No-signaling-in-time as a condition for macrorealism and flavor oscillations

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Motivations

- The notion of Macroscopic realism (Macrorealism) tries to encode our intuition of macroscopic world[1]
- Violations of macrorealism can be tested by means of Leggett-Garg inequalities (LGIs): temporal analogue of Bell inequalities
- Violations of LGIs in neutrino oscillations have been proved by using the MINOS data[2]
- Bell vs LGIs: Bell inequalities are necessary and sufficient for local realism[3], while LGIs are only necessary but not sufficient for macrorealism

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[1] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985)
[2] J. Formaggio, D. Kaiser, M. Murskyj, and T. Weiss, Phys. Rev. Lett. **117**, 050402 (2016)
[3] A. Fine, Phys. Rev. Lett. **48**, 291 (1982).

Motivations

- A necessary and sufficient condition can be formulated in terms of two set of *equalities*[4]: non-signaling in time (NSIT) and arrow of time (AoT)
- We computed NSIT/AoT in the case of two-flavor neutrino oscillations in the wave-packet formalism[5] and in the case of meson oscillations[6]
- We compared NSIT/AoT with LGIs: NSIT/AoT reveal violations of macrorealism hidden by LGIs

[4] L. Clemente and J. Kofler, Phys. Rev. A **91**, 062103 (2015).
[5] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Eur. Phys. J. C **83** , 688 (2023)
[6] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Phys. Rev. A **109**, 062209 (2024)

Macrorealism

Macrorealism

Macrorealism[7]:

- **Macrorealism per se: A macroscopic object is always in one of the available states, regardless of the measurement process**
- **Noninvasive measurability: It is in principle possible to measure the state of the system without affecting its dynamical evolution**

We measure a dichotomic observable $O(t)$ at three equidistant times $t_n = nt$, $n = 0, 1, 2$.

[7] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857-860 (1985)

Macrorealism violation in neutrino oscillations

Leggett–Garg inequalities (LGIs):

$$\mathcal{L}_1(t_0, t_1, t_2) = 1 + C_{01} + C_{12} + C_{02} \geqslant 0$$
$$\mathcal{L}_2(t_0, t_1, t_2) = 1 - C_{01} - C_{12} + C_{02} \geqslant 0$$
$$\mathcal{L}_3(t_0, t_1, t_2) = 1 + C_{01} - C_{12} - C_{02} \geqslant 0$$
$$\mathcal{L}_4(t_0, t_1, t_2) = 1 - C_{01} + C_{12} - C_{02} \geqslant 0$$

with

$$C_{ij} = \langle O(t_i) O(t_j) \rangle$$

must be fulfilled in macrorealistic systems.

Wigner form of LGIs

In ters of joint probabilities $P(O_i, O_j)$, Wigner form of LGIs (WLGIs)[8]

$$
\begin{aligned}
\mathcal{W}_1(t_0,t_1,t_2) &= P(O_1,O_2) - P(-O_0,O_1) - P(O_0,O_2) \leqslant 0 \\
\mathcal{W}_2(t_0,t_1,t_2) &= P(O_0,O_2) - P(O_0,-O_1) - P(O_1,O_2) \leqslant 0 \\
\mathcal{W}_3(t_0,t_1,t_2) &= P(O_0,O_1) - P(O_1,-O_2) - P(O_0,O_2) \leqslant 0
\end{aligned}
$$

Both LGIs and WLGIs are just necessary conditions for macrorealism[9]

[8]D. Saha, S. Mal, P. K. Panigrahi, D. Home, Phys. Rev. A, **91**, 032117 (2015)
[9]L. Clemente and J. Kofler, Phys. Rev. A **91**, 062103 (2015).

NSIT/AoT conditions

Necessary and sufficient condition is given by a combination of NSIT

$$\text{NSIT}^{(1)}: \quad P(O_2) = \sum_{O_1} P(O_1, O_2)$$

$$\text{NSIT}^{(2)}: \quad P(O_0, O_2) = \sum_{O_1} P(O_0, O_1, O_2)$$

$$\text{NSIT}^{(3)}: \quad P(O_1, O_2) = \sum_{O_0} P(O_0, O_1, O_2)$$

and AoT

$$\text{AoT}^{(1)}: \quad P(O_0, O_1) = \sum_{O_2} P(O_0, O_1, O_2)$$

$$\text{AoT}^{(2)}: \quad P(O_0) = \sum_{O_1} P(O_0, O_1)$$

$$\text{AoT}^{(3)}: \quad P(O_1) = \sum_{O_2} P(O_1, O_2)$$

NSIT/AoT for neutrino oscillations

Neutrino oscillations: plane-waves

Flavor states (in relativistic limit)[10]

$$|\nu_\sigma(t)\rangle \;=\; \sum_j U^*_{\sigma j}|\nu_j(t)\rangle$$

Mass states evolve as

$$|\nu_j(t)\rangle \;=\; e^{-iE_j t}|\nu_j(0)\rangle\,, \qquad E_j \;=\; \sqrt{p^2+m_j^2} \;\approx\; p+\frac{m_j}{2E},$$

