Towards a more robust intermittency analysis technique in heavy ion collisions: achievements and challenges.

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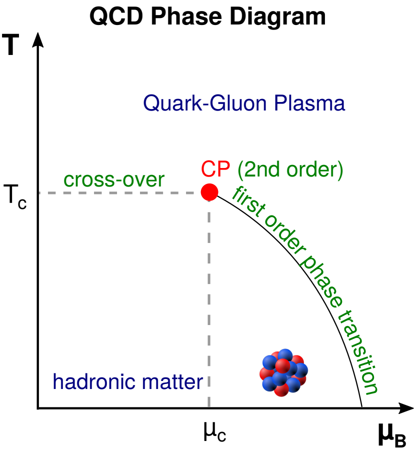
Outline

The phase diagram of QCD

- Phase diagram of strongly interacting matter in T and μ_B $\Rightarrow$
- Phase transitions from hadronic matter to quark-gluon plasma:
  - Low μ_B & high T $\rightarrow$ cross-over (lattice QCD)
  - High μ_B & low T $\rightarrow$ 1st order (effective models)

  $\Rightarrow$ 1st order transition line ends at Critical Point (CP) $\rightarrow$ 2nd order transition
- At the CP: scale-invariance, universality, collective modes $\Rightarrow$ good physics signatures



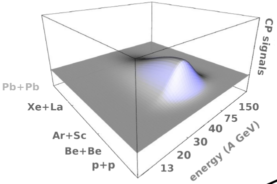
- Detection of the QCD Critical Point (CP): Main goal of many heavy-ion collision experiments (in particular the SPS NA61/SHINE experiment)
- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

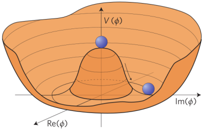
Critical Observables & the Order Parameter (OP)

CP observables

Event-by-event (global) fluctuations:
Variance, skewness, kurtosis –
sensitive to experimental acceptance

Local:
density fluctuations of OP
in transverse space
(stochastic fractal)

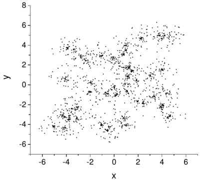




Chiral condensate
$\sigma(\boldsymbol{x}) = \langle \bar{q}(\boldsymbol{x}) q(\boldsymbol{x}) \rangle$

coupling $\Updownarrow$ induced critical fluctuations*

Net baryon density
$n_B(\boldsymbol{x})$



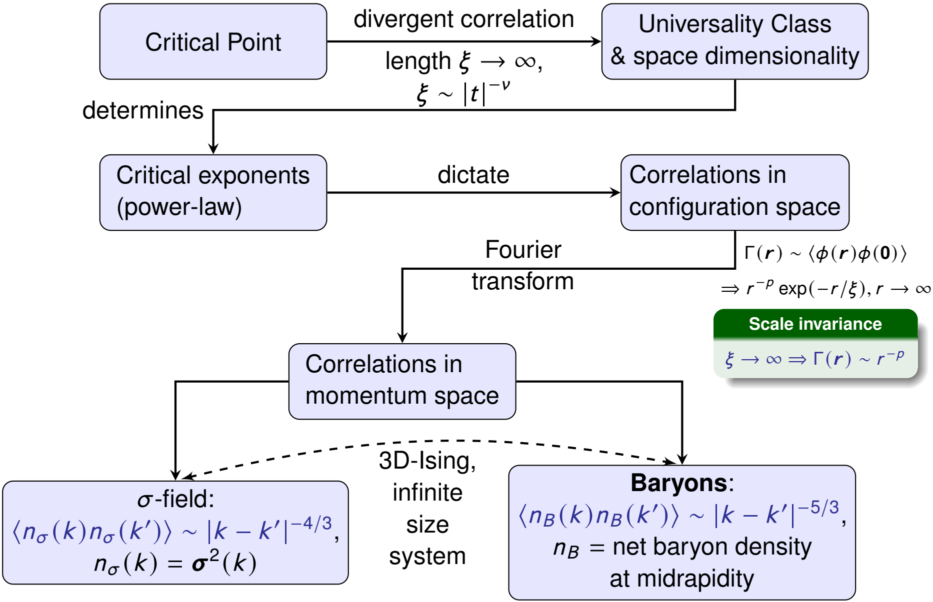
Order parameter

A quantity that:

- is $= 0$ in disordered phase (QGP)
- is $\neq 0$ in ordered phase (hadrons)

*[Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

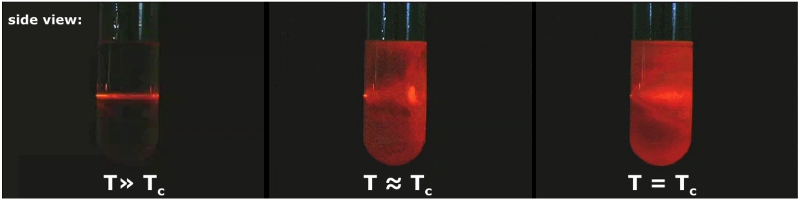
Self-similar density fluctuations near the CP



[Antoniou *et al*, Nucl. Phys. A **693** 799–824 (2001)]

[Antoniou *et al*, PRL **97**, 032002 (2006)]

Observing power-law fluctuations through intermittency



[Csorgo, Tamas, PoS CPOD2009 (2009) 035]

Experimental observation of local, power-law distributed fluctuations of net baryon density

$\Downarrow$

Intermittency in transverse momentum space at mid-rapidity (Critical opalescence in ion collisions)

[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

- Net proton density carries the same critical fluctuations as the net baryon density, and can be substituted for it.

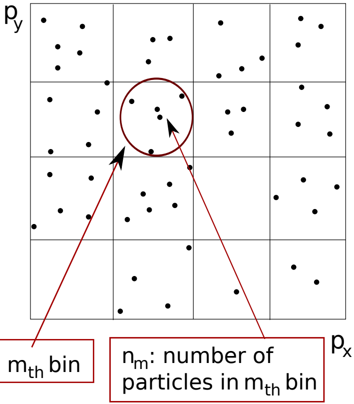
  [Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]
- Furthermore, antiprotons can be ignored (their multiplicity is negligible compared to protons), and we can analyze just the proton density.

Observing power-law fluctuations: Factorial moments

- Pioneered by Białas and others, as a method to detect non-trivial dynamical fluctuations in high energy nuclear collisions
- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



[A. Bialas and R. Peschanski, *Nucl. Phys. B* **273** (1986) 703-718]
[A. Bialas and R. Peschanski, *Nucl. Phys. B* **308** (1988) 857-867]
[J. Wosiek, *Acta Phys. Polon.* **B 19** (1988) 863-869]
[A. Bialas and R. Hwa, *Phys. Lett.* **B 253** (1991) 436-438]
[Z. Burda, K. Zalewski, R. Peschanski, J. Wosiek, *Phys. Lett. B* **314** (1993) 74-78]

$p_{x,y}$ range in present analysis:
$-1.5 \leq p_{x,y} \leq 1.5$ GeV/c
$M^2 \sim 10\,000$

Background subtraction – the correlator $\Delta F_2(M)$

- Background of **non-critical pairs** must be **subtracted** from experimental data;

Partitioning of pairs into critical/background

$$\langle n(n-1)\rangle = \underbrace{\langle n_c(n_c-1)\rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b-1)\rangle}_{\text{background}} + \underbrace{2\langle n_b n_c\rangle}_{\text{cross term}}$$

$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \cdot \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \cdot \underbrace{\lambda(M)}_{\text{ratio } \frac{<n>_b}{<n>_d}} \cdot (1-\lambda(M))\, f_{bc}$$

