Standard Model at large hypercharge

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Goals

- To make progress in multi-loop calculations
- To compute various RG functions needed for BSM model-building, precision calculation at colliders and critical phenomena

Example: $m^2\phi^2$

Coupling $m^2(\mu)$ depends on the scale so to run it we need to know anomalous dimension of the operator ϕ^2

We will compute anomalous dimensions for ϕ^n family beyond state-of-the-art of perturbation theory

If the model has continuous global symmetry

Composite operators family **charged** under the symmetry

$$\phi^n(x) = \underbrace{\phi(x) \times \phi(x) \times \phi(x) \times ...}_{n \text{ times}}$$

Or you can construct singlet (**uncharged**) operators

$$\phi^n(x) = [\phi(x)\bar{\phi}(x)]^{n/2}$$

Consider model with U(1) global symmetry

Badel, Cuomo, Monin, Rattazzi 2019

$$L = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\lambda}{4} \left(\bar{\phi} \phi \right)^2$$

The operators $\phi^Q(x)$ and $\bar{\phi}^Q(x)$ carry U(1) charge $+Q(-Q)$

Rescale the field $\phi \to \frac{\phi}{\sqrt{\lambda}}$

$$L_{new} = \frac{1}{\lambda} \left(\partial_\mu \bar{\phi} \partial^\mu \phi + \frac{1}{4} (\bar{\phi}\phi)^2 \right)$$

Now consider correlators of $\phi^Q(x)$

$$\langle \bar{\phi}^Q(x_f) \phi^Q(x_i) \rangle \sim \int D\bar{\phi} D\phi \;\; \phi^Q(x_f) \phi^Q(x_i) e^{-\frac{S}{\lambda}}$$

For **λ**<<1 dominated by the extrema of S

Bring field insertions to the exponent

$$\langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle \sim \int D\bar{\phi}D\phi \; e^{-\frac{S_{eff}}{\lambda}}$$

$$S_{eff} = \int d^dx\left[\partial\bar{\phi}\partial\phi + \frac{1}{4}(\bar{\phi}\phi)^2 + \lambda Q(\log\phi(x_f) + \log\phi(x_i)\right]$$

For $\lambda Q \ll 1$ perturbation theory works (expand around) $\phi = 0$

For $\lambda Q \gg 1$ expand around new saddles

λ<<1 so Q >>1 to have new saddles and keep λQ=fixed

$$S_{eff} = \int d^d x \left[\partial\bar{\phi}\partial\phi + \frac{1}{4}(\bar{\phi}\phi)^2 + \lambda Q(\log\phi(x_f) + \log\phi(x_i)\right]$$

E.O.M

$$\partial^2\phi(x) - \frac{1}{2}\phi^2(x)\bar{\phi}(x) = -\frac{\lambda Q}{\bar{\phi}(x_f)}\delta^{(d)}(x - x_f),$$

$$\partial^2\bar{\phi}(x) - \frac{1}{2}\phi(x)\bar{\phi}^2(x) = -\frac{\lambda Q}{\phi(x_i)}\delta^{(d)}(x - x_i).$$

- can be solved perturbatively but technically challenging

- If we are at the fixed point of RG, however, we can use the power of conformal invariance

In a CFT

$$\langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle_{CFT} = \frac{1}{|x_f - x_i|^{2\Delta_{\phi^Q}}}$$

Physical critical exponents

$$\Delta_{\phi^Q} \equiv Q\left(\frac{d-2}{2}\right) + \gamma_{\phi^Q}$$

We will use conformal invariance just to simplify the calculation.
Results will be valid also for non-conformal theories

Goal : compute $\Delta_{\phi^Q} \equiv Q\left(\frac{d-2}{2}\right) + \gamma_{\phi^Q}$

We expect scaling dimensions to take the form:

$$\Delta_Q = \sum_{k=-1} \frac{\Delta_k(\lambda_0 Q)}{Q^k}$$

Δ_k is (k+1)-loop correction to the saddle point equation

We will compute Δ_{-1} and Δ_0

We will use conformal invariance just to simplify the calculation.
Results will be valid also for non-conformal theories

