Do we exist because of a gravitational Anomaly?

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EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

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Workshop on the Standard Model & Beyond, 26 August -3 September 2024

1. Outline

- ❖ The (string-inspired) Model : Chern-Simons gravity with gravitational axions
- ❖ Origin of Gravitational Anomalies
- ❖ The role of primordial gravitational waves in inducing gravitational anomaly condensates – weak quantum gravity estimates
- ❖ Induced Linear-axion monodromy inflation of Running-Vacuum-Model (RVM) type (metastable de Sitter vacuum, compatible with swampland)
- ❖ Gravitational anomaly condensates and matter-antimatter asymmetry (i.e. reason for our existence)
- ❖ Conclusions: a flash of the entire cosmological history of this Universe

2. The Model:

String-inspired Chern-Simons gravity

Gross and Sloan, Metsaev and Tseytlin

Massless Gravitational multiplet of (closed) strings:

spin 0 scalar (dilaton Φ)
spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)
spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R + 2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}\,e^{-4\Phi}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \dots\Big)$$

$\kappa^2 = 8\pi\, G$

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\, B_{\nu\rho]} \quad \Rightarrow \quad \boxed{\boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c} \wedge \mathbf{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c} \wedge \omega^c_{\ d} \wedge \omega^d_{\ a}, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

Gross and Sloan, Metsaev and Tseytlin

Stringy gravitational Axions + torsion

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout

$$\boxed{\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \underbrace{\frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho}}_{\text{Contorsion}} \neq \overline{\Gamma}^{\mu}_{\rho\nu}}$$

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generalised curvature

quantum torsion →

gravitational axion b "dual" to H torsion

Campbell, Duncan, Kaloper, Olive
Svrcek, Witten

$$\boxed{\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}}$$

Contorsion

Stringy gravitational Axions + torsion

Gross and Sloan, Metsaev and Tseytlin

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Contorsion

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Φ = constant throughout, e.g. → 0

Green, Schwarz

$$S_B = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left[-R + 2\,\cancel{\partial_\mu\Phi\,\partial^\mu\Phi}\right] - \frac{1}{6\kappa^2}\,e^{-\cancel{4\Phi}}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \dots\right)$$

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\,B_{\nu\rho]} \quad\Rightarrow\quad \boxed{\boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\left(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\right)}$$

$$\Omega_{3\mathrm{L}} = \omega^a{}_c \wedge \mathrm{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathbf{dA} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$

$\alpha' = M_s^{-2}$

Modified Bianchi Constraint

$$\boxed{\varepsilon_{abc}{}^{\mu}\,\nabla_\mu H^{abc} = \frac{\alpha'}{32}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathscr{G}(\omega,\mathbf{A})}$$

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Φ = constant throughout, e.g. → 0

Green, Schwarz

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Implement in path-integral as a field theory δ(...) via Lagrange multiplier b(x) pseudoscalar (axion-like) field (Kalb-Ramond (KR) Axion) becomes dynamical after H-torsion integration

CP -invariant

pseudoscalar

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \overline{\boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})}\Big) = \int Db\, \exp\Big[i \int d^4x \sqrt{-g}\, \frac{1}{\sqrt{3}} b(x) \Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$= \int Db\, \exp\Big[-i \int d^4x \sqrt{-g} \Big(\partial^\mu b(x)\, \frac{1}{\sqrt{3}} \epsilon_{\mu\nu\rho\sigma}\, \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x \sqrt{-g} \Big[-\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\, \alpha'}{96\, \kappa \sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \Big) + \dots \Big]$$

CP -invariant

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \overline{\boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu}} - \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db \exp\Big[i\int d^4x\sqrt{-g}\,\frac{1}{\sqrt{3}}b(x)\Big(\varepsilon^{\mu\nu\rho\sigma}H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega,\mathbf{A})\Big)\Big]$$

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KR-axion anomalous
CP-conserving interaction with gravity

cf. classically in 4 dim:
b-field "dual" to H-torsion

$$\boxed{-3\sqrt{2}\,\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ **Axial Current**

KR-axion anomalous CP-conserving interaction

torsion

cf. classically in 4 dim:
b-field "dual" to H-torsion

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

The Model

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x \sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\right) J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x \sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

$$\mathcal{F}^d = \varepsilon^{abcd} e_{b\lambda}\partial_a e^\lambda_c,$$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots \Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\right) J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

$$\mathcal{F}^d = \varepsilon^{abcd}\, e_{b\lambda}\partial_a e^\lambda_c\, ,$$

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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All fermion species

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

3. Gravitational (Chern-Simons) Anomalies

NB:

Anomalies in Quantum Field Theory:

Classical Symmetry → Conserved Current

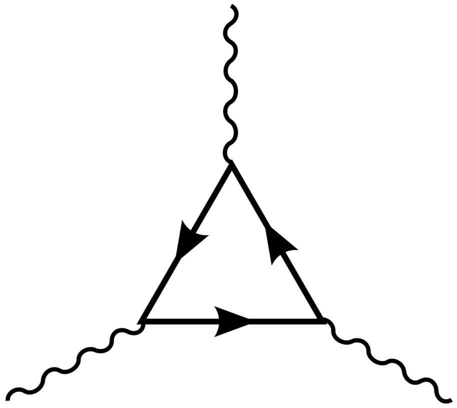
Quantum Theory: Failure of current conservation in ANY REGULARIZATION of the quantum theory

or equivalently:

Path-Integral measure NOT INVARIANT under symmetry transformation

Fujikawa

OF INTEREST HERE: GAUGE & GRAVITATIONAL CHIRAL ANOMALIES



CHIRAL FERMIONIC LOOP in graphs with 1+D/2 external legs (gauge fields or gravitons) in D- space-time dimensions D=4 → triangular graphs

Alvarez-Gaume, Witten

NB: Mixed Anomalies (Gravitational + Gauge)

$$\nabla_\mu J^{5\mu} = \mathrm{c}_1 \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)$$

gravitational covariant derivative

$$\boxed{J^{5\mu} = \sum_j \overline{\psi}_j \gamma^\mu \gamma^5 \psi_j}$$

Axial Current

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\pi} R^{\lambda\pi}{}_{\rho\sigma}, \qquad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma}, \qquad \varepsilon^{\mu\nu\rho\sigma} = \frac{\mathrm{sgn}(g)}{\sqrt{-g}}\, \epsilon^{\mu\nu\rho\sigma}$$

Anomaly terms are total derivatives:

$$\sqrt{-g}\left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right) = \sqrt{-g}\, \mathcal{K}^\mu_{\mathrm{mixed}}(\omega)_{;\mu} = \partial_\mu \left(\sqrt{-g}\, \mathcal{K}^\mu_{\mathrm{mixed}}(\omega) \right)$$

$$= 2\,\partial_\mu \left[\epsilon^{\mu\nu\alpha\beta}\, \omega_\nu^{ab} \left(\partial_\alpha \omega_{\beta ab} + \frac{2}{3} \omega_{\alpha a}{}^{c}\, \omega_{\beta cb} \right) - 2\epsilon^{\mu\nu\alpha\beta} \left(A^i_\nu\, \partial_\alpha A^i_\beta + \frac{2}{3} f^{ijk} A^i_\nu A^j_\alpha A^k_\beta \right) \right]$$

NB: Mixed Anomalies (Gravitational + Gauge)

$$\nabla_\mu J^{5\mu} = c_1 \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)$$

gravitational covariant derivative

$$J^{5\mu} = \sum_j \overline{\psi}_j \gamma^\mu \gamma^5 \psi_j$$ Axial Current

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\pi} R^{\lambda\pi}{}_{\rho\sigma}, \quad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

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Anomaly terms are total derivatives – can couple to axion-like fields *b*

$$S_B^{\rm eff} \ni \frac{1}{f_b} \int d^4x \sqrt{-g} \Big[\, b(x) \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right) + \dots \Big]$$

Axion coupling

e.g. gravitational axion $f_b = 96 \sqrt{\frac{3}{2} \frac{\kappa}{\alpha'}} = 96 \sqrt{\frac{3}{2}} \frac{M_s^2}{M_{\rm Pl}}$

NB: **Mixed Anomalies (Gravitational + Gauge)**

$$\nabla_\mu J^{5\mu} = c_1 \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)$$

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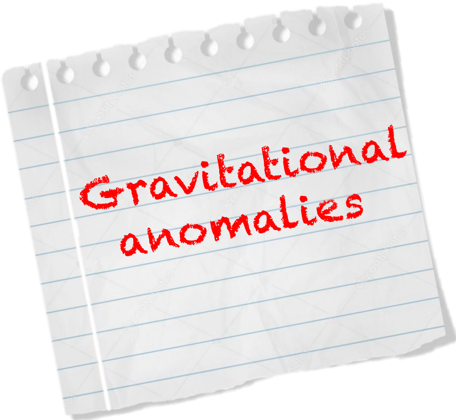
Contributions to Stress tensor YES NO

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x\,\sqrt{-g}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big)$$

Spoils conservation of stress tensor (**diffeomporphism invariance affected in quantum theory**)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x\sqrt{-g}\,b\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\Big] = 4\int d^4x\sqrt{-g}\,\mathcal{C}^{\mu\nu}\,\delta g_{\mu\nu} = -4\int d^4x\sqrt{-g}\,\mathcal{C}_{\mu\nu}\,\delta g^{\mu\nu}$$

Cotton tensor

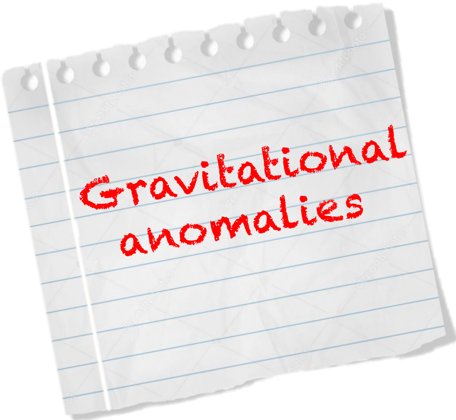
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma\left(\varepsilon^{\sigma\mu\alpha\beta}R^\nu{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta}R^\mu{}_{\beta;\alpha}\right) + v_{\sigma\tau}\left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu}\right)\Big] = -\frac{1}{2}\Big[\left(v_\sigma\,\widetilde{R}^{\lambda\mu\sigma\nu}\right)_{;\lambda} + (\mu \leftrightarrow \nu)\Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma},\;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $\qquad g_{\mu\nu}\,\mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



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Spoils conservation of stress tensor (diffeomporphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

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$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma},\quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy

Traceless $\qquad g_{\mu\nu}\,\mathcal{C}^{\mu\nu} = 0$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\boxed{\kappa^2 T^{\mu\nu}_{\rm matter\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0}$$

Diffeomoprphism invariance breaking by gravitational anomalies ?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}}$$