- If $\lambda(M) \lesssim 1$ **(dominant background)** $\Rightarrow$ **cross term negligible** & $F_2^{(b)}(M) \sim F_2^{\text{mix}}(M)$ (**Critical Monte Carlo*** **simulations**), **then:**

$$\Delta F_2(M) \simeq F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

Intermittency **restored** in $\Delta F_2(M)$:

$$\Delta F_2(M) \sim (M^2)^{\varphi_2}\,,\ M \gg 1$$

$\Rightarrow$

φ_2: **intermittency index**

Theoretical prediction* for φ_2

$$\varphi_{2,cr}^{(p)} = \frac{5}{6}\ (0.833\ldots)$$

*[Antoniou *et al*, PRL **97**, 032002 (2006)]

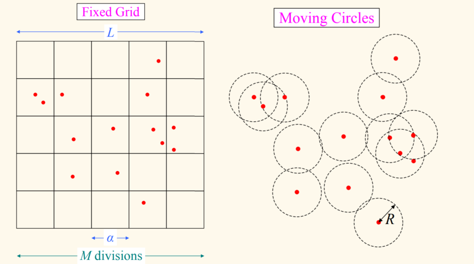
The correlation integral C(R) as an aid to intermittency

- A computationally faster alternative to lattice averaging on a fixed grid, the **correlation integral** is defined as:

$$C(R) = \frac{2}{\langle N_{mul}\,(N_{mul}-1)\rangle_{ev}} \left\langle \sum_{\substack{i,j \\ i<j}} \Theta\left(|x_i - x_j| \leq R\right) \right\rangle_{ev}$$

[P. Grassberger and I. Procaccia (1983). "Measuring the strangeness of strange attractors". Physica. 9D: 189–208]

[F. K. Diakonos and A. S. Kapoyannis, Eur. Phys. J. C **82**, 200 (2022)]



- $F_2(M)$ can be obtained from $C(R)$, or vice-versa, by the relations:

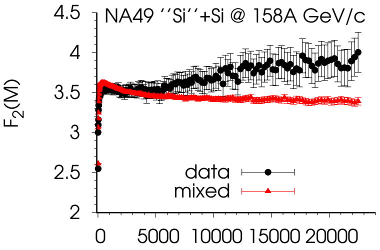
$$C(R_M) = \frac{\langle N_{mul}\rangle_{ev}^2}{\langle N_{mul}\,(N_{mul}-1)\rangle_{ev}} \frac{F_2(M)}{M^2}$$

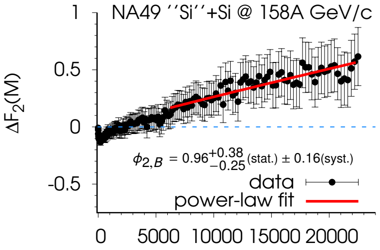
$$F_2(M) = \frac{\langle N_{mul}\,(N_{mul}-1)\rangle_{ev}}{\langle N_{mul}\rangle_{ev}^2} M^2 C(R_M),$$

where $\pi R_M^2 = a^2$.

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV – protons

- **Factorial moments of proton transverse momenta** analyzed at **mid-rapidity**





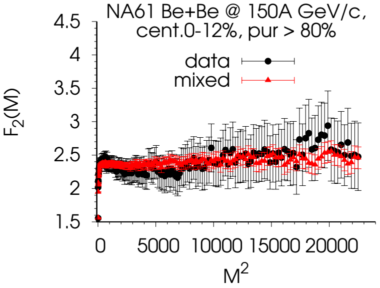
M^2 ⟵——— **number of cells** ———⟶ M^2

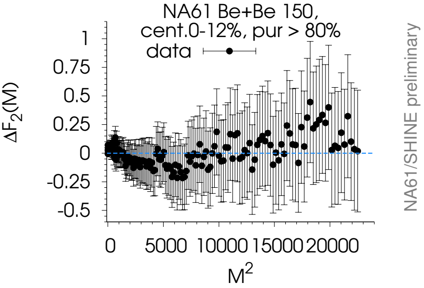
[T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

- $F_2(M)$, $\Delta F_2(M)$ errors estimated by the **bootstrap method**
  [W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]
- Fit with $\Delta F_2^{(e)}(M\ ;\ \mathcal{C}, \phi_2) = 10^{\mathcal{C}} \cdot \left(\frac{M^2}{M_0^2}\right)^{\phi_2}$, for $M^2 \geq 6000$ ($M_0^2 \equiv 10^4$)
- **Evidence** for intermittency in "Si"+Si – but **large statistical errors**.

NA61/SHINE intermittency: ^{7}Be + ^{9}Be @ $\sqrt{s_{NN}} \simeq 17$ GeV

- Intermittency analysis is pursued within the framework of the **NA61/SHINE experiment**, inspired by the **positive**, if ambiguous, **NA49 Si+Si** result.
  [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]
- **Two NA61/SHINE systems** were initially examined:
  ^{7}Be + ^{9}Be and **^{40}Ar + ^{45}Sc @ 150A GeV/c ($\sqrt{s_{NN}} \simeq 17$ GeV)**

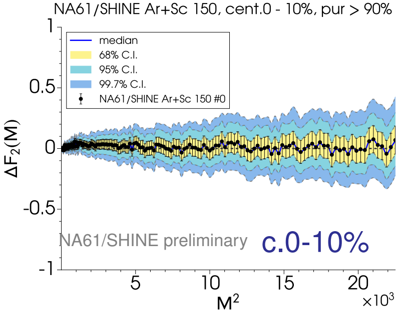


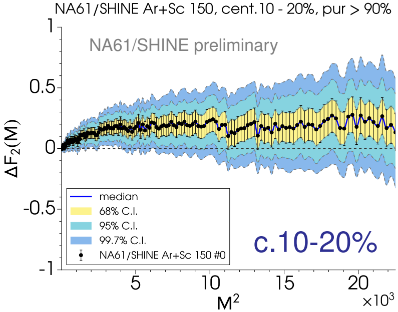


- $F_2(M)$ of data and mixed events **overlap** $\Rightarrow$
- Subtracted moments $\Delta F_2(M)$ **fluctuate around zero** $\Rightarrow$
  No intermittency effect is observed in **Be+Be**.

NA61/SHINE $^{40}Ar + ^{45}Sc$ @ $\sqrt{s_{NN}} \simeq 17$ GeV

- **First indication** of **intermittency** in **mid-central Ar+Sc 150A GeV/c** collisions (**CPOD2018**); In **2019**, an **extended event statistics set** was analysed;
- A **scan in centrality** was performed (**maximum range: 0-20% most central**), as centrality may influence the system's **freeze-out temperature**;
- **Event statistics:** $\sim$ **400K events** per **10% centrality interval**;





- **Some signal indication** in **c.10-20% ("mid-central")**, but this **(final) correlated bin analysis** was **inconclusive**;
- Afterwards, NA61/SHINE turned to **independent bin analysis** (more on this later. . .)

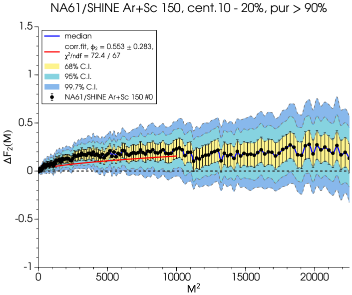
Challenges in proton intermittency analysis

1. **Particle species**, especially **protons**, **cannot** be **perfectly identified** experimentally; candidates will always contain a **small percentage of impurities**;
2. **Experimental momentum resolution** sets a **limit** to how **small a bin size (large M)** we can probe;
3. A **finite (small) number** of **usable events** is available for analysis; the **"infinite statistics" behaviour** of $\Delta F_2(M)$ must be **extracted** from these;
4. **Proton multiplicity** for medium-size systems is **low** (typically $\sim$ **2 – 3 protons** per event, in the window of analysis) – and the demand for **high proton purity lowers** it still more;
5. M-bins are **correlated** – the **same events** are used to calculate **all $F_2(M)$**! This **biases fits** for the **intermittency index ϕ_2**, and makes **confidence interval estimation hard**.

