An inflationary cosmology from (AdS) wormholes

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arXiv:2403.17046 and in progress (+ I. Gialamas)

Quantum Gravity, Strings and the Swampland

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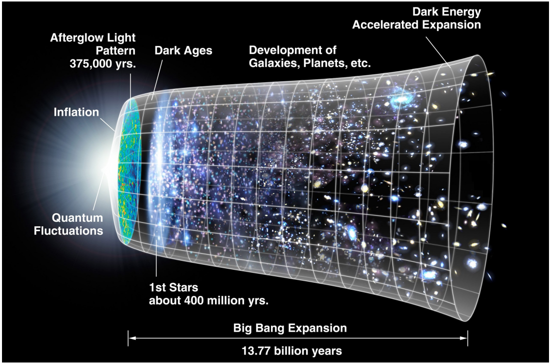
The History of our Universe

- Our Universe is currently expanding
- It is "Hot" ($T \simeq 2.73$ K)
- Extremely uniform at large scales $\delta T/T \sim 10^{-5}$



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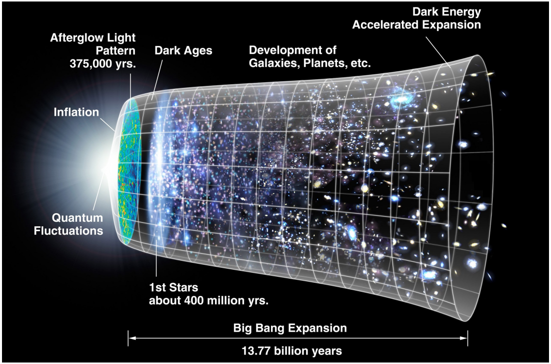
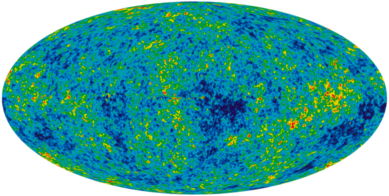
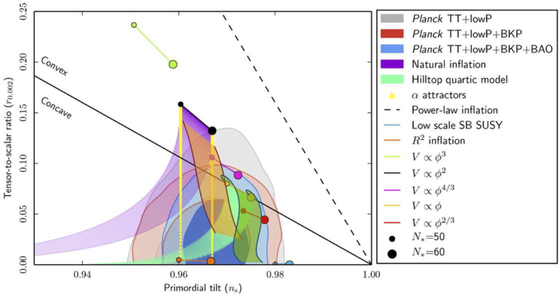


But how did it all start?



Features of the cosmic evolution

- Flatness "problem" - Universe is nearly flat, homogeneous and isotropic
- Horizon "problem" - causally disconnected regions of spacetime very similar
- Monopole "problem" - No exotic relics (ex: monopoles) around
- Production of primordial perturbations that are nearly scale invariant
- Inflation is a theory that can adequately explain these features (+more)



Pertinent Questions

- What gave rise to the initial conditions/state of inflation?
- Initial singularity - Our physical laws cease to work
- Do we really need a complete theory of quantum gravity to understand these?
- Is there any (approximate) way to compute (estimate) probabilities and features of the early universe Cosmology?

The Wheeler - DeWitt equation and "Quantum Cosmology"

- [Hartle and Hawking] gave one such appealing proposal for computing the "Wavefunction of the Universe"
- Based on the so called [Wheeler DeWitt] (WDW) equation
- In this approach one uses the canonical (Hamiltonian) formalism of general relativity and promotes the constraints expressing diffeomorphism invariance to quantum operators annihilating the wavefunction

Canonical formalism and constraints

- Using the [Arnowitt-Deser-Misner] decomposition

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

N is called the "lapse", N^i is the "shift" vector and g_{ij} is the spatial metric on a slice Σ