Flavor oscillation formula:

$$P_{\sigma\to\rho}(t) \;=\; \left|\langle\nu_\rho(t)|\nu_\sigma(0)\right|^2 \;=\; \sum_{j,k} U_{\rho j}U_{\sigma k}U^*_{\rho k}U^*_{\sigma j}\exp\left(-i\frac{\Delta m^2_{jk}}{2E}t\right)$$

[10] S. M. Bilenky and B. Pontecorvo, Phys. Rept. **41**, 225 (1978)

Neutrino oscillations: wave packets

Neutrino wave packets[11]

$$|\nu_\sigma(x,t)\rangle \;=\; \sum_j U^*_{\sigma j}\,\psi_j(t,x)\,|\nu_j\rangle$$

Gaussian wave packet

$$\psi_j(t,x) \;=\; \left(\sqrt{2\pi}\sigma_x\right)^{-\frac{1}{2}} e^{i(px-E_j t)} e^{-\frac{(x-v_j t)^2}{4\sigma_x^2}} \; .$$

v_j are group-velocities. Computing the flavor transition and integrating over x

$$P_{\sigma\to\rho}(t) = \sum_{j,k} U_{\rho j}U_{\sigma k}U^*_{\rho k}U^*_{\sigma j} \exp\left(-i\frac{\Delta m^2_{jk}}{2E}t\right) \exp\left(-\frac{(\Delta m^2_{jk})^2\, t^2}{32E^2\sigma_x^2}\right)$$

[11] C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D **44**, 3635 (1991)

Two-flavor case

Two-flavor mixing matrix

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Oscillation probability

$$P_{\sigma\to\rho}(t) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2}{2E}t\right), \quad \sigma \neq \rho$$

Wave-packet case

$$P_{\sigma\to\rho}(t) = \frac{\sin^2(2\theta)}{2}\left(1 - e^{-\left(\frac{t}{L^{coh}}\right)^2}\cos\left(\frac{\Delta m^2}{E}t\right)\right)$$

$L^{coh} = \frac{4\sqrt{2}\,E}{|\Delta m^2|}\sigma_x$

NSIT/AoT for neutrinos

NSIT for neutrinos[12] ($O_0 = e$, $O_1 = O_2 = \mu$)

$$\begin{aligned}
\text{NSIT}^{(1)}: \quad P_{e\to\mu}(2t) &= 2P_{e\to\mu}(t)P_{e\to e}(t) \\
\text{NSIT}^{(2)}: \quad P_{e\to\mu}(2t) &= 2P_{e\to\mu}(t)P_{e\to e}(t) \\
\text{NSIT}^{(3)}: \quad P_{e\to\mu}(t)P_{\mu\to\mu}(t) &= P_{e\to\mu}(t)P_{\mu\to\mu}(t)
\end{aligned}$$

Only one non-trivial relation

$$\mathcal{N}(t) \equiv P_{e\to\mu}(2t) - 2P_{e\to\mu}(t)\,P_{e\to e}(t) = 0$$

The AoT conditions are all trivially satisfied.

[12] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Eur. Phys. J. C **83** , 688 (2023)

NSIT: plane waves vs wave packets

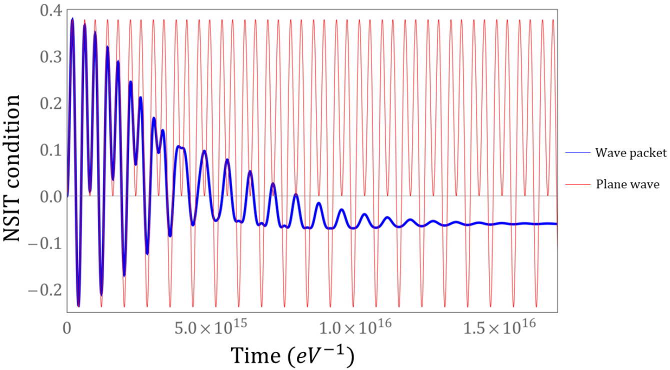


Figure 1: The function $\mathcal{N}$ for the plane-wave (red) and the Gaussian wave-packet (blue) approach. The values have been taken from the MINOS experiment, with $\sin^2\theta = 0.314$, $\Delta m^2 = 7.92 \times 10^{-5}$ eV^2, $E = 10$ GeV and $\sigma_x = 0.5$ GeV^{-1}.

Residual quantumness at large times/distances.