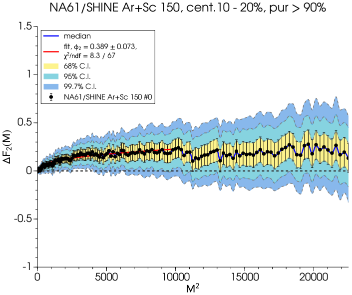
Intermittency analysis tools: correlated fit

- Possible to perform **correlated fits for ϕ_2**, with M-correlation matrix **estimated via bootstrap**;

Correlated fit



Uncorrelated fit



- **Replication of events** means bootstrap sets are **not independent** of the original: **magnitude of variance and covariance** estimates **can be trusted**, but central values will be **biased** to the original sample;
- **Correlated fits for ϕ_2** are known to be **unstable**;

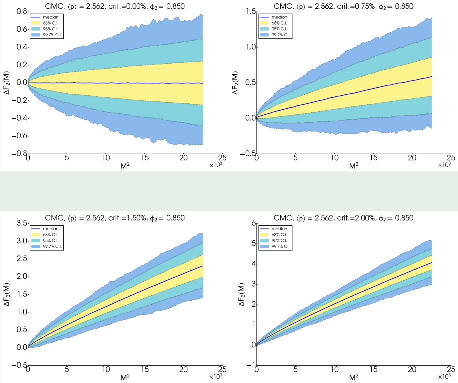
  [B. Wosiek, APP B21, 1021 (1990); C. Michael, PRD49, 2616 (1994)]
- The approach of **independent bins decimates** event statistics.

  [NA61/SHINE Collaboration, EPJC 83 (2023) 881; EPJC 84 (2024) 7, 741]

Intermittency analysis tools: Monte Carlo model scan

Avoid fitting, use model weighting!

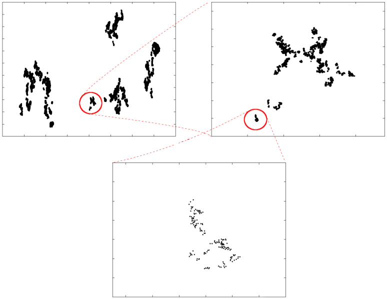
- **Build Monte Carlo models** incorporating **background & fluctuations**;
- **Compare** them against **experimental moments $\Delta F_2(M)$**;
- Models are parametrized in **critical exponent strenght (ϕ_2 value)**, **critical component (% of critical to total protons)**, and possibly other parameters (e.g. detector effects);
- Ideally, a **wide scan** of **model parameters** should be performed against the experimental data.



Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only **protons** produced;
  - **One cluster** per event, produced by sampling random Lévy walk of **adjustable dimension d_F**, e.g.: $\boldsymbol{d_F^B = 1/3 \Rightarrow \phi_2 = 1 - d_F^B/2 = 5/6}$
  - **Lower / upper bounds** of Lévy walks $p_{min,max}$ plugged in;
  - Cluster center **adjustable** to **experimental set mean proton p_T** per event;
  - **Poissonian** proton multiplicity distribution.

Lévy walk example



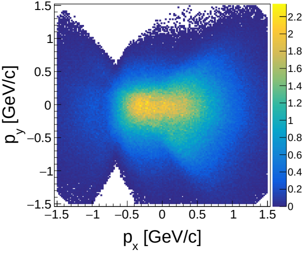
Input parameters (example)

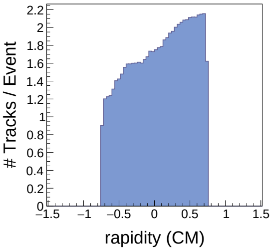
Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}
Value	$0.1 \to 1$	$800 \to 1200$	$\langle p \rangle_{\text{non-empty}}$

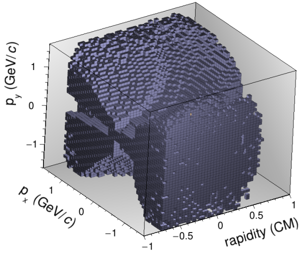
*[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

CMC – background simulation & detector effects

- **Non-critical background** simulation: **replace critical tracks** by **uncorrelated (random) tracks**, with **fixed probability:** $\mathcal{P}_{track} = 1 - \mathcal{P}_{crit}$, where $\mathcal{P}_{crit}$ is the **percentage** of **critical component**;
- p_T **distribution** of background tracks **plugged in** to **match experimental data**;
- y_{CM} **rapidity value** generated **orthogonal** to p_T, matching **experimental distribution**;
- p_T, y_{CM}, **quality & acceptance cuts** applied, modeled after **NA61/SHINE cuts;** [NA61/SHINE Collaboration, EPJC 83 (2023) 881]

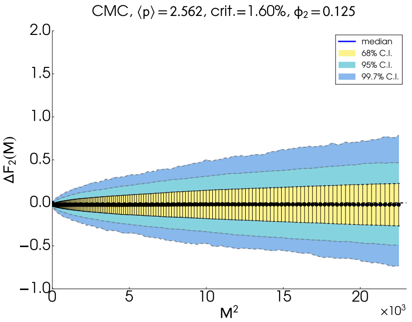


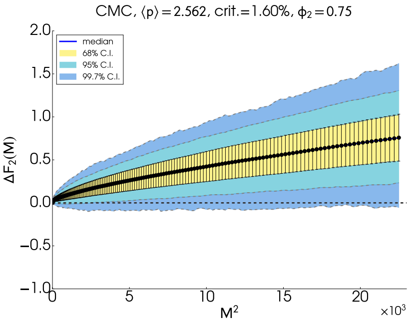




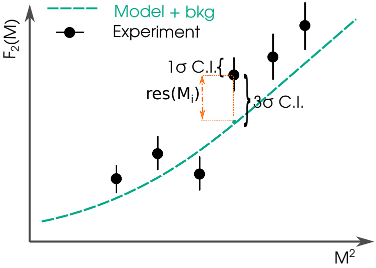
CMC scan $\Delta F_2(M)$ – examples

- Results shown for **CMC $\Delta F_2(M)$**, with **$\langle p \rangle = 2.562$**, corresponding to **SHINE Ar+Sc @ 150A GeV/*c*, cent.10-20%**;
- **2 settings**:
  1. **$\phi_2 = 0.125$, crit.% = 1.60%**;
  2. **$\phi_2 = 0.750$, crit.% = 1.60%**;
- For each setting, **~ 8K independent samples** of **~ 400K events** are generated; event statistics selected to **match SHINE data**.





