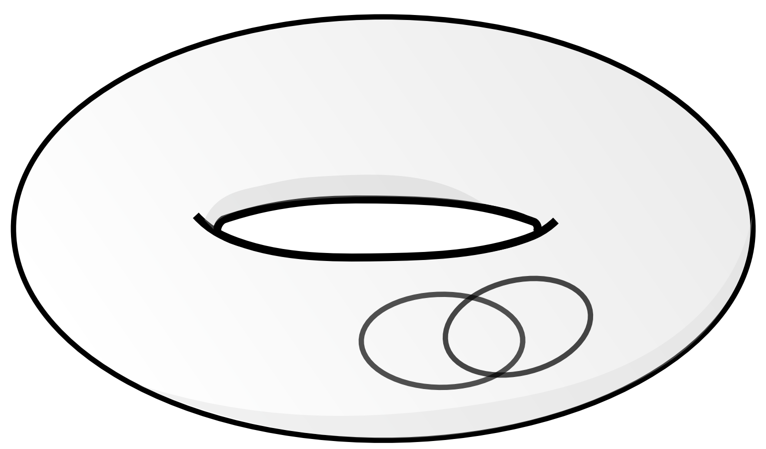
c.f also [Li,Palti,Petri '23][Palti,Petri '24]

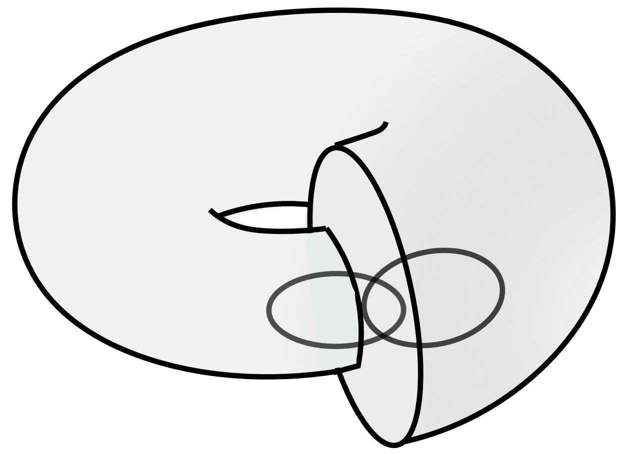
Non-geometric backgrounds

Described locally by Riemannian geometry with fluxes. However, **transition functions** are allowed to be **T-dualities**.

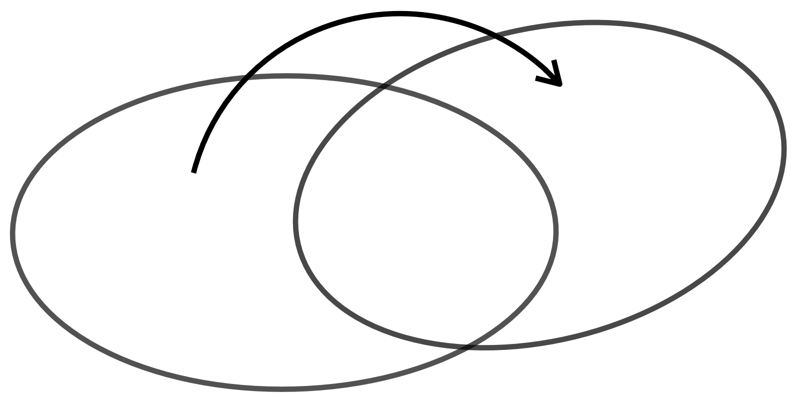
[Dabholkar, Hull ’02&‘05; Flournoy, Wecht, Williams ’04…]

Manifold, e.g. a torus vs. non-geometric background

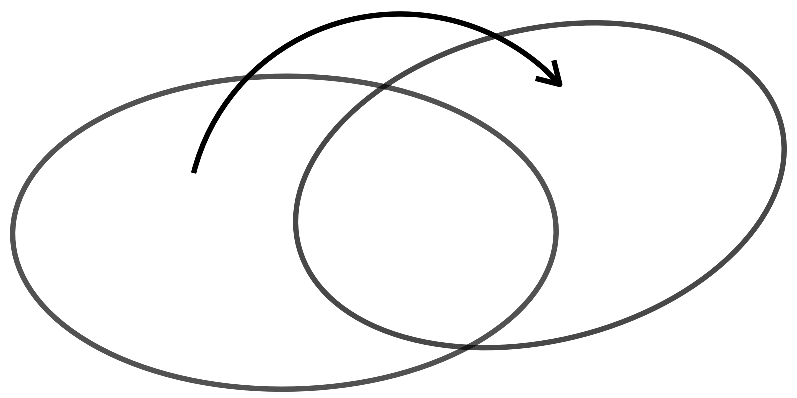


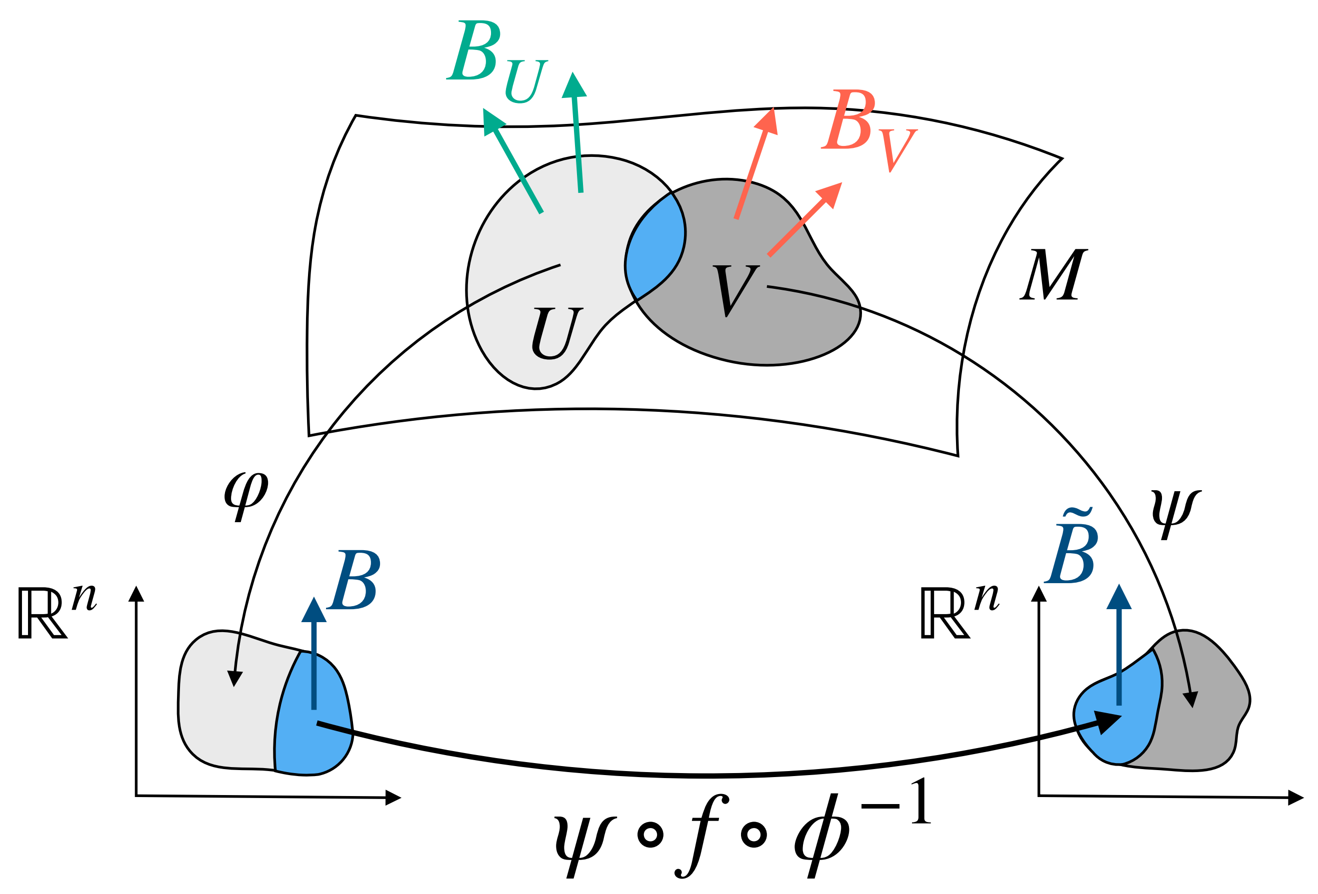


diffeomorphism



… and T-duality





$$f \in \begin{cases} Diff(M) & \text{: Riemannian} \\ Diff(M) \cup \text{T-duality} & \text{: non-geometric} \end{cases}$$