- Starting from the Einstein Hilbert ($+$ matter) action

$$S = \frac{1}{2\kappa}\int d^4x\,\sqrt{|g|}\,R^{(4)} \,+\, S^{matter}$$

the canonical Hamiltonian can be written in the form

$$H_c = \int_\Sigma d^3x\sqrt{g}\,(NH + N^i H_i)$$

$$H = 2\kappa g^{-1}\left(g_{ik}g_{jl}\pi^{kl}\pi^{ij} - \frac{1}{2}(g_{ij}\pi^{ij})^2\right) - \frac{1}{2\kappa}R^{(3)} + H^{matter}$$

$$\pi^{ij} = \frac{\delta S}{\delta \dot{g}_{ij}}\,, \qquad H_i = -2g_{ij}D_k\frac{\pi^{jk}}{\sqrt{g}} + H_i^{matter}$$

where D_i is the g_{ij} covariant derivative and we indicate possible additional matter contributions

Constraints and the Wheeler DeWitt equation

- Diffeomorphism invariance $\Rightarrow$ The physical states/configurations are independent of the choice of lapse and shift (N, N^i)
- This leads to constraints `[Dirac]` $\Rightarrow H,\ H_i = 0$
- Let us also consider as matter a scalar field ϕ (that will play the role of the inflaton)
- At the quantum level one has to impose the constraints, acting as operators on the wavefunctions

$$\widehat{H}_{WDW}(\pi_{ij}, g_{ij}; \pi_\phi, \phi)\,\Psi_\Sigma(g_{ij}, \phi) = 0\,, \qquad \widehat{H}_i(\pi_{ij}, g_{ij}; \pi_\phi, \phi)\,\Psi_\Sigma(g_{ij}, \phi)$$

$$\hat{\pi}_{ij}\Psi_\Sigma(g_{ij}, \phi) = -i\frac{\delta}{\delta g_{ij}}\Psi_\Sigma(g_{ij}, \phi)\,, \qquad \hat{\pi}_\phi\Psi_\Sigma(g_{ij}, \phi) = -i\frac{\delta}{\delta \phi}\Psi_\Sigma(g_{ij}, \phi)$$

- These equations are not really well defined
  $\Rightarrow$ There exists a "minisuperspace" ansatze/truncation that is better defined and leads to self-adjoint operators

  Fortunately the isotropy and homogeneity of the universe makes this ansatze physically relevant

Minisuperspace and the No Boundary Proposal

- The WDW equation makes sense in the reduced minisuperspace ansatze

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_\Sigma^2\,, \quad \phi = \phi(t)$$

- In this case $\widehat{H}_i\Psi_\Sigma = 0$ automatically and $\widehat{H}_{WDW}\Psi_\Sigma = 0$ becomes well defined
- One has to supplement appropriate "boundary" conditions
- The [Hartle - Hawking] No Boundary (NB) proposal posits that one has to make an excursion to Euclidean signature and consider compact metrics with no boundary at early times
- The state/wavefunction that one obtains in this way is also called the [Bunch - Davies] or Euclidean vacuum (the analogue of the Minkowski vacuum in a Cosmological setting i.e. $\Lambda > 0$)
- There is also an alternative [Linde - Vilenkin] Tunelling (T) proposal (probability influx/outflux in the superspace boundaries), that we shall contrast it with

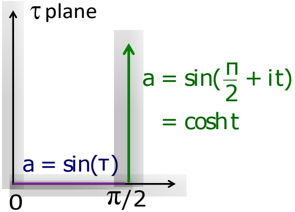
The simplest example: Empty de Sitter

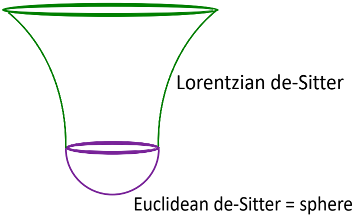
Consider the Einstein Hilbert action with positive cosmological constant

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(R-2\Lambda)\,, \qquad \Lambda > 0$$

that admits an empty de Sitter solution

The [Hartle - Hawking] proposal classically describes a (complex) metric
- half of Euclidean de-Sitter glued to half of Lorentzian de-Sitter -





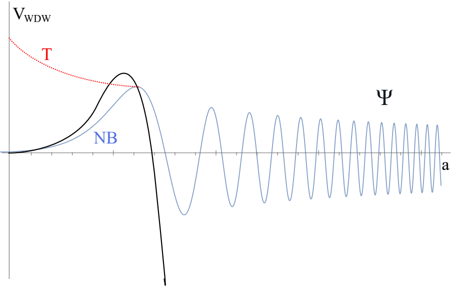
$$ds^2 = d\tau^2 + \sin^2\tau\, d\Omega_3^2 \longrightarrow ds^2 = -dt^2 + \cosh^2 t\, d\Omega_3^2$$

