Iwate Medical University

EISA
European Institute for Sciences and Their Applications
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Taming the little SUSY hierarchy with the quiver supersymmetric standard model

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科研費
KAKENHI

KO, JPS meeting March 2022
KO, Planck2023 June 2023
KO in progress

Introduction

- SUSY Standard Model looks suffering from the little hierarchy problem.
  - EW symmetry breaking（100 GeV) vs SUSY breaking (gluino 2 TeV、stop 1 TeV over)
  - fine-tuning of O(0.1) %
- Any attempt realizing natural multi-TeV supersymmetry would be welcome for future LHC run/FCC
- <u>We propose Quiver Supersymmetric Standard Model +“Sliding 3rd Generation”</u>

Little SUSY hierarchy problem

EW symmetry breaking

SUSY

$$\frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2$$

SUSY breaking

$$\delta \approx \frac{m_Z^2}{2m_{H_u}^2}$$

0.5 %
for 1 TeV

Renormalization Group Equation

stop > 1 TeV

$$\delta m_{H_u}^2 = \frac{1}{16\pi^2}\left[-6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2 S + 6|y_t|^2\left(m_{H_u}^2 + \underline{m_{Q_3}^2 + m_{\bar{U}_3}^2 + |A_t|^2}\right)\right]\ln\left(\frac{Q}{M}\right)$$

Even a RG scale ambiguity can generates a few hundred GeV correction!

Little SUSY hierarchy problem

Lesson:

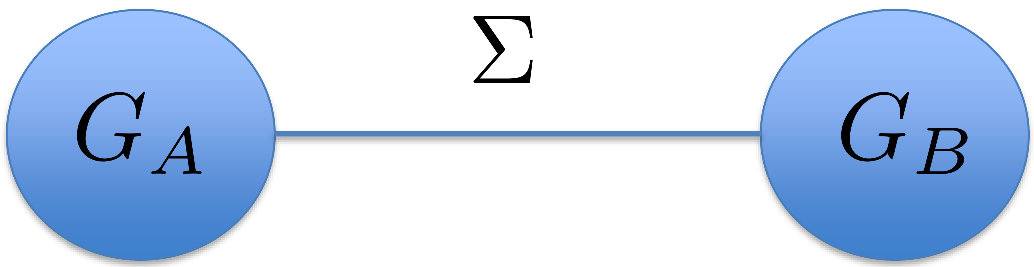
The Higgs field should not touch the SUSY breaking via gauge and Yukawa interactions until just before the gaugino, squark and slepton decouple.

$$\delta m_{H_u}^2 = \frac{1}{16\pi^2}\left[-6g_2^2|M_2|^2 - \overset{=0}{\frac{6}{5}g_1^2|M_1|^2} + \frac{3}{5}g_1^2 S \right.$$
$$\left. + 6|y_t|^2\left(m_{H_u}^2 + \underbrace{m_{Q_3}^2 + m_{\overline{U}_3}^2 + |A_t|^2}_{=0}\right)\right]\ln\left(\frac{Q}{M}\right)$$

Quiver (SUSY) Standard Model

C.Csaki, J.Erlich, C.Grojean, G.Kribs (2001)
H-C. Cheng, D.E.Kaplan, M. Schmaltz, W.Skiba (2001)
.....Many others

$$G_A \times G_{B(SM)} \rightarrow G_{SM} \qquad (G_A \supset G_{SM})$$



$$V_H = \frac{g_A V_A - g_B V_B}{\sqrt{g_A^2 + g_B^2}}$$

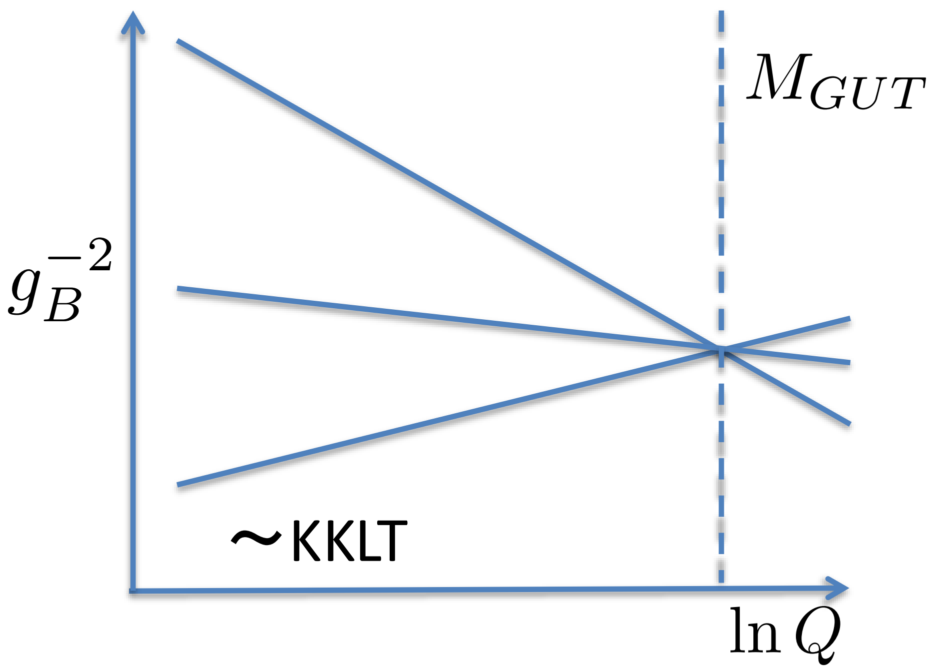
$$V_{SM} = \frac{g_B V_A + g_A V_B}{\sqrt{g_A^2 + g_B^2}}$$