- ▷ **“Inevitable”** in string theory
- ▷ **Moduli stabilisation**

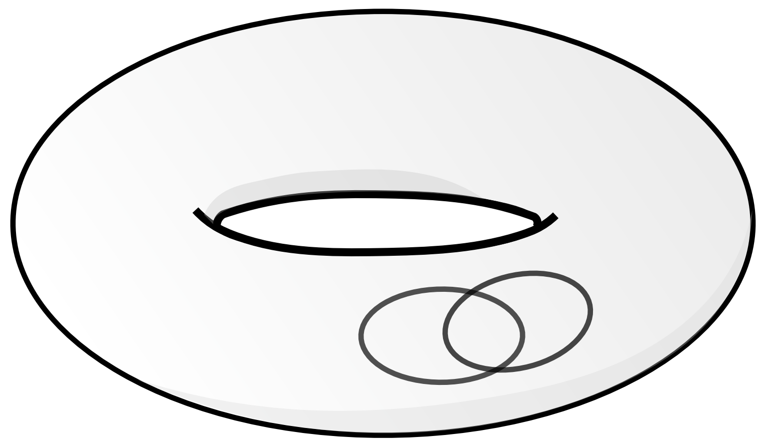
  [Becker, Becker, Blumenhagen, Lüst, Plauschinn, Shelton, Taylor, Vafa, Wecht, Walcher, …]

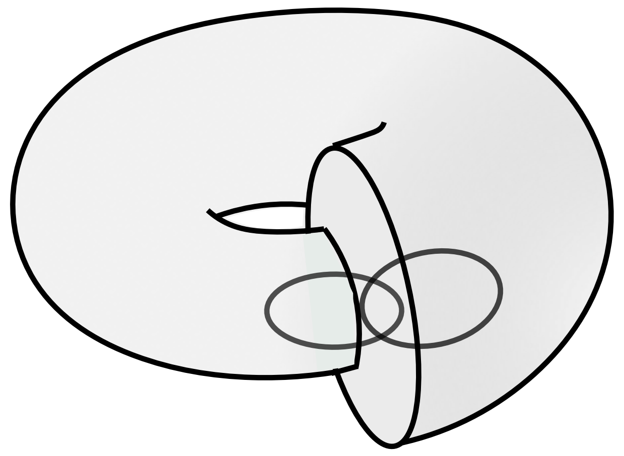
Non-geometric backgrounds

Described locally by Riemannian geometry with fluxes. However, **transition functions** are allowed to be **T-dualities.**

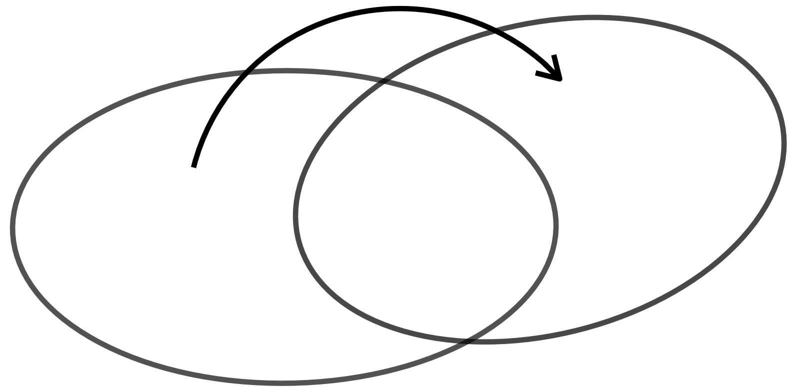
[Dabholkar, Hull ’02&‘05; Flournoy, Wecht, Williams ’04…]

Manifold, e.g. a torus vs. non-geometric background

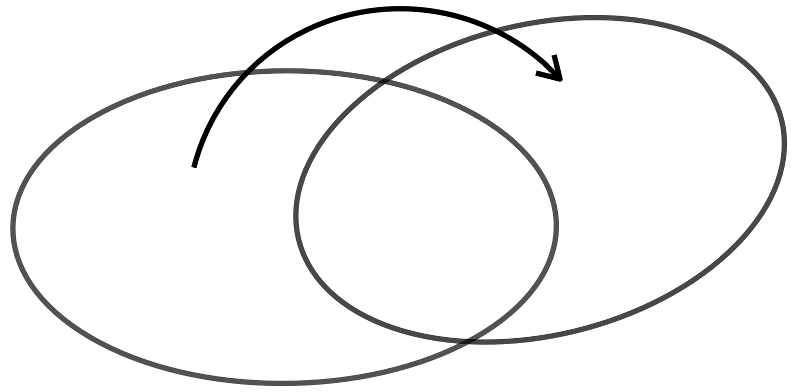


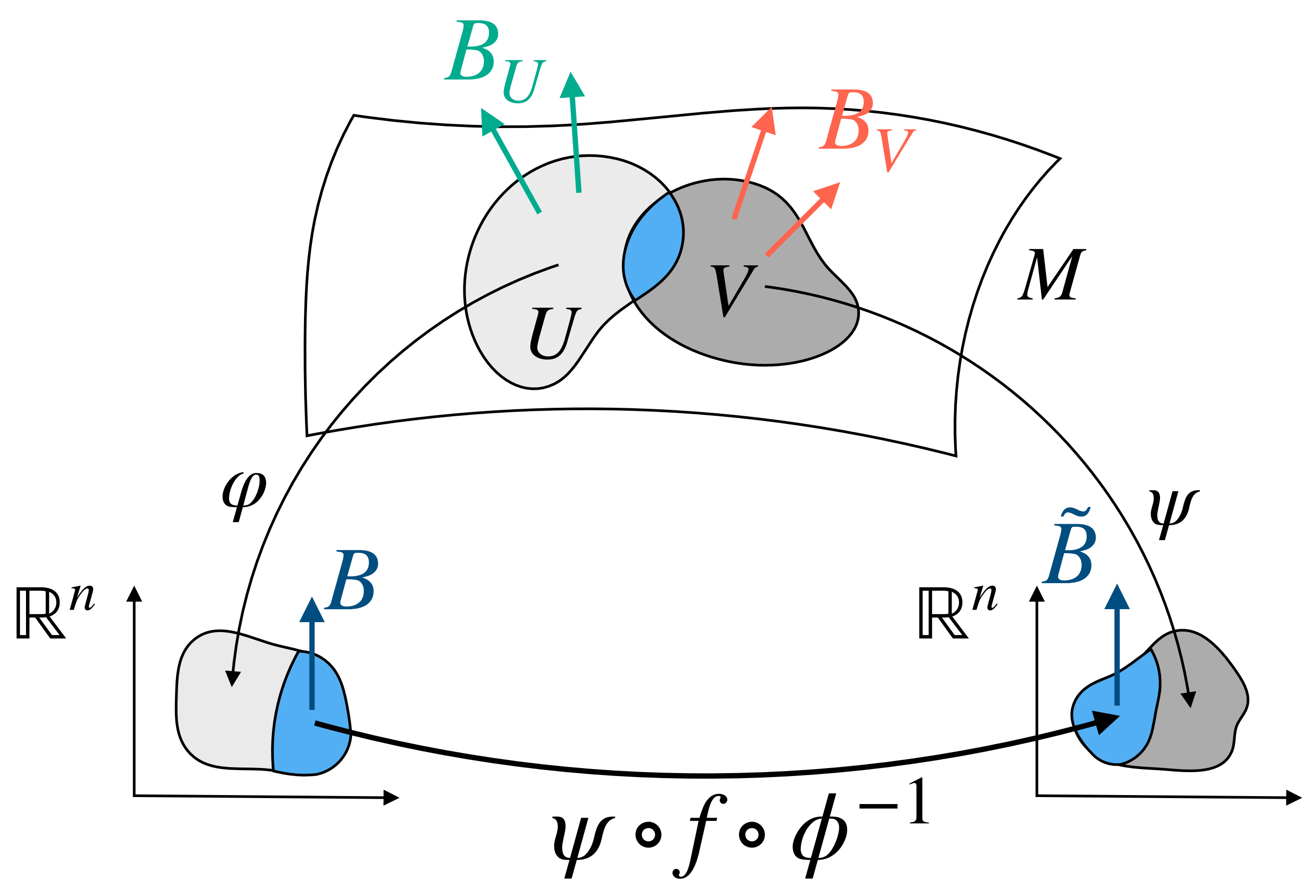


diffeomorphism



… and T-duality





$$f \in \begin{cases} Diff(M) & \text{: Riemannian} \\ Diff(M) \cup \text{T-duality} & \text{: non-geometric} \end{cases}$$

- ▷ **“Inevitable”** in string theory
- ▷ **Moduli stabilisation**
  [Becker, Becker, Blumenhagen, Lüst, Plauschinn, Shelton, Taylor, Vafa, Wecht, Walcher, …]

In addition to standard fluxes, there are **non-geometric fluxes**, called Q- and R-fluxes.