Weighting models: Goodness-of-fit function



- Calculate the **residuals** for each **bin M_i** between model & experiment:

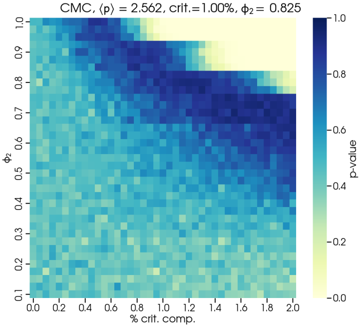
$$res(M_i) \equiv \frac{F_2^{\text{exper.}}(M_i) - F_2^{\text{model}}(M_i)}{1\sigma},$$

$\boldsymbol{\sigma}$ ~ **uncertainties** (e.g. by **bootstrap**);

- **Weight models** by **χ^2 metric**:

$$\chi^2 = \sum_i res^2(M_i) \Rightarrow$$

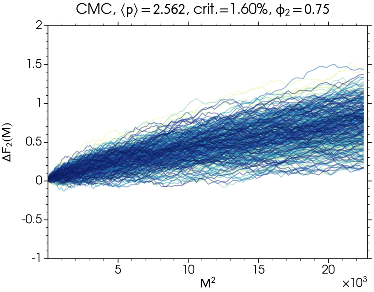
$$\text{Model Weight} \sim e^{-\frac{\chi^2}{2}}$$

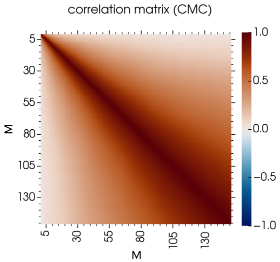


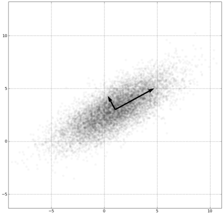
- **Scan parameter space**, weighting models on a **grid**.

Handling bin correlations through PCA

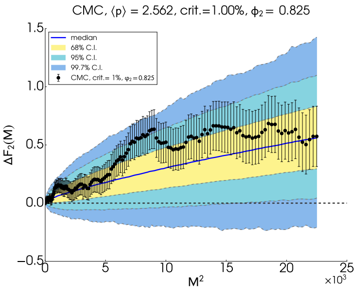
- While **CMC samples (events) are independent**, ***M*-bins** in a sample **are not**; they are **strongly correlated**;
- Additionally, there are $\sim$ **150 bins**, i.e. **dimensions** to consider, and we have $\sim$ $\boldsymbol{N_s}$ **= 8K independent samples** – **too few** to probe the **joint distribution**;
- We need to **reduce the effective dimensionality** and **untangle correlations**;
- We can do this via **Principal Component Analysis (PCA)**: **center and scale** sample points in ***M*-space**, then **rotate** the axes to make **independent linear combinations** of ***M*-bins**. Finally, keep **only few** significant components.

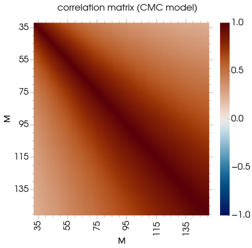


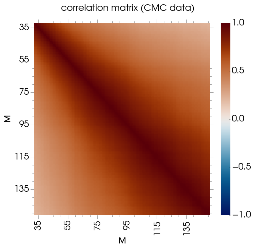


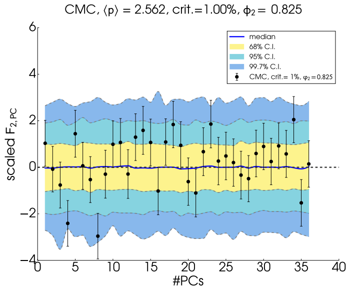


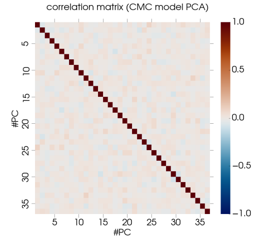
Performing PCA on CMC vs itself

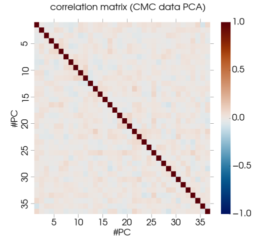










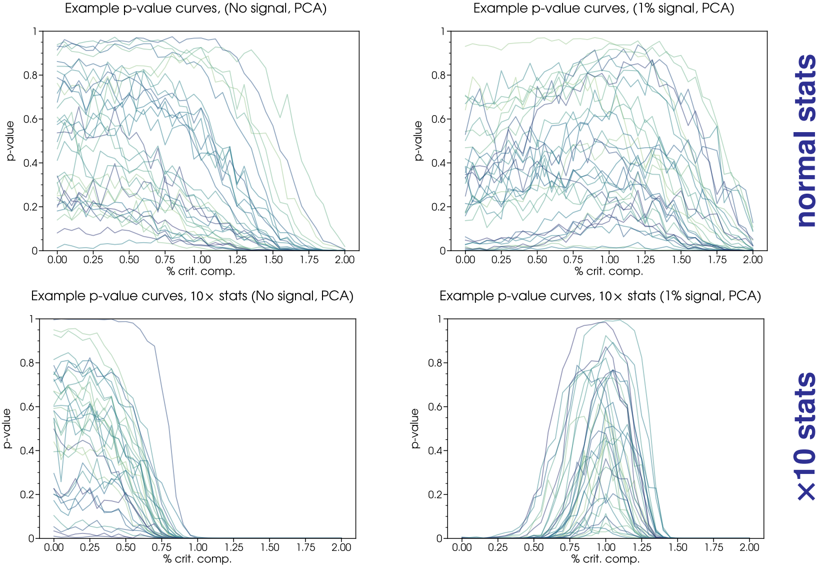


- PCA **decouples bins**; χ^2 of **CMC sample vs itself** can be **summed per PC**.

CMC model scan & the role of event statistics

- In order to illustrate the behavior of **CMC samples vs CMC models**, it is useful to focus on one **single row** of the **parameter grid**, roughly corresponding to **critical $\phi_2 = 5/6$**;
- We choose $\phi_2 = 0.825$, and plot the **p-values** along this row of $\sim$ **1600 samples** (CMC simulations) of:
  1. CMC with crit. comp. = 0%, $\phi_2 = 0.1$ (no signal case)
  2. CMC with crit. comp. = 1%, $\phi_2 = 0.825$ (signal case)
- The result is $\sim$ **1600 curves of p-value** as a function of **% crit. comp.**, and we study their behavior: **peaks** correspond to **more likely models (% crit. comp.)**; the **width of a peak** indicates the **resolution (power) of our scan** in telling models apart;
- We then repeat the scan with $\sim$ **1600 CMC samples** of $\sim$ **4M events** each, i.e. **10×** the usual Ar+Sc @ 150 GeV/c **SHINE event statistics**;
- We want to investigate **the effect of larger event statistics** on the **accuracy of the scan**; ideally, the model scan should produce curves **narrowly peaked** around the **true % crit. comp.**

Scan of models – normal & 10× stats



- **Almost twice** the **% crit. comp. resolution** for **10×** statistics!

Conclusions & Outlook

- **Proton intermittency analysis** is a **promising tool** for **detecting the critical point** of strongly interacting matter; however, **large uncertainties** and **bin correlations** cannot be handled by the **conventional analysis methods**;
- The **Principal Component Analysis technique** is able to **handle statistical** & **systematic uncertainties**, as well as **bin correlations**, **without sacrificing event statistics**;
- We build suitable **Monte Carlo models** and **evaluate them against data** via a **scan in parameter space**; **rotation** from **original bins** to **principal components** ensures that **bin correlations** do **not** invalidate the analysis;

Conclusions & Outlook

- **Critical Monte Carlo (CMC) investigation** of **PCA analysis performance** indicates that **large $F_2(M)$ fluctuations** are **inherent** to **proton intermittency analysis**; thus, the effect of **event statistics** on **model parameter scan resolution** is **decisive**;
- We **cannot** distinguish between **no-signal** and **reasonable ($\leq$ 2%) levels of critical component**, with a statistics of $\sim$ **500K events**;
- We **could** discern as weak as a $\sim$ **1% critical component** from **no-signal** with **10×** statistics, i.e. $\sim$ **5M events**; this is **well within reach** of e.g. the **NA61/SHINE experiment**;
- **Results** of this study **soon to appear** in **methodological paper(s)**, along with extensive discussion; **stay tuned! :-)**

Thank You!