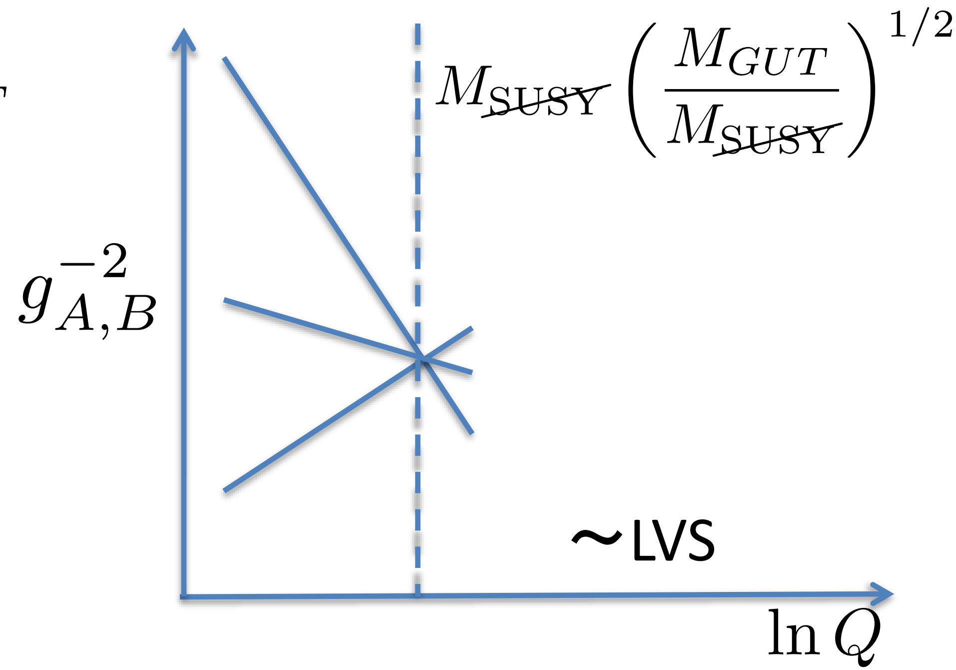
$$\frac{1}{g_{SM}^2} = \frac{1}{g_A^2} + \frac{1}{g_B^2}$$

Quiver (SUSY) Standard Model

$$G_A \times G_{B(SM)} \to G_{SM}$$

$G_A = SU(5)$ $\qquad\qquad$ $G_A = G_{SM}$





Severe constraint from
Landau poles

N.Arkani-Hammed, A. Cohen, H.Georgi (2001)

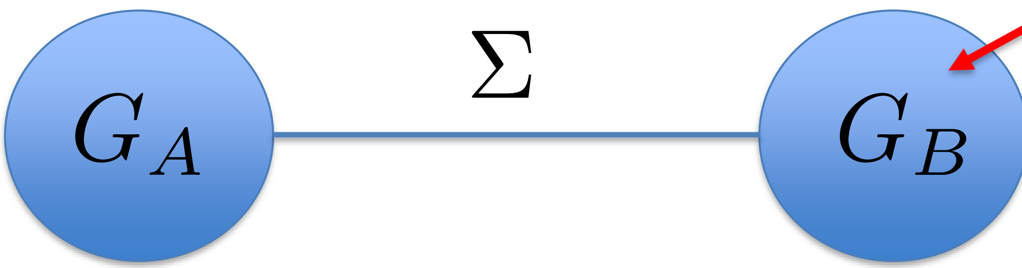
Quiver (SUSY) Standard Model

$$G_A \times G_{B(SM)} \to G_{SM}$$

$H_u\, H_d$

Gaugino Mass

$M_A \neq 0$



$M_B = 0$

~~SUSY~~ SUSY

Deconstructed Gaugino mediation (Tree)

$$\frac{1}{2}M_A\overline{\lambda^c_A}\lambda^c_A = \frac{1}{2}M_A\frac{\overline{(g_A\lambda^c_H + g_B\lambda^c_{SM})}(g_A\lambda_H + g_B\lambda_{SM})}{g_A^2 + g_B^2}$$

Sequestering of the gauge interaction

G_A

$H'_u \quad H'_d$

$M_A \neq 0$

$\Sigma \quad \overline{\Sigma}$

G_B

$Q \quad \overline{U} \quad \overline{D} \quad L \quad \overline{E}$

$Q \quad \overline{U} \quad \overline{D} \quad L \quad \overline{E}$

$Q \quad \overline{U} \quad \overline{D} \quad L \quad \overline{E}$

$H_u \quad H_d$

$M_B = 0$

$m_0^2 \neq 0$

$(M_{B-L} \neq 0)$

$m_H^2 = 0$

$\langle \Sigma \rangle = \langle \overline{\Sigma} \rangle \approx M_A$

Decouple before the loop corrections

Sequestering of the top Yukawa

KO (2020) JPS meeting

Scalar mass

$$\mathcal{W}_B \supset$$

$m_0^2 \neq 0$	Q'	$\overline{U}'$	$\lambda\, X\, (Q'\,\overline{Q} + \overline{U}'\,U)$
	$\overline{Q}$	U	
			$\mu_V\, (Q\,\overline{Q} + \overline{U}\,U)$
$m_0^2 = 0$	Q	$\overline{U}$	

$$y_t\, H_u\, Q\, \overline{U}$$

$$\langle X \rangle \approx M_A$$

Sequestering of the top Yukawa

$$Q_H \qquad \overline{U}_H$$

$$\overline{Q} \qquad U$$

$$\mu'_V \left(Q_H \overline{Q} + \overline{U}_H U\right)$$

$$\mu_V'^{\,2} = \lambda^2 \langle X \rangle^2 + \mu_V^2$$

$$m^2_{Q,\overline{U}} \neq 0 \qquad \boxed{Q_{SM} \quad \overline{U}_{SM}}$$

$$y_t^{SM}\, H_u\, Q_{SM}\, \overline{U}_{SM}$$

Sliding 3rd generation !

Sliding 3rd Generation

Sequestering the top Yukawa interaction from the SUSY breaking

$$M_{GUT} \to M_{\cancel{SUSY}}$$

$$\cancel{SUSY}$$

$$M_t = \begin{array}{cc} & \begin{array}{ccc} U & Q & Q' \end{array} \\ \begin{array}{c} \overline{Q} \\ \overline{U} \\ \overline{U}' \end{array} & \begin{bmatrix} & \mu_{Q_V} & \lambda_Q v_X \\ \mu_{U_V} & \lambda_t v_u & \\ \lambda_U v_X & & \end{bmatrix} \end{array}$$

$$\cancel{SUSY}$$

$$M_t = \begin{array}{cc} & \begin{array}{ccc} U & Q_H & Q_{SM} \end{array} \\ \begin{array}{c} \overline{Q} \\ \overline{U}_H \\ \overline{U}_{SM} \end{array} & \begin{bmatrix} & \mu'_{Q_V} & \\ \mu'_{U_V} & \lambda_t^H v_u & * \\ & * & y_t^{SM} v_u \end{bmatrix} \end{array}$$

$$v_X \approx \cancel{SUSY}$$

Sliding 3rd Generation

$$\mu_V = \mu' \cos\theta \qquad \lambda\langle X\rangle = \mu' \sin\theta$$

$$\begin{aligned} Q_H &= \cos\theta\, Q + \sin\theta\, Q' \\ Q_{SM} &= -\sin\theta\, Q + \cos\theta\, Q' \end{aligned}$$

$$y_t^{SM} = y_t \sin^2\theta$$

$$m^2_{Q_H} = m^2_{Q'} \sin^2\theta \qquad m^2_{Q_{SM}} = m^2_{Q'} \cos^2\theta$$