Basic example: T-duality chain of T^3 with H-flux $\quad H_{ijk} \rightarrow f_{ij}{}^{k} \rightarrow Q_i{}^{jk} \rightarrow R^{ijk}$

Are these valid backgrounds for quantum gravity?

Non-geometric backgrounds

Generically the action of non-geometric backgrounds is **ill-defined** in standard **NSNS frame**

$$\theta \simeq \theta + 2\pi$$
$$g_{\mu\nu}(\theta + 2\pi) \neq g_{\mu\nu}(\theta)$$

- How to obtain previous reduction procedure?
- ? How to obtain **metric & potential** on moduli space
- Consistent picture under **T-duality**?

Non-geometric backgrounds

$$\theta \simeq \theta + 2\pi$$
$$g_{\mu\nu}(\theta + 2\pi) \neq g_{\mu\nu}(\theta)$$

Generically the action of non-geometric backgrounds is **ill-defined** in standard **NSNS frame**

- ▷ How to obtain previous reduction procedure?
- ? ▷ How to obtain **metric & potential** on moduli space
- ▷ Consistent picture under **T-duality**?

$$V(\varphi^a) \sim \int \mathrm{d}^n y \sqrt{h_0} \left(\mathscr{R}(h) - \frac{1}{12} H^2 + 4(\partial \Phi_y)^2 \right)$$

$$\tilde{V}(\varphi^a) \sim \int \mathrm{d}^n y \sqrt{\tilde{h}_0} \left(\mathscr{R}(\tilde{h}) - \frac{1}{4} Q^2 + 4(\partial \tilde{\Phi}_y)^2 \right)$$

Perform the **field redefinition:** $(h + B)^{-1} = (\tilde{h}^{-1} + \beta)$ ⟶ $\mathscr{L}_\beta = \mathscr{L}_{NSNS} + \partial(\ldots)$ … **β-supergravity action**

[Andriot,Larfors,Lüst,Patalong ’11]

Non-geometric backgrounds

Generically the action of non-geometric backgrounds is **ill-defined** in standard **NSNS frame**

$$\theta \simeq \theta + 2\pi$$
$$g_{\mu\nu}(\theta + 2\pi) \neq g_{\mu\nu}(\theta)$$

?
- ▷ How to obtain previous reduction procedure?
- ▷ How to obtain **metric & potential** on moduli space
- ▷ Consistent picture under **T-duality**?

$$V(\varphi^a) \sim \int d^n y \sqrt{h_0}\Big(\mathscr{R}(h) - \frac{1}{12}H^2 + 4(\partial\Phi_y)^2\Big)$$

$$\tilde{V}(\varphi^a) \sim \int d^n y \sqrt{\tilde{h}_0}\Big(\mathscr{R}(\tilde{h}) - \frac{1}{4}Q^2 + 4(\partial\tilde{\Phi}_y)^2\Big)$$

Perform the **field redefinition:** $(h+B)^{-1} = (\tilde{h}^{-1} + \beta)$ ⟶ $\mathscr{L}_\beta = \mathscr{L}_{NSNS} + \partial(\ldots)$ … **β-supergravity action**

[Andriot,Larfors,Lüst,Patalong '11]

Crucial to **use β-supergravity** for consitency of **non-geometry backgrounds & geometric duals:**

> A **consistent picture** between a (globally) non-geometric space and its geometric dual
> - i.e. matching moduli spaces, potentials and towers of states -
> can be established **only after moving to the β-frame**, where the background is well-defined.

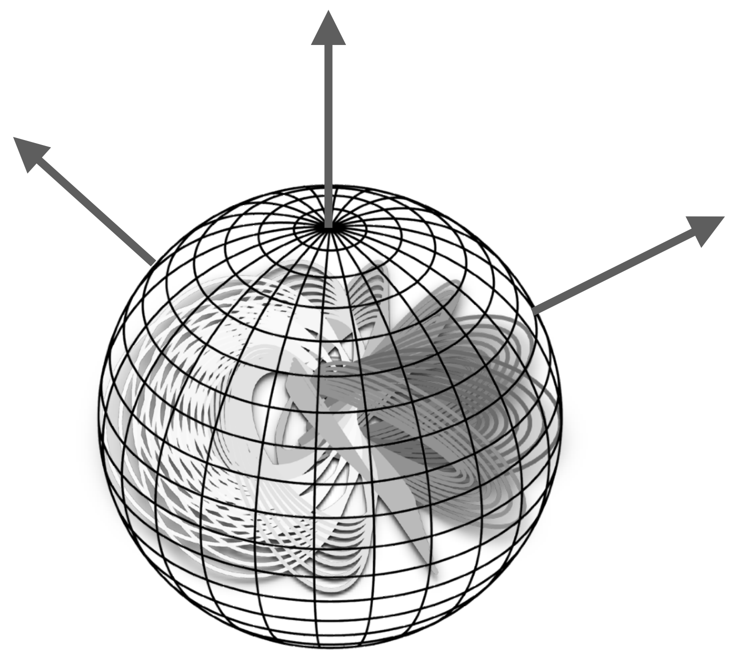
[Demulder, Lüst, TR '23]

▸ **Distance Conjecture:**

$$\gamma_{RR} = \frac{3}{R^2}$$

$$V(R;k) = -\frac{3}{2R^2} + \frac{k^2}{R^6}$$

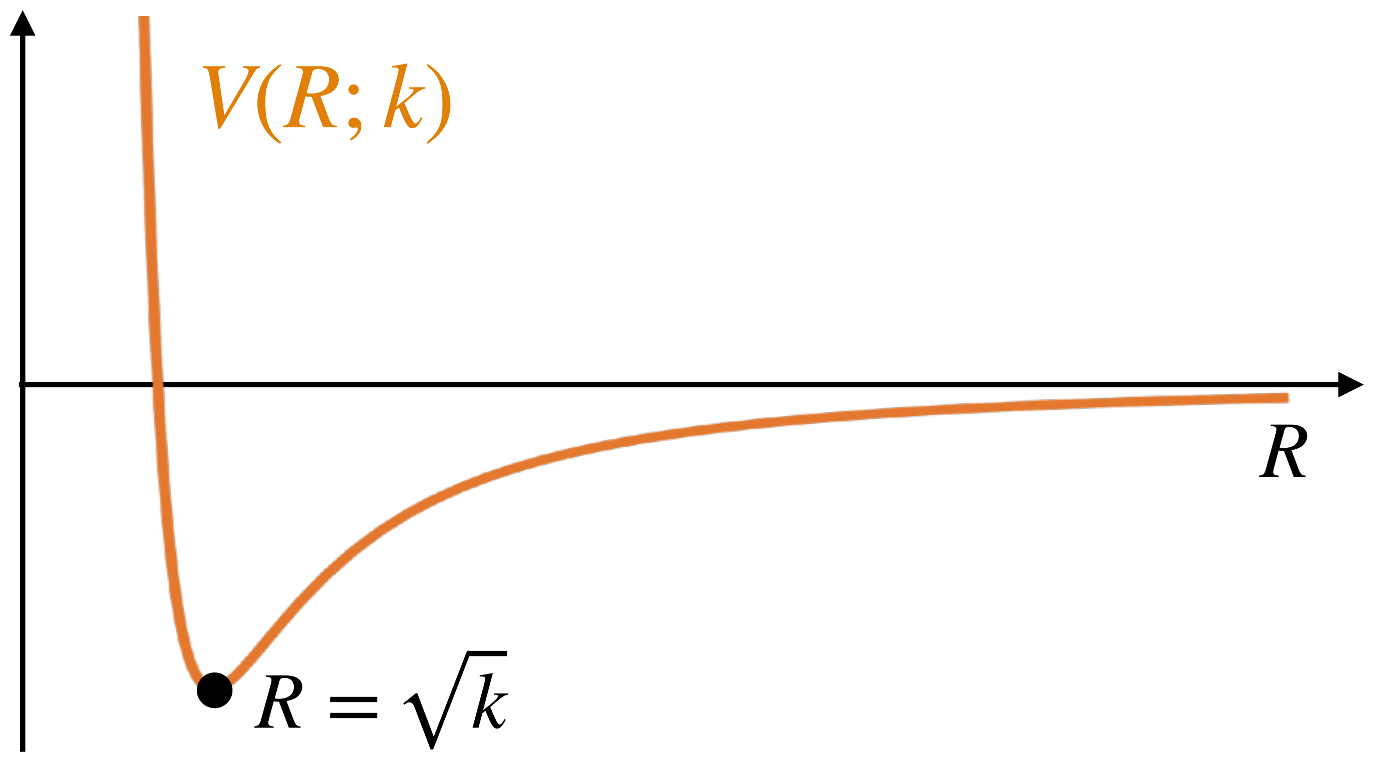
$\mathbf{S^3_R}$ with $\mathbf{[H] = k}$



Apparent inconsistency: S^3 with appropriately tuned H-flux is **valid string background** and therefore should be in the **Landscape**

However there is **no tower of light states** for R → 0 which is an infinite distance limit

…need to **take into account scalar potential**



[Demulder, Lüst, TR '23]

In **effective field theories** that **can be lifted to a theory of quantum gravity** in the UV, a **divergence in the scalar potential** emerges when approaching an **infinite locus point** for which the target space geometry **cannot give rise to a light tower of states**.