- The minisuperspace WDW equation (positive cc./no matter) reads

$$\left(\widehat{\pi}_a^2 + a^2 - \frac{\Lambda}{3}a^4\right)\Psi_\Sigma(a) = 0 \qquad \widehat{\pi}_a = -i\kappa\frac{d}{da}$$

- To understand its semi-classical properties - convenient to employ a "WKB" ansatze ($\kappa = 8\pi G_N \hbar \to 0$)

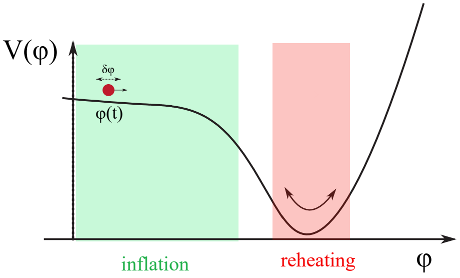
$$\Psi^L_\Sigma(a) = A_L e^{iS_L/\kappa} + B_L e^{-iS_L/\kappa}, \qquad \Psi^E_\Sigma(a) = A_E e^{S_E/\kappa} + B_E e^{-S_E/\kappa}$$



- For large a the wavefunction is oscillatory (Lorentzian), while for small a it has an exponential increasing/decreasing behaviour (Euclidean)
- The No Boundary proposal selects the increasing branch and the wavefunction vanishes at zero a -
The Tunneling/[Vilenkin] proposal would select the decreasing branch

- One can include the presence of the scalar inflaton field ϕ
- We assume a slow roll approximation for the potential $V(\phi)$ in the inflationary region

$$\epsilon_V \equiv \frac{M_P^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 \ll 1\,, \quad \eta_V \equiv \frac{M_P^2}{8\pi}\frac{V_{\phi\phi}}{V} \ll 1$$



- The WDW wavefunction now depends on two arguments i.e. $\Psi_\Sigma(a,\phi)$

- Under the slow roll approximation for the potential $V(\phi)$ one finds the semi-classical (WKB) No Boundary/Tunneling wavefunctions $(\tilde{V} = \kappa V/3)$

$$\Psi_{NB}(a,\phi) \simeq P_{NB}^{1/2}\,\Re\left(e^{iS_L(a,\phi)}\right), \quad P_{NB} = e^{-S_E(\phi)}$$

$$\Psi_T(A,\phi) \simeq P_T^{1/2}\left(e^{-iS_L(a,\phi)}\right), \quad P_T = e^{+S_E(\phi)},$$

$$S_E(\phi) = -\frac{8\pi^2}{\kappa\tilde{V}(\phi)}, \quad S_L(a,\phi) \simeq \frac{8\pi^2(a^2\tilde{V}(\phi)-1)^{3/2}}{\kappa\tilde{V}(\phi)}$$

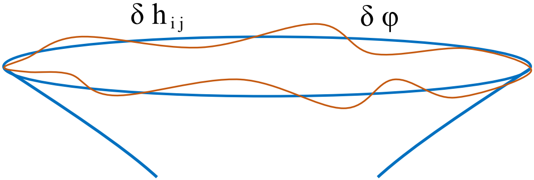
- S_E is the on-shell action of Euclidean de-Sitter (sphere)
  S_L is the on-shell action in the Lorentzian-oscillatory region $a^2\tilde{V}(\phi) > 1$

- The value of the inflaton/size of the sphere are at horizon crossing during inflation (ϕ_*, a_*)

No Boundary and slow roll inflation: Fluctuations

[Halliwell - Hawking]

- It is also possible to describe (inhomogeneous) fluctuations of the fields $\phi(\Omega) = \phi_* + \delta\phi(\Omega)\,,\ g_{ij}(\Omega) = g^*_{ij} + \delta h_{ij}(\Omega)$ etc.



- The No Boundary proposal predicts the correct spectrum of primordial perturbations with a Gaussian suppression factor

$$|\Psi_{NB}(\phi)|^2 \sim e^{-S_E(\phi_*)} \prod_{modes} \exp\left(-\delta\phi_{mode}\, C_{mode}\, \delta\phi_{mode}\right)$$

(it describes the analogue of a Cosmological "vacuum")

- In the Tunneling proposal such fluctuations are unsuppressed $(- \leftrightarrow +)$...