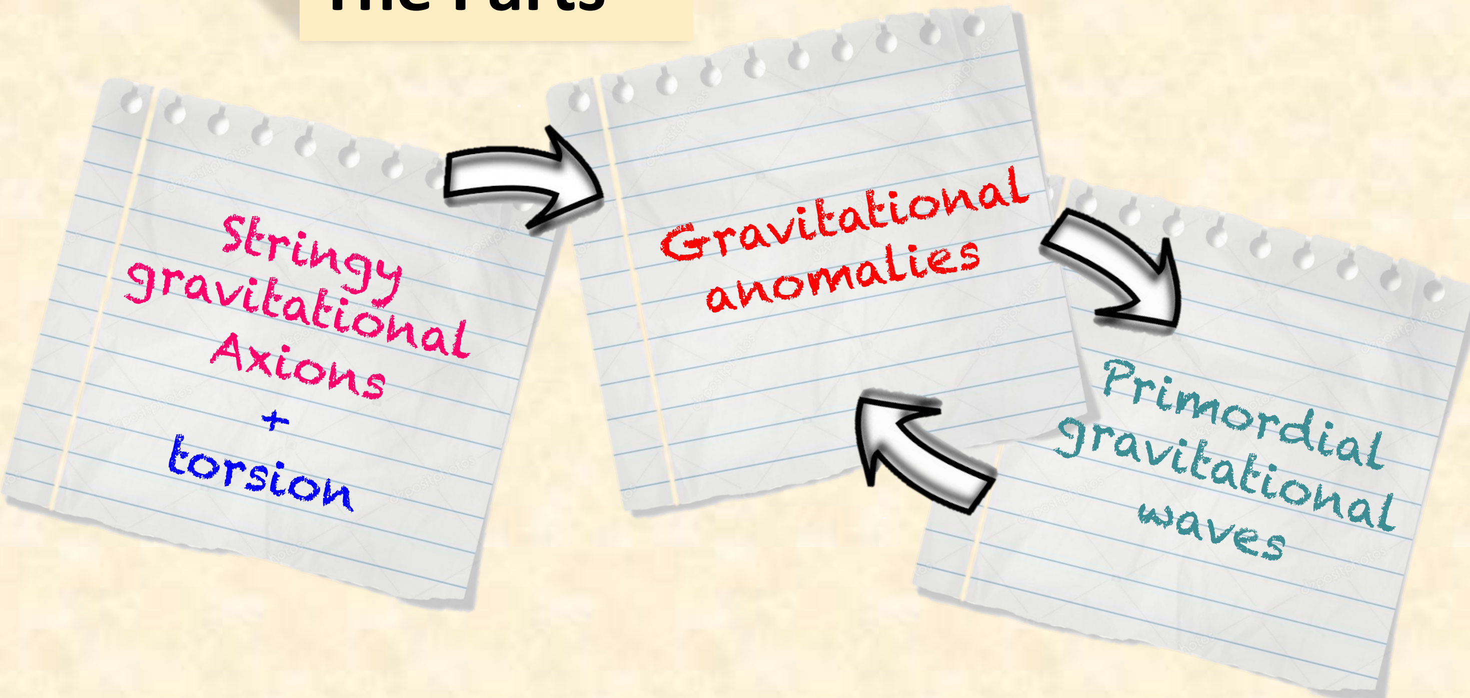
$$v_\sigma \equiv \partial_\sigma b$$

$$\boxed{\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0}$$

No problem with diffeo

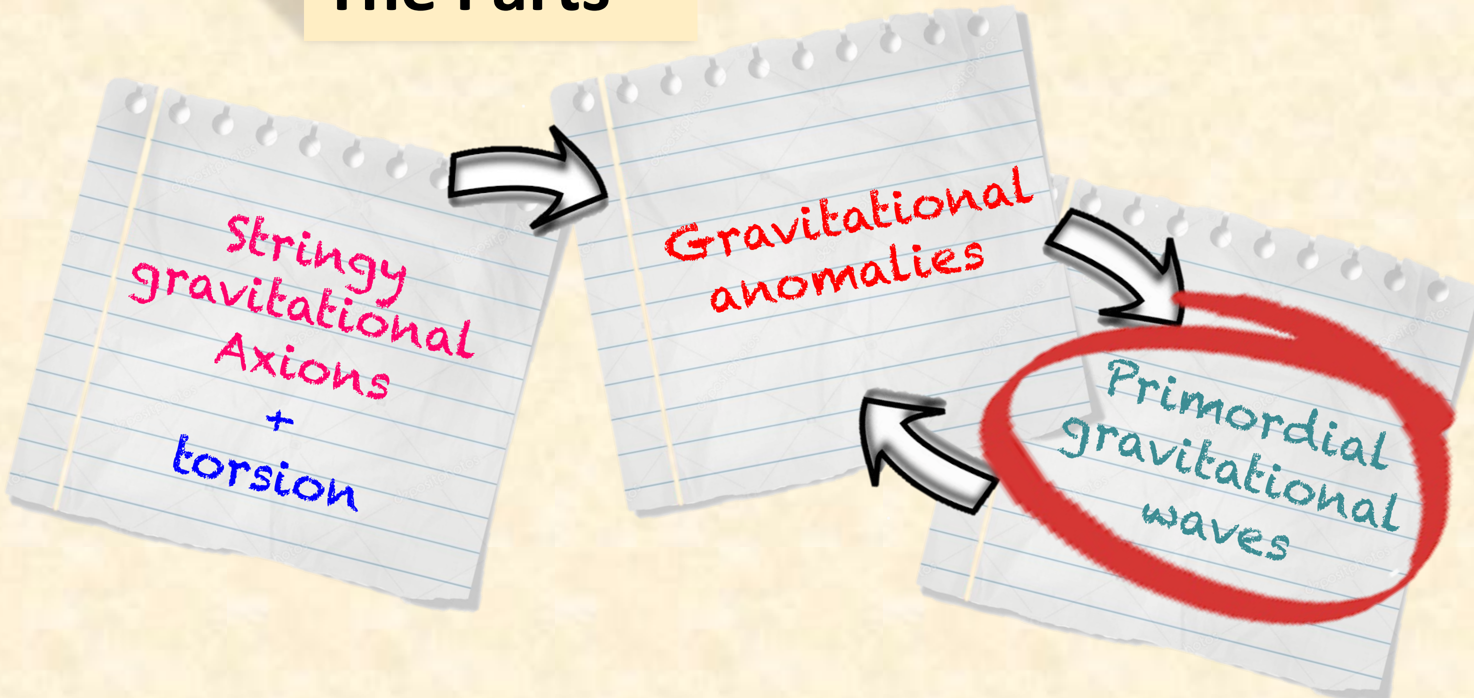
Conserved Modified stress-energy tensor

The Parts



Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves

The Parts


Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$
$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

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Chiral Fermionic matter & radiation fields are supposed to be generated by the decay of the false (running) vacuum (cf below) at the end of inflation

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only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,\cancel{R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}} + \dots\Big]$$

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No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

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only gravitational d.o.f. (b, $g_{\mu\nu}$)

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c.f. Zeldovich but for baryons in his model

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Basilakos, NEM, Solà (2019-20)

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Primordial Gravitational Waves

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

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Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

NEM, Solà
EPJ-ST
(2020)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

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Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(i) merging of primordial Black Holes formed from collapse of massive brane/stringy defects

NEM, Solà EPJ-ST (2020)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[\ldots \frac{1}{\ldots} R + \frac{1}{\ldots}\partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \ldots\Big]$$
$$= \int \ldots b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \ldots\Big]\,,$$

Carr, Garcia-Bellido,
Khlopov, Malomed, Zeldovich,
Hawking, Page,
Ricotti, Ostriker, Mack,
Clese, Fleury, Kuhnel,
Peloso, Sandstand, Unal,
Sendouda, Yokoyama, ...

Primordial Gravitational Waves
Potential Origins in pre-inflationary era?
(i) merging of primordial Black Holes formed
from collapse of massive brane/stringy defects

NEM, Solà
EPJ-ST
(2020)

Ellis, NEM, Nanopoulos,
Sakellariadou, Elghozi, Yusaf....

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$
\begin{aligned}
S_B^{\rm eff} &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big] \\
&= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,
\end{aligned}
$$

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(ii) Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino or gaugino)

NEM, Solà EPJ-ST (2020)

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Basilakos, NEM, Solà (2019-20)

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Zeldovich, Kobzarev, Okun, Kibble, Vilenkin, Sikivie, Gelimini, Gleiser, Kolb, ...

Lalak, Ovrut, Lola, G. Ross, Thomas

Takahashi, Yanagida, Yonekura

Primordial Gravitational Waves

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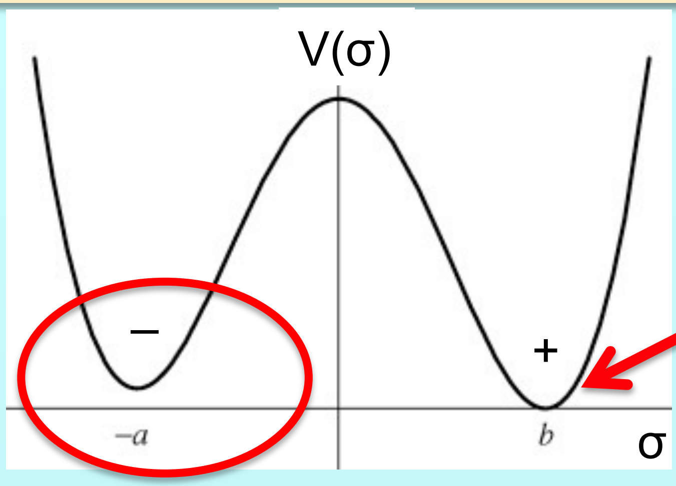
NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Zeldovich, Kobzarev, Okun, Kibble, Vilenkin, Sikivie, Gelimini, Gleiser, Kolb, ...

Lalak, Ovrut, Lola, G. Ross, Thomas

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Primordial Gravitational Waves

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NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Non-trivial if GW present

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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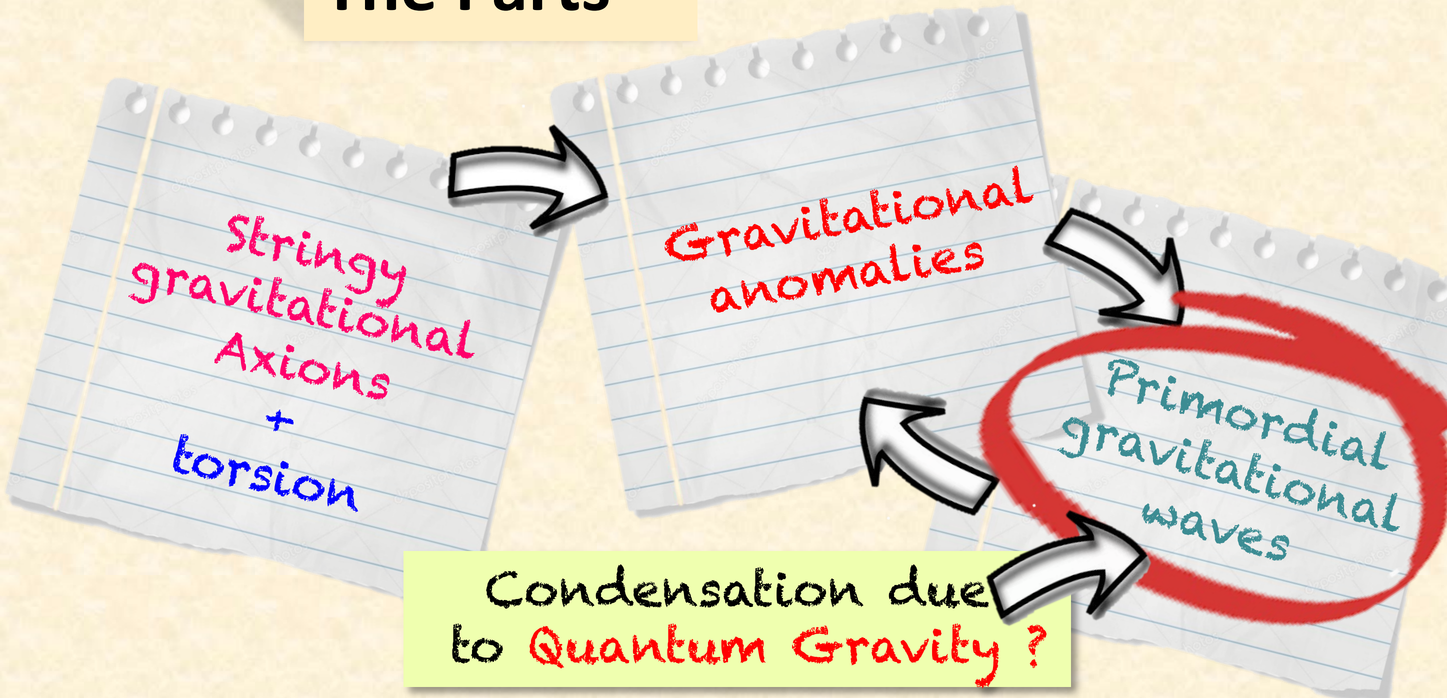
Primordial Gravitational Waves

3. Gravitational waves & Grav. Anomaly condensates

The Parts



Basilakos, NEM, Solà

Dorlis, NEM, Vlachos

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves →
Condensate < …> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

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$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Cosmological-Constant-like

Condensate < …> of Gravitational Anomalies

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quantum ordered

$$\sqrt{-g}\,\mathcal{K}^\mu(\omega)_{;\mu}$$

Effective action contains **CP violating axion-like coupling**

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$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

$b(x)=b(t)$

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

$$\boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int^{\mu}\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\,\Theta + O(\Theta^3)}$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1}, \quad \dot{b} \equiv db/dt, \quad a(t) \sim e^{Ht}$$

H ≈ const. (inflation)

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EFT

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Revisiting-Reestimating the Calculation of Condensate

In weak quantum-gravity framework

Dorlis, NEM, Vlachos
arXive:2403.09005 [gr-qc],
PRD in press

Dorlis, NEM, Vlachos
2404.18741 (PoS Corfu 2023)

Improvement on approximations made in previous works

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

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In weak quantum-gravity framework

Dorlis, NEM, Vlachos arXive:2403.09005 [gr-qc], PRD in press

Dorlis, NEM, Vlachos 2404.18741 (PoS Corfu 2023)

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD in press

$$S^{\rm eff} = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS}\right], \quad A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$2 < R_{CS} > = - \langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle = - \int DbDg_{\mu\nu} e^{-S^{\rm eff}} R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} = \text{constant}$$

WE DO NOT KNOW THE FULL QG THEORY!