That is, the **potential signals pathological infinite distance loci** in the scalar field space.

A different perspective

? **In which sense is $R = 0$ at infinite distance in the presence of a (divergent) scalar potential?**

- Is the point “**reachable**” in the presence of a divergent **potential**?
- What is a “**good**” notion of **distance**?

A different perspective

? **In which sense is $R = 0$ at infinite distance in the presence of a (divergent) scalar potential?**

- ▷ Is the point "**reachable**" in the presence of a divergent **potential**?
- ▷ What is a "**good**" notion of **distance**?

There is a plethora of works, examining the notion of distance in various settings.

Very recently taking into account a scalar potential:

- ▷ [Mohseni, Montero, Vafa, Valenzuela '24]
- ▷ [Debusschere, Tonioni, van Riet '24]
- ▷ [De Biasio '22]

A different perspective

? **In which sense is $R = 0$ at infinite distance in the presence of a (divergent) scalar potential?**

- ▷ Is the point **"reachable"** in the presence of a divergent **potential**?
- ▷ What is a **"good"** notion of **distance**?

There is a plethora of works, examining the notion of distance in various settings.

Very recently taking into account a scalar potential:

- ▷ [Mohseni, Montero, Vafa, Valenzuela '24]
- ▷ [Debusschere, Tonioni, van Riet '24]
- ▷ [De Biasio '22]

…another somewhat **alternative aporach**: **Ricci flow**

[Kehagias, Lüst, Lüst '19]

Ricci flow & Ricci flow Conjecture

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton ’82]

Ricci flow & Ricci flow Conjecture

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton '82]

Ricci flow Conjecture: Take a family of metrics $g_{\mu\nu}(t)$ **satisfying Ricci flow equation**. There is a **infinite tower of states** becoming massless when **approaching a fixed point** $\partial_t g_{\mu\nu}(t)|_{t=t_0} = 0$ at **infinite distance**. [Kehagias, Lüst, Lüst '19]

Ricci flow & Ricci flow Conjecture

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton '82]

Ricci flow Conjecture: Take a family of metrics $g_{\mu\nu}(t)$ **satisfying Ricci flow equation**. There is a **infinite tower of states** becoming massless when **approaching a fixed point** $\partial_t g_{\mu\nu}(t)|_{t=t_0} = 0$ at **infinite distance**. [Kehagias, Lüst, Lüst '19]

$$\Delta_g \sim \int_{\tau_i}^{\tau_f} \left(\frac{1}{V_M} \int_M \sqrt{(g)} g^{mn} g^{op} \frac{\partial g_{mo}}{\partial \tau} \frac{\partial g_{np}}{\partial \tau} \right)^{1/2} \mathrm{d}\tau$$

AdS: $\mathcal{R}_{\mu\nu}(0) = \Lambda_0 \hat{g}_{\mu\nu}$ ⟶ $\Lambda(t) = \frac{\Lambda_0}{(1-2\Lambda_0 t)}$

fixed point with $\Lambda = 0$ as $t \to \infty$

I: $\Delta_g \simeq \log(1-2\Lambda_0 t) \to \infty$

II: $\Delta_R \equiv \log\left(\frac{\mathcal{R}(0)}{\mathcal{R}(t)}\right) \sim \log(1-2\Lambda_0 t) \to \infty$

[Kehagias, Lüst, Lüst '19]

Ricci flow Conjecture $\Longrightarrow$ AdS Distance Conjecture

Distance from Entropy functionals

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton '82]

Including the dilation leads to **Perelmans combined flow** [Perelman '02]

$$\begin{cases} \partial_t g_{\mu\nu}(t) &= -2\mathcal{R}_{\mu\nu}(t) \\ \partial_t \phi(t) &= -\frac{1}{2}\mathcal{R}(t) - \Delta\phi(t) + 2(\nabla\phi(t))^2 \end{cases}$$

Distance from Entropy functionals

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton '82]

Including the dilation leads to **Perelmans combined flow** [Perelman '02]

$$\begin{cases} \partial_t g_{\mu\nu}(t) &= -2\mathcal{R}_{\mu\nu}(t) \\ \partial_t \phi(t) &= -\frac{1}{2}\mathcal{R}(t) - \Delta\phi(t) + 2(\nabla\phi(t))^2 \end{cases}$$

…gradient with respect to "Entropy Functional"

$$\mathcal{F}(g,f) = \int_M \mathrm{d}^d x \sqrt{-g}\, e^{-2\phi} \left(\mathcal{R} + 4(\nabla\phi)^2 \right)$$

$$\int_M \mathrm{d}^d x \sqrt{-g}\, e^{-2\phi} = 1$$

Distance from Entropy functionals

Ricci flow equation: $$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -2\mathcal{R}_{\mu\nu}(t)$$ [Hamilton '82]

Including the dilation leads to **Perelmans combined flow** [Perelman '02]

$$\begin{cases} \partial_t g_{\mu\nu}(t) &= -2\mathcal{R}_{\mu\nu}(t) \\ \partial_t \phi(t) &= -\frac{1}{2}\mathcal{R}(t) - \Delta\phi(t) + 2(\nabla\phi(t))^2 \end{cases}$$

…gradient with respect to "Entropy Functional"

$$\mathcal{F}(g,f) = \int_M \mathrm{d}^d x \sqrt{-g}\, e^{-2\phi}\left(\mathcal{R} + 4(\nabla\phi)^2\right) \qquad \int_M \mathrm{d}^d x \sqrt{-g}\, e^{-2\phi} = 1$$

Conjecture: The **distance in field space $\Delta_{\mathcal{F}}$** along the combined flow is determined by the **entropy functional $\mathcal{F}(g,f)$**.

$\mathcal{F} = 0$ **lies at infinite distance** and is accompanied by an **infinite tower of massless states** and

$$\Delta_{\mathcal{F}} \simeq \log\left(\frac{\mathcal{F}_i}{\mathcal{F}_f}\right)$$

[Kehagias, Lüst, Lüst '19]

Generalized Ricci flow & Distance Conjecture

[Demulder, Lüst, TR] work in progress

?

Can we define notion of distance for generic internal manifold using generalized Ricci flow?
What are the implications for the Swampland Distance Conjecture in presence of a potential?

Generalized Ricci flow & Distance Conjecture

[Demulder, Lüst, TR] work in progress

?

Can we define notion of distance for generic internal manifold using generalized Ricci flow?
What are the implications for the Swampland Distance Conjecture in presence of a potential?

▷ Purely **NSNS internal background**: …can consider generalized Ricci flow [Oliynyk,Suneeta,Woolgar '05]

$$\mathcal{F}^H(g,H,\phi) = \int_M \mathrm{d}V\, e^{-2\phi}\left(\mathcal{R} - \frac{1}{12}|H|^2 + 4|\nabla\phi|^2\right) \longrightarrow \begin{cases} \frac{\partial g_{ij}}{\partial t} = -\left(\mathcal{R}_{ij} + \nabla_i\nabla_j\phi - \frac{1}{4}H_{ikl}H_j^{\ kl}\right) \equiv -\beta^g_{ij} \\ \frac{\partial H_{ijk}}{\partial t} = \dots \\ \dots \end{cases}$$

Generalized Ricci flow & Distance Conjecture

[Demulder, Lüst, TR] work in progress

? **Can we define notion of distance for generic internal manifold using generalized Ricci flow?**
What are the implications for the Swampland Distance Conjecture in presence of a potential?

