Euclidean Wormholes in Holography

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arXiv:1903.05658,2110.14655 w. P. Betzios and Elias Kiritsis
and in progress w. P. Betzios and Ji-Hoon Lee

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Why to study (Euclidean) Wormholes

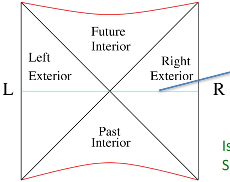
Wormholes are interesting (exotic) solutions of GR + matter

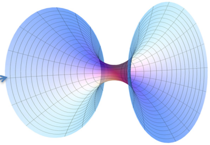
- Proposed physical effects due to wormholes
  - They lead to a non-trivial topology of space(time)
  - Connect the black hole interior with exterior? - Implications on the information paradox?
  - Related to Cosmologies (Bang-Crunch universes) upon analytic continuation
- Different types of wormholes
  - Lorentzian vs Euclidean
  - Macroscopic multi-boundary geometries (saddles) vs. Microscopic "gas of wormholes"
  - Different characteristic scales
    $L_P \ll L_W \sim L_{AdS}$ vs. $L_P \leq L_W \ll L_{AdS}$
- Our main focus will be macroscopic (Euclidean) wormholes in the context of holography (AdS/CFT)
- Plan of the talk
  - Introduction
  - Bulk Perspective
  - Dual QFT models
  - $\mathcal{N} = 4$ Wilson loops and type IIB "bubling" wormholes
  - Summary and Future directions

Introduction

Lorentzian wormholes or "ER = EPR"

- Einstein - Rosen Bridge: Connects the two sides of the eternal black hole





Israel Maldacena
Susskind

- Wormhole = Einstein Podolski Rosen pair of two black holes in a particular entangled state of two non-interacting QFT's:

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

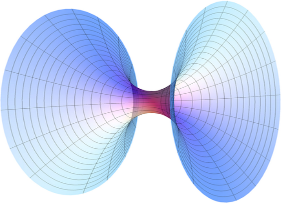
- Large amounts of entanglement can give rise

  to a geometric connection!

- We cannot communicate a message between the two sides
- Traversable Wormholes: Lorentzian signature solutions for which the null energy condition is violated $\Rightarrow$ Signals can pass through the wormhole
- Local interactions that couple the two boundary QFTs $\int d^d x \mathcal{O}_L(x) \mathcal{O}_R(x)$

  [Gao-Jafferis-Wall ...]

Euclidean Wormholes (saddles)



- There is no Lorentzian time, only Euclidean space
- To have such solutions, one needs locally negative Euclidean Energy to support the throat from collapsing
- Such energy can be provided by axionic fields or "magnetic" fluxes

- Several solutions in different dimensions/setups (some can be embedded in the standard model + gravity)
  - a subset of those is perturbatively stable [Marolf-Santos ...]

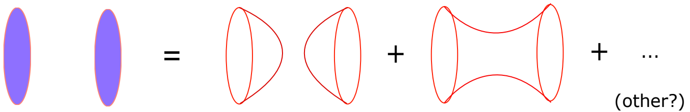
- There is a further reason why Euclidean wormholes are interesting: They are related to cosmology[see P.Betzios talk]

Holographic comments

- No time $\Rightarrow$ No entaglement in the usual sense
- Naively: different QFTs on $\partial\mathcal{M} = \cup_i \partial\mathcal{M}_i \Rightarrow$ Cross-correlations factorise
- Common Bulk dictates otherwise $\Rightarrow$ Some form of interaction?
- Global symmetries for the boundary theories? $\leftrightarrow$ A common Bulk "Gauss Law constraint"

The factorisation problem: $Z(J_1, J_2) \neq Z_1(J_1)Z_2(J_2)$

[Maldacena - Maoz (2004) ...]



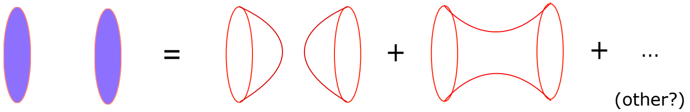
Possible resolutions in the literature :

- The QGR path integral corresponds to an average: $\langle Z(J_1)Z(J_2)\rangle \Rightarrow$ Several options [...]
- Explicit averaging over ensembles of CFT's - (Unitarity crisis)
- In canonical AdS/CFT there is a single theory with fixed parameters
- Approximate statistical averaging ("ETH" - "Quantum Chaos") $\Rightarrow$ "Statistical wormholes" from complicated/almost random Hamiltonians [...]
- Consistency with $\mathcal{N}=4$ planar integrability? $\Rightarrow$ Observables/states above the BH threshold [Schlenker - Witten ...]