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD in press

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WE DO NOT KNOW THE FULL QG THEORY!

Only Estimate at Stationary points of S^{eff}

$$\frac{\delta S^{\text{eff}}}{\delta b} = \frac{\delta S^{\text{eff}}}{\delta g_{\mu\nu}} = 0$$

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD in press

$$S^{\rm eff} = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS}\right], \quad A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$2\langle R_{CS}\rangle = -\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = -\int Db\,Dg_{\mu\nu}\, e^{-S^{\rm eff}}\, R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} = \text{constant}$$

WE DO NOT KNOW THE FULL QG THEORY!

EFT approach in expanding universe

Only Estimate at Stationary points of $S^{\rm eff}$

$$ds^2 = -dt^2 + \alpha^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \qquad h_{ij} = h_+\epsilon_{ij}^{(+)} + h_\times\epsilon_{ij}^{(\times)} \qquad [h_{ij}] = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GW tensor perturbations

$$\lambda = R, L$$

$$h_{ij}(t,\vec{x}) = h_L\,\epsilon_{ij}^{(L)} + h_R\,\epsilon_{ij}^{(R)} = \sum_{\lambda=L,R} h_\lambda(t,\vec{x})\epsilon_{ij}^{(\lambda)},$$

Chiral GW waves needed for $R_{CS} \neq 0$

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD in press

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GW tensor perturbations

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Chiral GW waves needed for $R_{CS} \neq 0$

$$h_{ij} = \kappa \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \frac{\psi_{\lambda,\vec{k}}(\eta)}{\alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{CS}(\eta)}} \epsilon^\lambda_{ij}, \qquad L \to R \quad \text{and} \quad \vec{k} \to -\vec{k}.$$

$$L_{CS}(\eta) = k\xi, \qquad \xi = \frac{4Ab'\kappa^2}{\alpha^2}, \qquad b' = db/d\eta \qquad \left[\epsilon^{(R)}_{ij}\right] = \frac{1}{\sqrt{2}}\left(\left[\epsilon^{(+)}_{ij}\right] + i\left[\epsilon^{(\times)}_{ij}\right]\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \left[\epsilon^{(L)}_{ij}\right]^\dagger$$

conformal time

Canonical quantization of weak gravity

Introduce complex scalar fields

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \widetilde{\psi}_{L,\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \widetilde{\psi}_{L,\vec{k}},$$

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Quantum operators (creation,annihilation acting on appropriate vacuum state |0> (Bunch-Davis)

$$\hat{\widetilde{\phi}}_{\vec{k}}(\eta) = \widetilde{v}_{\vec{k}} \hat{\alpha}^-_{\vec{k}} + v^*_{-\vec{k}} \hat{b}^+_{-\vec{k}},$$

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Lyth, Rodriguez, Quimbay

$$u_{L,\vec{k}} = \kappa \frac{\widetilde{v}_{\vec{k}}}{z_{L,\vec{k}}}, \quad u_{R,\vec{k}} = \kappa \frac{v_{\vec{k}}}{z_{R,\vec{k}}}.$$

$$z_{\lambda,\vec{k}}(\eta) = \alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{CS}(\eta)}$$

$$\left[\hat{\alpha}^-_{\vec{k}}\,,\,\hat{\alpha}^+_{\vec{k}'}\right] = \left[\hat{b}^-_{\vec{k}}\,,\,\hat{b}^+_{\vec{k}'}\right] = \delta^{(3)}(\vec{k}-\vec{k}')$$

& the rest zero

$$\boxed{\langle R_{CS}\rangle = \frac{2i}{\alpha^4}\left[\langle \partial_z^2 h_L \partial_z h'_R\rangle + \langle h''_L \partial_z h'_R\rangle - \langle \partial_z^2 h_R \partial_z h'_L\rangle - \langle h''_R \partial_z h'_L\rangle\right]}$$

$$\langle \ldots \rangle = \langle 0|\ldots|0\rangle$$

Trivial for non-chiral GW

Keep **ALL terms** (further to approximations made in **Lyth, Rodriguez, Quimbay**)

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD in press

Canonical quantization of weak gravity

Introduce complex scalar fields

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \widetilde{\psi}_{L,\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \widetilde{\psi}_{L,\vec{k}},$$

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(creation,annihilation
acting on appropriate
vacuum state |0>
(Bunch-Davis)

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Lyth, Rodriguez, Quimbay

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& the rest zero

$$\boxed{\langle R_{CS}\rangle = \frac{2}{\alpha^4}\int^{\alpha\mu} \frac{d^3\vec{k}}{(2\pi)^3} l_{\vec{k}} \left[k^3\left(u_{L,\vec{k}} u^{*\prime}_{L,\vec{k}} - u_{R,\vec{k}} u^{*\prime}_{R,\vec{k}}\right) + k\left(u''_{R,\vec{k}} u^{*\prime}_{R,\vec{k}} - u''_{L,\vec{k}} u^{*\prime}_{L,\vec{k}}\right)\right]}$$

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μ = low-energy UV cutoff ~ M_s

Canonical quantization of weak gravity

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& the rest zero

$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$\boxed{\langle R_{CS}\rangle^I = -\frac{A}{\pi^2}\frac{\dot{b}_I}{M_{\rm Pl}}\left(\frac{H_I}{M_{\rm Pl}}\right)^3 \mu^4 < 0,}$$

$H_I \approx$ *constant during inflation*

$$\langle \ldots \rangle = \langle 0| \ldots |0\rangle$$

μ = low-energy UV cutoff ~ M_s

Canonical quantization of weak gravity

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$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \widetilde{\psi}_{L,\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \widetilde{\psi}_{L,\vec{k}}\,,$$

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& the rest zero

If you *have N_I sources* of GW Linear superposition of GW perturbations

$$\boxed{\langle R_{CS}\rangle_I^{total} = -\mathcal{N}_I\,\frac{A\kappa^4\mu^4}{\pi^2}\,\dot{b}_I H_I^3}$$

$H_I \approx$ constant during inflation

$$\langle \ldots \rangle = \langle 0| \ldots |0\rangle \qquad \kappa^{-1} = M_{Pl}$$

μ = low-energy UV cutoff ~ M_s

The Parts

Basilakos, NEM, Solà

Homogeneity & Isotropy

Solutions (backgrounds) to the Eqs of Motion

$$\boxed{\alpha' = M_s^{-2}}$$

$$S_B^{eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big] \,,$$

$$\boxed{\partial_\alpha \Big[\sqrt{-g} \Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^\alpha(t) \Big) \Big] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^0 \sim \text{constant}}$$

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μ = low-energy UV cutoff ~ M_s

FLRW spacetime

$$\Big(A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}\Big)$$

Planck Data $H/M_{\rm Pl} \lesssim 10^{-5}$

time evolution of Anomaly

$$\boxed{\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\Big[-3Ht\Big(1 - 0.5\, N_I\, 10^{-4}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\Big(\frac{\mu}{M_s}\Big)^4\Big)\Big]}$$

Homogeneity & Isotropy

Solutions (backgrounds) to the Eqs of Motion

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to ensure approximately constant anomaly current during inflation

$$\mathcal{K}^0 = \text{const.}$$

Spontaneous LV solution ⚠

$$\boxed{N_I \sim O(10^{14})}$$

@ the beginning of RVM inflation

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\Big)\Big] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk} \approx \textit{constant}$$

Spontaneous LV solution (constant spatial components of H-torsion)

Parametrisation

@ end of Inflationary era

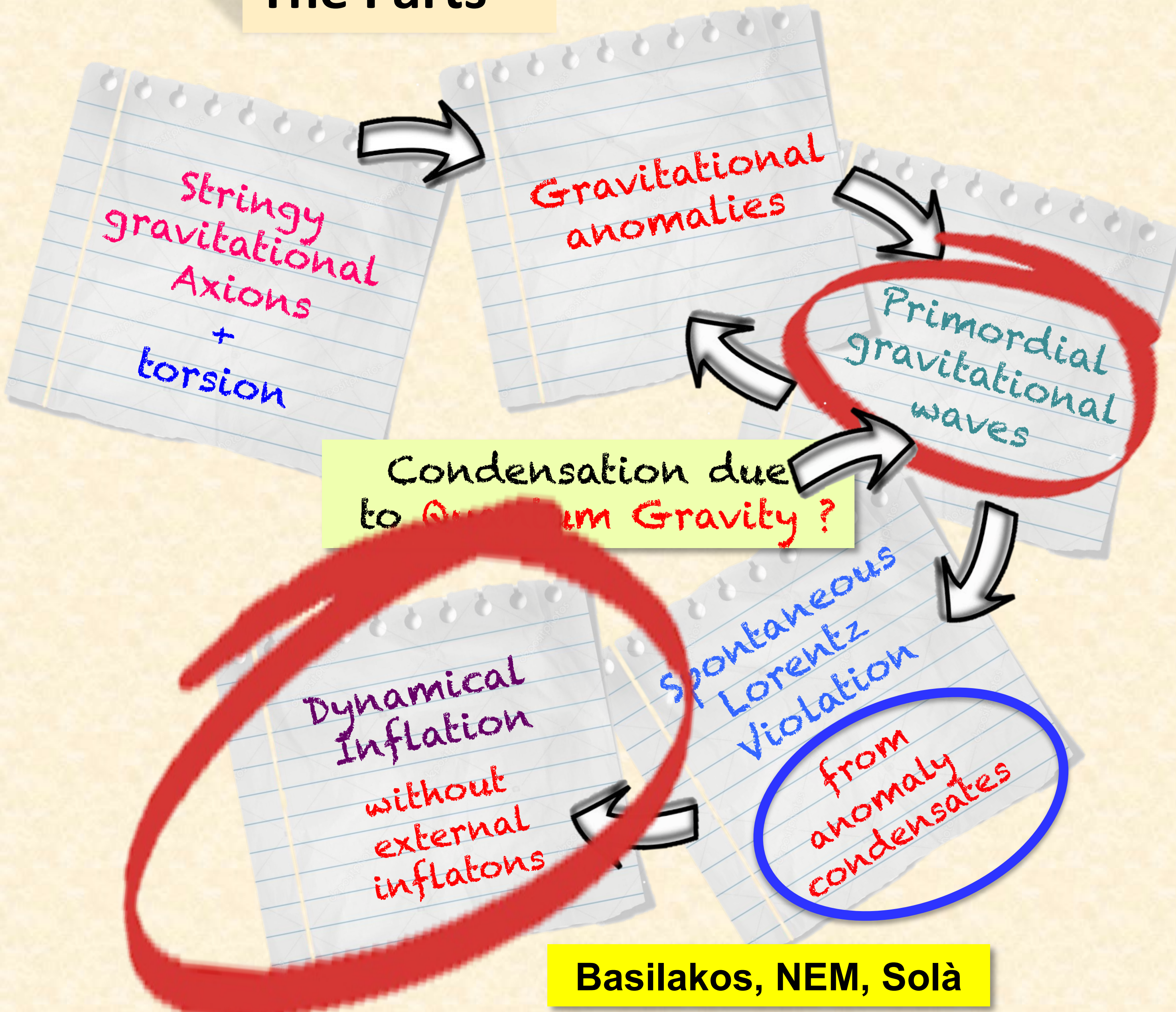
$$\boxed{\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

How can we estimate ε ?