▷ Purely **NSNS internal background**: …can consider generalized Ricci flow [Oliynyk,Suneeta,Woolgar '05]

$$\mathcal{F}^H(g,H,\phi) = \int_M \mathrm{d}V\, e^{-2\phi}\left(\mathcal{R} - \frac{1}{12}|H|^2 + 4|\nabla\phi|^2\right) \longrightarrow \begin{cases} \frac{\partial g_{ij}}{\partial t} = -\left(\mathcal{R}_{ij} + \nabla_i\nabla_j\phi - \frac{1}{4}H_{ikl}H_j^{\ kl}\right) \equiv -\beta^g_{ij} \\ \frac{\partial H_{ijk}}{\partial t} = \dots \\ \dots \end{cases}$$

Metric & potential on moduli space both derived from **background data** on internal space

⟶ what are the implications of defining **distance in presence of potential** according to $\Delta_{\mathcal{F}^H} \simeq \log\left(\frac{\mathcal{F}^H_i}{\mathcal{F}^H_f}\right)$

[Demulder, Lüst, TR; ongoing]

Generalized Ricci flow & Distance Conjecture

[Demulder, Lüst, TR] work in progress

? **Can we define notion of distance for generic internal manifold using generalized Ricci flow?**
What are the implications for the Swampland Distance Conjecture in presence of a potential?

▷ Purely **NSNS internal background**: …can consider generalized Ricci flow [Oliynyk,Suneeta,Woolgar '05]

$$\mathcal{F}^H(g,H,\phi) = \int_M \mathrm{d}V\, e^{-2\phi}\left(\mathcal{R} - \frac{1}{12}|H|^2 + 4|\nabla\phi|^2\right) \longrightarrow \begin{cases} \frac{\partial g_{ij}}{\partial t} = -\left(\mathcal{R}_{ij} + \nabla_i\nabla_j\phi - \frac{1}{4}H_{ikl}H_j^{\ kl}\right) \equiv -\beta^g_{ij} \\ \frac{\partial H_{ijk}}{\partial t} = \dots \\ \dots \end{cases}$$

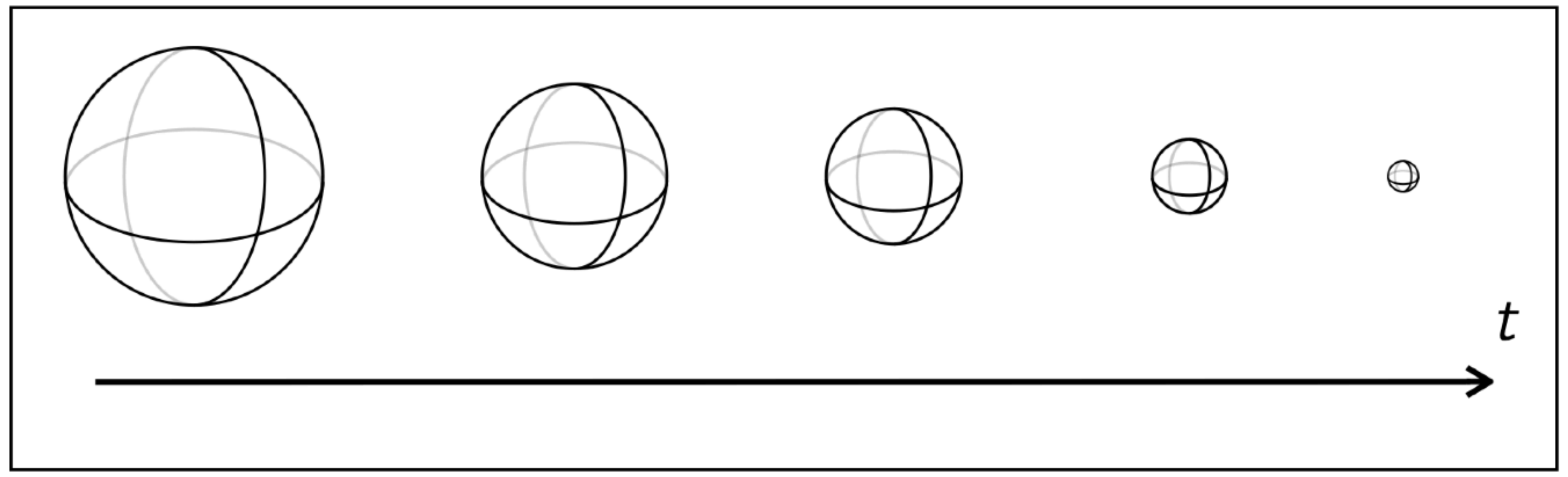
Metric & potential on moduli space both derived from **background data** on internal space

⟶ what are the implications of defining **distance in presence of potential** according to $\Delta_{\mathcal{F}^H} \simeq \log\left(\frac{\mathcal{F}^H_i}{\mathcal{F}^H_f}\right)$

[Demulder, Lüst, TR; ongoing]

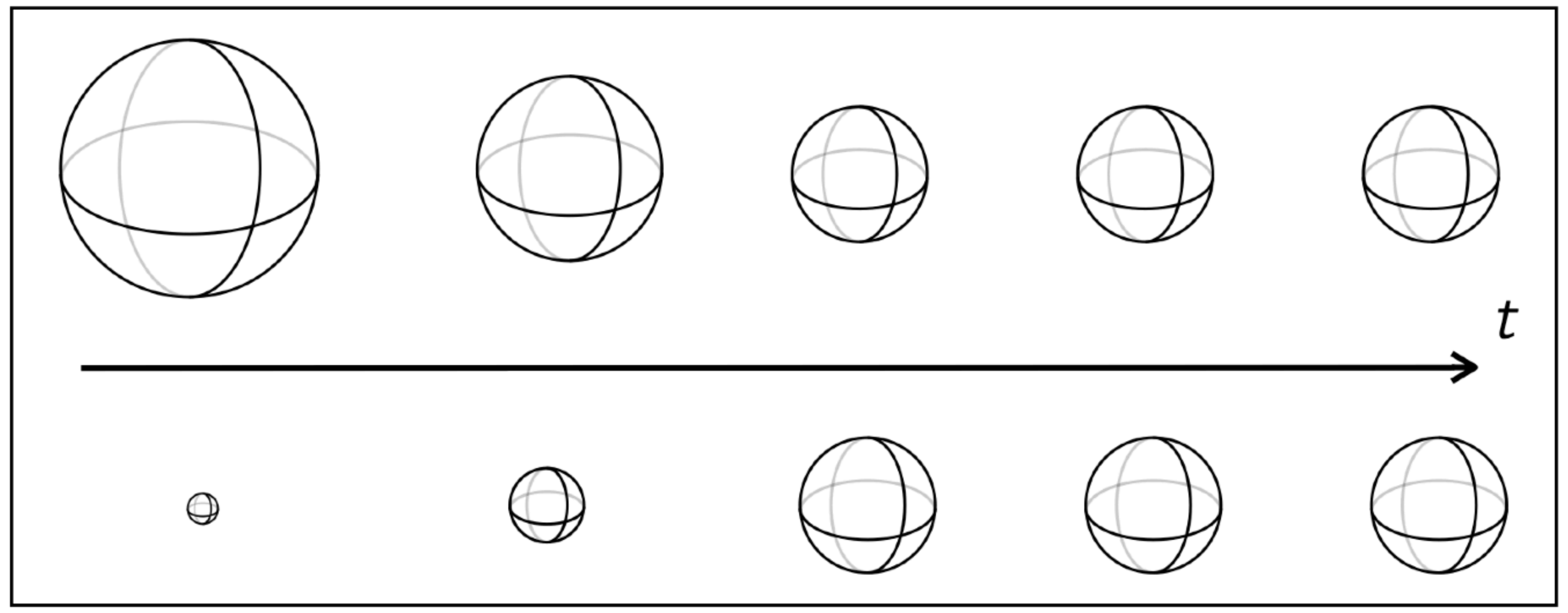
$\mathbf{S^3_r}$

$$\partial_t r(t) = -\frac{1}{2r(t)}$$



$\mathbf{S^3_r}$ **with H-flux [h]**

$$\partial_t r(t) = -\frac{1}{2r(t)} + \frac{h^2}{2r(t)^5}$$



Generalized Ricci flow & Distance Conjecture

[Demulder, Lüst, TR] work in progress

Can we define notion of distance for generic internal manifold using generalized Ricci flow?
What are the implications for the Swampland Distance Conjecture in presence of a potential?

