Quantum particles in noncommutative spacetime: An identity crisis

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September 18, 2024
Corfu Summer Institute 2024 - Workshop on Noncommutative and Generalized Geometry in String theory, Gauge theory and Related Physical Models

QUANTUM GRAVITY PHENOMENOLOGY

Are we at the dawn of quantum gravity phenomenology?

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Progress in Particle and Nuclear Physics 125 (2022) 103948

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

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Review

Quantum gravity phenomenology at the dawn of the multi-messenger era—A review

Check for updates

A. Addazi [1,2], J. Alvarez-Muniz [3], R. Alves Batista [4], G. Amelino-Camelia [5,6],
V. Antonelli [7,8], M. Arzano [5,6], M. Asorey [9], J.-L. Atteia [10], S. Bahamonde [11,136],

this review, appeared in 2022, has by now almost 350 citations on inspire...

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- Modified energy-momentum dispersion relations (time delays, modified thresholds)
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some works scattered in the literature argued for such possibility

(Jackson and Hogan, "A New Spin on Quantum Gravity," Int. J. Mod. Phys. D **17**, 567-570 (2008) [arXiv:hep-th/0703133 [hep-th]]; Swain, "Exotic Statistics for Ordinary Particles in Quantum Gravity," Int. J. Mod. Phys. D **17**, 2475-2484 (2009) [arXiv:0805.2373 [gr-qc]])

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I will argue that one of the most studied effective frameworks for QG phenomenology, **non-commutative deformations of Poincaré symmetries**, leads to modifications of the usual Fock space picture in QFT

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- **"Quantum Minkowski space-time"** described by a **non-commutative algebra of functions** of coordinates belonging to a **Lie algebra** which becomes abelian in the $\kappa \to \infty$ limit
- The **four-momenta** describing the particle kinematics become **coordinates on a non-abelian Lie group**

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- Upon quantization relativistic particles are described by a **non-commutative field theory** with $\mathfrak{sl}(2,\mathbb{R})$ coordinates (Freidel and Livine, Phys.Rev.Lett. **96** (2006))

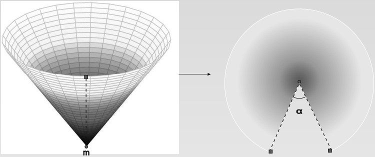
$$[X_\mu, X_\nu] = \frac{i}{\kappa}\epsilon_{\mu\nu\lambda}X_\lambda$$

(see also 't Hooft, Class. Quant. Grav. **13**, 1023-1040 (1996))

Group momentum space from 3d gravity

Curved momentum space in *flatland*

- Point particles → *conical space* with deficit angle $\alpha = 8\pi Gm$



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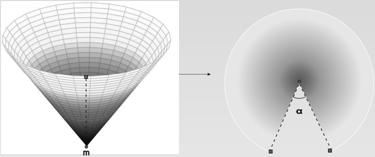

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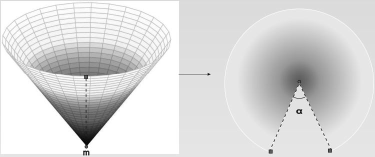

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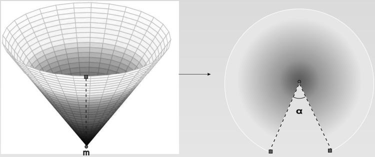

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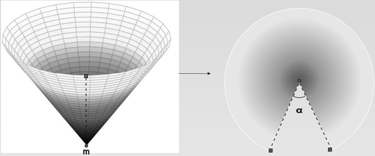

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Bottomline: **momenta are elements of** $SL(2,\mathbb{R})$

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Hopf algebra notions "built in" in everyday quantum theory..

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In Hopf algebraic lingo: **non-trivial co-product** ΔP^μ and **antipode** of $S(P^\mu)$

Embedding coordinates and non-trivial coproduct

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Fix choice of $\mathcal{P}^\mu$: parametrize group elements by **coordinates** p^μ