Minimal Model

$$\mathcal{W}_B = \sum_{i=1,2,3} \left(\lambda_i^u H_u Q_i \overline{U}_i + \lambda_i^d H_d Q_i \overline{D}_i + \lambda_i^e H_d L_i \overline{E}_i\right)$$

$$+ \mu H_u H_d$$

$$+ \mu_F \left(Q_3\, \overline{Q} + \overline{U}_3\, U + \overline{D}_3\, D + L_3\, \overline{L} + \overline{E}_3\, E\right)$$

$$+ \lambda_V\, X \left(Q'\, \overline{Q} + \overline{U}'\, U + \overline{D}'\, D + L'\, \overline{L} + \overline{E}'\, E\right)$$

$$+ \frac{1}{3!}\, \kappa\, X^3$$

Sliding 3rd Generation

$$\mathcal{W}_\Sigma = \sum_{i=2,3} \left[\lambda_i^A\, \mathrm{tr}(\Sigma_i\, A_i\, \overline{\Sigma}_i) + \lambda_i^S\, S_i\, \mathrm{tr}(\Sigma_i\, \overline{\Sigma}_i)\right] + \frac{1}{3!}\, \kappa_i^X\, S_i^3$$

+ spectator for the unification

Nontrivial Vacuum $\langle \Sigma \rangle \neq 0$ $\langle X \rangle \neq 0$ due to the SUSY breaking

RG running

Radiative “quiver” symmetry breaking

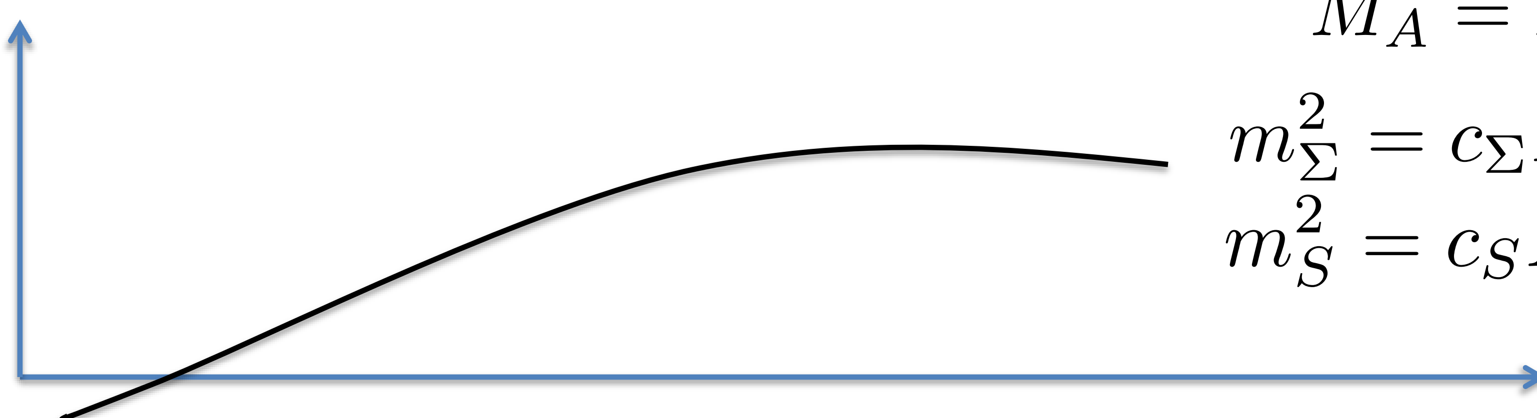
$$\mathcal{W}_\Sigma = \sum_{i=2,3} \left[\lambda_i^A \,\mathrm{tr}(\Sigma_i\, A_i\, \overline{\Sigma}_i) + \lambda_i^S\, S\, \mathrm{tr}(\Sigma_i\, \overline{\Sigma}_i)\right] + \frac{1}{3!}\,\kappa^X\, S^3$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = m_\Sigma^2 \left(|\Sigma|^2 + |\overline{\Sigma}|^2\right) + m_S^2 |S|^2 + (\mathrm{A\ terms})$$