Semiclassical computation

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$S(\phi_0) \downarrow \Delta_{-1}$

$\frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) \downarrow \Delta_0$

Semiclassical method

Badel, Cuomo, Monin, Rattazzi 2019

Double scaling limit : $\lambda \to 0, \quad Q \to \infty, \quad \lambda Q = fixed$

- Tune QFT to the (perturbative) fixed point (WF or BZ type)
- Map the theory to the cylinder $\mathbb{R}^d \to \mathbb{R} \times S^{d-1}$
- Exploit operator/state correspondence for the 2-point function to relate anomalous dimension to the energy

$$\langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle_{CFT} = \frac{1}{|x_f - x_i|^{2\Delta_{\phi^Q}}} \qquad E = \Delta_{\phi^Q}/R$$

- To compute this energy, evaluate expectation value of the evolution operator in an arbitrary state with fixed charge Q

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$$\langle Q|e^{-HT}|Q\rangle \overset{T\to\infty}{=} \bar{N}e^{-E_{\phi^Q}T}$$

as long as there is an overlap between |Q> and the ground state, the latter will dominate for $T \to \infty$

To study system at fixed charge thermodynamically we have:

$$H \to H + \mu Q$$

μ is chemical potential

Back to our U(1) global symmetry model

$$L = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\lambda}{4} \left(\bar{\phi} \phi \right)^2$$

In d=4-ε there is an IR WF fixed point

$$\lambda^* = \frac{3}{10} \epsilon + \cdots$$

Map the theory to the cylinder:

$$S_{cyl} = \int d^d x \sqrt{-g} \left(g_{\mu\nu} \partial^\mu \bar{\phi} \partial^\nu \phi + m^2 \bar{\phi} \phi + \frac{\lambda}{4} (\bar{\phi} \phi)^2 \right)$$

$$m^2 = \left(\tfrac{d-2}{2R} \right)^2$$

stemming from the coupling to Ricci scalar

Classical solution: $$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$$\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}$$

$$S_{eff} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \Big(\frac{1}{2}(d\rho)^2 + \frac{1}{2}\rho^2 (d\chi)^2 + \frac{m^2}{2}\rho^2 + \frac{1}{16}\rho^4 + \mu^2 f^2 \Big)$$

Stationary solution: $\rho = f$ $\chi = -i\mu\tau$

$$\mu^2 - m^2 = \frac{\lambda}{4} f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

$$m^2 = \left(\tfrac{d-2}{2R}\right)^2$$

Plug the solution into the action:

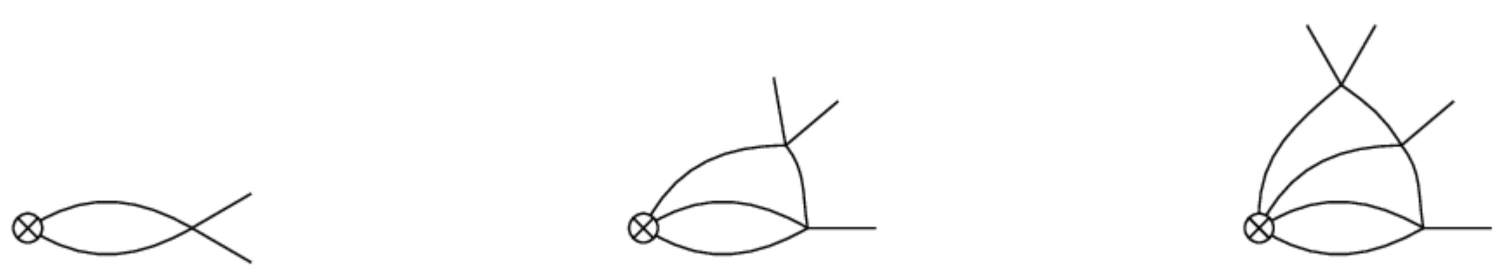
$$S_{eff} R = E_{-1} R = \Delta_{-1}$$

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}}$$

$$x \equiv 6\lambda Q$$

$$\frac{\Delta_{-1}}{\lambda_*} \overset{\lambda Q \ll 1}{=} Q\left[1 + \frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right) - \frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right)^2 + \cdots\right]$$