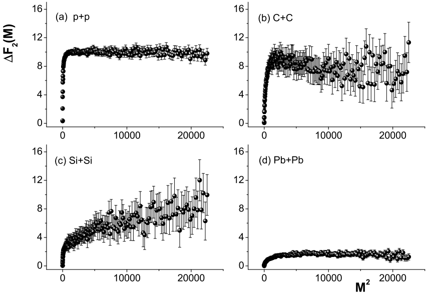
Acknowledgements

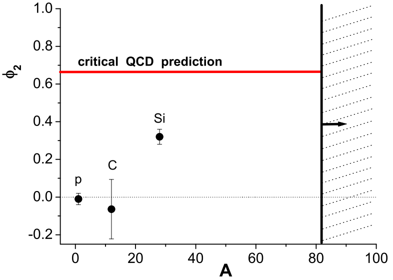
This work was supported by the National Science Centre, Poland under grant no. 2014/14/E/ST2/00018.

Backup Slides

Backup Slides Outline

- **3 sets of NA49 collision systems** at **158A GeV/c ($\sqrt{s_{NN}} \simeq 17$ GeV)**
  [T. Anticic *et al*, Phys. Rev. C **81**, 064907 (2010); T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015)]
- **Intermittent behaviour ($\phi_2^{(\sigma)} \simeq 0.35$)** of **dipion pairs ($\pi^+, \pi^-$)** in **transverse momentum space** observed in **central Si+Si collisions** at **158A GeV**.

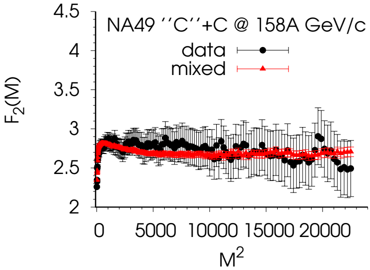


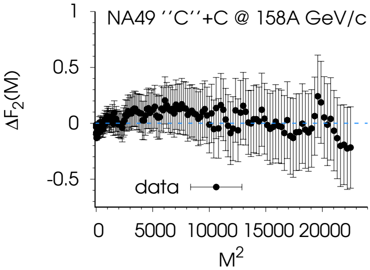


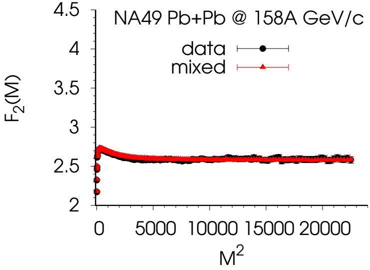
[T. Anticic *et al*, Phys. Rev. C **81**, 064907 (2010)]

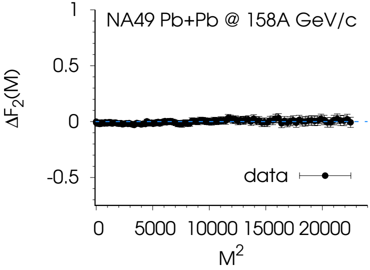
- **No such** power-law behaviour observed in **central C+C and Pb+Pb** collisions at **the same energy**.

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV





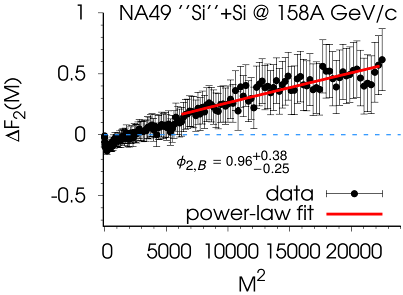


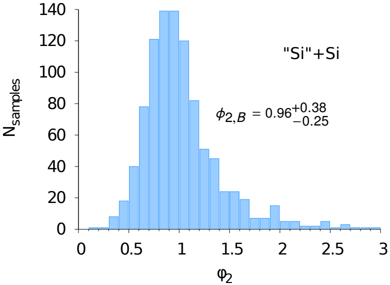


- No intermittency detected in the "C"+C, Pb+Pb datasets.

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV

- Evidence for intermittency in “Si”+Si – but large statistical errors.
- Distribution of φ_2 values, $P(\varphi_2)$, and confidence intervals for φ_2 obtained by fitting individual bootstrap samples [B. Efron, *The Annals of Statistics* **7**,1 (1979)]

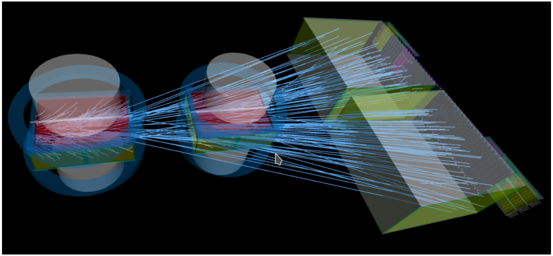




- Bootstrap distribution of ϕ_2 values is highly asymmetric (due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$).
- Uncorrelated fits used, but errors between M are correlated!
- Estimated intermittency index: $\phi_{2,B} = 0.96^{+0.38}_{-0.25}(\text{stat.}) \pm 0.16(\text{syst.})$

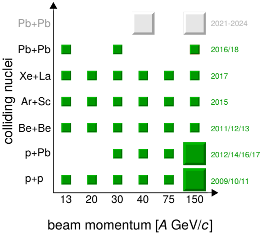
  [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

The NA61/SHINE experiment



- Fixed-target, high-energy collision experiment at CERN SPS;
- Reconstruction & identification of emitted protons in an extended regime of rapidity, with precise evaluation of their momentum vector;
- Centrality of the collision measured by a forward Projectile Spectator Detector (PSD);

- Direct continuation of NA49
- Search for Critical Point signatures



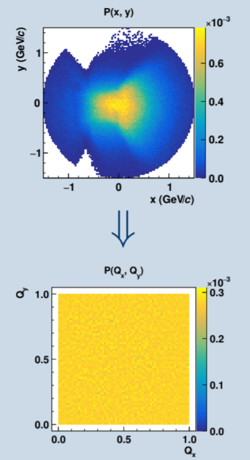
Independent bin analysis with cumulative variables

- **M-bin correlations** complicate uncertainties estimations for $\Delta F_2(M)$ & ϕ_2; one way around this problem is to use **independent bins** – a **different subset** of events is used to calculate $F_2(M)$ for **each** M;
- **Advantage:** correlations are no longer a problem; **Disadvantage:** we **break up statistics**, and can only **calculate** $F_2(M)$ for **a handful of bins**.
- Furthermore, instead of p_x and p_y, one can use **cumulative quantities:** [Bialas, Gazdzicki, PLB 252 (1990) 483]

$$Q_x(x) = \int_{min}^{x} P(x)dx \bigg/ \int_{min}^{max} P(x)dx;$$

$$Q_y(x,y) = \int_{ymin}^{y} P(x,y)dy \bigg/ P(x)$$

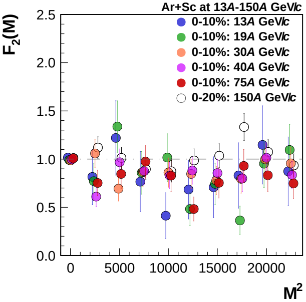
  - transform any distribution into **uniform** one $(0,1)$;
  - **remove the dependence** of F_2 on the shape of the **single-particle distribution**;
  - approximately **preserves ideal power-law** correlation function. [Antoniou, Diakonos, https://indico.cern.ch/event/818624/]

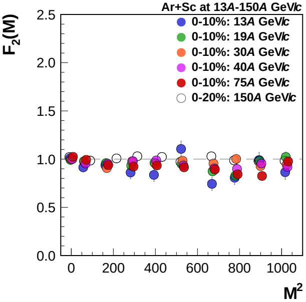


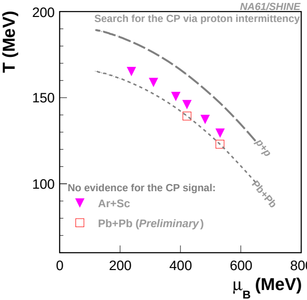
(example for 0-5% Ar+Sc at 150A GeV/c)