The "statistical wormholes" need not be saddles of (SU)GRA eoms

The factorisation problem

[Betzios - Kiritsis - OP (19 - 21)], see also related work by [Van Raamsdonk et. al. (20-22)]



A straightforward but subtle resolution for wormhole saddles:

- Interactions between holographic QFT's $\Rightarrow$ UV soft - IR strong
- Could the Schwinger functional acquire the form (S some "sector")

$$Z(J_1, J_2) = \sum_S e^{w(S)} Z_S^{(QFT1)}(J_1) Z_S^{(QFT2)}(J_2)$$

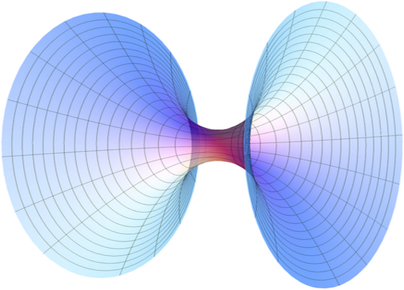
  for a unitary/reflection positive system?
- Cross correlators $\Rightarrow$ averages of lower point correlators in individual subsystems
  $\Rightarrow$ No $1-2$ cross correlator singularities

Bulk perspective

Types of solutions

Betzios-Kiritsis-OP '19

We studied Euclidean solutions with two asymptotic AdS boundaries (bottom-up)

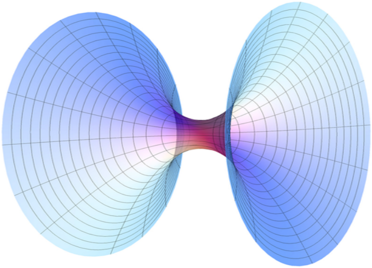


- We analysed examples in different dimensions
- And different matter content

- We found universal features for various observables irrespective of dimensions

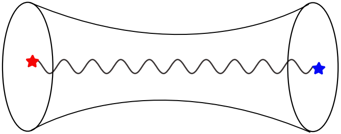
Local observables: Two boundary correlators

[Betzios - Kiritsis - OP (19)]

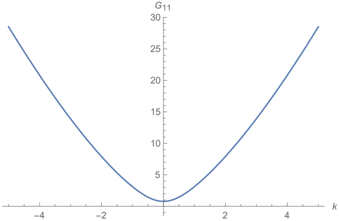


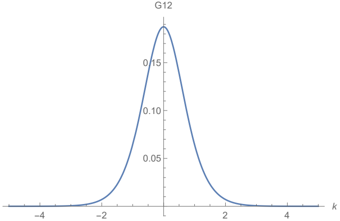
- To unravel the physics of Euclidean wormhole saddles in holography we should further study observables/correlation functions
- Correlators for local boundary (scalar) operators $\mathcal{O}_1(x)$, $\mathcal{O}_2(x)$
  $\Rightarrow$ Study the (2nd order) bulk fluctuation equation for the dual bulk (scalar) field $\phi(z, x)$

- We have two boundaries, where we can insert operators or sources
- The extra freedom provides for two types of correlation functions, either on a single boundary such as $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ or $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$, or cross-correlators across the two boundaries such as $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$



Scalar Correlators: Universal properties





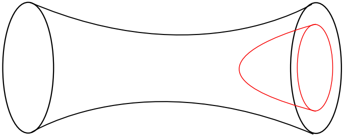
- The $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ have a similar behaviour in the UV as when there is only one boundary (power law divergence)
- In the IR they saturate to a constant positive value
- The cross correlator $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ goes to zero in the UV and has a finite maximum in the IR
- In position space ($EAdS_2$) they behave as $\sim 1/\sinh^{2\Delta_+}(\tau)$ and $\sim 1/\cosh^{2\Delta_+}(\tau)$ respectively $\Rightarrow$ No short distance singularity for the cross-correlator
- The qualitative behavior of the correlators is the same for all the types of solutions $\Rightarrow$ Universality

Non-local observables: Wilson Loops

[Betzios - Kiritsis - OP (19), Refined in: Betzios - OP (23)]

- Wilson loop observables $W(C) = \text{Tr}\left(\mathcal{P}\exp\ i\oint_C A_\mu dx^\mu\right)$ refine the analysis of [Schlenker - Witten (2022)] that studied the compressibility properties of various boundary cycles C in the wormhole bulk
- In holography: Find the string worldsheet ending on the corresponding loop C on a boundary (if it exists) and minimize its area

- Simplest observable: expectation value of a single Wilson loop $\langle W(C)\rangle$



Universal features:

- Large loops on the boundary penetrate further in the bulk and we can probe the IR properties of the boundary dual
- Typically we find an Area law behaviour in the IR

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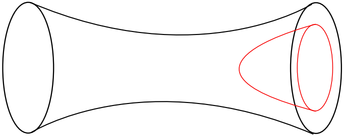

Universal features:

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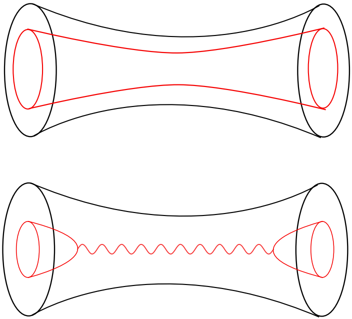
- If the EW geometry contains a non-contractible (thermal) cycle $C_\beta : S^1_\beta$, then there is no bulk surface ending on it, so that $\langle W_P(C_\beta)\rangle = 0$
- Again reminiscent of some kind of confining behaviour (center symmetry) In contrast with the BH cigar for which $\langle W_P(C_\beta)\rangle \neq 0$ (deconfinement)

Wilson Loop correlators (universal results)



- Study loop cross-correlators $\langle W(C_1)W(C_2)\rangle$, the two loops residing on different boundaries
- As we shrink the boundary loops, we find that the leading configuration of lowest action is the one for two disconnected loops
- In the regime of large Wilson loops, the leading contribution originates from a single surface connecting the two loops having a cylinder topology $S^1 \times R$
- Large loops $\Rightarrow$ Strong IR cross-coupling

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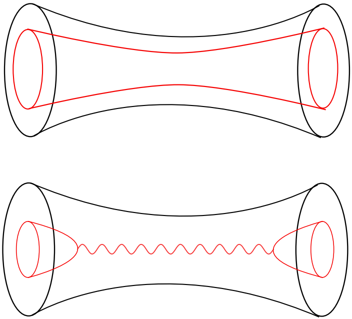

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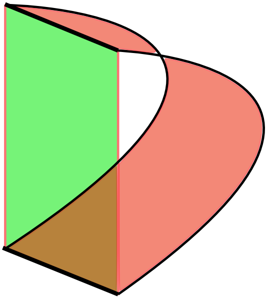
- Large loops $\Rightarrow$ Strong IR cross-coupling
- In the presence of a a non-contractible (thermal) cycle $C_\beta : S^1_\beta$, we find only a connected cylindrical bulk surface $(\langle W_P(C_\beta^{(1)})W_P(C_\beta^{(2)})\rangle \neq 0)$
- Consistent with unbroken diagonal center symmetry ex: $Z_N^{(1)} \times Z_N^{(2)} \to Z_N^{diag.}$ "cross-confining behaviour" - diagonal singlets

Dual QFT models

Tripartite BQFT construction

[van Raamsdonk (20) - (22)], [Betzios - Kiritsis - OP (21)]

- Two d-dim (holographic) BQFT's on Σ coupled through a $d+1$-dim intermediate ("messenger") theory on $I \times \Sigma$



- Consider a system for which $c_{d+1} \ll c_d$
- We would like the system to flow to a gapped/confining theory in the IR
- The geometric idea: The dual bulk gravity can localise on $d+1$-dim EOW branes that bend and connect in the IR [van Raamsdonk]

- We focus in the case where the messenger theory is (quasi) topological ($TQFT_{d+1}$) $\Rightarrow$ No contamination from $d+2$ bulk perturbative modes, natural gap in the IR ... [Betzios - Kiritsis - OP]
- Integrate out $TQFT_{d+1}$ $\Rightarrow$ The Schwinger functional does become

$$Z_{system} = \sum_S e^{w(S)} Z_S^{(BQFT_1)}(J_1) Z_S^{(BQFT_2)}(J_2)$$

Solvable microscopic tripartite model $(2d - 1d)$

[Betzios - Kiritsis - OP (21), Betzios - OP (23)]

- Consider a generalised YM in $2d$ (τ, z) with BF action

$$S_{gYM} = \frac{1}{g_{YM}^2} \int_\Sigma \mathrm{Tr}\, BF + \frac{\theta}{g_{YM}^2} \int_\Sigma \mathrm{Tr}\, B\, d\mu - \frac{1}{2g_{YM}^2} \int_\Sigma \mathrm{Tr}\, \Phi(B)\, d\mu$$

where $F = dA + A \wedge A$

- Couple it with two $1d$ $U(N)$ gauged matrix quantum mechanics theories $M_{1,2}(\tau)$ at the endpoints of an interval I $(z = \pm L)$

$$S_{MQM_{1,2}} = \int d\tau \mathrm{Tr} \left(\frac{1}{2} (D_\tau M_{1,2})^2 - V(M_{1,2}) \right) , \; D_\tau M_{1,2} = \partial_\tau M_{1,2} + i[A_\tau^{1,2}, M_{1,2}]$$

$A_\tau(\tau, z = \pm L) = A_\tau^{1,2}(\tau)$ is the value of the 2d gauge field on the two boundaries

- Solvable system: $2d$ YM - ($\Phi(B) = B^2$) coupled to two Gaussian MQM $(V(M_{1,2}) = \frac{1}{2} M_{1,2}^2)$

"Entangling" the representations

- Place the system on $I \times S^1$ (cylinder) of length $2L$ and circumference β