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momentum coordinates obey a **non-abelian composition rule**

$$p^\mu(g_1 g_2) = v(p_2)\, p_1^\mu + u(p_1)\, p_2^\mu + \frac{1}{\kappa}\epsilon^{\mu\nu\sigma} p_{1\nu} p_{2\sigma}$$

$$p_1^\mu \oplus p_2^\mu = p_1^\mu + p_2^\mu + \frac{1}{\kappa}\epsilon^{\mu\nu\sigma} p_{1\nu} p_{2\sigma} + \mathcal{O}(1/\kappa^2) \neq p_2^\mu \oplus p_1^\mu$$

reflecting a **non-trivial co-product for translation generators**

$$\Delta P^\mu = P^\mu \otimes \mathbb{1} + \mathbb{1} \otimes P^\mu + \frac{1}{\kappa}\epsilon^{\mu\nu\sigma} P_\nu \otimes P_\sigma + \mathcal{O}(1/\kappa^2)$$

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THE MODEL: κ-**Poincaré algebra**:

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$\Rightarrow$ use **quantum groups tools** to *deform* symmetries introducing a **UV energy-scale** κ

κ-deformation

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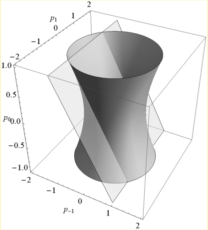
κ-four-momenta: coordinates on **Lie group** $AN(3)$ obtained form the **Iwasawa decomposition** of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4



embedding coordinates

$$\boxed{-p_0^2 + \vec{p}^{\,2} + p_{-1}^2} = \kappa^2, \quad p_0 + p_{-1} > 0$$

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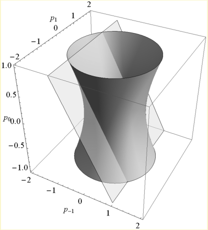

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- $\mathfrak{an}(3)$ Lie algebra: κ-Minkowski **"non-commutative space-time"**

$$[X_0, X_a] = \frac{i}{\kappa} X_a\,,\ [X_a, X_b] = 0$$

Lorentz transformations of $AN(3)$ momentum space

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For **two momenta**

$$(gh)' = \Lambda g h \Lambda_{gh}^{'-1}$$

and $(gh)' \neq g'h'$:

$$(gh)' = \Lambda g \Lambda_g^{'-1}\, \Lambda'_g\, h \Lambda_{gh}^{'-1} = g'\, \Lambda'_g\, h \Lambda_{gh}^{'-1}$$

leading to a **deformed co-product** for Lorentz generators
(MA and Kowalski-Glikman, Phys. Rev. D **107**, no.6, 065001 (2023) [arXiv:2212.03703 [hep-th]])

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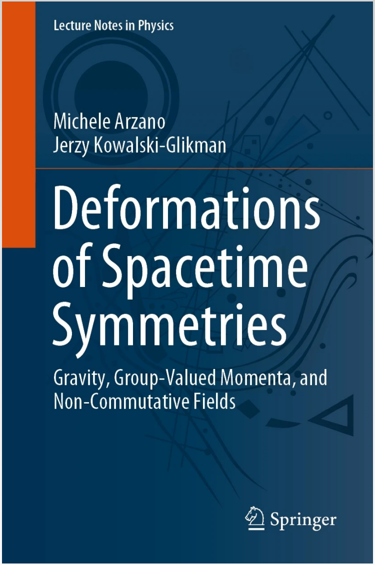
In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

For further reading...



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In QFT **Fock space** is given by (anti-)symmetrized tensor prods of one-particle states

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if the particles are indistinguishable, **swapping the factors** *in the tensor product describing their state should* **lead to another state which is indistinguishable** *from the original one, i.e., with the* **same quantum numbers!!**

Braiding multi-particle states

Way out (?): $|g_1\rangle \otimes |g_2\rangle \rightarrow |g_1 g_2 g_1^{-1}\rangle \otimes |g_1\rangle$ **"flux metamorphosis"** (Bais, NPB **170**, 32 (1980))

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So, can we build a Fock space?

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and the final state has again **total momentum** $p^\mu(g_1 g_2)$!

Any superposition of states obtained from an **arbitrary number of braidings** of the initial state $|g_1\rangle \otimes |g_2\rangle$ is a legitimate candidate for a **two-particle state with total momentum** $p^\mu(g_1 g_2)$!

- Loss of Fock space description?
- Multiparticle states defined in terms of "equivalence classes" of tensor products?
- Only **total momentum** of the system is a **well-defined quantum number**?

Open questions

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- How does one obtain on **ordinary Fock space** description in the limit $\kappa \to \infty$?
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Fascinating challenges for the non-commutative community!


THANK YOU!