An important problem of the No Boundary proposal

- Given the wavefunction, we can also compute the probability for a specific "history"/realisation of the Universe, via its norm $P = |\Psi|^2$

$$|\Psi_{NB}(\phi)|^2 \simeq \exp\left(-S_E(\phi)\right) = \exp\left(\frac{M_P^4}{V(\phi)}\right)$$

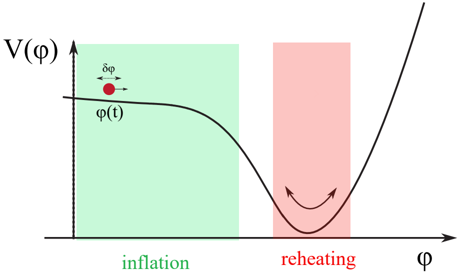
- This comes from the leading piece of the wavefunction

- It leads to an important problem for the No Boundary proposal in the context of inflation (See the reviews by [Lehners, Maldacena])

The problem

- Remember the current cosmological constant problem

$$\frac{M_P^4}{V(\phi_{now})} \simeq 10^{120}$$



- The problem with the No Boundary proposal is exponentially worse!

$$P_{NB} = |\Psi_{NB}(\phi)|^2 \simeq \exp\left(-S_E(\phi)\right) = \exp\left(\frac{M_P^4}{V(\phi)}\right)$$

- It gives an overwhelming probability ($P_{NB} \gg 1$) for an empty cold universe, with the smallest allowed number for the cosmological constant
- In the inflationary context it predicts the least number of e-folds
- The issue stems from the fact that the on-shell action for the positively curved Euclidean de-Sitter is negative

Ideas to evade this problem

- The Tunneling wavefunction [Linde - Vilenkin] evades this issue ($P_T \simeq e^{+S_E}$), but does not describe correctly the cosmological fluctuations beyond minisuperspace (they get enhanced)
- Selection rule or anthropic reasoning [Hartle - Hawking - Hertog ...]
- The gravitational path integral is not very well defined - non-renormalizability and the conformal mode problem - we need to understand it in a Picard-Lefschetz fashion and define an appropriate (steepest descend) contour in field space.
  Can this help to solve the problem?
- Change entirely the assumptions/setup giving rise to our Cosmology [PB - Papadoulaki]

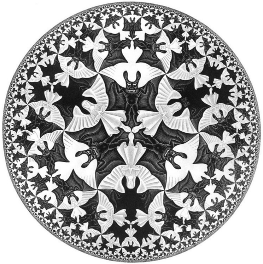
The No Boundary proposal and AdS/CFT

There is a case where the analogue of the No Boundary proposal works perfectly well: The AdS/CFT correspondence ($Z_{QGR}^{AdS} = Z_{CFT}^{\partial AdS}$)

- ex: Global $EAdS_4$ (regular interior $\leftrightarrow$ N.B.) and the sphere partition function

$$ds^2_{H_4} = L^2_{AdS}(d\tau^2 + \sinh^2\tau d\Omega_3^2)$$

$$e^{-S_E} \sim Z_{CFT}(S^3)\,, \quad S_E = \frac{L^2_{AdS}}{2G_N}$$



- Both sides can be computed and agree. For example in ABJM (finite-N) [Kapustin-Willet-Yaakov, Drukker-Marino-Putrov ...]
- Here it is crucial that the on-shell action of AdS is positive (after performing holographic renormalization)
- No direct relation to Cosmology (with a simple $\tau = it$)

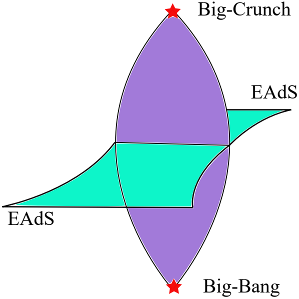
Euclidean Wormholes and Bang-Crunch Cosmologies

AdS/CFT context: [Maldacena-Maoz (04), PB-Gaddam-Papadoulaki (17) + Kiritsis (19-21), Van Raamsdonk et. al. (20-23) ...]