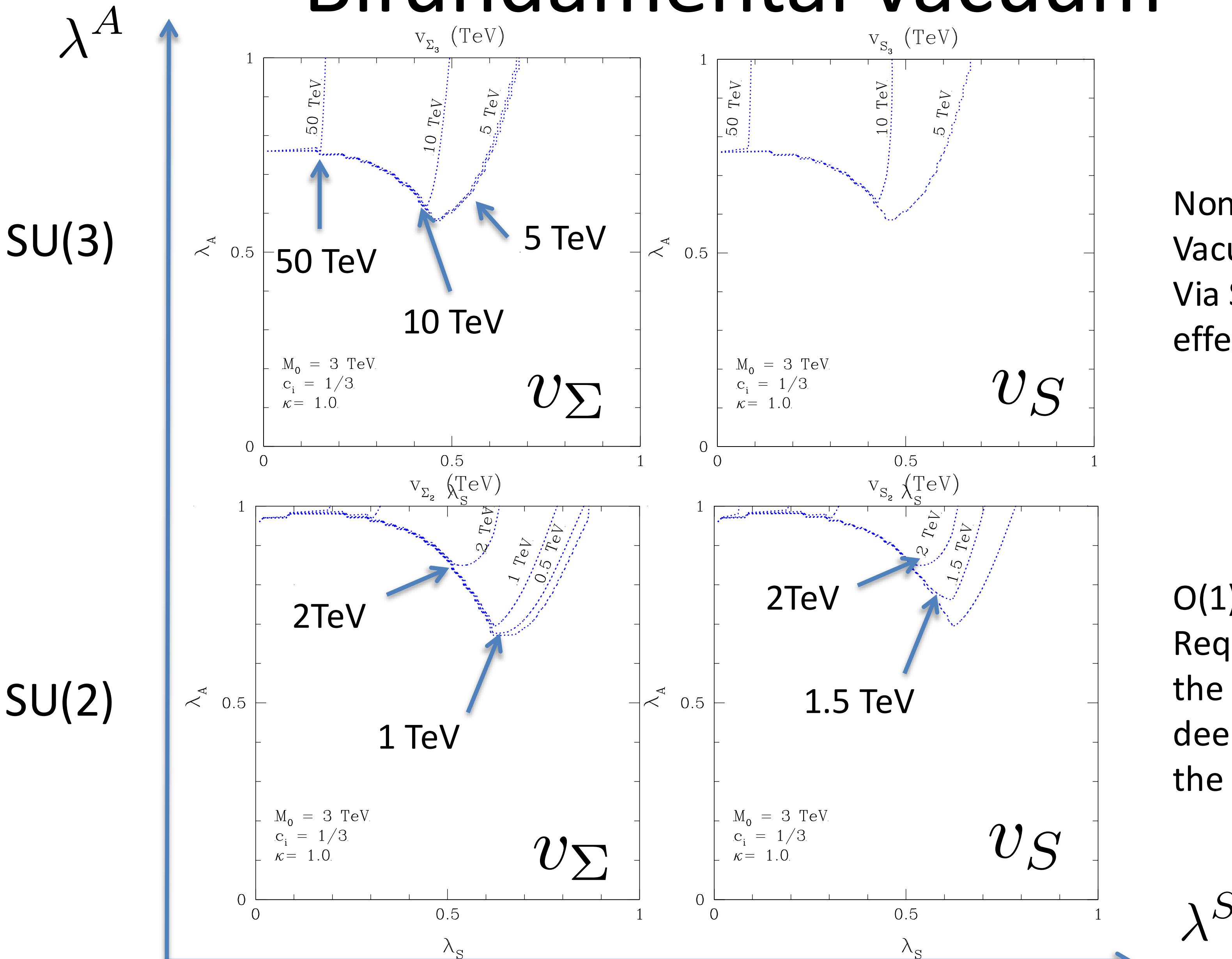
$$M_A = M_0$$

$$m_\Sigma^2 = c_\Sigma M_0^2$$

$$m_S^2 = c_S M_0^2$$



Bifundamental vacuum

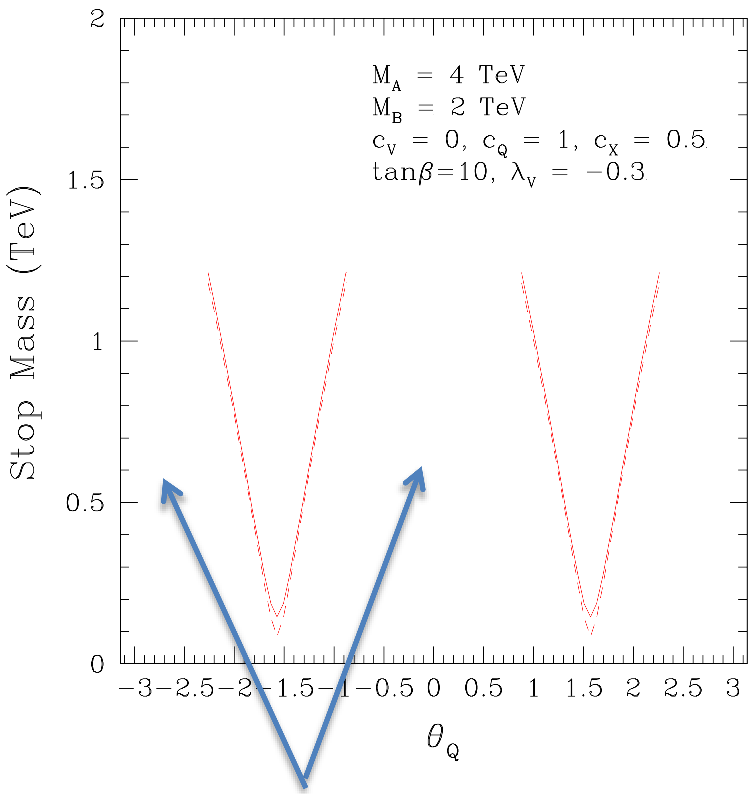


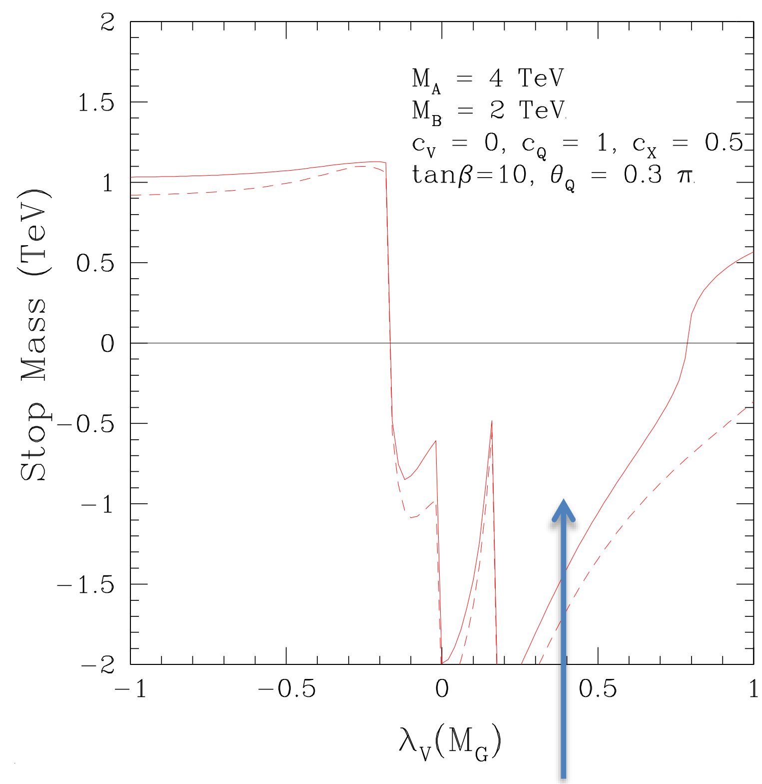
Non-trivial
Vacuum
Via SUSY breaking
effect