Classical computation resums infinite number of Feynman diagrams



Leading quantum correction:

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$$\rho = f + r(x) \qquad \chi = -i\mu\tau + \frac{\pi(x)}{\sqrt{2}f}$$

$$S^{(2)} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \left[\frac{1}{2}(\partial r)^2 + \frac{1}{2}(\partial \pi)^2 - 2i\mu\, r\partial_\tau \pi + (\mu^2 - m^2)r^2\right]$$

One relativistic (Type I) Goldstone boson (the conformal mode=phonon) and one massive state

$$\omega_\pm^2(\ell) = J_l^2 + 3\mu^2 - m^2 \pm \sqrt{4J_l^2\mu^2 + (3\mu^2 - m^2)^2}$$

$$J_l^2 = \ell(\ell + d - 2)/R^2$$

Energy= sum of zero point energies

$$\Delta_0 = \frac{R}{2}\sum_{\ell=0}^{\infty} n_\ell \big[\omega_+(\ell) + \omega_-(\ell)\big]$$

The MSbar renormalized result reads:

$\lambda Q \ll 1 \quad :$

$$\Delta_0 = -\frac{3\lambda_* Q}{(4\pi)^2} + \frac{\lambda_*^2 Q^2}{2(4\pi)^4} + \cdots$$

Compare with:

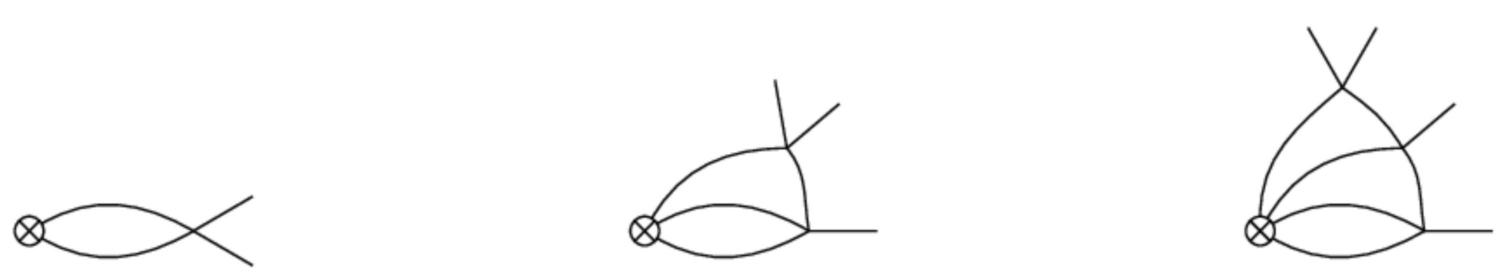
$$\frac{\Delta_{-1}}{\lambda_*} \;=\; Q\Big[1 + \frac{1}{2}\Big(\frac{\lambda_* Q}{16\pi^2}\Big) - \frac{1}{2}\Big(\frac{\lambda_* Q}{16\pi^2}\Big)^2 + \cdots\Big]$$

Charged operators family result

$$\Delta_{\phi^Q} \equiv Q\left(\frac{d-2}{2}\right) + \gamma_{\phi^Q} \quad = \Delta_{-1} + \Delta_0 + \cdots$$

$$\gamma_{\phi^Q} = Q\left[\frac{\lambda_*}{16\pi^2}\,\frac{(Q-1)}{2} - \left(\frac{\lambda_*}{16\pi^2}\right)^2 \frac{2Q^2 - 2Q - 1}{4}\right] + \cdots$$