4. Grav. Anomaly Condensates & inflation

The Parts



Basilakos, NEM, Solà

The Parts

String
gravitati
Axio
+
torsio

Gravitational
anomalies

Primordial
gravitational
waves

Condensation due
to Qu um Gravity

Spontan
Lor on
Vio

from
anomaly
condensates

Dynamical
Inflation
without
external
inflatons

Axion-monodromy-
like Linear inflation
(cf. string theory)

Dorlis, NEM, Vlachos

Torsion axions, Condensates & Inflation

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Primordial string Universe Gravitational Waves (GW): e.g. from collapse of (rotating) primordial black holes (PBH) sourced by Torsion-induced axion field *b(x)*

$$\Box b \propto R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}$$

constant

can induce **condensates** of gravitational Chern-Simons terms $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$

Compute using weak (perturbative)
Quantum gravity techniques
With GW perturbation modes

Condensates lead to linear axion potentials

$$V(b) \ni b(x)\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Lyth, Rodriguez, Qiumbay
Alexander, Peskin, Sheikh-Jabbari
Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

(cf. string/brane theory linear axion monodromy potentials:
Silverstein, Mc Allister, Westphal, …, but here different origin)

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

Linear axion potential and Running Vacuum Model (RVM) Inflation

Dynamical System approach to inflation

$$3H^2 = \kappa^2 \left(\frac{\dot{b}^2}{2} + V(b) \right)$$

$$2\dot{H} + 3H^2 = -\kappa^2 \left(\frac{\dot{b}^2}{2} - V(b) \right)$$

$$\ddot{b} + 3H\dot{b} + V_{,b} = 0$$

$$x' = -\frac{3}{2}\left[2x - x^3 + x\left(y^2 - 1\right) - \frac{\sqrt{2}}{\sqrt{3}}\lambda y^2\right]$$

$$y' = -\frac{3}{2}y\left[-x^2 + y^2 - 1 + \frac{\sqrt{2}}{\sqrt{3}}\lambda x\right]$$

$$\lambda' = -\sqrt{6}\left(\Gamma - 1\right)\lambda^2 x$$

$$\lambda = -\frac{V_{,b}}{\kappa V} \quad \text{and} \quad \Gamma = \frac{V V_{,bb}}{V_{,b}^2}$$

$$V_{,b} \equiv \frac{\delta V}{\delta b}$$

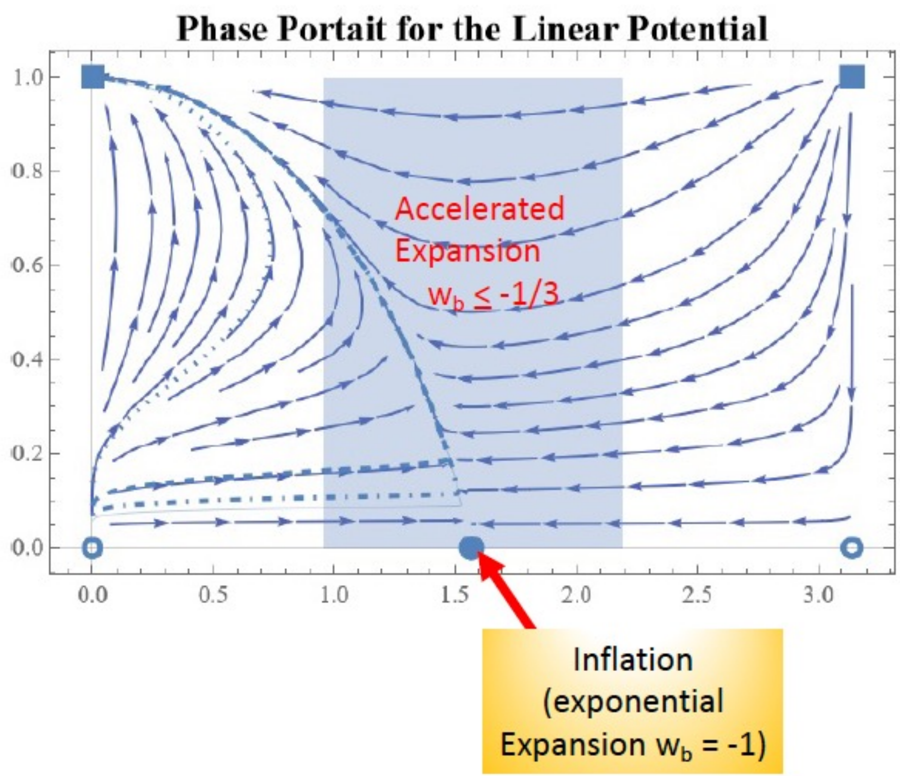
$$x = cos\varphi \ , \ y = sin\varphi \quad \Rightarrow$$

$$\boxed{\begin{aligned} \varphi' &= \left(3\cos\varphi - \frac{\sqrt{6}}{2}\lambda\right)\sin\varphi \\ \lambda' &= -\sqrt{6}(\Gamma - 1)\lambda^2\cos\varphi \end{aligned}}$$

$$\zeta = \frac{\lambda}{\lambda + 1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2 \cos\varphi$$

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]



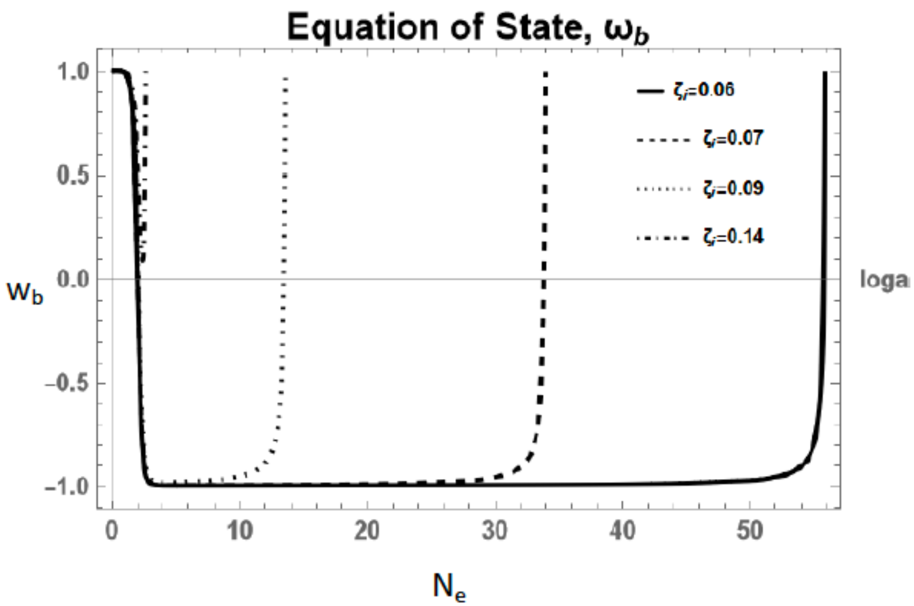
$\zeta - \varphi$ plane.

$$\zeta = \frac{\lambda}{\lambda + 1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$
$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2\cos\varphi$$

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced

by **quantum graviton** fluctuations of chiral (left-right asymmetric) GW type



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$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

String EFT:
$\mu = M_s = (\alpha')^{-1/2}$

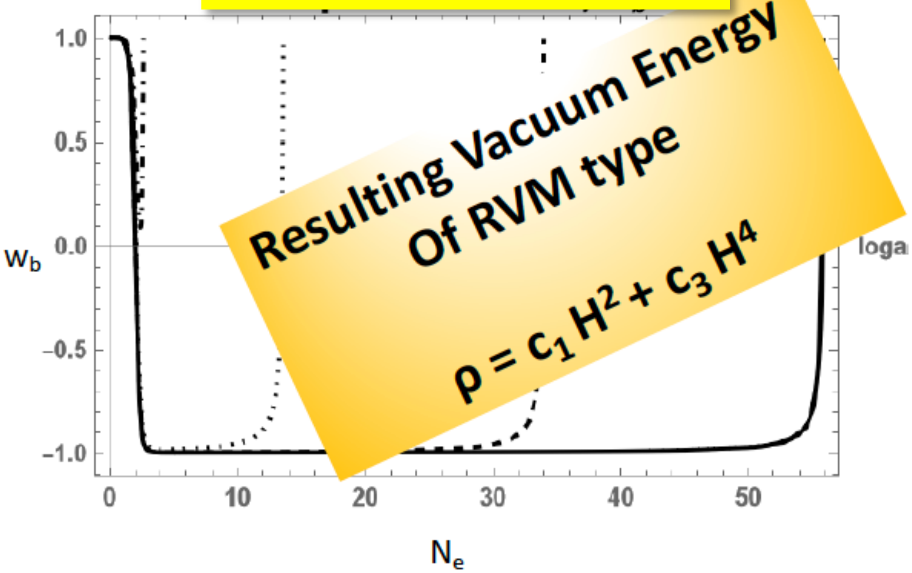
Evolution of equation of state for the orbits of the linear b-potential phase space

For some initial value of φ_i inflation with e-foldings $N_e > 50$ is achieved for $\zeta < 0.06$ (**inflation → saddle point**)

$$\zeta = \frac{\lambda}{\lambda + 1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2\cos\varphi$$

Basilakos, NEM, Solà



Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

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$$-\mathrm{Re}\,\langle R_{CS}\rangle^{total} = \mathcal{N}_I \frac{A\kappa^4\mu^4}{\pi^2}\dot{b}_I H_I^3$$

Hubble

GW sources

UV cutoff of graviton modes

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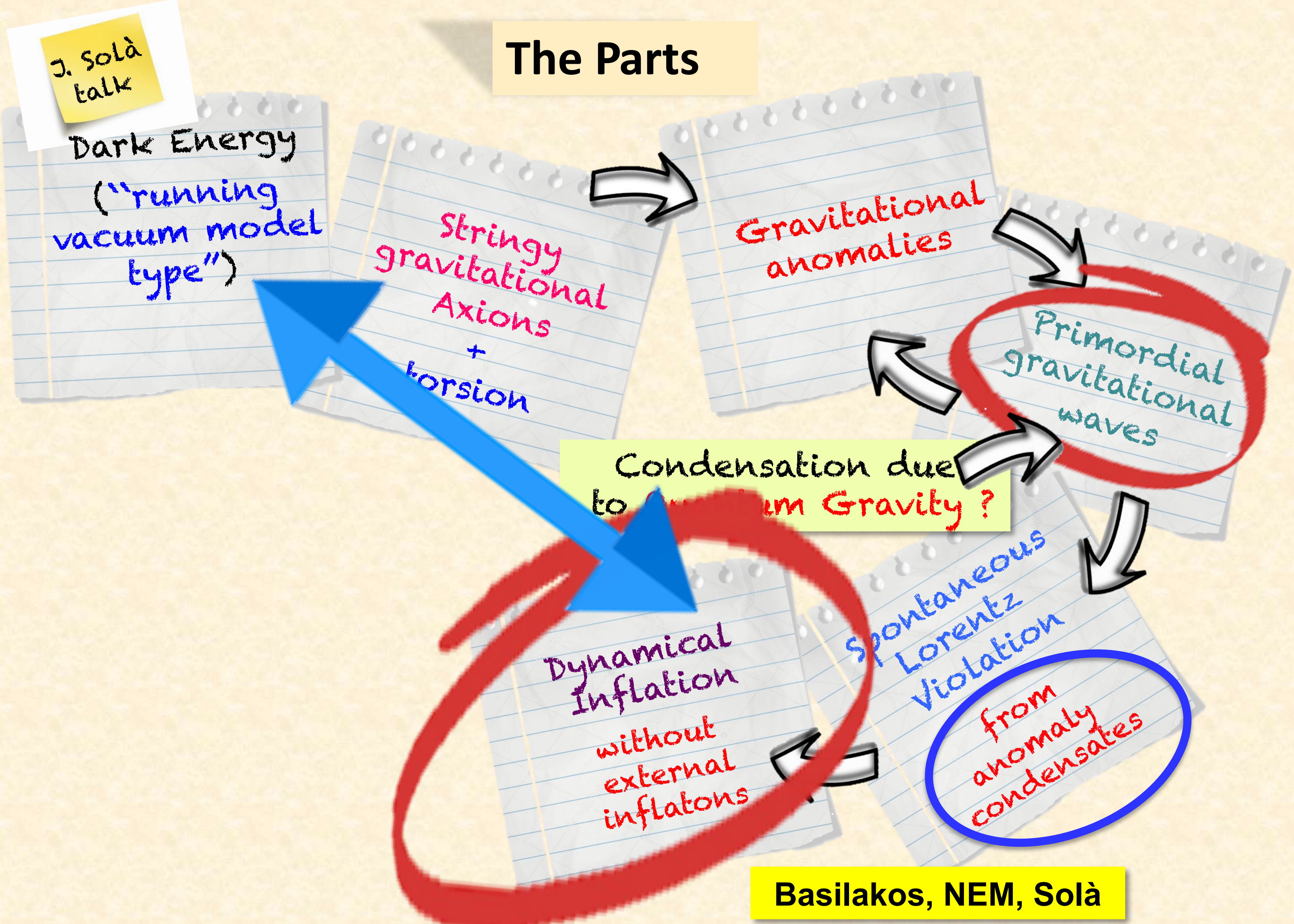
The Parts