O(1) coupling is
Required for
the vacuum
deeper than
the origin

Stop mass

It's not easy to lift the stop mass to multi-TeV



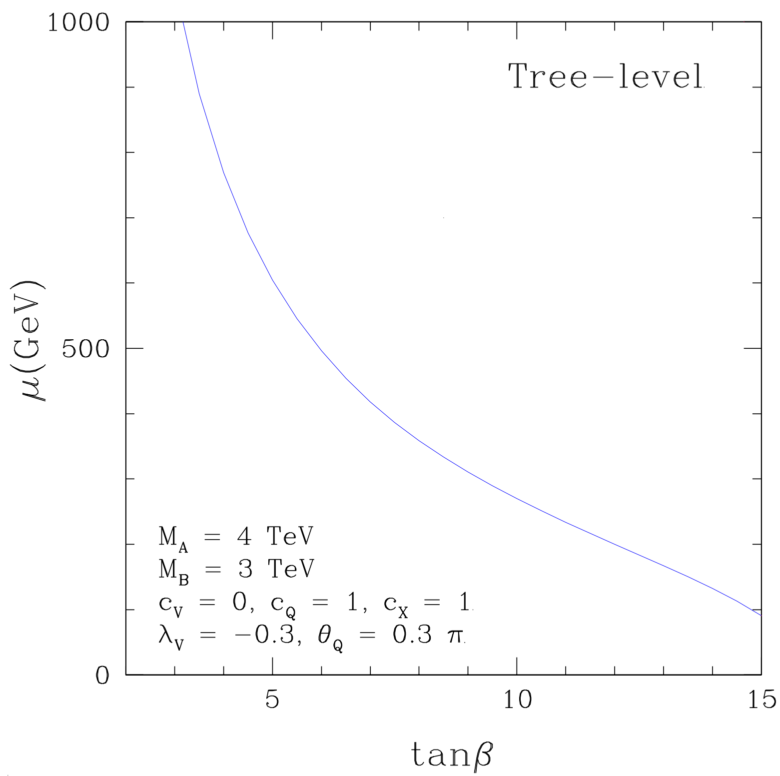


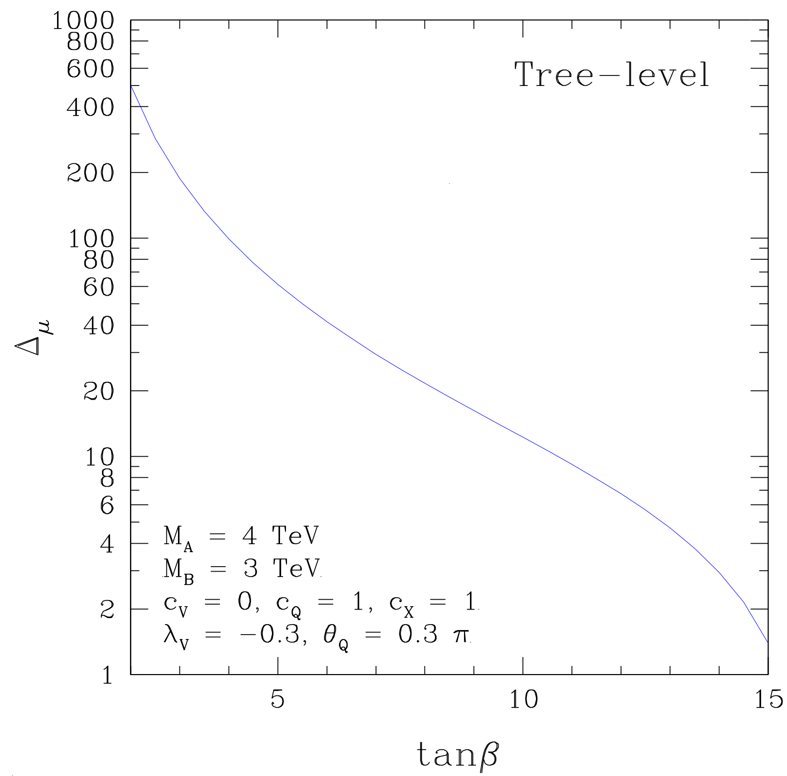
The SM top Yukawa is fixed and a small mixing hits the Landau pole

SUSY breaking by the singlet F term generates mixing with $\overline{Q}^*$ and reduce the stop mass.

EW symmetry breaking (Tree-level)

$$\frac{1}{2}m_Z^2 = -|\mu|^2 - \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{1-\tan^2\beta} \qquad \Delta_\mu = \frac{\partial \ln m_Z^2}{\partial \ln \mu} \approx \frac{2\mu^2}{m_Z^2}$$





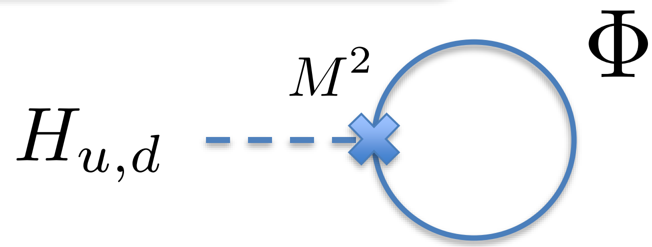
$$m_{H_u}^2(M_G) = 0 \quad m_{H_d}^2(M_G) = M_B^2$$

RG due to the bottom Yukawa

Effective potential and Tadpole

$$V_{eff} = \frac{1}{64\pi^2} Str M^4 \left(\ln \left(\frac{M^2}{Q^2_{\text{renorm}}} \right) - \frac{3}{2} \right)$$

$$T_{u,d} = -\frac{\partial V_{eff}}{\partial H_{u,d}}$$



$$\frac{1}{2} m_Z^2 = -|\mu|^2 - \frac{(m^2_{H_d} - T_d/v_d) - (m_{H_u} - T_u/v_u) \tan^2\beta}{1 - \tan^2\beta}$$

$$\approx T_u/v_u - m^2_{H_u} - |\mu|^2 + \frac{m^2_{H_d}}{\tan^2\beta}$$

Effective potential and tadpole

$$\color{red}{T_{u,d}} = -\frac{\partial V_{eff}}{\partial H_{u,d}}$$

$$= -\frac{1}{32\pi^2}\sum_i (-)^{2S_i}\frac{\partial \hat{M}_i^2}{\partial H_{u,d}}\hat{M}^2{}_i\left(\ln\left(\frac{\hat{M}_i^2}{Q^2_{\mathrm{renorm}}}\right)-1\right)$$

$$UU^\dagger = \mathbf{1}$$

$$\frac{\partial \hat{M}_i^2}{\partial H_{u,d}} = \frac{\partial}{\partial H_{u,d}}\left(UMU^\dagger\right)_{ii}$$

$$= \left(U\frac{\partial M}{\partial H_{u,d}}U^\dagger\right)_{ii}$$

Neutralino Mass

$$\chi_L^0 = \Big[\; \overbrace{\lambda_Y^A, \quad \lambda_Y^B}^{\text{U(1) Gaugino}}, \quad \overbrace{\tilde{G}_{\Sigma_2}, \quad \tilde{G}_{\Sigma_3}}^{\text{Bifundamental}}, \quad \overbrace{\lambda_2^A, \quad \lambda_2^B}^{\text{SU(2) Gaugino}}, \quad \overbrace{\tilde{G}^0_{\Sigma_2}}^{\text{Bifundamental}}, \quad \tilde{H}_d, \quad \tilde{H}_u \;\Big]$$