- In AdS/CFT there is an example that gives rise to FRW cosmologies: Two boundary Euclidean AdS wormholes ($' = d/d\tau$)

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3^2 \,, \quad a''(0) > 0 \,, \, a'(0) = 0 \,, \, a(\tau \to \pm\infty) \sim e^{H|\tau|}$$

- These Euclidean Wormholes are NOT related to Black Holes (horizons) via analytic continuation - Instead:



- Their radial analytic continuation $\tau = it$ gives rise to Bang - Crunch Cosmologies (Remember that Λ is negative)

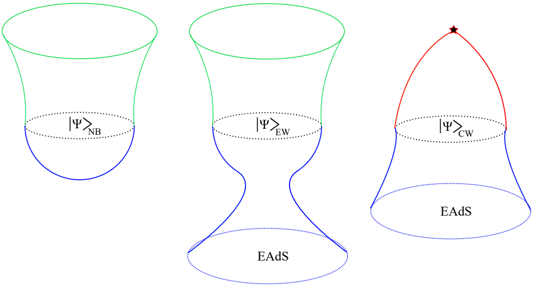
$$ds^2 = -dt^2 + a^2(t) d\Omega_3^2$$
$$\ddot{a}(0) < 0 \,, \quad \dot{a}(0) = 0$$

A new proposal for the wavefunction of the Universe

- An issue with these geometries is that upon analytic continuation they inevitably crunch and do not allow for a period of inflation

- Our idea [PB - Papadoulaki (24)] : Combine features of both anti-de Sitter and de-Sitter - we need a Euclidean wormhole geometry that is asymptotically EAdS that transitions into EdS near its throat

- By cutting it in half we can "glue" to it an expanding Lorentzian Universe

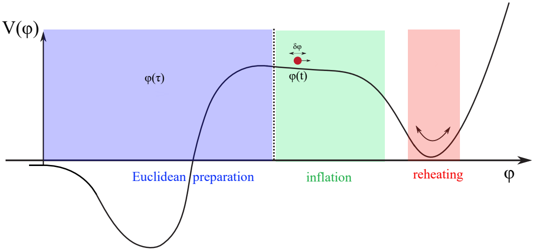


"Wineglass" AdS wormholes

- We shall call (half of) these geometries "wineglass" (half) wormholes
- Their defining properties: They should asymptote to a EAdS space: $a(\tau \to \pm\infty) \sim \exp(H_{AdS}|\tau|)$ and in addition

$$a''(0) < 0\,, \quad a'(0) = 0\,, \quad a(0) = a_{\mathsf{max}}\,, \quad \phi'(0) = 0$$

  so that a_{max} is a local maximum of the scale factor
- These are also good initial conditions for a subsequent inflationary evolution (since $\ddot{a}(0) > 0$)
- An example of a scalar potential that can support all these features



A model for "wineglass" AdS wormholes

- A simple model: Consider an Einstein-scalar-axion system ($\kappa \equiv M_{Pl}^{-2}$)

$$S_E = \int d^4x \sqrt{g_E} \left(-\frac{1}{2\kappa} R + \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + V(\phi) + \frac{1}{12 f_\alpha^2} H_{\mu\nu\rho} H^{\mu\nu\rho} \right)$$

and the spherically symmetric and homogeneous ansatze (q is a constant axion charge)

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3^2 \,, \quad \phi(\tau) \,, \quad H_{ijk} = q \epsilon_{ijk}$$

One finds the two independent EOMs ($Q^2 \equiv q^2/2f_\alpha^2$)

$$\frac{a'^2}{a^2} - \frac{1}{a^2} + \frac{\kappa}{3} \left(V(\phi) - \frac{\phi'^2}{2} \right) + \frac{\kappa Q^2}{3a^6} = 0 \,,$$

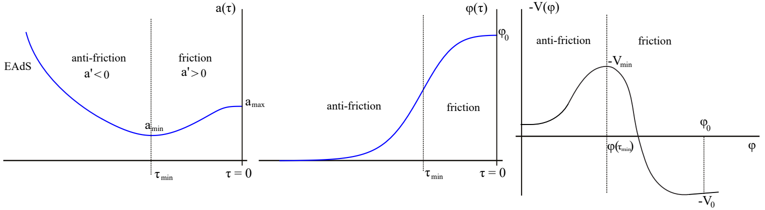
$$\phi'' + 3 \frac{a' \phi'}{a} - \frac{dV}{d\phi} = 0 \,,$$