▷ Purely **NSNS internal background**: …can consider generalized Ricci flow [Oliynyk,Suneeta,Woolgar '05]

$$\mathcal{F}^H(g,H,\phi) = \int_M \mathrm{d}V\, e^{-2\phi}\left(\mathcal{R} - \frac{1}{12}|H|^2 + 4|\nabla\phi|^2\right) \longrightarrow \begin{cases} \frac{\partial g_{ij}}{\partial t} = -\left(\mathcal{R}_{ij} + \nabla_i\nabla_j\phi - \frac{1}{4}H_{ikl}H_j^{\ kl}\right) \equiv -\beta^g_{ij} \\ \frac{\partial H_{ijk}}{\partial t} = \dots \\ \dots \end{cases}$$

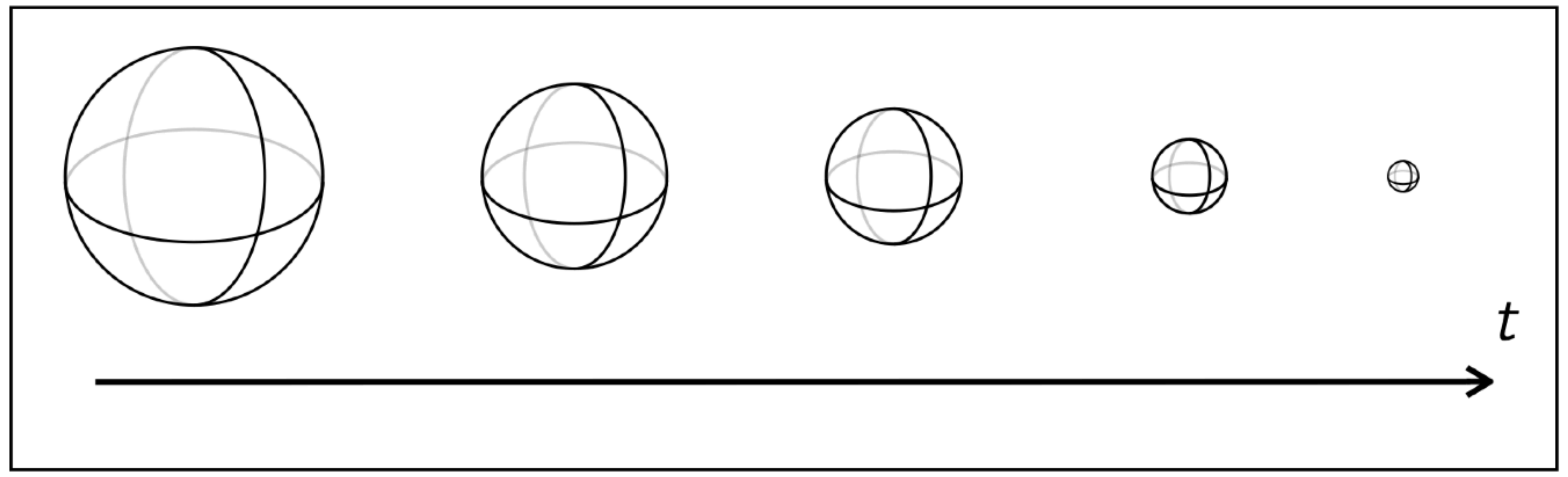
Metric & potential on moduli space both derived from **background data** on internal space

⟶ what are the implications of defining **distance in presence of potential** according to $\Delta_{\mathcal{F}^H} \simeq \log\left(\frac{\mathcal{F}^H_i}{\mathcal{F}^H_f}\right)$

[Demulder, Lüst, TR; ongoing]

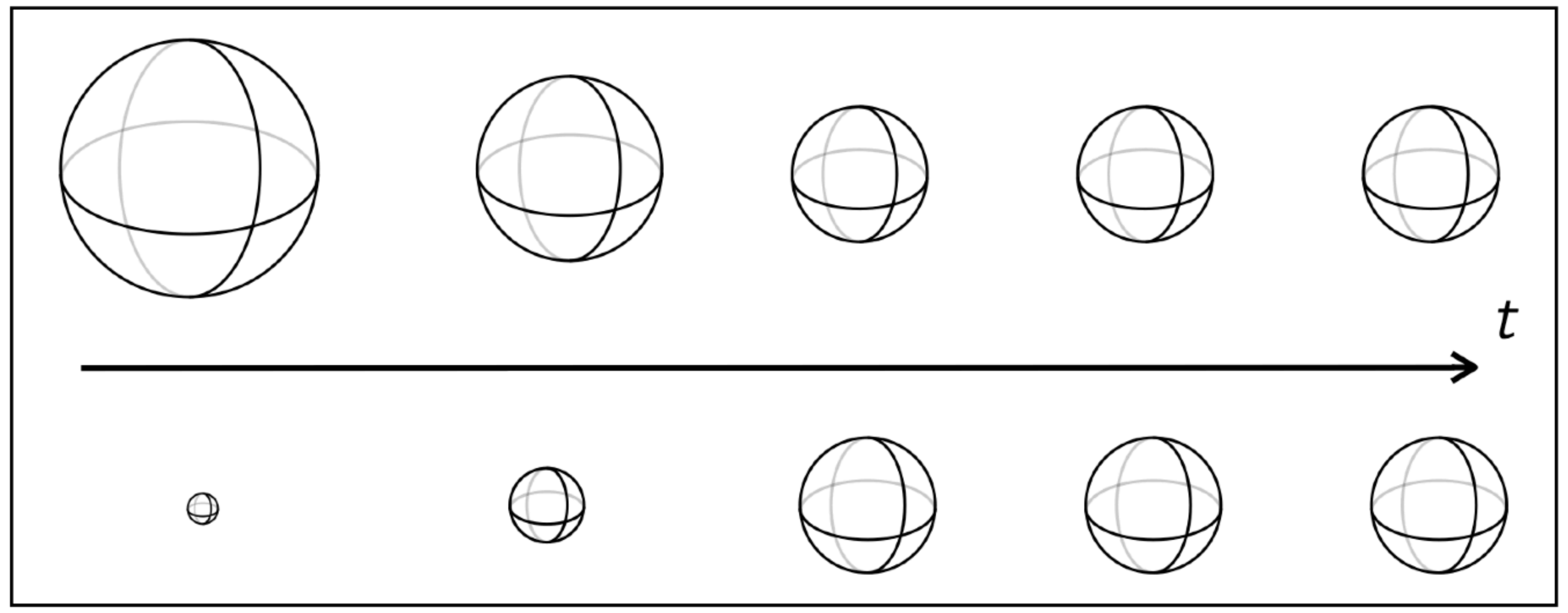
$\mathbf{S^3_r}$

$$\partial_t r(t) = -\frac{1}{2r(t)}$$



$\mathbf{S^3_r}$ **with H-flux [h]**

$$\partial_t r(t) = -\frac{1}{2r(t)} + \frac{h^2}{2r(t)^5}$$



▷ Adding **RR-fluxes**?
▷ **Non-geometric flow**?

[Demulder, Lüst, TR; ongoing]

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

?

- ▷ More **realistic setups: full 10d backgrounds**, e.g. $AdS_5 \times S^5, AdS_4 \times T^6$ with fluxes,…
- ▷ Flux variations & potential: **on-shell vs off-shell** [Li,Palti,Petri ’23][Palti,Petri ’24]
- ▷ **Deformations** and **generalized T-duality** (Poisson-Lie T-duality)
- ▷ **Distance** in presence of **potential** and **generalized Ricci flow** [Kehagias, Lüst, Lüst ’19] [Mohseni, Montero,Vafa, Valenzuela ’24] [Debusschere, Tonioni, van Riet ’24]
- ▷ **Truly non-geometric spaces** and the Swampland?

work in progress

Summary & Conclusions

- ▷ Studied **Distance Conjecture** for **curved compact spaces (with fluxes)**
- ▷ **Invariance** of the metric on moduli space **under (abelian) T-duality**
- ▷ Interplay of **scalar potential** and Distance Conjecture & **absence of tower of states**
- ▷ First step towards **non-geometric backgrounds** and associated distance on moduli space

- ▷ More **realistic setups: full 10d backgrounds**, e.g. $AdS_5 \times S^5, AdS_4 \times T^6$ with fluxes,...
- ▷ Flux variations & potential: **on-shell vs off-shell** [Li,Palti,Petri '23][Palti,Petri '24]
- ▷ **Deformations** and **generalized T-duality** (Poisson-Lie T-duality)
- ▷ **Distance** in presence of **potential** and **generalized Ricci flow** [Kehagias, Lüst, Lüst '19] [Mohseni, Montero,Vafa, Valenzuela '24] [Debusschere, Tonioni, van Riet '24]
- ▷ **Truly non-geometric spaces** and the Swampland?

work in progress

?