$$M_{\chi^0} = \begin{bmatrix} M_Y^A & -g_Y^A\, v_{\Sigma_2} & \frac{2}{3} g_Y^A\, v_{\Sigma_3} & & & & \\ & g_Y^B\, v_{\Sigma_2} & -\frac{2}{3} g_Y^A\, v_{\Sigma_3} & & & -\frac{g_Y^B}{\sqrt{2}}\, {\color{red} v_d} & \frac{g_Y^B}{\sqrt{2}}\, {\color{red} v_u} \\ & -\lambda_S\, v_S & & & & & \\ & & -\lambda_S\, v_S & & & & \\ & & & M_2^A & g_2^A\, v_{\Sigma_2} & & \\ & & & & -g_2^B\, v_{\Sigma_2} & \frac{g_2^B}{\sqrt{2}}\, {\color{red} v_d} & -\frac{g_2^B}{\sqrt{2}}\, {\color{red} v_u} \\ & & & & -\lambda_S\, v_S & & \\ & & & & & & -\mu \end{bmatrix}$$

$$M_{\chi^0}^T = M_{\chi_0}$$

Chargino Mass

$$\chi_L^- = [\ \overbrace{\lambda_L^{A-},\quad \lambda_L^{B-}}^{\text{SU(2) Gaugino}},\quad \overset{\text{Bifundamental}}{\tilde{G}_L^-},\quad \tilde{H}_d^-\ ]$$

$$\chi_R^- = [\ \lambda_L^{A+\,c},\quad \lambda_L^{B+\,c},\quad \tilde{G}_L^{+\,c},\quad \tilde{H}_u^{+\,c}\ ]$$

$$\overline{\chi_R^-}\, M_{\chi^+}\, \chi_L^- = \overline{\chi_R^-} \begin{bmatrix} M_2^A & & g_2^A\, v_{\Sigma_2} & \\ & & -g_2^B\, v_{\Sigma_2} & g_2^B\, v_d \\ g_2^A\, v_{\Sigma_2} & -g_2^B\, v_{\Sigma_4} & -\lambda_S\, v_S & \\ & g_2^B\, v_u & & \mu \end{bmatrix} \chi_L^-$$

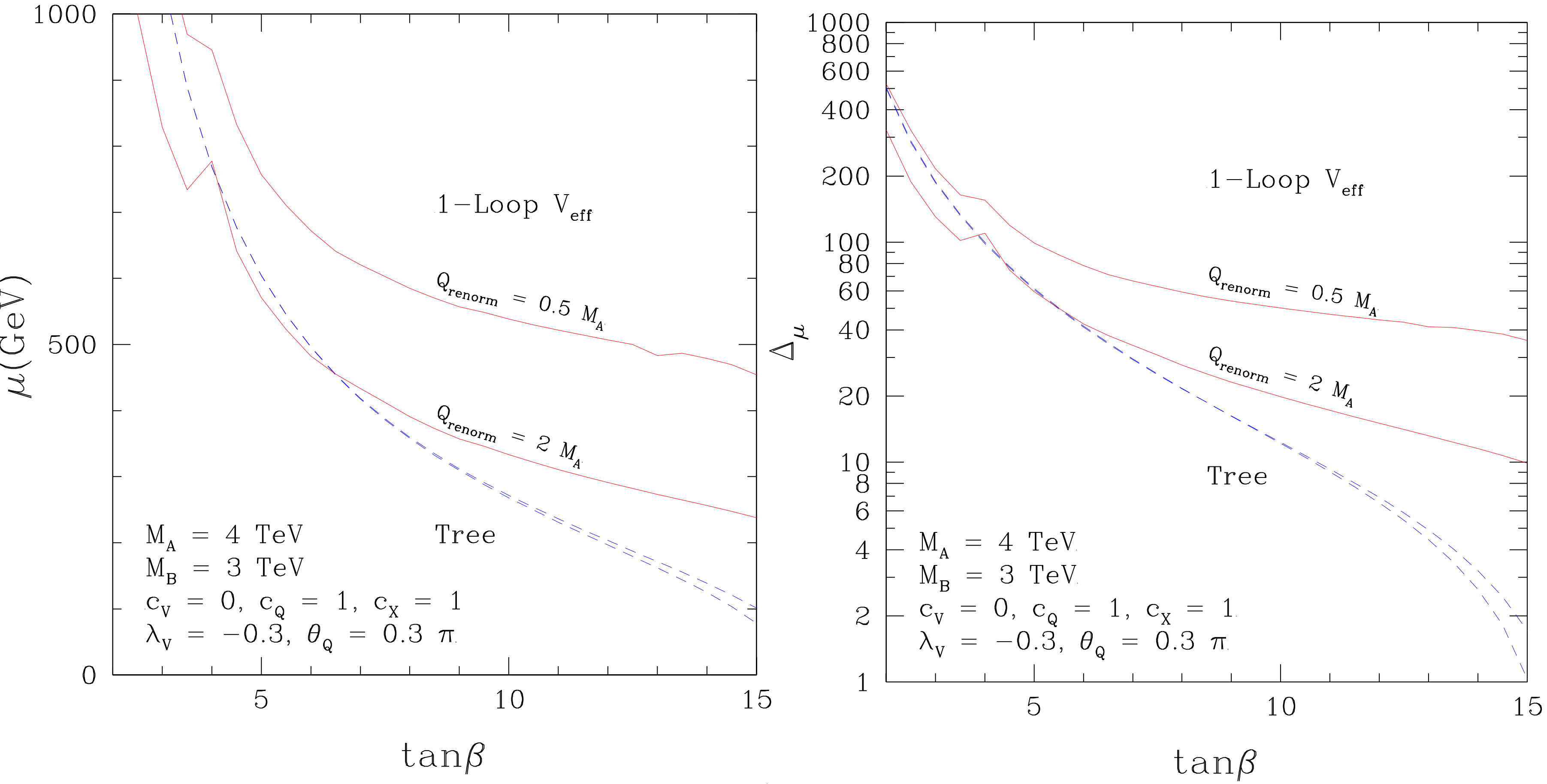
Stop Mass

$$\Psi_{\tilde{t}} = \left[\begin{array}{cccccc} Q', & \overline{Q}^*, & Q, & \overline{U}'^*, & U, & \overline{U}^* \end{array}\right]$$