J. Solà talk
Dark Energy
("running vacuum model type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves
Condensation due
to
um Gravity ?
Spontaneous Lorentz Violation
from anomaly condensates
Dynamical Inflation
without external inflatons
Basilakos, NEM, Solà

NB: Metastable Inflationary Vacua

Dorlis, NEM, Vlachos 2404.18741 (PoS Corfu 2023)

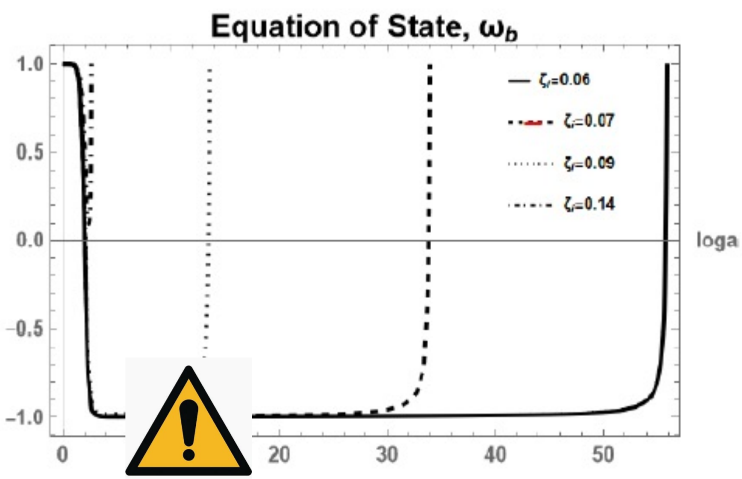
Compatibility with swampland (Ooguri, Vafa, …)

CONDENSATE WITH CUTOFF GRAVITON MODES HAS IMAGINARY PARTS

(ENVIRONMENT OF MODES WITH MOMENTA ABOVE THE CUTOFF μ)

Consistent with swampland ⟷ INSTABILITY OF CONDENSATE PHASE → FINITE LIFE TIME OF THIS PHASE

→ CONSISTENT WITH *50 e-FOLDINGS* AS STEMS FROM DYNAMICAL SYSTEM ANALYSIS !



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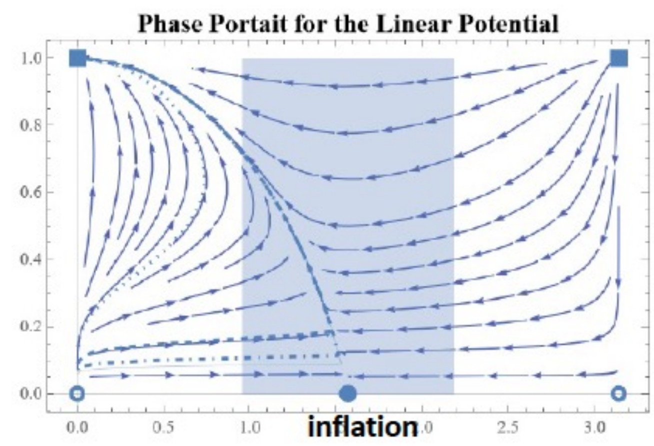

NB: **Metastable Inflationary Vacua**

Dorlis, NEM, Vlachos
2404.18741 (PoS Corfu 2023)

Compatibility with swampland (Ooguri, Vafa, …) ⚠

CONDENSATE WITH CUTOFF GRAVITON MODES HAS IMAGINARY PARTS

(ENVIRONMENT OF MODES WITH MOMENTA ABOVE THE CUTOFF μ)

Consistent with swampland ⟷ INSTABILITY OF CONDENSATE PHASE → FINITE LIFE TIME OF THIS PHASE

→ CONSISTENT WITH *50 e-FOLDINGS* AS STEMS FROM DYNAMICAL SYSTEM ANALYSIS !

$$\mathrm{Im}\left(\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}\rangle\right) = \frac{16A\dot{b}\mu^7}{7M_{\rm Pl}^4\,\pi^2}\left[1+\left(\frac{H_I}{\mu}\right)^2\left(\frac{21}{10}-6\left(\frac{A\mu\dot{b}}{M_{\rm Pl}^2}\right)^2\right)\right]$$

They induce imaginary parts in the Hamiltonian of GW perturbations

$$\mathrm{Im}\,(\mathcal{H}) = \int d^3x\,\frac{1}{2}A\,b\,\mathrm{Im}\left(\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}\rangle\right) \approx V_{dS}^{(3)}\,\frac{8bA^2\dot{b}\mu^7}{7M_{\rm Pl}^4\,\pi^2}\,.$$

$V_{dS}^{(3)}$ denotes the de Sitter 3-volume $\quad V_{dS}^{(3)}T^E = \dfrac{24\pi^2}{M_{\rm Pl}^2\Lambda}, \quad \Lambda \approx 3H_I^2$ $\quad$ *T^E = Euclidean time*

NB: **Metastable Inflationary Vacua**

Dorlis, NEM, Vlachos
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Life time of Inflationary vacua

$$\boxed{\tau \sim (\mathrm{Im}\mathcal{H})^{-1}}$$

To ensure $T^E \sim (50-60) H_I^{-1}$ (**phenomenologically consistent inflation duration**)

$$H_I \tau \sim \frac{7 H_I^2 M_{\rm Pl}^6}{64 b A^2 \dot{b} M_s^7} \left(H_I T^E\right) \sim 10^{-2} \left(\frac{M_{\rm Pl}}{M_s}\right)^3 \cdot (H_I T^E)$$

$$\boxed{\frac{M_s}{M_{\rm Pl}} \lesssim 0.215}$$

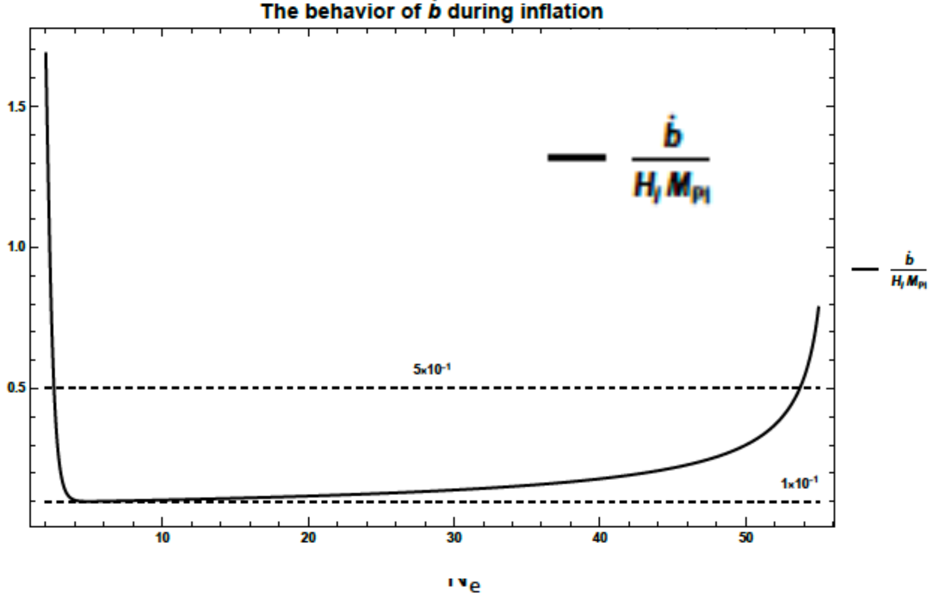
Restriction on the string scale

The rate of KR axion background during Inflation

$$\zeta = \frac{\lambda}{\lambda+1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$
$$\zeta' = -\sqrt{6}(\Gamma-1)\zeta^2\cos\varphi$$

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

The behavior of $\dot{b}$ during inflation



The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced

by **quantum graviton** fluctuations of chiral (left-right asymmetric) GW type

$$\text{-Re}\,\langle R_{CS}\rangle^{total} = \mathcal{N}_I\,\frac{A\kappa^4\mu^4}{\pi^2}\,\dot{b}_I H_I^3$$

Hubble

GW sources

UV cutoff of graviton modes

$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

String EFT:
$\mu = M_s = (\alpha')^{-1/2}$

$$\dot{b}_I \sim 10^{-1} H_I M_{\rm Pl}$$

Evolution of equation of state for the orbits of the linear b-potential phase space

For some initial value of φ_i inflation with e-foldings $N_e > 50$ is achieved for $\zeta < 0.06$ (**inflation → saddle point**)

$$\dot{b}_I \simeq \text{constant}$$

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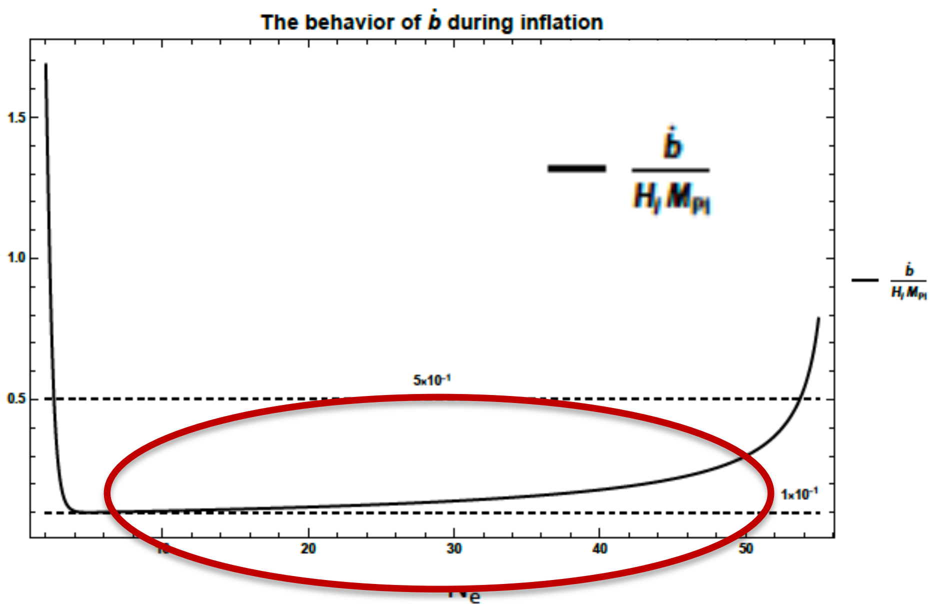

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$$\boxed{\dot{b}_I \simeq \text{constant}}$$

Hence

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\Big)\Big]=0 \quad\Rightarrow\quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}}$$

$$\dot{\bar{b}}\sim\varepsilon_{ijk}H^{ijk} \approx \textit{constant}$$

$$\dot{b}_I\sim 10^{-1}H_I M_{\rm Pl}$$

Spontaneous LV solution (constant spatial components of H-torsion)

Parametrisation

$\varepsilon = O(10^{-2})$

@ end of Inflationary era

$$\boxed{\dot{\bar{b}}\sim\sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H\sim 0.14\,M_{\rm Pl}\,H}$$

$$H=H_{\rm infl}\simeq\text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

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Undiluted KR axion background at the end of Inflation