Perfect agreement for coloured terms with diagrammatics



Recover perturbative expansion

	1-loop	**2-loop**	**3-loop**	
Δ_{-1}	$Q^2\lambda_0$	$Q^3\lambda_0^2$	$Q^4\lambda_0^3$	$\ldots.$
Δ_0	$Q\lambda_0$	$Q^2\lambda_0^2$	$Q^3\lambda_0^3$	$\ldots.$
Δ_1		$Q\lambda_0^2$	$Q^2\lambda_0^3$	$\ldots.$
Δ_2			$Q\lambda_0^3$	$\ldots.$
$\vdots$				

Uncharged operators: Real scalar (Z2 symmetry)

2408.01414
OA, Bersini, Sannino

$$\phi = \frac{\rho}{\sqrt{2}} \cancel{e^{i\chi}}$$

$$S_{eff} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \left(\frac{1}{2}(d\rho)^2 + \cancel{\frac{1}{2}\rho^2(d\chi)^2} + \frac{m^2}{2}\rho^2 + \frac{1}{16}\rho^4 + \cancel{\mu^2 f^2} \right)$$

Stationary solution:

Charged	**Uncharged**
$\rho = f \qquad \chi = -i\mu\tau$	$\frac{d^2\rho}{dt^2} + m^2\rho + \lambda\rho^3 = 0$
$\mu^2 - m^2 = \frac{\lambda}{4} f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}}$	$2\pi^2 \int_0^{\mathcal{T}} \left(\frac{d\phi}{dt}\right)^2 dt = 2\pi n$
	$\rho(t) = \sqrt{n}\, x_0 \, \mathrm{cn}(\omega t \vert m)$

Local U(1) model

$$S = \int d^D x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D_\mu \phi + \frac{\lambda (4\pi)^2}{6} (\bar{\phi}\phi)^2 \right)$$

$$D = 4 - \epsilon \qquad\qquad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

- Perturbative WF fixed point at 1-loop reads

$$\lambda^* = \frac{3}{20}\left(19\epsilon \pm i\sqrt{719}\epsilon\right) \, , \qquad a_g^* = \frac{3}{2}\epsilon \qquad\qquad a_g = \frac{e^2}{(4\pi)^2}$$

complex!

Homogeneous ground state ansatz

$$\rho(x) = f, \quad \chi(x) = -i\mu\tau, \quad A_\mu = 0$$

Homogeneous ground state

$$\rho(x) = f, \quad \chi(x) = -i\mu\tau, \quad A_\mu = 0$$

From EOM

$$\mu^2 - m^2 = \frac{\lambda}{4} f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

Plugging into Seff.

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}}$$

The same as in U(1) global case

$$x \equiv 6\lambda Q$$

$$\rho(x) = f + r(x)$$
$$\chi(x) = -i\mu\tau + \frac{\pi(x)}{f}$$

Add gauge-fixing and ghost terms

$$\delta S = \frac{1}{2}\int d^d x \left(G^2 + \mathcal{L}_{\text{ghost}}\right), \qquad G = \frac{1}{\sqrt{\xi}}(\nabla_\mu A^\mu + ef\pi)$$

and expand S_{eff} to quadratic order

$$\mathcal{L}^{(2)}_{\text{eff}} = \frac{1}{2}A_\mu\left(-g^{\mu\nu}\nabla^2 + \mathcal{R}^{\mu\nu} + \left(1-\frac{1}{\xi}\right)\nabla^\mu\nabla^\nu + (ef)^2 g^{\mu\nu}\right)A_\nu$$
$$+ \frac{1}{2}(\partial_\mu r)^2 - \frac{1}{2}2(m^2-\mu^2)r^2 + \frac{1}{2}(\partial_\mu\pi)^2 - \frac{1}{2\xi}(ef)^2\pi^2$$
$$-2i\mu\, r\partial_\tau\pi - 2if\mu r A^0 + ef\left(1-\frac{1}{\xi}\right)A_\mu\partial^\mu\pi + \bar{c}[-\nabla^2 + (ef)^2]c$$