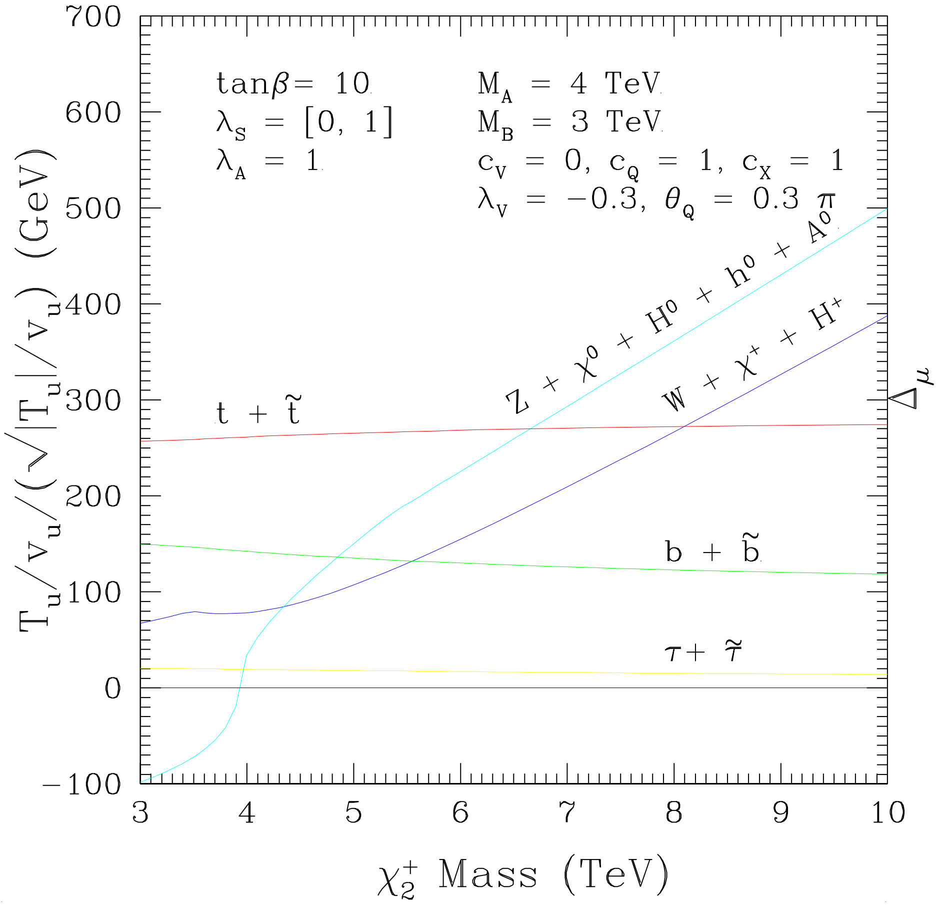
$$M^2_{\tilde{t}} = \begin{bmatrix} \begin{array}{c} m^2_{Q'} + \lambda_Q^2 v_X^2 \\ +\Delta_{U_L} \end{array} & \begin{array}{c} \lambda_Q(A_Q v_X \\ + F_X) \end{array} & \mu_Q \lambda_Q v_X & & & \\ & \begin{array}{c} m^2_{\overline{Q}} + \mu_Q^2 \\ +\lambda_Q^2 v_X^2 \\ -\Delta_{U_L} \end{array} & B_Q \mu_Q & & & \mu_Q \lambda_t v_u \\ & & \begin{array}{c} \mu_Q^2 + \lambda_t^2 v_u^2 \\ +\Delta_{U_L} \end{array} & & \mu_U \lambda_t v_u & \begin{array}{c} \lambda_t(A_t v_u \\ - \mu v_d) \end{array} \\ & & & \begin{array}{c} m^2_{\overline{U}'} + \lambda_U^2 v_X^2 \\ +\Delta_{\overline{U}} \end{array} & \begin{array}{c} \lambda_U(A_U v_X \\ + F_X) \end{array} & \mu_U \lambda_U v_X \\ & & & & \begin{array}{c} m^2_{\overline{U}} + \mu_U^2 \\ +\lambda_U^2 v_X^2 \\ -\Delta_{\overline{U}} \end{array} & B_U \mu_U \\ & & & & & \begin{array}{c} \mu_U^2 + \lambda_t^2 v_u^2 \\ +\Delta_{\overline{U}} \end{array} \end{bmatrix}$$

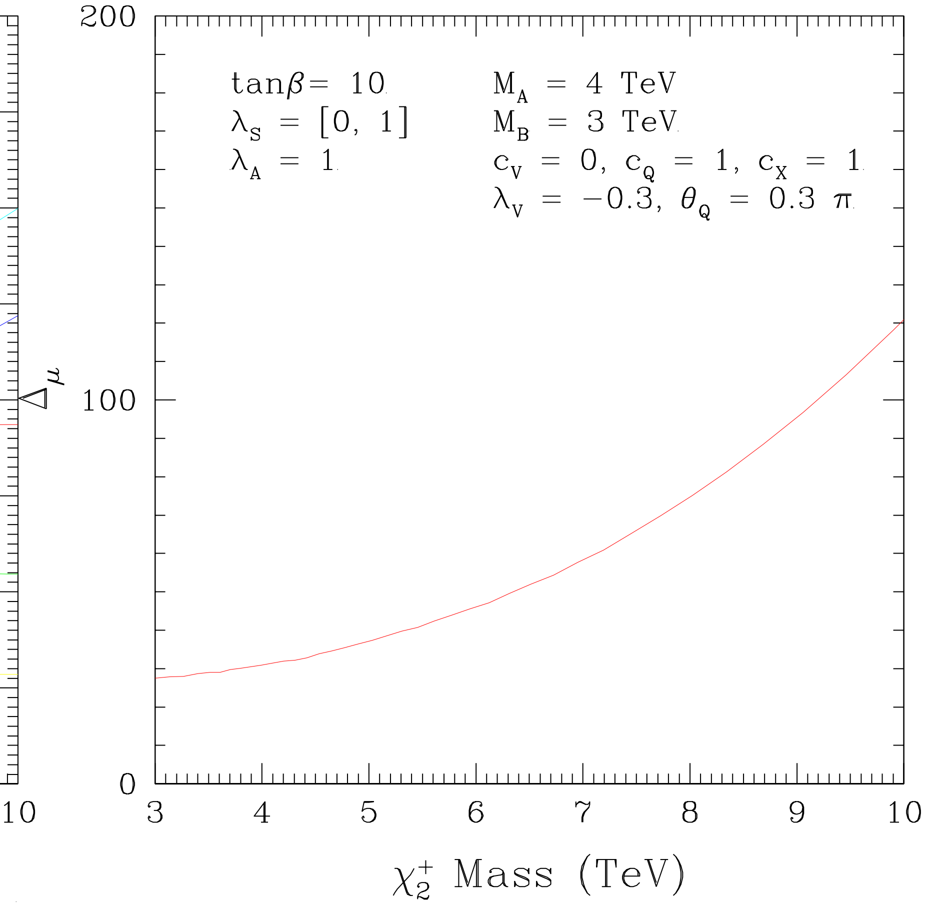
Δ_{U_L} $\Delta_{\overline{U}}$: D-term contribution $\qquad M^\dagger_{\tilde{t}} = M_{\tilde{t}}$