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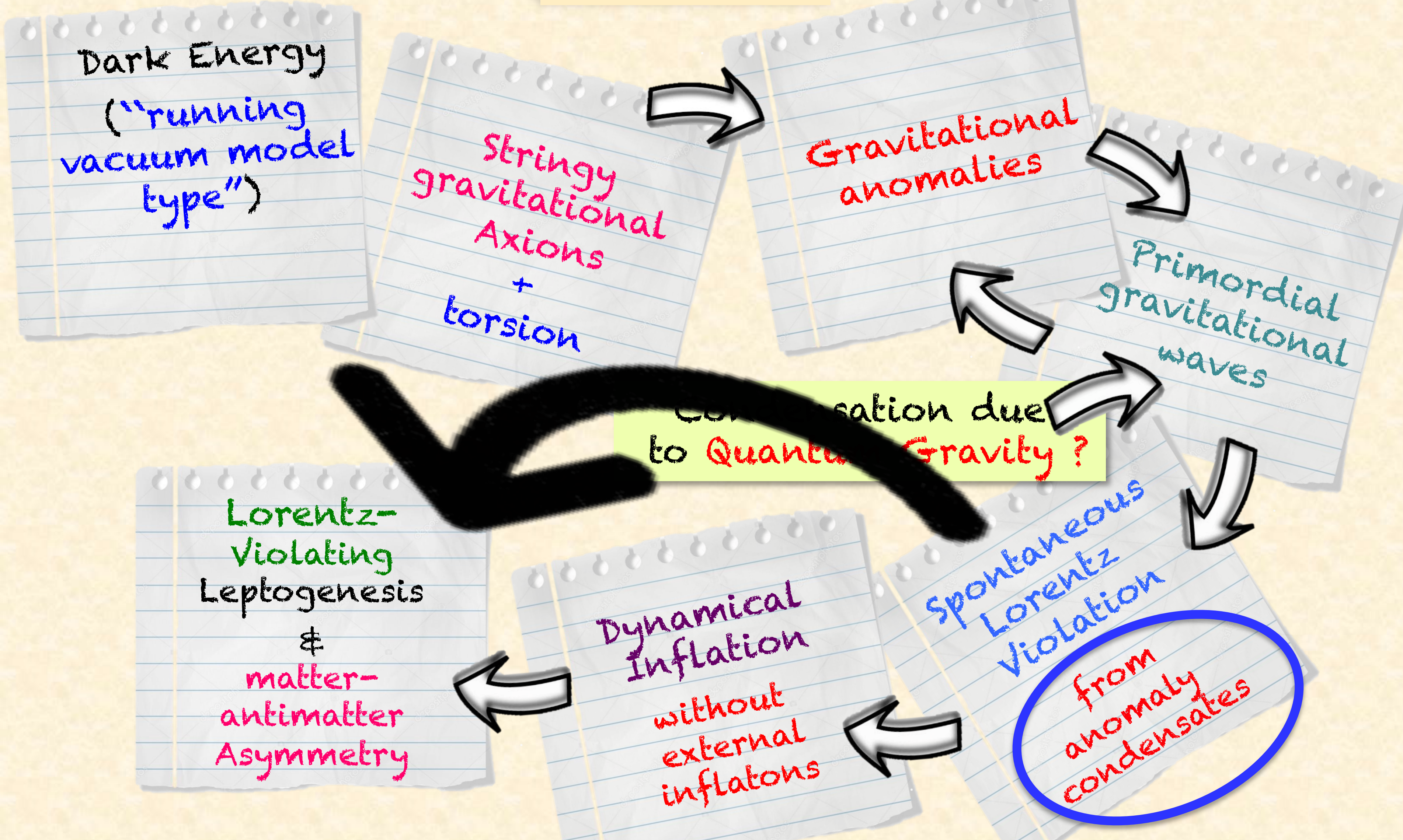
Important for Leptogenesis @ radiation era

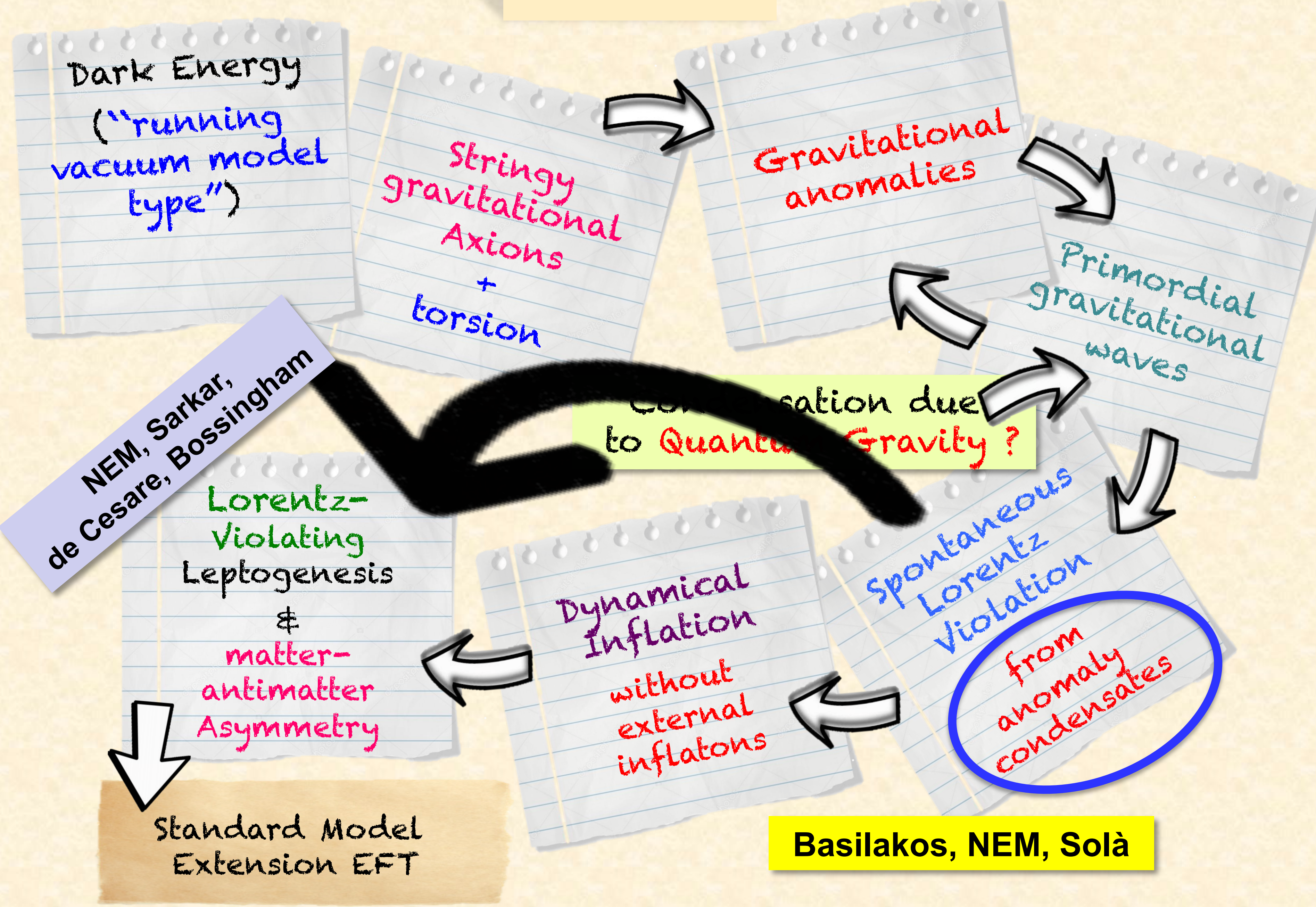
5. Post-inflationary RVM era & Matter-antimatter asymmetry

The Parts



Basilakos, NEM, Solà

The Parts


Dark Energy
("running vacuum model type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves
sation due
to Quantu Gravity ?
NEM, Sarkar,
de Cesare, Bossingham
Lorentz-Violating Leptogenesis
&
matter-antimatter Asymmetry
Dynamical Inflation
without external inflatons
Spontaneous Lorentz Violation
from anomaly condensates
Standard Model Extension EFT
Basilakos, NEM, Solà

The Parts

Dark Energy
("running vacuum model type")

Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

...ation due
... Gravity ?

We exist because of Anomalies !

NEM, Sarkar, de Cesare, Bossingham

Lorentz-Violating Leptogenesis
&
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Standard Model Extension EFT

Basilakos, NEM, Solà

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \kappa\, b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j;$$ Chiral current, including RHN

$$\partial_\mu\Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}J^{5\mu} - \sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\Big]\Big) = \text{“chiral U(1) anomalies”}.$$

Possibly also QCD type

Eqs of Motion for b-field → $\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies”},$

Scale factor $a(t) \sim T^{-1}$

Possibly also QCD

$$\boxed{\dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

Viewed as sufficiently slow moving to induce Leptogenesis

Bossingham, NEM, Sarkar (2018)

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Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM, Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Scale factor ***a(t) ~ T***$^{-1}$

Possibly also QCD

$$\boxed{\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

sufficiently slowly varying during leptogenesis (brief) epoch → qualitatively similar to approximately const. background

Bossingham, NEM, Sarkar

Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T \gg T_{EW}$

$$\boxed{\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.}$$

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T \gg T_{EW}$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy RHN interact with axial (approximately) constant background

with only temporal component $B_0 \neq 0$ $\quad B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

STANDARD MODEL EXTENSION EFT

Lorentz & CPT Violation

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

$$\mathcal{L} = \tfrac{1}{2} i\bar{\psi}\Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \bar{\psi}M\psi,$$

$$M \equiv m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \tfrac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

Spontaneous Violation of Lorentz Symmetry
(LV coefficients are v.e.v. of tensor-valued field quantities)
$B_0 \approx$ constant is H-torsion background in our model

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

Early Universe
T >> T_{EW}

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy RHN interact with axial constant background

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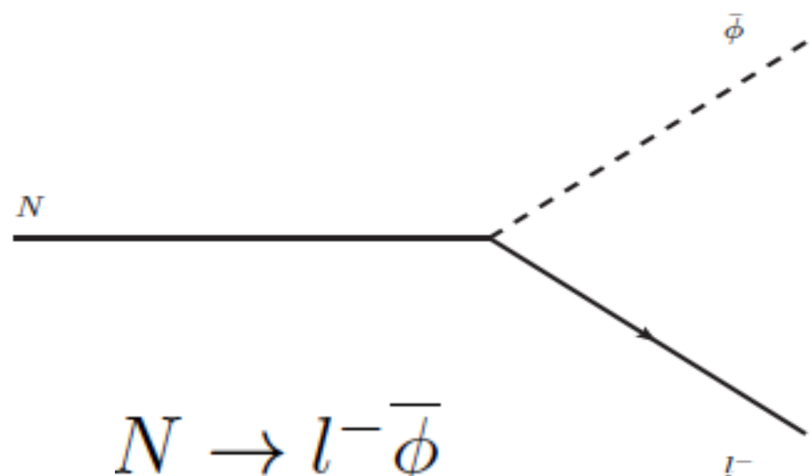
Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$$N \to l^+\phi \qquad\qquad N \to l^-\overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Constant B_0 Background

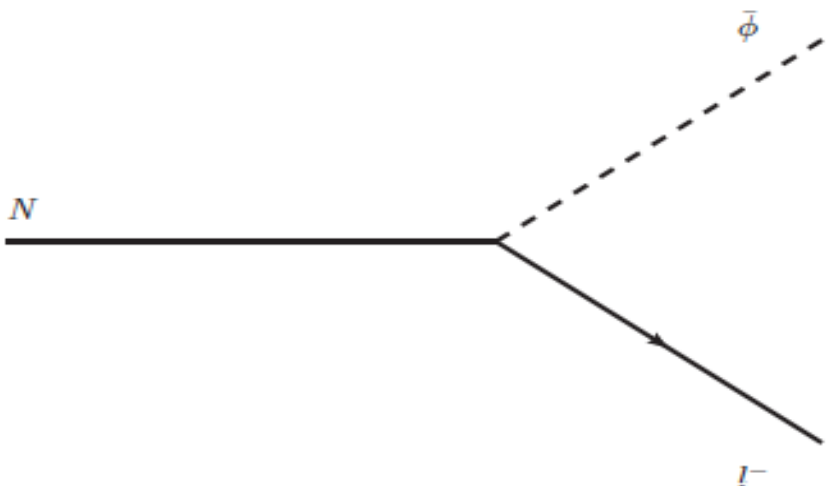
CPT Violation

Early Universe
$T \gg T_{EW}$

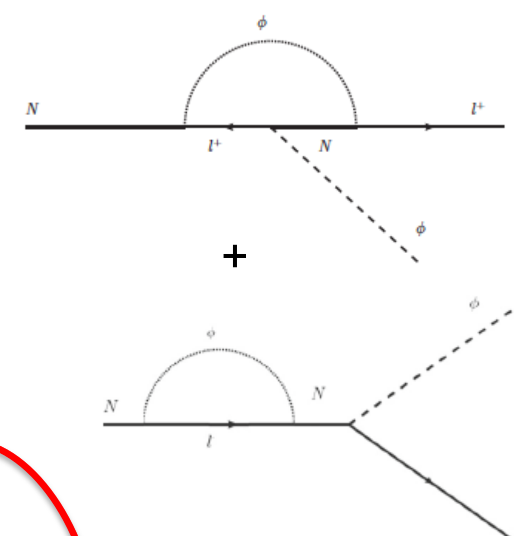
Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

CPT Violation

Early Universe
$T \gg T_{EW}$

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

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CPT Violation

Early Universe
T > 10^5 GeV

Constant $B^0 \neq 0$ background

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$$Y_k \sim 10^{-5}$$

$$m \geq 100\text{TeV} \rightarrow$$

$$B^0 \sim 1\text{MeV}$$

$$T_D \simeq m \sim 100\ \text{TeV}$$

consistent with :
light neutrino masses in SM +
stability of Higgs vacuum

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

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Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

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Early Universe
$T > 10^5$ GeV

CPT Violation

nt $B^0 \neq 0$
round

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\ldots$$

$\simeq 10^{-8}$

Solving
system
of Boltzm
eqs

$\sim 1\mathrm{MeV}$

$T_D \simeq m \sim 100$ TeV

This Leptogenesis scenario can be embedded/extend existing scenarios of Leptogenesis (Pilaftsis, Deppisch, Underwood, …)

Also can be accommodated within the νMSM (Shaposhnikov, Asaka, Blanchet, Canetti, Drewes, Gorbunov, Laine, Boyarski, Ruchaiskiy, Tkachev …)

Simila… of magnitude estimates
if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

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CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

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Produce Lepton asymmetry

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Baryogenesis

?