Explicit 3-loop result for ϕ^Q

2210.10685
OA, Bednyakov, Bersini, Panopoulos, Pikelner

$$\gamma_Q^{(1)}(\lambda, a_g, \xi) = \underbrace{\frac{\lambda}{3}Q^2}_{\text{leading}} \underbrace{- Q\left(3a_g + \frac{\lambda}{3}\right)}_{\text{sub-leading}} + a_g Q^2 \xi$$

$$\gamma_Q^{(2)}(\lambda, a_g) = \underbrace{-\frac{2\lambda^2}{9}Q^3}_{\text{leading}} + \underbrace{\left(a_g^2 - \frac{4a_g\lambda}{3} + \frac{2\lambda^2}{9}\right)Q^2}_{\text{sub-leading}} + \left(\frac{7a_g^2}{3} + \frac{4a_g\lambda}{3} + \frac{\lambda^2}{9}\right)Q$$

$$\gamma_Q^{(3)}(\lambda, a_g) = \underbrace{\frac{8\lambda^3}{27}Q^4}_{\text{leading}} + \underbrace{Q^3\left[\frac{4a_g\lambda^2}{3}(3-2\zeta_3) - \frac{8a_g^2\lambda}{3}(1+3\zeta_3) + 4a_g^3(9\zeta_3-1) + \frac{2\lambda^3}{27}(16\zeta_3-17)\right]}_{\text{sub-leading}}$$
$$+ Q^2\left[\frac{29a_g^2\lambda}{6} + a_g^3(95-108\zeta_3) + \frac{\lambda^3}{18}(57-64\zeta_3) - \frac{4a_g\lambda^2}{9}(31-30\zeta_3)\right] + Q\left[\frac{13a_g^2\lambda}{6} + \frac{2a_g\lambda^2}{9}(49-48\zeta_3) - \frac{2\lambda^3}{27}(31-32\zeta_3) - a_g^3\left(\frac{3251}{54} - 72\zeta_3\right)\right]$$

Perfect agreement for the leading and subleading terms with large-Q results!

Yukawa interactions: NJLY model

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$$\mathcal{L}_{\rm NJLY} = \partial_\mu \bar\phi \partial^\mu \phi + \bar\psi_j \partial\!\!\!/ \psi^j + g \bar\psi_{Rj} \bar\phi \psi_L^j + g \bar\psi_{Lj} \phi \psi_R^j + \frac{\lambda}{24} \left(\bar\phi \phi\right)^2$$

$$\phi = f e^{i\chi}$$

$$\chi = -i\mu\tau$$

Remove phases from Yukawa term via:

$$\psi_L \to \psi_L \, e^{\mu\tau/2} \,, \qquad \psi_R \to \psi_R \, e^{-\mu\tau/2}$$

Classically: $\psi^{cl}_{L,R} = 0$ → Δ_{-1} is again U(1) model result

Quadratic in fluctuations:

$$S^{(2)} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \Big[\frac{1}{2}(\partial r)^2 + \frac{1}{2}(\partial \pi)^2 - 2i\mu \, r \partial_\tau \pi + (\mu^2 - m^2) r^2$$
$$+ i\mu \, \bar\psi_j \gamma^0 \psi^j + \bar\psi^j \nabla\!\!\!\!/_{\mathcal{M}} \psi^j + g \, f \bar\psi_{Lj} \psi^j_R + g \, f \bar\psi_{Rj} \psi^j_L \Big]$$

Gaussian integral

$$\int \mathcal{D}r \mathcal{D}\pi \mathcal{D}\bar\psi \mathcal{D}\psi \, e^{-S^{(2)}} = \frac{\det \, F}{\det \, B}$$

Fermionic dispersions

$$\omega_{f\pm}(\ell) = \sqrt{\frac{3g^2\left(\mu^2 - m^2\right)}{8\pi^2\lambda} + \left(\frac{\mu}{2} + \lambda_{f\pm}\right)^2}$$