- The $2d$ YM amplitude on the cylinder is

$$Z_{YM}(U_1, U_2) = \sum_R \chi_R(U_1)\chi_R(U_2^\dagger) e^{-L\frac{g_{YM}^2}{N}C_R^{(2)} + i\theta C_R^{(1)}}$$

  and depends on the two asymptotic holonomies $U_{1,2} = \exp \oint d\tau \, A_\tau^{1,2}$ (zero modes of the gauge field)

- R a $U(N)$ representation, $C_R^{(1,2)}$ its Casimirs and $\chi_R(U)$ are $U(N)$ characters/wavefunctions at the ends of the cylinder

- Integrate out $M_{1,2}$ to obtain the (twisted) MQM partition functions $Z_{1,2}^{MQM}(U_{1,2};\beta) = \int DM_{1,2} \, \langle U_{1,2} M_{1,2} U_{1,2}^\dagger \,|\, M_{1,2} \rangle_{H.Osc.}$

- Couple the $2d$ YM amplitude $Z_{YM}(U_1, U_2)$ to the two MQM partition functions $Z_{1,2}^{MQM}(U_{1,2};\beta)$ and integrate over the zero modes $U_{1,2}$

"Entangling" the representations

- The complete partition function on $I \times S^1$ is

$$Z_{system} = \sum_R e^{-L \frac{g_{YM}^2}{N} C_R^{(2)} + i\theta C_R^{(1)}} Z_R^{MQM_1}(\beta) Z_R^{MQM_2}(\beta)\,,$$

$$Z_R^{MQM}(\beta) = \mathrm{Tr}_{\mathcal{H}_R} e^{-\beta \hat{H}_R^{MQM}} = \int DU \chi_R(U) Z^{MQM}(U;\beta)$$

  with β the S^1 size and $\mathcal{H}_R$ the Hilbert space of MQM in a fixed representation R [Kazakov, Klebanov ...]

- The two MQM representations R are correlated/"entangled"

  $\sum_R \Rightarrow$ is a form of "averaging", consistent with unitarity (reflection positivity) for a single (tripartite) quantum mechanical system
  $\Rightarrow$ What we previously called "the sectors S"

- No approximation (such as ETH or coarse graining) or averaging over theories involved!

- The allowed representations in the sum are center symmetric, so indeed $g_c^{(1)} \times g_c^{(2)} \to g_c^{(diag.)}$ [Betzios - OP (23)]

$\mathcal{N} = 4$ Wilson loops and type IIB "bubbling" wormholes

Wilson loops in $\mathcal{N}=4$ SYM

- The $2d/1d$ model is reminiscent of SUSY localization computations of line/defect operators in $\mathcal{N}=4$ SYM [Wang, Komatsu, Dedushenko,...]
- Idea: correlate representations of ($1/2$-BPS) Wilson loops W_R in higher dimensional examples that have known semiclassical holographic duals. Here: Consider two (non-interacting) copies of $\mathcal{N}=4$ SYM and a correlated observable

$$\sum_R e^{w(R)} \langle W_R \rangle_1 \langle W_R \rangle_2 \quad W_R = \mathrm{Tr}_R P \exp\left[i \oint ds (i A_\mu \dot{x}^\mu + \vec{n} \cdot \vec{\Phi} |\dot{x}|)\right]$$

- A single $1/2$-BPS Wilson loop in the representation R is computed via localization resulting in a Hermitean matrix integral [Pestun ...]

$$\langle W_R \rangle = \langle \mathrm{Tr}_R (e^M) \rangle_M = \frac{1}{Z} \int DM e^{-\frac{2N}{\lambda} \mathrm{Tr} M^2} \chi_R \left(e^M\right)$$

- We would like to understand the limit where the operator is "very heavy" and backreacts strongly in the dual geometry
- We need to consider representations $R: \{R_1, ..R_N\}$ with $O(N^2)$ boxes and the highest weights $R_i \sim O(N)$

The type IIB backreacted geometries

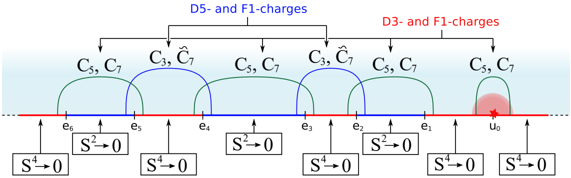
- The geometry dual to a backreacted loop in rep R, has an $SO(2,1) \times SO(3) \times SO(5)$ isometry [D'Hoker-Estes- Gutperle, ...]