EW symmetry breaking (1-loop)

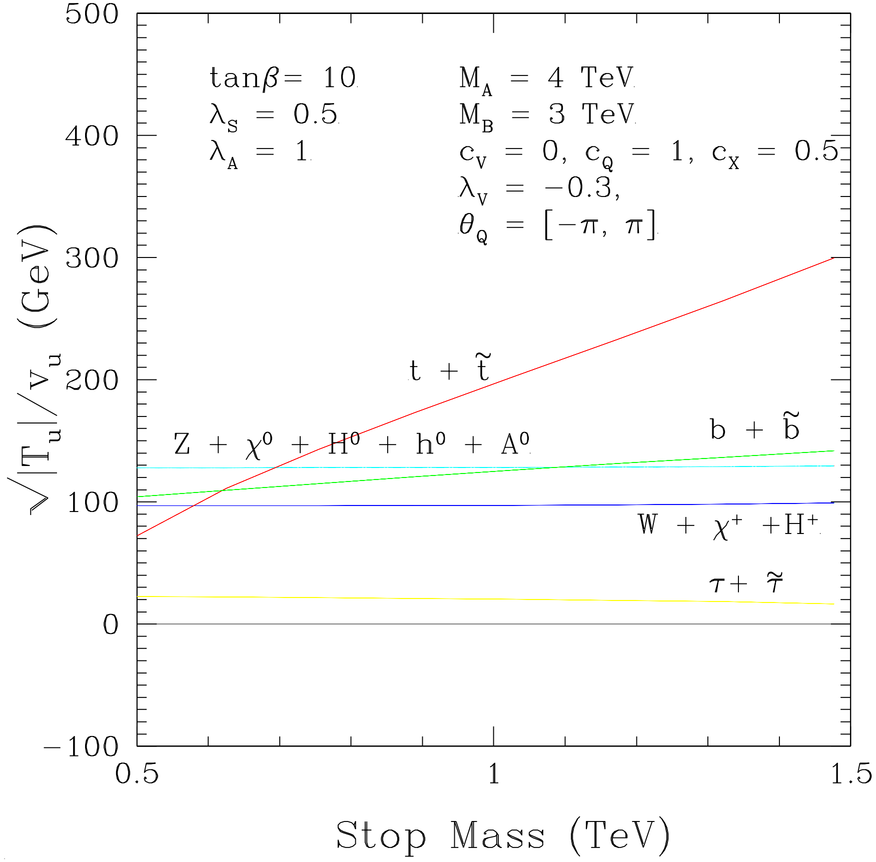


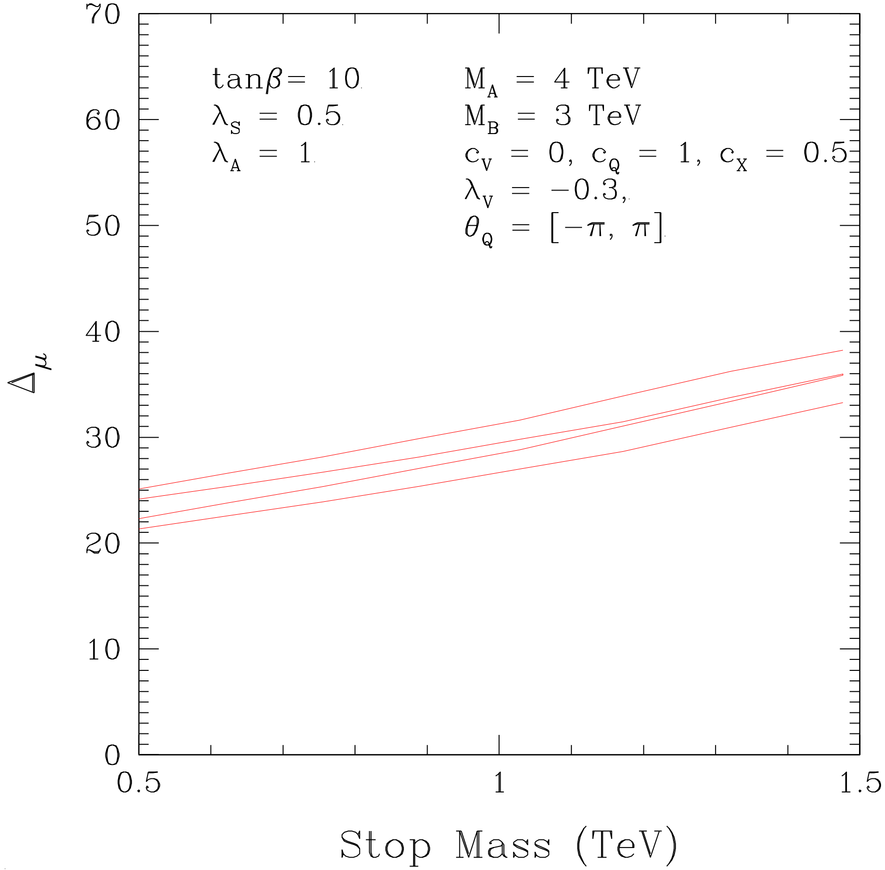
Gaugino mass and tadpole





Stop mass and tadpole





Summary

- We analysed a model with a natural SUSY little hierarchy combining the quiver supersymmetric SM and sliding 3rd generation.
- We can obtain required bifundamental vaccums with a relatively simple superpotential.
- Landau poles and non-holomorphic mixing due to SUSY breaking appears to be obstacles to lift the stop mass to multi-TeV.
- Tadpoles due to the multi-TeV gaugino are small.
- Tadpoles due to the stop dominate the cause of fine-tuing (<~3-5%), a factor improvement could have a strong impact.

Thank You