Eigenvalues of the Laplacian on the sphere

Leading quantum correction

$$\Delta_0 = \frac{1}{2}\sum_{\ell=0}^{\infty}\left[n_\ell(\omega_+(\ell) + \omega_-(\ell)) - N_f n_{f,\ell}(\omega_{f+}(\ell) + \omega_{f-}(\ell))\right]$$

$$\Delta_0^{(f)} = Q\left(\frac{g^2}{8\pi^2} - \frac{3g^4}{32\pi^4\lambda}\right) + Q^2\left(\frac{g^2\lambda}{12\pi^2} - \frac{g^4}{32\pi^4}\right) + Q^3\left(\frac{g^6\zeta(3)}{64\pi^6} - \frac{g^2\lambda^2}{18\pi^2} + g^4\lambda\frac{1-3\zeta(3)}{48\pi^4}\right)$$

$+ \ldots.$

Standard model

SU(3)xSU(2)xU(1) local symmetry

- To NLO in semiclassical expansion SU(3) does not enter
- Add fermions with full CKM structure

$$\mathcal{L}_{\text{Yukawa}} = -4\pi \left(Y_u^{ij} \left(\mathcal{Q}_i^L H^c \right) u_j^R + Y_d^{ij} \left(\mathcal{Q}_i^L H \right) d_j^R + Y_l^{ij} \left(L_i^L H \right) l_j^R \right)$$

Standard model: Higgs family $H^Q(x)$

2312.12963
OA, Bersini, Panopoulos, Sannino, Wang

$$\Delta_Q = Q + \left\{\frac{1}{3}\lambda Q^2 + \left[N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l - \frac{3}{4}g'^2 - \frac{\lambda}{3}\right]Q\right\} - \left\{\frac{2}{9}\lambda^2 Q^3 - \left[2N\mathcal{Y}_{uu} + 2N\mathcal{Y}_{dd} + 2\mathcal{Y}_{ll} - \frac{2}{3}\lambda(N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l) - \frac{1}{3}\lambda g'^2 + \frac{g'^4}{16} + \frac{\lambda^2}{9}\right]Q^2 + C_{22}Q\right\} + \left\{\frac{8}{27}\lambda^3 Q^4 + \left[\frac{1}{16}g'^6(9\zeta(3)-1) - \frac{1}{6}g'^4\lambda(1+3\zeta(3)) + \frac{1}{3}g'^2\lambda^2(3-2\zeta(3)) + \frac{4}{27}\lambda^3(9\zeta(3)-8) + \frac{4}{27}\left(3N\left(\lambda^2\mathcal{Y}_u - 3\lambda\mathcal{Y}_{uu} + 9\zeta(3)(\lambda\mathcal{Y}_{uu} - 2\mathcal{Y}_{uuu})\right) + 3N\left(\lambda^2\mathcal{Y}_d - 3\lambda\mathcal{Y}_{dd} + 9\zeta(3)(\lambda\mathcal{Y}_{dd} - 2\mathcal{Y}_{ddd})\right) + 3\left(\lambda^2\mathcal{Y}_l - 3\lambda\mathcal{Y}_{ll} + 9\zeta(3)(\lambda\mathcal{Y}_{ll} - 2\mathcal{Y}_{lll})\right)\right)\right]Q^3 + C_{23}Q^2 + C_{33}Q\right\} + \mathcal{O}\left(\kappa_I^4 Q^5\right) . \quad (7.1)$$

$$\mathcal{Y}_f = (4\pi)^2 \mathrm{Tr} Y_f Y_f^\dagger , \quad \mathcal{Y}_{ff} = (4\pi)^4 \mathrm{Tr}(Y_f Y_f^\dagger)^2 , \quad \mathcal{Y}_{fff} = (4\pi)^6 \mathrm{Tr}(Y_f Y_f^\dagger)^3 , \quad f = u, d, l .$$