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + f_4^2 ds^2_{S^4} + 4\rho^2 dz d\bar{z}$$

  where $z, \bar{z}$ parametrise a Riemann surface Σ and $f_{1,2,4}(z,\bar{z})$, $\rho(z,\bar{z})$. The Wilson loop is on the S^1 boundary of the AdS_2 disk

- The solution also contains a non-trivial dilaton and 3-cycles/5-cycles/7-cycles with RR/RR/NSNS fluxes supporting them ($D5/D3/F1$)

- Everything is determined by two harmonic functions $h_{1,2}(z,\bar{z})$. $h_2 = 0$ determines the boundary of Σ and h_1 contains the data of the "bubbling" geometry (cuts $\leftrightarrow$ fluxes + singularity $\leftrightarrow$ asymptotic $AdS_5 \times S^5$ region)



Bubbling Wormholes $\equiv$ multiple singularities on $\partial\Sigma$

- The matrix model resolvent $2\omega(z) = V'_{cl}(z) - y(z) = ih_2(z) - ih_1(z)$ completely determines the dual SUGRA geometry
- $h_{1,2}$ need to have common singularities on $\partial\Sigma$. Near such singularities the metric asymptotes to $AdS_5 \times S^5$
- We found solutions with more than one singularities/asymptotic regions, still preserving the regularity conditions of [D'Hoker-Estes- Gutperle, ...]
- The simplest such Σ corresponds to a disk with two cuts and two singularities [Betzios, Ji Hoon Lee, OP]

$$h_1(z) = i\frac{2}{\lambda z}\sqrt{(z^2 - e_{min}^2)(z^2 - e_{max}^2)} + cc.\,, \quad h_2(z) = i\frac{2}{\lambda}\left(z - \frac{e_{min}e_{max}}{z}\right) + cc.$$

Matrix model dual of Σ wormhole with two S^4 boundaries

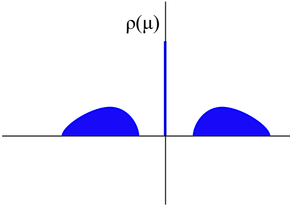
- The dual matrix model spectral curve needs two cuts and two singularities
- The correct field theoretic observable is an "analogue of the Dirac-δ" for two $1/2$-BPS loop operators on two copies of $\mathcal{N}=4$ $\Rightarrow$ We "glue" the two copies of $\mathcal{N}=4$ on the great S^1 where the loops are placed

$$\langle \det\left(I\otimes I - e^{M_1}\otimes e^{M_2}\right)^{-1}\rangle_{1,2} = \sum_R \langle \chi_R(e^{M_1})\rangle_1 \, \langle \chi_R(e^{M_2})\rangle_2$$

- This can be analysed as a coupled two matrix model $\Rightarrow$ The planar resolvent describes precisely our wormhole solution!

$$\omega(z) = \frac{2}{\lambda}\left(z - \frac{ab}{z}\right) - \frac{2}{\lambda z}\sqrt{(z^2-b^2)(z^2-a^2)}$$

$$a = \tfrac{1}{2}(\sqrt{3}-1)\sqrt{\lambda}\,, \qquad b = \tfrac{1}{2}(\sqrt{3}+1)\sqrt{\lambda}$$



Summary and Future

Summary and Future Directions

Summary

- We proposed a general class of microscopic models for Euclidean Wormholes, in terms of BQFTs coupled via a higher dimensional TQFT
- These models are reflection positive and do not require any ad hoc averaging (over couplings/ensembles of CFTs or otherwise)
  - no deviation from the usual holographic prescription and rules

  There is though a resulting sum over representations of the gauge group after we integrate out the "messenger" TQFT
- This makes the resulting field theoretic correlators to be compatible with dual computations on wormhole saddles
- We found that similar models can also arise by considering heavy correlated observables in otherwise decoupled QFTs

  We analysed the case of correlated Wilson loops between copies of $\mathcal{N} = 4$ SYM. They give rise to "bubbling" wormhole geometries in IIB
- In the $1/2$-BPS case we have exact control on both sides of the duality but the boundaries touch on one dimensional $S^1 \subset S^4$'s (similar to Janus)

A Hilbert space interpretation of our constructions

- For Lorentzian wormholes (eternal BH): $\mathcal{H} = \mathcal{H}_{CFT1} \otimes \mathcal{H}_{CFT2}$ and

$$|\Psi\rangle_{TFD} = \frac{1}{Z}\sum_n e^{-\frac{\beta}{2}E_n}|E_n\rangle_1 \otimes |E_n\rangle_2$$

- This correlates the energies of the two subsystems

- Our proposed models for Euclidean wormholes: Correlate ("entangle") $U(N)$ representations and not energies as in the TFD

- Realisation I: Presence of gauge constraints (messenger TQFT) - the Hilbert space is reduced into $\mathcal{H} = \sum_R \mathcal{H}^1_R \otimes \mathcal{H}^2_R$. One could think this in terms of states

$$|\Psi\rangle_{RD} = \sum_R e^{w(R)}|R\rangle_1 \otimes |R\rangle_2$$

- Realisation II: Consider insertions of "heavy" operators that correlate the copies with a similar representation theoretic "entanglement" (ex: Wilson loops W_R in $\mathcal{N} = 4/IIB$)