Thank you!

BACKUP SLIDES

T-duality and change in topology

[Giveon, Kiritsis ‘94; Bouwknegt, Evslin, Mathai ‘04; Alvarez, Alvarez-Gaume, Lozano ’05, ….]

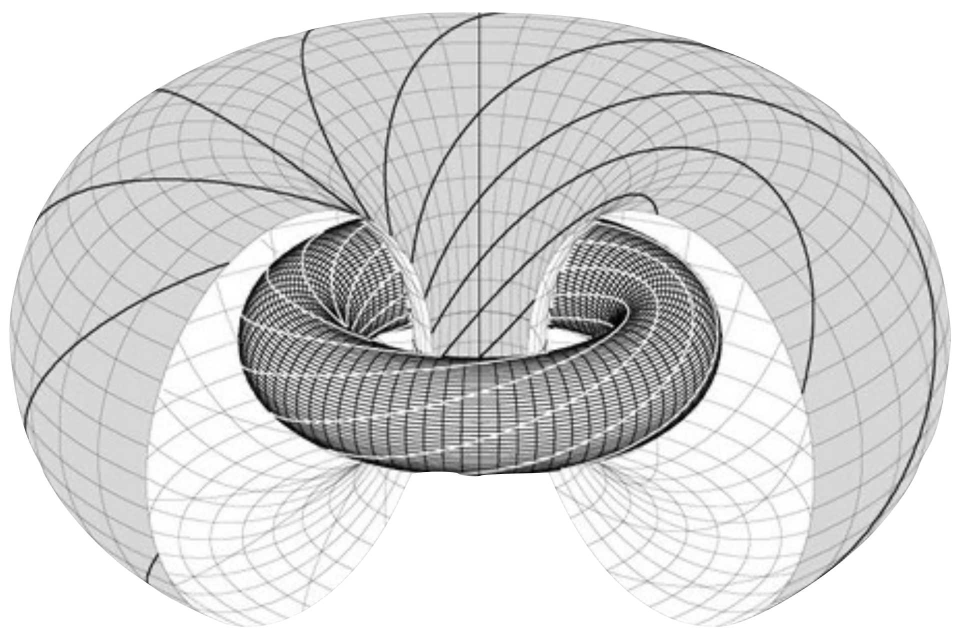
T-duality does not only affect the geometry locally but **can also affect its global structure**

T-duality exchanges *H*-flux and the **first Chern class**

$$\text{fluxes} : \imath_k \mathcal{H} \quad \longleftrightarrow \quad \text{topology} : c_1(E_k)$$

Concrete example:

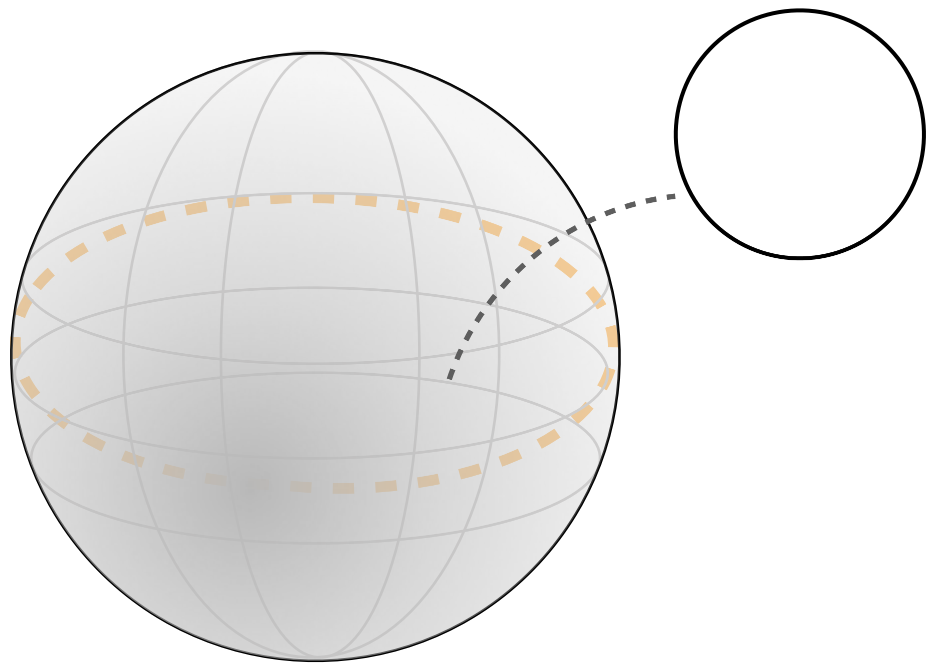
S^3 in the Hopf fibration



Abelian T-duality

$\longleftrightarrow$

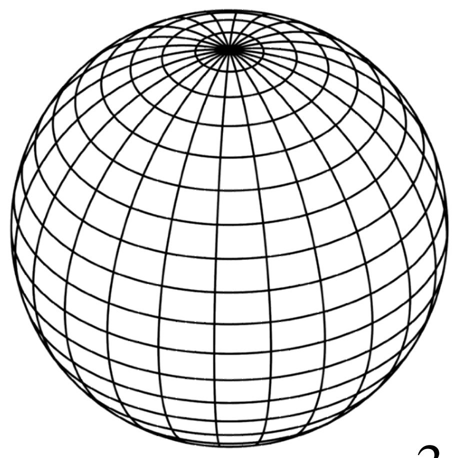
Trivial fibration $S^2 \times S^1$



$\pi_1(S^3) = 1\,, \quad [H] = 0$

$\pi_1(S^2 \times S^1) = 0\,, \quad [H] = 1$

NATD of S^3



S^3_R

NATD

$$V(\varphi^a) \sim \int \mathrm{d}^n y \sqrt{h_0}\left(\mathcal{R}(h) - \frac{1}{12}H^2 + 4(\partial\Phi_y)^2\right)$$

$$(h+B)^{-1} = (\hat{h}^{-1} + \beta)$$

$$\hat{V}(\varphi^a) \sim \int \mathrm{d}^n y \sqrt{\hat{h}_0}\left(\mathcal{R}(\hat{h}) - \frac{1}{4}Q^2 + 4(\partial\hat{\Phi}_y)^2\right)$$

$$h = \frac{1}{R^2\chi}\begin{pmatrix} R^4+\phi^2 & \phi\psi & \theta\phi \\ \phi\psi & R^4+\psi^2 & \theta\psi \\ \theta\phi & \theta\psi & R^4+\theta^2 \end{pmatrix}, \; B = \frac{1}{\chi}\begin{pmatrix} 0 & -\theta & \psi \\ \theta & 0 & -\phi \\ -\psi & \phi & 0 \end{pmatrix}$$

β-gravity

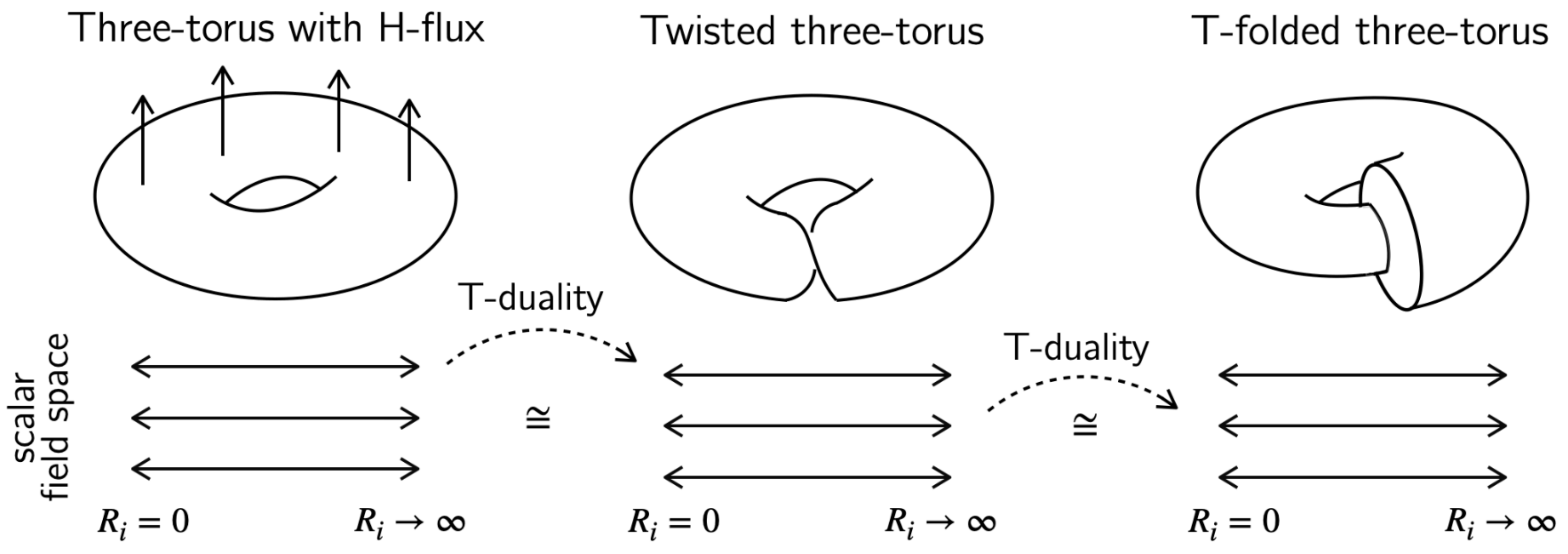
[Andriot,Larfors,Lüst,Patalong '11]

$$\hat{h} = R^{-2} I_3 , \; \beta = \begin{pmatrix} 0 & -\theta & \psi \\ \theta & 0 & -\phi \\ -\psi & \phi & 0 \end{pmatrix}$$