SHINE ^{40}Ar + ^{45}Sc independent bin proton intermittency

- **No signal** indicating the **critical point**







$$1^2 \leq M^2 \leq 150^2$$

$$1^2 \leq M^2 \leq 32^2$$

number of subdivisions in cumulative transverse momentum space

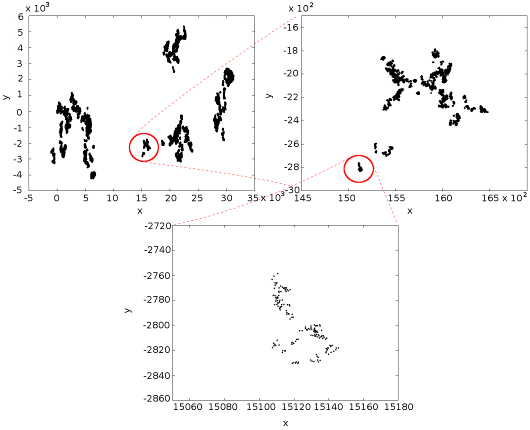
[NA61/SHINE, EPJC 83 (2023) 881]

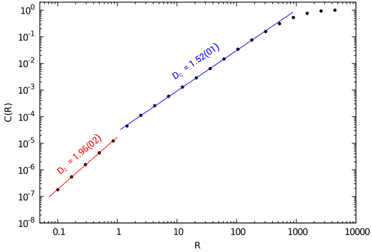
[NA61/SHINE, EPJC 84 (2024) 7, 741]

Simulating fractal sets through random Lévy walks

- In D-dimensional space, we can simulate a fractal set of dimension d_F, $D-1 < d_F < D$, through a random walk with step size Δr distribution:

$$Pr(\Delta r > \Delta r_0) = \begin{cases} 1, & \text{for } \Delta r_0 < \Delta r_d \\ C\,\Delta r_0^{\,-d_F}, & \text{for } \Delta r_d \leq \Delta r_0 \leq \Delta r_u \\ 0, & \text{for } \Delta r_0 > \Delta r_u \end{cases}$$



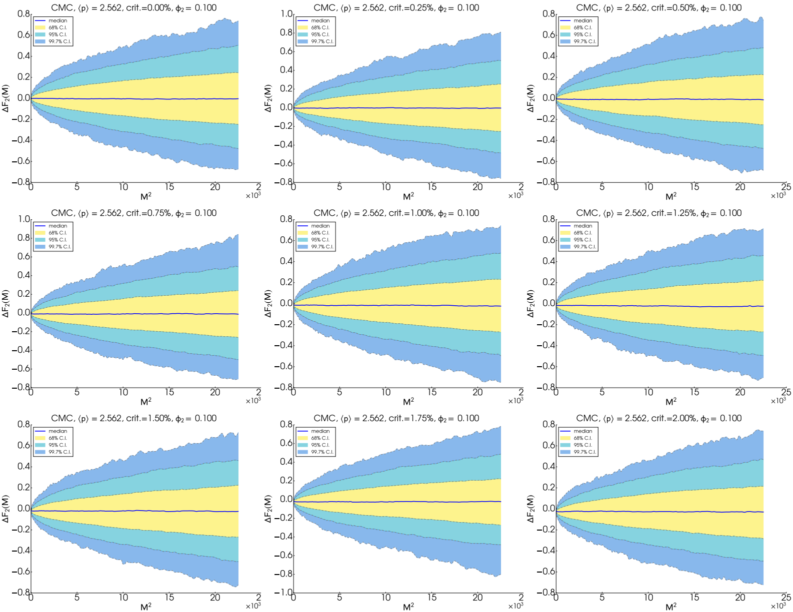


- The result is a set of fractal correlation dimension,

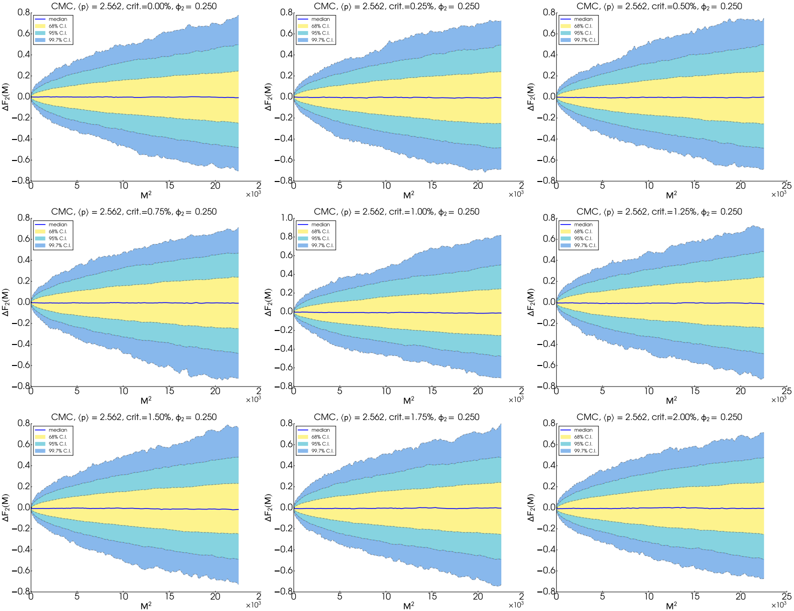
$$C(R) = \frac{2}{N(N-1)} \sum_{\substack{i,j \\ i<j}} \Theta(R - |\boldsymbol{x}_i - \boldsymbol{x}_j|)$$

CMC model scan (zoomed)

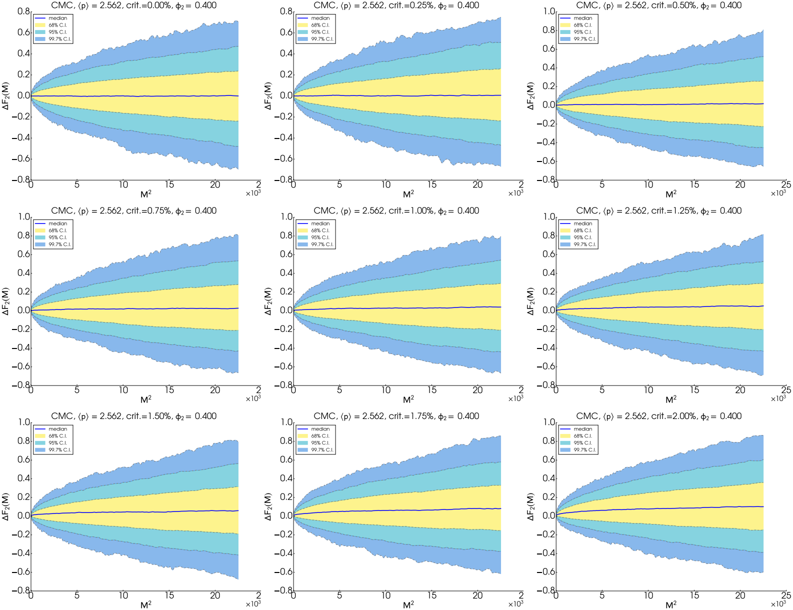
CMC model scan (zoomed) – $\phi_2 = 0.10$