- The EOM for the scalar field describes a particle moving in the potential $-V(\phi)$ with an (anti)-friction term $3a'\phi'/a$

- We consider a potential $V(\phi)$ with a local maximum at $\phi = 0$ i.e. $V(\phi) \sim -1 + m^2\phi^2/2$ with $m^2 < 0$

- This leads to a renormalization group flow driven by a relevant operator with conformal dimension $\Delta = 3/2 + \sqrt{9/4 + m^2} < 3$

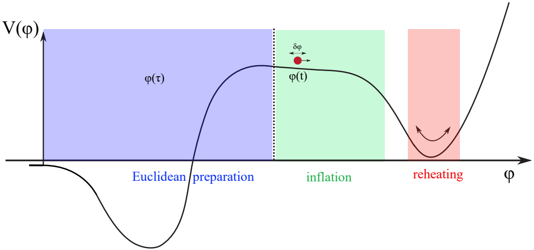
- The Euclidean evolution of the scale factor and the scalar field in $-V(\phi)$



- The Euclidean manifold initially shrinks ($a' < 0$/anti-friction) and then expands ($a' > 0$/friction) causing the ϕ particle to first accelerate and then stop at ϕ_0. (Desirable to stop as early as possible...)

Subsequent Lorentzian evolution

- The potential should also contain a slow roll region for $\phi > \phi_0$, so that the Universe can subsequently inflate/expand in Lorentzian time



- Our proposal can accommodate various options consistent with the latest experimental constraints on inflation ex. [Planck] - incorporated in the shape of the potential

Evading the issue of the No Boundary proposal

- To compute the semi-classical probability and compare with the No-Boundary proposal $(P = |\Psi|^2 \simeq e^{-S_E})$
  $\Rightarrow$ evaluate the Euclidean wormhole on-shell action

$$S_E^{\text{on-shell}} = 4\pi^2 \int_{UV}^{0} d\tau \left(\frac{2Q^2}{a^3} - a^3 V(\phi) \right) + S_{GH}^{UV} + S_{c.t.}^{UV} \, ,$$

- The EAdS UV boundary contains the Gibbons-Hawking S_{GH}^{UV} as well as boundary counterterms $S_{c.t.}^{UV}$ that one needs to add in order to perform holographic renormalization
- Either numerically or analytically using thin/thick wall approximations one typically finds a positive on-shell action for the wormhole
- As in other Holographic examples, due to the AdS asymptotics we have a well defined probability $(P \simeq e^{-S_E} < 1)$ and the issue of the No Boundary proposal can be evaded : The Universe prefers to "nucleate" high up in the potential and then follows the slow roll trajectory

Future

A phenomenological model (SM + GR)

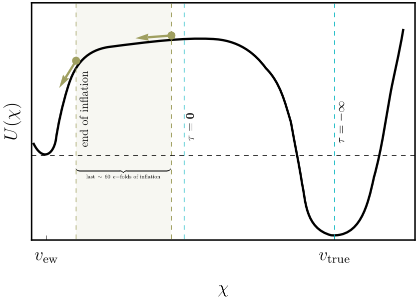
In progress [P.B. - I. Gialamas - O. Papadoulaki]

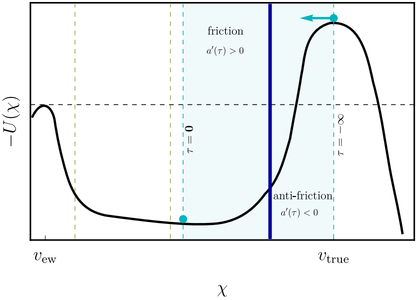
- Replace the contribution of the axion, with radiation density $\sim 1/a^4$ arising from the SM gauge fields
- The Higgs boson is the only experimentally observed scalar particle in nature and could perhaps also play the role of the inflaton
- A class of models of inflation that conform very well with experimental data : "Higgs Inflation" [Bezrukov - Shaposhnikov ...]
- These models include a non-minimal coupling term $\sim \xi\phi^2 R$ to the Einstein-Higgs action (Jordan-frame action)
- Going back to Einstein frame ($g_{\mu\nu} = e^{2\Omega}\tilde{g}_{\mu\nu}$, $\phi(\chi)$) one finds a potential of the slow roll type at large χ and of the Higgs type at small χ
- Current experimental data of the Higgs and Top mass [PDG ...] favor SM metastability $\Rightarrow$ the Higgs effective potential turns negative at high energies/field values!
- These lead to a phenomenological model with the desired properties!