- Future Realisation? An effective constraint on the Hilbert space could arise dynamically in the IR ("cross-confinement"/diagonal IR singlets: $U(N) \times U(N) \to U_{diag.}(N)$)

Future Directions

- The MQM non-singlet sectors are also relevant for black hole physics and involve similar sums over representations ($c = 1$ MQM). Connections? [Kazakov et al., Betzios - OP]
- Other top down constructions embeddable in critical string theory
- Less (super)symmetric but still controllable examples of correlated loops or tripartite systems
- Understand better the Lorentzian continuations of our field theoretic setups and their holographic duals (Bang/Crunch Cosmologies) - a $\Lambda < 0$ alternative to the dS/CFT correspondence? see talk by P.Betzios, [P. Betzios, OP]
- Study (target space) Euclidean wormhole backgrounds in string theory from the worldsheet perspective (WZW cosets?)

Thank you!

Intuitive understanding of the 2MM: Two component gas

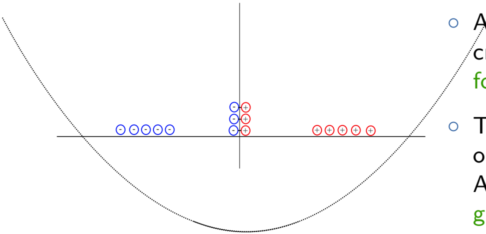
- The 2MM saddle point equations describe two types of particles

$$-\frac{4N_1}{\lambda_1}\mu_i^{(1)} - \sum_{k=1}^{N_2} \frac{2}{\sinh(\mu_i^{(1)} + \mu_k^{(2)})} + \sum_{j\neq i} \frac{2}{\mu_i^{(1)} - \mu_j^{(1)}} = 0\,,$$

$$-\frac{4N_2}{\lambda_2}\mu_k^{(2)} - \sum_{i=1}^{N_1} \frac{2}{\sinh(\mu_i^{(1)} + \mu_k^{(2)})} + \sum_{j\neq k} \frac{2}{\mu_k^{(2)} - \mu_j^{(2)}} = 0$$

  with an $1-1$ and $2-2$ repulsion and $1-2$ attraction to "mirror" points

- There is an overall Gaussian attractive potential $\Rightarrow$ This leads to a paired $1-2$ condensate at the origin (the additional pole of the planar resolvent)



  - After lots of pairs condense, they create a repulsive effective potential for the rest of the eigenvalues
  - The rest of the eigenvalues distribute on two opposite sides of the origin. At large-N they form two cuts, giving rise to the wormhole resolvent

- The n-point cross-correlator takes the general form

$$\langle O_{i_1}(\tau_{i_1}) \ldots \tilde{O}_{i_2}(\tau_{i_2}) \ldots \rangle = \sum_R \langle O_{i_1}(\tau_{i_1}) \ldots \rangle_1^R \langle \tilde{O}_{i_2}(\tau_{i_2}) \ldots \rangle_2^R \, e^{-L \frac{g_{YM}^2}{n} C_R^{(2)} + i\theta |R|}$$

where i_1 refers to the first and i_2 to the second MQM subsystem

- This correlator generically only depends separately on the differences $\tau_{i_1} - \tau_{j_1}$ and $\tau_{i_2} - \tau_{j_2}$ and not on time differences that mix the $1,2$ sub-indices, or O_{i_1} with $\tilde{O}_{i_2}$ operators
- No short distance singularities in the cross-correlators!
- The absence of short distance singularities in the cross correlators is a *robust-universal* feature of dual wormhole backgrounds

4D Einstein - Yang - Mills Solutions

[Hosoya-Ogura'89]

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{16\pi G} R + \Lambda + \frac{1}{4g_{YM}^2} \left(F_{\mu\nu}^a\right)^2 \right)$$

- The metric is $ds^2 = dr^2 + \left(B \cosh(2r) - \frac{1}{2}\right) d\Omega_3^2,\ r \in [-\infty, \infty]$
- with $B = \sqrt{\frac{1}{4} - r_0^2 H^2}$, $r_0^2 = 4\pi G / g_{YM}^2,\ H^2 = 8\pi G \Lambda / 3$
- The minimum size of the throat is $r_{min}^2 = B - \frac{1}{2}$
- The throat is supported by a background gauge field A^α: the Meron configuration ("half-instanton")
- Using Euler angles

$$d\Omega_3^2 = \frac{1}{4}\left(dt_1^2 + dt_2^2 + dt_3^2 + 2\cos\, t_1\, dt_2 dt_3 \right) = \frac{1}{4}\omega^a \omega^a$$
$$0 \le t_1 < \pi, \quad 0 \le t_2 < 2\pi, \quad -2\pi \le t_3 < 2\pi$$

$$A^a = \tfrac{1}{2}\omega^a = \tfrac{1}{2} g^{-1} dg\,, \qquad \text{with} \quad F^a = \frac{1}{8}\epsilon^{abc}\omega^b \wedge \omega^c$$

ω^a is the Maurer-Cartan form of $SU(2)$

Dual geometry?