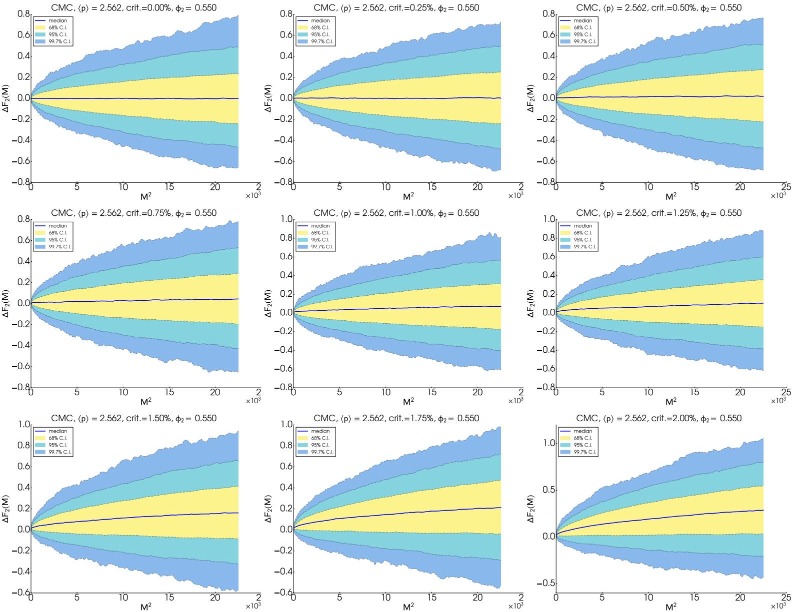
CMC model scan (zoomed) – $\phi_2 = 0.25$



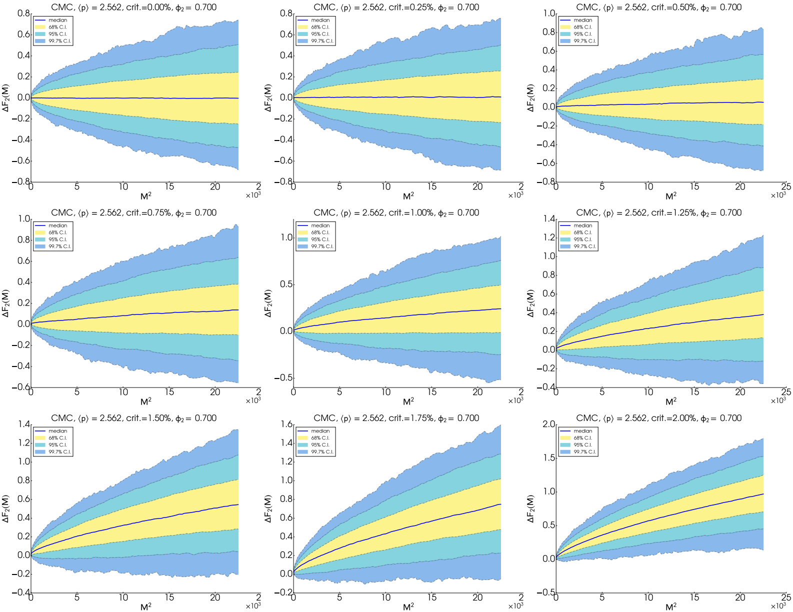
CMC model scan (zoomed) – $\phi_2 = 0.40$



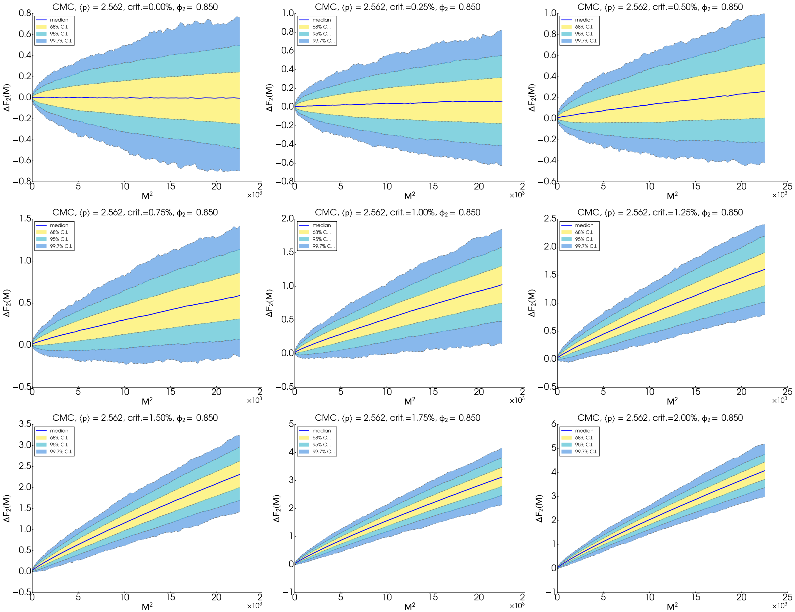
CMC model scan (zoomed) – $\phi_2 = 0.55$



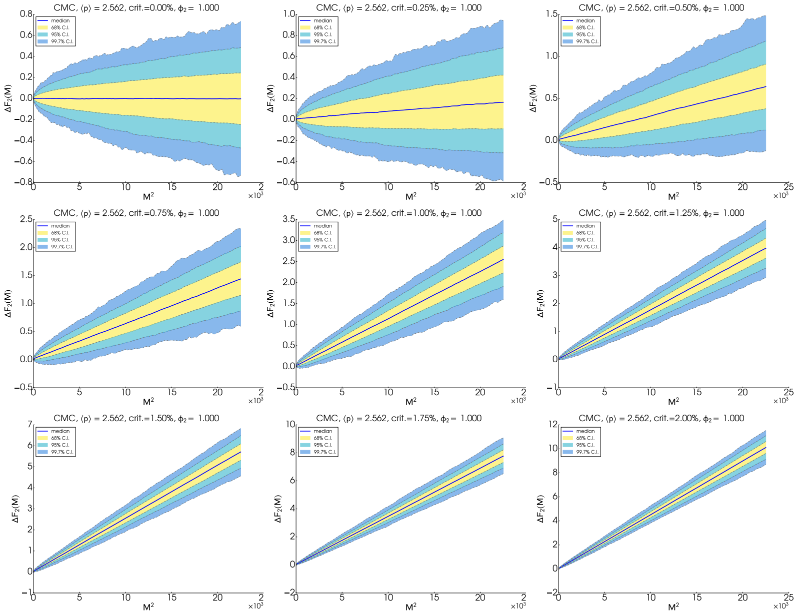
CMC model scan (zoomed) – $\phi_2 = 0.70$



CMC model scan (zoomed) – $\phi_2 = 0.85$



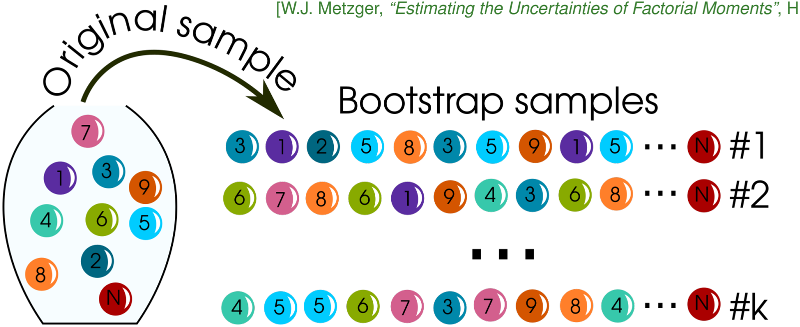
CMC model scan (zoomed) – $\phi_2 = 1.00$



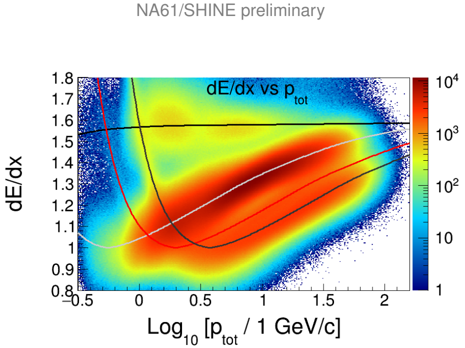
Intermittency analysis tools: the bootstrap