A phenomenological model (SM + GR)

In progress [P.B. - I. Gialamas - O. Papadoulaki]

- The one-loop Higgs effective potential (in the Einstein frame)





Correlators and embedding to Holography

- Bulk correlators at $\tau = 0$ can be computed from the wavefunction using

$$\int D\phi \, |\Psi_{\tau=0}|^2 \, \phi(0, \vec{x}_1)...\phi(0, \vec{x}_n)$$

  Later time/Cosmological correlators are computed using the in/in formalism [Weinberg ...] or evolving the wavefunction in Lorentzian

- Our construction is amenable to a possible Holographic interpretation and embedding (EAdS)

- It points to the existence of a Euclidean QFT (or a pair) whose correlators encode the physics of the inflating Cosmology [PB - Papadoulaki - Kiritsis, Van Raamsdonk et al. ...]

- A possible tension of the slow roll region of the potential with swampland criteria and bounds? [Vafa et al. ...]

- We would like to realize our setup in a top-down string theory construction if possible

Thank you!

Summary

- We proposed a new type of wavefunction for the universe computed from the gravitational path integral, with asymptotically $EAdS$ boundary conditions

- In the semiclassical limit, it describes a Euclidean (half)-wormhole geometry. If the scale factor acquires a local maximum at the surface of reflection symmetry, it gives rise to an expanding universe upon analytic continuation to Lorentzian signature

- For this to happen, our class of models contain a non-trivial scalar potential $V(\phi)$ that takes both positive and negative values

- Our proposal evades some issues of the No Boundary proposal, leading to a well defined probability $P \simeq e^{-S_E} < 1$. It can also favor a long-lasting period of inflation - (for certain scalar potentials)

- It also raises the interesting possibility of describing the physics of inflating cosmologies and their perturbations within the context of holography (AdS/CFT)

Future Directions

- We would like to perform a thorough WKB analysis of the two-parameter (a, ϕ) WDW equation (turning points, caustics etc...)
- It is important to understand whether the resulting (half)-wormhole wavefunction is normalisable or not
- Analyse the spectrum of fluctuations around such wormholes
- Embed our setup in holography. A UV complete microscopic model of Euclidean wormholes? [PB - Papadoulaki - Kiritsis, Van Raamsdonk ...]
- Understand what our (half)-wormholes correspond to from a dual field theory perspective
- A related simpler question [PB - Gaddam - Papadoulaki ...]: What does opening up a hole in the center of EAdS and fixing bcs there mean for the holographic CFT?

WDW equation and normalizability of the wavefunction

- Issue II: The No-Boundary wavefunction is non-normalizable
- Our WDW equation is ($A = \log a$ avoids normal ordering issues)

$$\left[\frac{\partial^2}{\partial A^2} - \frac{\partial^2}{\partial \tilde{\phi}^2} + \left(\frac{12\pi^2}{\kappa}\right)^2 \left(e^{6A}\tilde{V}(\tilde{\phi}) - e^{4A} + \tilde{Q}^2\right)\right]\Psi = 0$$

with $\tilde{\phi} = \phi/M_{Pl}$, $\tilde{V} = \kappa V/3\,, \tilde{Q}^2 = \kappa Q^2/3$)

- Unfortunately we cannot solve this equation in closed form, but the work of [Hawking - Page] showed that a similar equation admits a discrete set of normalisable solutions/states
- Their idea is that semi-classical (half)-wormhole solutions are superpositions of these elementary states [Hawking - Page]
- If true this would mean that our (half)-wormholes would be described by a normalisable WDW wavefunction in contrast with the No Boundary wavefunction, but this remains to be checked