- The singlet sector of one gauged MQM (inverted oscillator/in the double scaling limit) is dual to $2d$ linear dilaton background of the $c = 1$-Liouville string
  $\Rightarrow$ A single asymptotic (weakly coupled) region of space
- Non trivial reps with few boxes in their Young diagrams are related to long strings - Large reps ("long string condensates") deform the background geometry, possibly creating black holes
  [Gaiotto, Maldacena, Kazakov-Kostov-Kutasov, Betzios-OP...]
- We studied the saddle point equations using a large representation limit (continuous Tableaux), in order to determine the corresponding geometric saddle $\rightarrow$ technically difficult, hard to reconstruct the dual metric
- However, we were able to prove the existence of different saddles some of which seem to correspond to disconnected and others to connected geometries (factorised vs. non-factorised contributions)

Further properties of wormhole saddle

- One can compare the free energy of the wormhole saddle with two disconnected $AdS_5 \times S^5$ spaces

$$\mathcal{F}_w - 2\mathcal{F}_{AdS} = -\frac{1}{2}\log\lambda$$

- The wormhole has lower free energy. (Indicative for its stability)

- One can also compute the expectation of probe Wilson loops. For example $W_f = \mathrm{Tr} e^M$

$$\langle W_f \rangle_{AdS} = \int_{-\infty}^{\infty} dz \rho_{AdS}(z) e^z = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

$$\langle W_f \rangle_{worm} = \frac{4}{\pi\lambda} \int_a^b \frac{dz}{z} \sqrt{(b^2 - z^2)(z^2 - a^2)} e^z$$

  It grows with a slower rate with λ wtr to the AdS example

- Interesting to extend this to observables with coordinate dependence, such as correlators of local operators and match with the gravity side

Geometric properties II: "conical excess" on Σ

- A second caveat: The bulk geometry exhibits an integer (4π) conical excess at the center of Σ

$$d\Sigma^2 \approx Cr^2(dr^2 + r^2 d\theta^2) = \frac{C}{4}(du^2 + 4u^2 d\theta^2)\,, \qquad \theta \in [0, 2\pi]$$

- The conical excess provides the negative energy to support the wormhole
- It is reminiscent of orientifolds ($O(7)$), but the branch locus is $AdS_2 \times S^2 \times S^4$ (also a large number of them for backreaction)
- Most deformations of $h_{1,2}(z)$ (within our half-BPS ansatze) turn the conical excess into a naked singularity
- We do not know a top down "resolution" of this conical excess in string theory - but perhaps it is only a "pathology" of the very (super)symmetric bulk ansatz we use
- The matrix model dual is perfectly well defined

Connecting the MM resolvent with the harmonic functions

- One can show that the matrix model resolvent is related to the two harmonic functions $h_{1,2}$ via ($y(z)$: "spectral - curve")

$$2\omega(z) = V_c'(z) - y(z)\,, \qquad \rho(z) = \frac{1}{2\pi}\Im y(z)\,, \quad z \in \mathcal{C}$$
$$h_1(z,\bar{z}) = \mathcal{A} + \overline{\mathcal{A}}\,, \qquad h_2(z,\bar{z}) = \mathcal{B} + \overline{\mathcal{B}}$$
$$iV_c'(z) = \frac{2i}{\lambda} z = \mathcal{B}(z)\,, \qquad iy(z) = \mathcal{A}(z)$$

- This means that it completely determines the properties of the dual SUGRA geometry

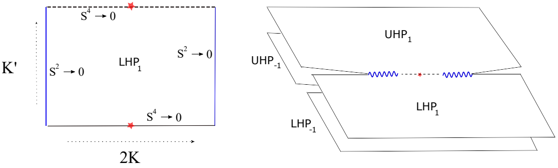
- $h_{1,2}$ need to have common singularities on $\partial\Sigma$. Near such singularities the metric asymptotes to $AdS_5 \times S^5$. ex:

$$h_1 = \frac{2i}{\lambda}\sqrt{z^2 - \lambda} + \text{c.c.}\,, \qquad h_2 = \frac{2i}{\lambda} z + \text{c.c.}$$

- For a single Wilson loop in any rep, there is only a single such singularity. The topology of the boundary is an S^4 and the half-BPS Wilson loop wraps a great $S^1 \subset S^4$

Wormholes $\equiv$ multiple singularities on $\partial\Sigma$

- We found solutions with more than one singularities/asymptotic regions, still preserving the regularity conditions of [D'Hoker-Estes- Gutperle, ...]
- The simplest such Σ corresponds to a disk with two cuts/singularities $\equiv$ a square with two singularities [Betzios, Ji Hoon Lee, OP]