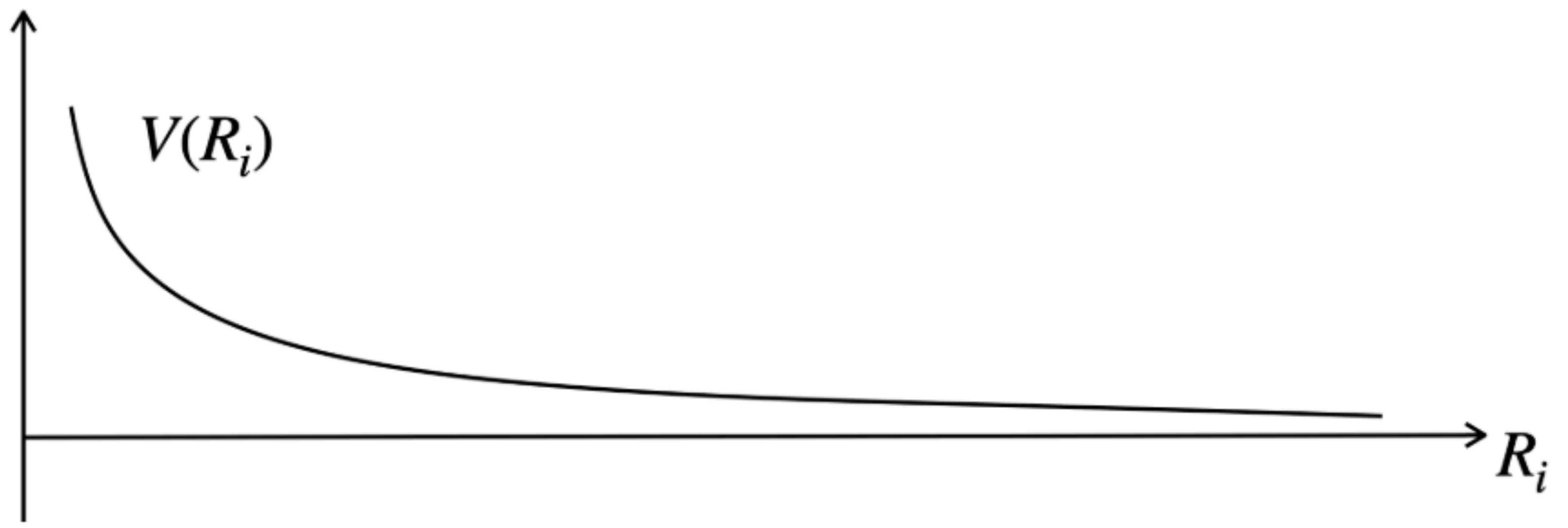
$\chi = R^4 + \phi^2 + \psi^2 + \theta^2$, and $\{\phi, \psi, \theta\}$ periodic angular coordinates

Q-flux: $Q_i^{\,jk} = \partial_i \beta^{jk}$

T-duality chain



	H^2	$\mathscr{R}(h)$	Q^2
$V(R_i)$:	$+\frac{h^2}{2}(R_1^2R_2^2R_3^2)^{-1}$	$+\frac{h^2}{2}\tilde{R}_1^2(R_2^2R_3^2)^{-1}$	$+\frac{h^2}{2}\tilde{R}_1^2\tilde{R}_2^2(R_3^2)^{-1}$



	modes	$R_i \to 0$	$R_i \to \infty$
$T^3_{\mathcal{H}\,\{R_1,R_2,R_3\}}$, $[\mathcal{H}] = k$	$w:\ \mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}$	$w:$ (light, light, light)	$w:$ (heavy, heavy, heavy)
	$p:\ nc\oplus nc\oplus nc$	$p:$ (~~heavy~~, ~~heavy~~, ~~heavy~~)	$p:$ (~~light~~, ~~light~~, ~~light~~)
$T^3_{\text{tw}\,\{R_1,R_2,R_3^{-1}\}}$, $[f] = k$	$w:\ \mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}_k$	$w:$ (light, light, ~~heavy~~)	$w:$ (heavy, heavy, ~~light~~)
	$p:\ nc\oplus nc\oplus\mathbb{Z}$	$p:$ (~~heavy~~, ~~heavy~~, light)	$p:$(~~light~~, ~~light~~, heavy)
$T^3_{Q\,\{R_1,R_2^{-1},R_3^{-1}\}}$, $[Q] = k$	$w:\ \mathbb{Z}\oplus\mathbb{Z}_k\oplus\mathbb{Z}_k$	$w:$ (light, ~~heavy~~, ~~heavy~~)	$w:$ (heavy, ~~light~~, ~~light~~)
	$p:\ nc\oplus\mathbb{Z}\oplus\mathbb{Z}$	$p:$ (~~heavy~~, light, light)	$p:$ (~~light~~, heavy, heavy)

T-duality chain of T^3 with H-flux is not a proper string background. Careful analysis reveals a lack of towers of states for some infinite distance points and therefore:

Without a completion, e.g. additional fluxes, the backgrounds of the **T-duality chain** lie in the **Swampland**.

[Demulder, Lüst, TR ’23]

$$S_{\rm DFT} = \int dx d\tilde{x}\, e^{-2d} \Big(\frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\,\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK} - 2\,\partial_M d\,\partial_N \mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d \Big)\,.$$

$$\begin{pmatrix} h - Bh^{-1}B & Bh^{-1} \\ -h^{-1}B & h^{-1} \end{pmatrix} = \mathcal{H} = \begin{pmatrix} \tilde{h} & \tilde{h}\beta \\ -\beta\tilde{h} & \tilde{h}^{-1} - \beta\tilde{h}\beta \end{pmatrix}$$

β-gravity

[Andriot,Larfors,Lüst,Patalong '11, Andriot et al '12,...]

$$\mathcal{L}_{\rm DFT}(\mathcal{E}, d)$$

$$\mathcal{L}_{\rm DFT}(g,b,\phi) = \mathcal{L}_{\rm DFT}(\tilde{g},\beta,\tilde{\phi}) = \mathcal{L}_{\rm DFT}(\mathcal{R},\check{\mathcal{R}}) + \partial(\dots) + \tilde{\partial}(\dots)$$

$$\tilde{\partial}=0$$

$$\mathcal{L}_{\rm NSNS} + \partial(\dots) = \tilde{\mathcal{L}}_0 + \partial(\dots) = \tilde{\mathcal{L}}_\beta + \partial(\dots)$$

$$\tilde{\mathcal{L}}_\beta = e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\Big(\mathcal{R}(\tilde{g}) + \check{\mathcal{R}}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}R^2 + 4(\beta^{mp}\partial_p\tilde{\phi} - \mathcal{T}^m)^2\Big)$$
$$\simeq \mathscr{R}(g) - \frac{1}{4}Q^2 - \frac{1}{2}\tilde{g}_{ij}Q_k{}^{jl}Q_l{}^{ki} + 4(\partial\tilde{\phi})^2 + \dots$$

$= \partial(\dots)$

$$\mathcal{L}_{\rm NSNS} = \mathcal{R}(h) - \frac{1}{12}\mathcal{H}^2 + 4(\partial\Phi_y)^2 = \frac{3}{2R^2} + 4R^2\mathcal{X}^{-2}\left(3R^4 + \phi^2 + \psi^2 + \theta^2\right) \qquad \mathcal{L}_\beta = \frac{3}{2R^2}$$