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CPT Violation

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Produce Lepton asymmetry

Equilibrated electroweak B+L violating sphaleron interactions

B-L conserved

$$L = \frac{2}{M} l_L l_L \phi\phi + \mathrm{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Environmental Conditions Dependent

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

CPTV Thermal L

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T > 1 GeV

NB:

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T \gg T_{EW}$

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NB: in our stringy models this mass **could be generated dynamically, e.g. through non-perturbative instanton effects that break shift-symmetry by coupling KR axions to right-handed neutrinos**

NEM, Pilaftsis

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a\left(\overline{\psi}_R^{\ C}\psi_R - \overline{\psi}_R\psi_R^{\ C}\right)\Big]\,,$$

NB:

NEM, Sarkar,
+ de Cesare, Bossingham

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$T \gg T_{EW}$

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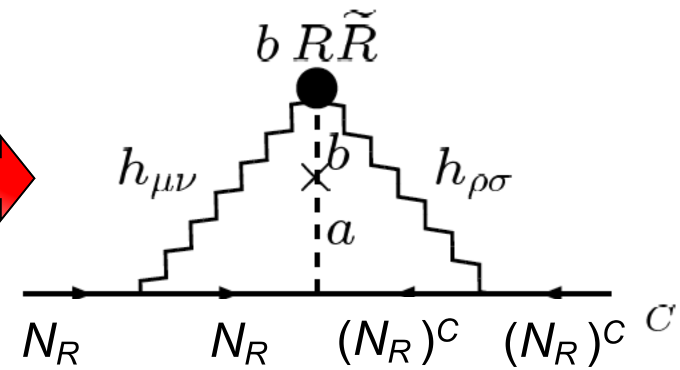
NB: in our stringy models this mass **could be generated dynamically, e.g. through non-perturbative instanton effects that break shift-symmetry by coupling KR axions to right-handed neutrinos**

NEM, Pilaftsis

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a\left(\overline{\psi}_R^{\,C}\psi_R - \overline{\psi}_R\psi_R^{\,C}\right)\Big],$$



Radiatively-induced RHN mass

6. The Whole:

Stringy-RVM

Cosmological Evolution

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase

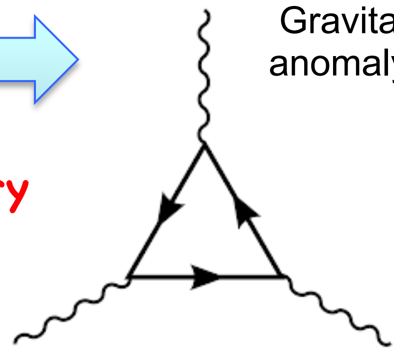
Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

From a pre-inflationary era after Big-Bang



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

chiral matter generation @ inflation exit

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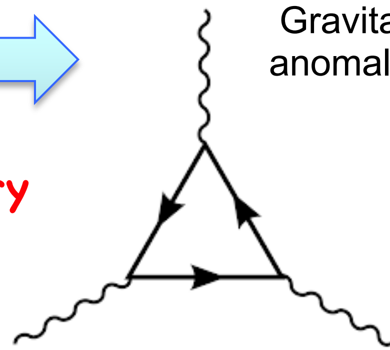

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Cancellation of GA

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Cosmic Time

forward direction

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

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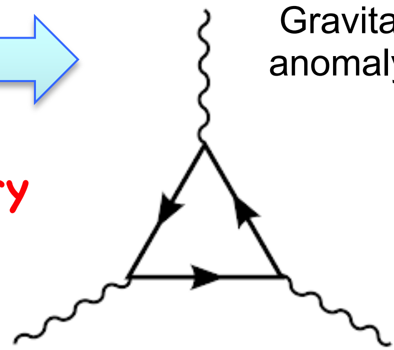
Primordial Gravitational Waves



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chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

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Big-Bang, pre-inflationary phase

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forward direction

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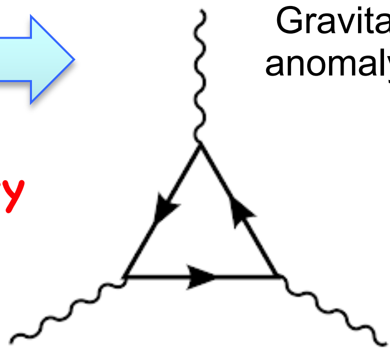

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Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

forward direction

Basilakos, NEM, Solà

KR mass: $m_b = \frac{\Lambda_{\rm QCD}^2}{\tilde{f}_b}$

$\frac{M_s}{M_{\rm Pl}} \lesssim 0.215$

Inflation pheno

$$V_b^{\rm QCD} \simeq \Lambda_{\rm QCD}^4 \Big(1 - \cos(\frac{b}{\tilde{f}_b})\Big) \, ,$$

$$\tilde{f}_b = \frac{3}{\pi^2}\sqrt{\frac{3}{2}\frac{\kappa}{\alpha'}} = \frac{3}{\pi^2}\sqrt{\frac{3}{2}}\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

$= 1.7 \times 10^{-2}\ M_{\rm Pl}$

$\Lambda_{\rm QCD} \sim 218\ \text{MeV}$

@ QCD era

$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\, \partial^\mu b - \tilde{f}_b^{-1}\ \frac{\alpha_s}{8\pi}\, b(x)\, G_{\mu\nu}^a\, \tilde{G}^{a\mu\nu}\Big]$$

$$m_b \simeq 1.15 \times 10^{-6}\, \text{eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential generation for b → axion Dark matter

Axion Dark Matter - **Adams, C.B. *et al* - arXiv:2203.14923**

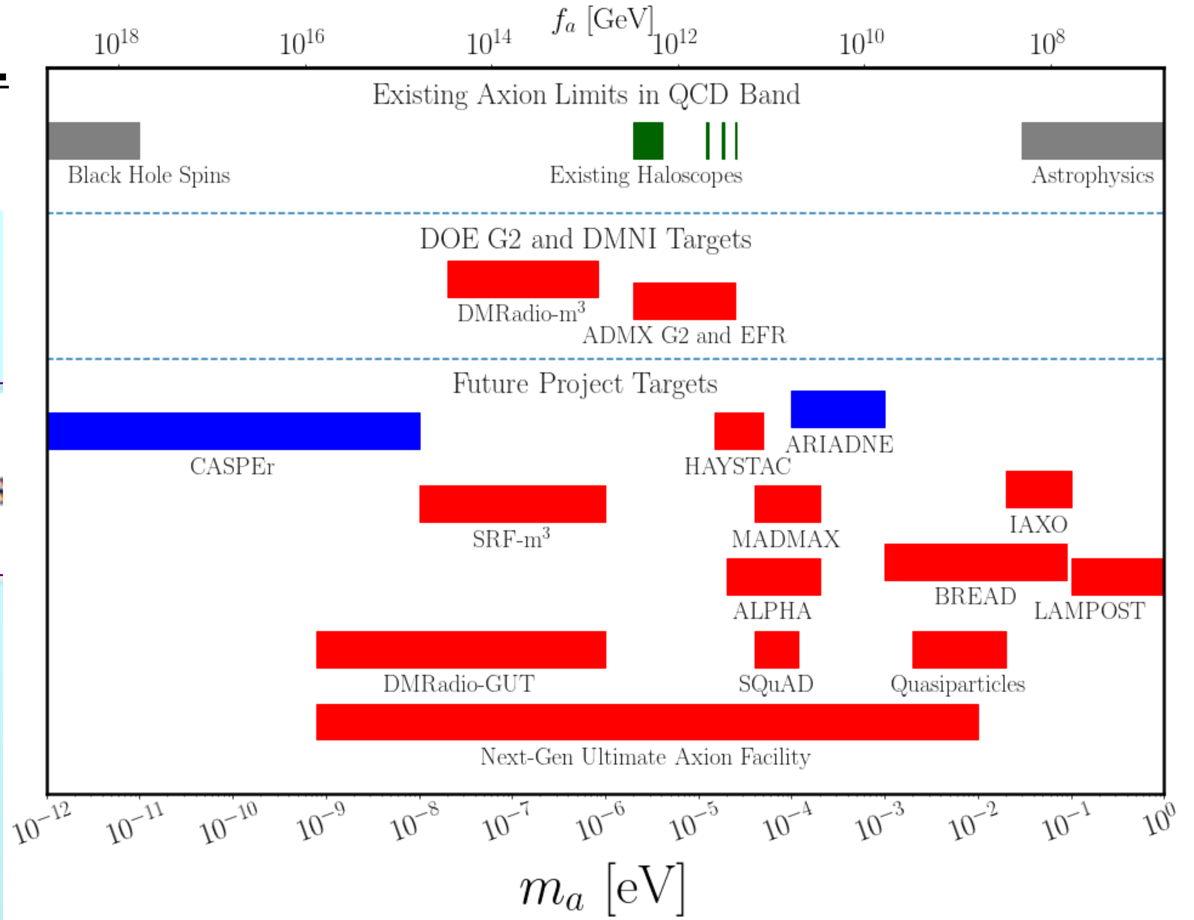
forward direction

$$V_b^{QCD} \simeq \Lambda^4_{QCD}\left(1 - \cos\right.$$

$$\Lambda_{QCD} \sim 218 \ \text{MeV}$$

@ QCD era

Compatible with QCD-Axion Phenomenology



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forward direction

Basilakos, NEM, Solà

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$= 1.7 \times 10^{-2}\ M_{\rm Pl}$

$$\Lambda_{\rm QCD} \sim 218 \ \text{MeV}$$

@ QCD era

Compatible with QCD-Axion Phenomenology

But in string theory the scale Λ in the instanton potential V_b might be different

$$m_b \simeq 1.15 \times 10^{-6}\,\text{eV}$$

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Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Basilakos, NEM, Solà

forward direction

Stringy gravitational Axions + torsion → Dark Matter

KR (gravitational or model-independent) axions connected to “torsion” in string theory → Geometrical origin of Dark Matter

Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

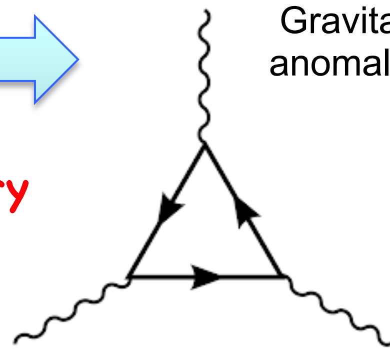
forward direction

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation

From a pre-inflationary era after Big-Bang

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\;\phi\,\overline{\ell}$$

Cancellation

OUTLOOK: Incorporate other model-dependent stringy axions → Axiverse
Interesting Cosmology (*eg* Marsh 2015)
could be ultralight → AION etc

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential generation for b → axion Dark matter