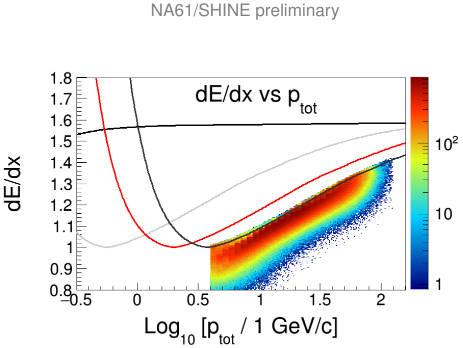
- Random **sampling** of events, **with replacement**, from the original set of events;
- *k* bootstrap samples ($k \sim 1000$) of the **same number of events** as the original sample;
- Each **statistic** ($\Delta F_2(M)$, ϕ_2) **calculated for bootstrap** samples as for the **original**; [B. Efron, *The Annals of Statistics* **7**,1 (1979)]
- **Variance of bootstrap values** estimates **standard error** of **statistic**.

[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]



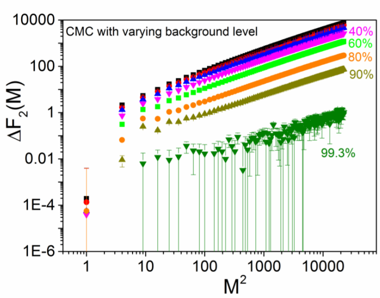
Proton selection

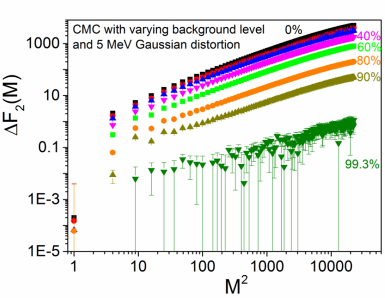


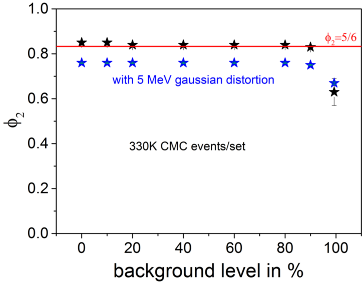


- Particle ID through energy loss dE/dx in the Time Projection Chambers (TPCs);
- Employ p_{tot} region where Bethe-Bloch bands do not overlap (3.98 GeV/c $\leq p_{tot} \leq$ 126 GeV/c);
- Mid-rapidity region ($|y_{CM}| < 0.75$) selected for present analysis.

Momentum resolution: effect on intermittency



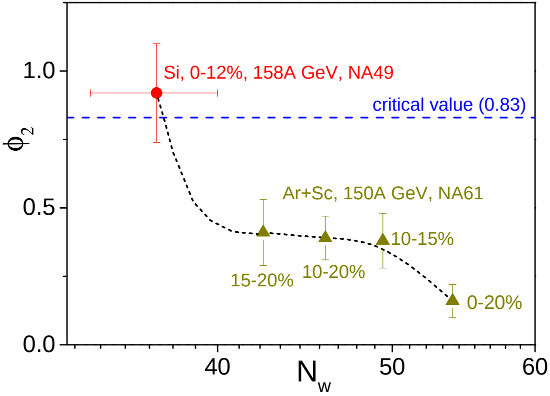




- **CMC + background + Gaussian noise (5 MeV radius)**;
- A **5 MeV Gaussian error** in p_x, p_y leads to ∼ **10% discrepancy** in the **value of** ϕ_2.
- For **very large backround** values **(> 99%)**, momentum resolution **matters little** to the **overall distortion**.

AMIAS on NA49 & NA61/SHINE data – ϕ_2 vs $N_{wounded}$

- ϕ_2 AMIAS confidence intervals calculated for NA49 & NA61/SHINE systems with indications of intermittency
- Corresponding mean number of participating (“wounded”) nucleons N_w estimated via geometrical Glauber model simulation



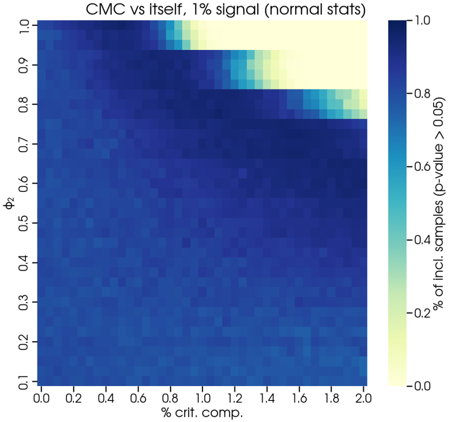
- Peripheral Ar+Sc collisions approach Si + Si criticality $\Rightarrow$ insight of how the critical region looks as a function of baryon density μ_B.
- Check theoretical predictions* for **narrow critical scaling region in T & μ_B**

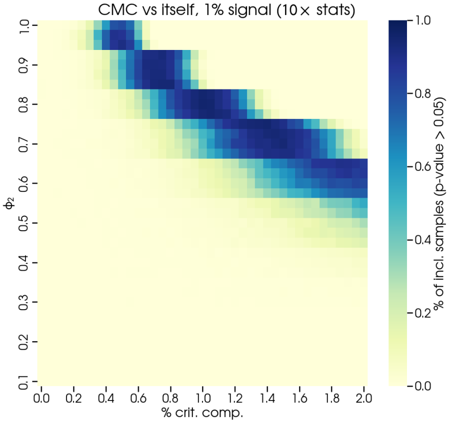
*[F. Becattini *et. al.*, PRC 90, 054907 (2014); N. G. Antoniou, F. K. Diakonos, J. Phys. G 46 (2019) 3, 035101]

[N. G. Antoniou (N. Davis) *et. al.*, *Decoding the QCD critical behaviour in A + A collisions*, NPA 1003 (2020) 122018]

Effect of event statistics on exclusion significance

- 2D plots comparing **CMC with 1% signal** to a **grid of models** in $\boldsymbol{\phi_2}$ and **% crit. comp.** (*"exclusion plots"*);
- Plotted: **percentage of samples** that **cannot** be excluded at $\boldsymbol{p}$**-value > 0.05** level;

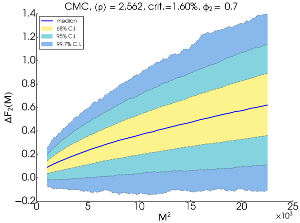


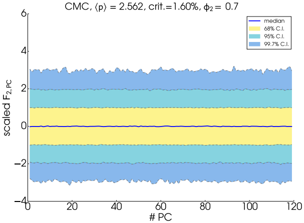


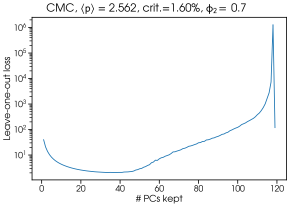
- **10× stats** can **exclude** many **more** scenarios than **normal stats**!

Selecting an optimal number of PCs

- We must select an optimal # of PCs; too few, and we lose information on the moments distribution; too many, and we retain noise from the particular set of samples;
- One criterion is to pick the # of PCs that minimizes the loss in reconstructing the original distribution from the PCs – but we have to be cautious!







- We use the $\Delta F_2(M)$ values of all but one M-bin to predict the missing value in one sample ("leave-one-out" predictor) using the model; then we aggregate the score over all samples;
- Scores are cross-validated in sub-samples for added confidence;
- About $\sim$ 35 components should be kept by leave-one-out metric.