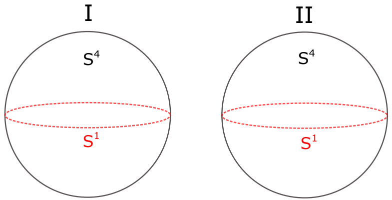


$$h_1(z) = i\frac{2}{\lambda z}\sqrt{(z^2 - e_{min}^2)(z^2 - e_{max}^2)} + cc.\,, \quad h_2(z) = i\frac{2}{\lambda}\left(z - \frac{e_{min}e_{max}}{z}\right) + c$$

- We also found more complicated solutions that can be mapped to regular polygons with $2n$ edges and n singularities, as well as solutions when Σ is an annulus

Geometric properties I: AdS_2 factor and "Janus"

- The two boundary wormhole geometry is a form of a double cover of $AdS_5 \times S^5$ (dilaton is still constant)
- There is a caveat: The geometry has an $EAdS_2$ factor with disk topology and its boundary S^1 is shared by all the AdS_5 asymptotic boundaries (Σ singularities) that have the topology of S^4
- This means that the would-be distinct S^4 boundaries are identified on a common S^1, in analogy with other Janus-type of solutions [D'Hoker, Estes, Gutperle, Bachas, Gomis, Assel ...]

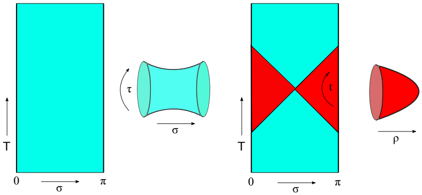


- Still it is possible to connect separate points on the two S^4's by traversing the bulk wormhole, without ever crossing the common S^1

An aside: Two boundary AdS_2 wormhole?

[Betzios - OP (23)]

- What about using global $EAdS_2$ that has two boundaries (cylinder)?



- In this case away from the Σ singularities the geometry is the two boundary $EAdS_2 \times S^4 \times S^2 \times R_2$
  (similar to the [Maldacena Milekhin Popov] wormhole geometries)
- At the Σ singularities, the former UV asymptotic S^4's are now replaced by $S^3 \times S^1$
- The two asymptotic S^1's of the cylinder $EAdS_2$ comprise the S^1's on the north and south poles of the S^3.
- Consistent with the fact that one needs to have a pair of Polyakov loops (around the S^1), sitting on the north and south poles of S^3 (Gauss-law)

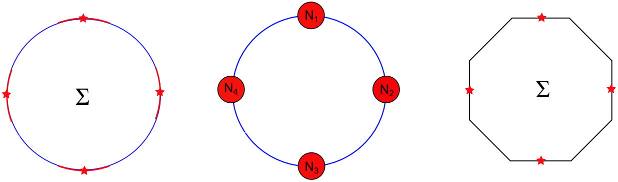
Matrix model dual of Σ wormhole with two S^4 boundaries

- The dual matrix model spectral curve needs two cuts and two singularities
- Use an "analogue of the Dirac-δ" for two $1/2$-BPS loop operators on two copies of $\mathcal{N}=4 \Rightarrow$ We "glue" the two copies of $\mathcal{N}=4$ Wilson loops

$$\langle \det \left(I \otimes I - e^{M_1} \otimes e^{M_2} \right)^{-1} \rangle_{1,2} = \sum_R \langle \chi_R(e^{M_1}) \rangle_1 \, \langle \chi_R(e^{M_2}) \rangle_2$$

  If the matrices were unitary this would have been a Weyl-invariant delta function

- This can be analysed as a coupled two matrix model or as a model in the space of highest weights R_i of R
- For the multi-boundary wormholes use an $\hat{A}_r$ necklace matrix chain and connect the nodes with determinant operators

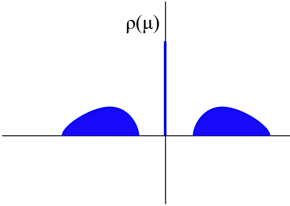


The resolvent at large-N and strong coupling

- At strong 't Hooft coupling the saddle point equations simplify in terms of only rational functions (similar to two coupled $O(2)$ models on a random lattice) [Kostov, Eynard ...]
- In this limit we can obtain an exact solution for the resolvent

$$\omega(z) = \frac{2}{\lambda}\left(z - \frac{ab}{z}\right) - \frac{2}{\lambda z}\sqrt{(z^2 - b^2)(z^2 - a^2)}$$

$$a = \tfrac{1}{2}(\sqrt{3}-1)\sqrt{\lambda}, \qquad b = \tfrac{1}{2}(\sqrt{3}+1)\sqrt{\lambda}$$



- The normalisability of the density of eigenvalues ($\int_{supp.} \rho(\mu) = 1$) fixes the end-points a, b in terms of the 't Hooft coupling λ
- The resulting harmonic functions $h_{1,2}$ correspond precisely to the ones we found in the gravitational description