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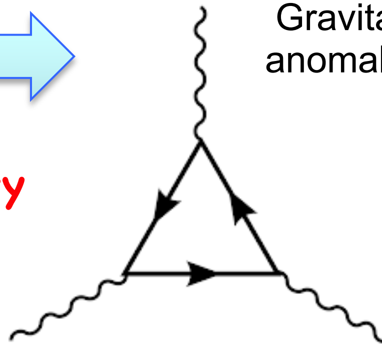

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Matter Era

Possible potential generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

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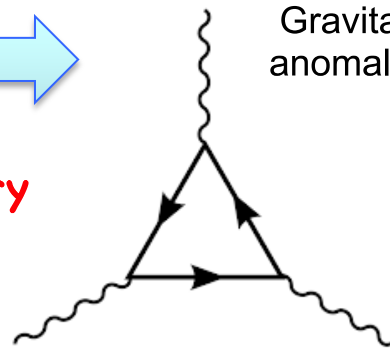

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From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto \boxed{\dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U}(1) \text{ anomaly terms}}$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,\mathrm{eV}$$

Consistent with current bounds on LV & CPTV

$B_0 < 10^{-2}$ eV,

$B_i < 10^{-22}$ eV

Matter Era

Possible potential generation for b -> axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

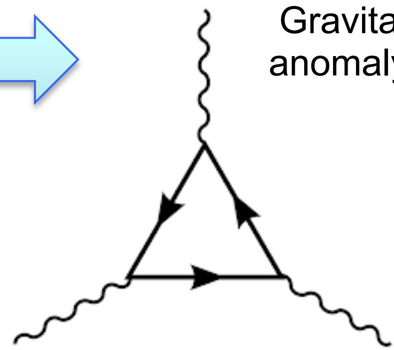
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation @ inflation exit

Cancellation of GA

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Need to understand Modern Era better

Matter Era

Dark matter

Modern de-Sitter Era

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Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

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Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gr Wa

Gravitational

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$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

Distinguishing feature from ΛCDM

Alleviate cosmological data tensions

$$\rho_{\rm RVM}^{\rm today}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu\left(\frac{H_0}{M_{\rm Pl}}\right)^2\right)$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\,M_{\rm Pl}^4$$

Gómez-Valent Solà

chiral matter generation

...lation exit

forward direction

Ra...

B_0...

Le...

RH...

N_I...

B-L...

Ma...

...ion ... matter

J. Solà talk

Modern de-Sitter Era

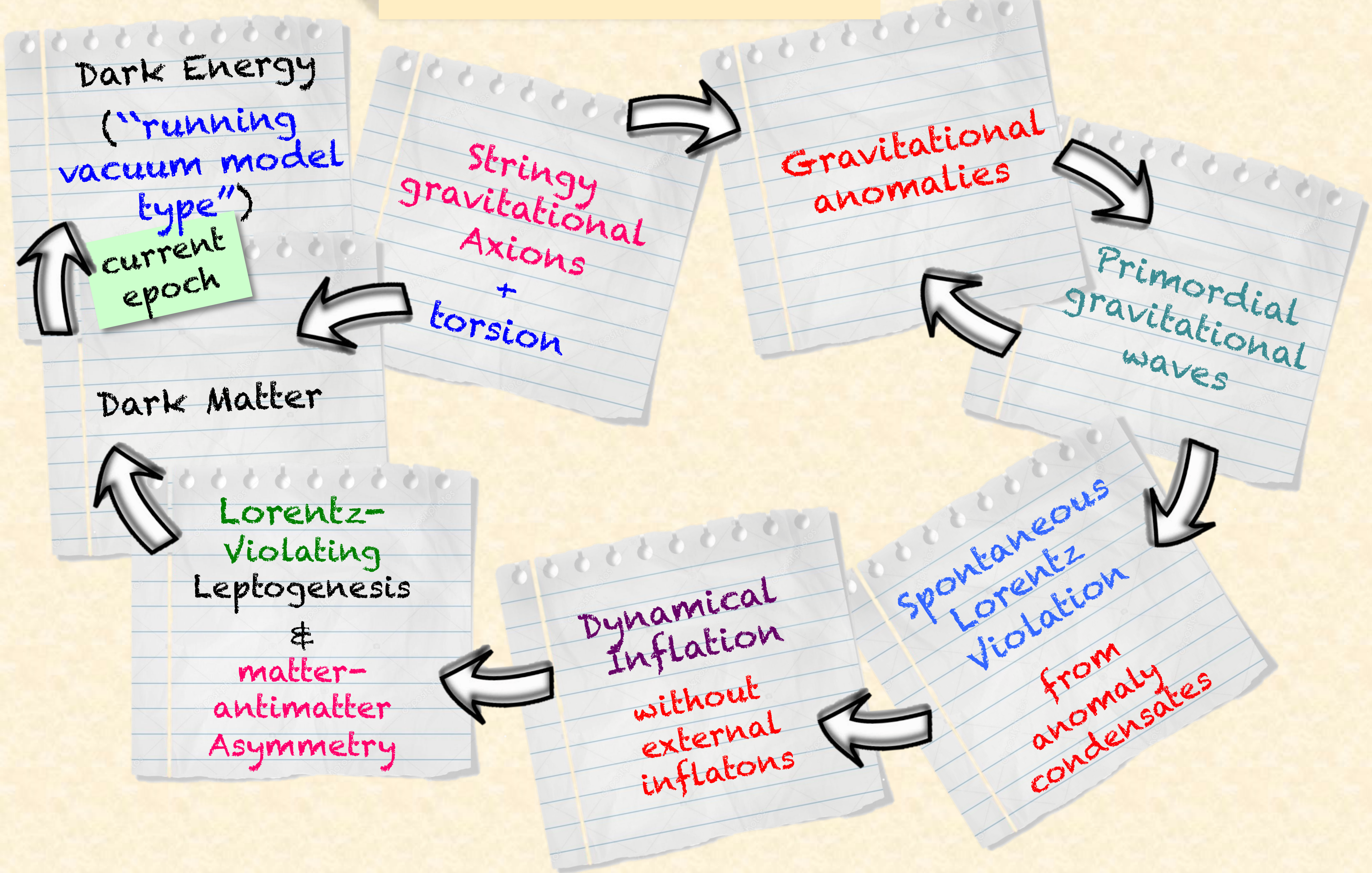
GA resurfacing

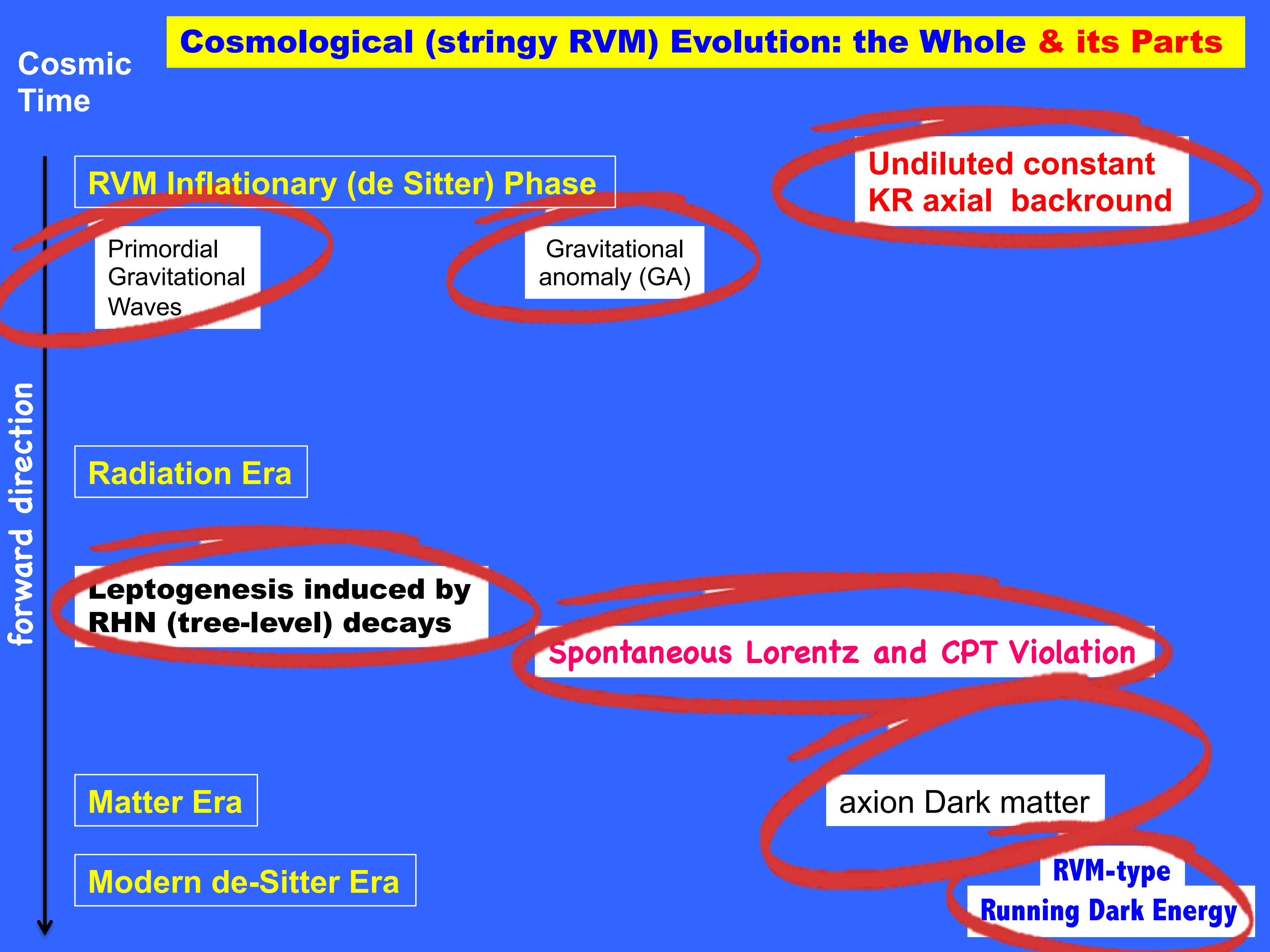
$$\ldots \varepsilon' M_{\rm Pl} H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

RVM-type Running Dark Energy

The Cosmic Evolution





Cosmological (stringy RVM) Evolution: the Whole & its Parts
Cosmic Time
forward direction
RVM Inflationary (de Sitter) Phase
Primordial Gravitational Waves
Gravitational anomaly (GA)
Undiluted constant KR axial backround
Radiation Era
Leptogenesis induced by RHN (tree-level) decays
Spontaneous Lorentz and CPT Violation
Matter Era
axion Dark matter
Modern de-Sitter Era
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Cosmic Time

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We exist because of Anomalies !

Paraphrasing C. Sagan:
We are "anomalously" made of star stuff

Radiation Era

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Spontaneous Lorentz and CPT Violation

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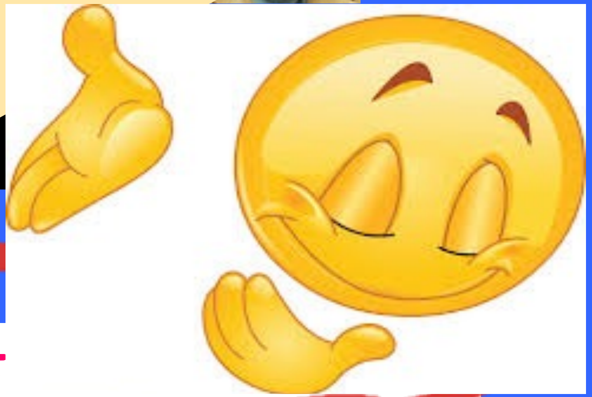

Matter E…

Modern de-Sitter Era

axion Dark matter

RVM-type
Running Dark Energy

Thank you !

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a microscopic (string-inspired) model for RVM Universe....

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(vi) Eur. Phys.J.ST 230 (2021),2077
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[c] Gómez-Valent, NEM, Solà
(viii) CQG 41 (2024) 1, 015026
[d] Dorlis, NEM, Vlachos
(ix) arXive:2403.09005 [gr-qc], PRD in press
(x) arXive: 2404.18741 (PoS Corfu 2023)

Links with :
spontaneous Lorentz violation (via (gravitational axion) backgrounds)
and
Matter-Antimatter Asymmetry
in theories with
Right-Handed Neutrinos

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(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558