Pheno application: Higgsplosion

Multi-boson production

$$h^* \to nh$$

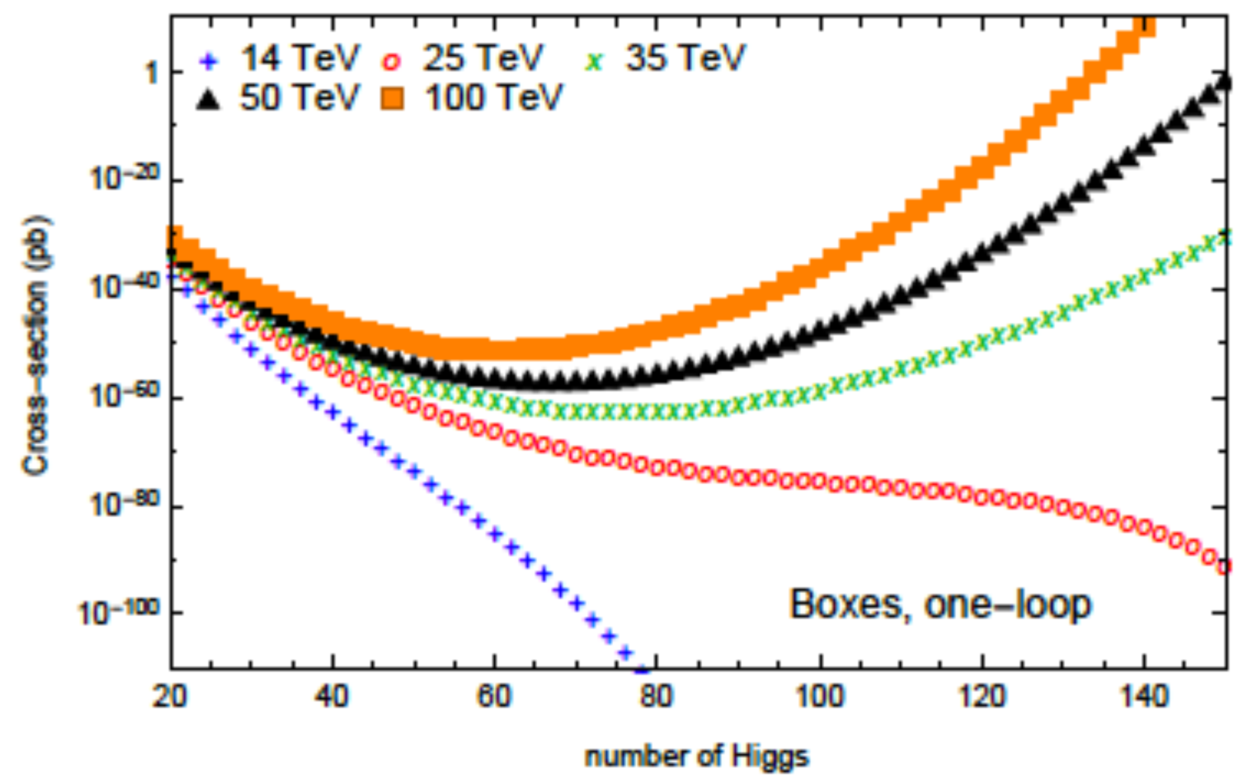
$$\lambda\phi^4$$

Consider the $1 \to n$ amplitude

$$A^{tree} = n!\lambda^{\frac{n-1}{2}} e^{-\frac{5}{6}En}$$

$$A = A^{tree} e^{B\lambda n}$$

$$\sigma(1 \to n) = e^{F(\lambda n, E)}$$



[Degrande, Khoze, Mattelaer, 2016]

$$n \approx \sqrt{s}/m$$

Other directions/aspects

- In a generic QFT, I showed how to semiclassically compute anomalous dimensions for operators ϕ^n needed for BSM model-building, precision calculation at colliders and critical phenomena

  - Large order behaviour of the series (resurgence)
  - Higher correlation functions
  - Condensed matter applications
  - Inhomogeneous ground state (operators with spin/derivatives)
  - Test dualities between different CFTs in their charged sectors
  - ……

Thank you!