LGIs for neutrino oscillations

Correlators

$$
\begin{aligned}
C_{01} &= P_{e\to e}(t) - P_{e\to\mu}(t) \\
C_{12} &= P_{e\to e}(t) - P_{e\to\mu}(t) \\
C_{02} &= P_{e\to e}(2t) - P_{e\to\mu}(2t)
\end{aligned}
$$

LGIs

$$
\begin{aligned}
\mathcal{L}_1(t) &= 2P_{e\to e}(t) + 2P_{e\to e}(2t) - 2P_{e\to\mu}(t) \geqslant 0 \\
\mathcal{L}_2(t) &= 2P_{e\to\mu}(t) - P_{e\to\mu}(2t) \geqslant 0 \\
\mathcal{L}_3(t) &= 2P_{e\to\mu}(2t) \geqslant 0 \\
\mathcal{L}_4(t) &= \mathcal{L}_3(t)
\end{aligned}
$$

NSIT vs LGIs

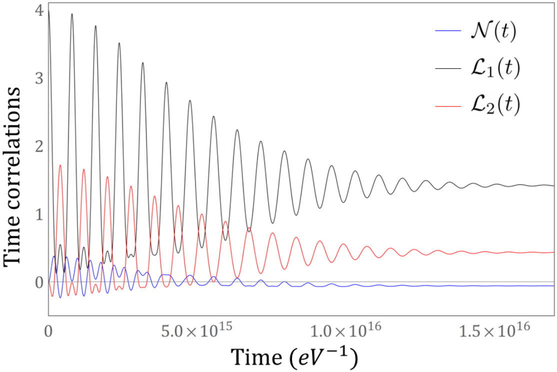


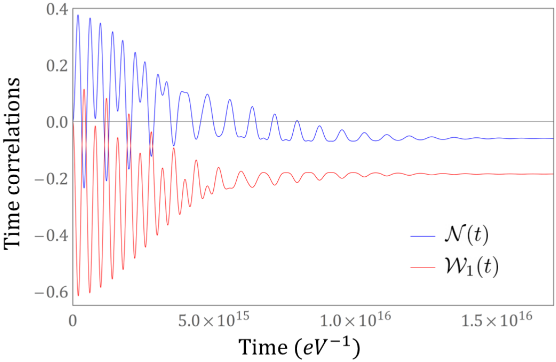
Figure 2: $\mathcal{N}(t)$ (blue) vs $\mathcal{L}_1(t)$ (black) and $\mathcal{L}_2(t)$ (red) as functions of time expressed in eV^{-1}. The values have been taken from the MINOS experiment, with $\sin^2\theta = 0.314$, $\Delta m^2 = 7.92 \times 10^{-5}$ eV^2, $E = 10$ GeV and $\sigma_x = 0.5$ GeV^{-1}.

At large times LGIs hide violation of macrorealism

WLGIs for neutrinos

WLGIs

$$\mathcal{W}_1(t) = P_{e\to e}(t)\,P_{\mu\to e}(t) - P_{\mu\to e}(2t) \leqslant 0$$
$$\mathcal{W}_2(t) = P^2_{e\to \mu}(t) - P_{e\to e}(2t) \leqslant 0$$
$$\mathcal{W}_3(t) = P_{e\to e}(t)\,P_{\mu\to e}(t) - P_{\mu\to e}(2t) \leqslant 0$$



NSIT vs WLGIs

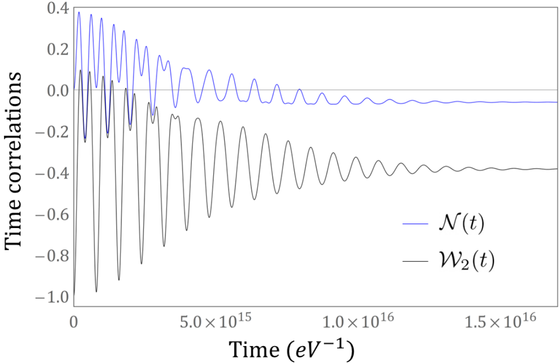


Figure 3: $\mathcal{N}(t)$ (blue) vs $\mathcal{W}_1(t)$ (red, first plot) and $\mathcal{W}_2(t)$ (black, second plot) as functions of time expressed in eV^{-1}. The values have been taken from the MINOS experiment, with $\sin^2\theta = 0.314$, $\Delta m^2 = 7.92 \times 10^{-5}\ \mathrm{eV}^2$, $E = 10$ GeV and $\sigma_x = 0.5\ \mathrm{GeV}^{-1}$.

NSIT/AoT for Kaon oscillations

Kaon oscillations

Flavor eigenstates as linear combination of mass eigenstates

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle + |K_L\rangle\right)$$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle - |K_L\rangle\right)$$

Survival/oscillation probabilities

$$P_{K^0\to K^0/\overline{K}^0}(t) = \frac{e^{-\Gamma\Delta t}}{2}\left(\cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) \pm \cos(\Delta m\Delta t)\right)$$

$$P_{\overline{K}^0\to \overline{K}^0/K^0}(t) = \frac{e^{-\Gamma\Delta t}}{2}\left(\cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) \pm \cos(\Delta m\Delta t)\right)$$

$\Delta m = m_L - m_S$, $\Gamma = \frac{\Gamma_S+\Gamma_L}{2}$ and $\Delta\Gamma = \Gamma_S - \Gamma_L$.

NSIT for Kaon oscillations

The observable is

$$\hat{O}^F = 2|F\rangle\langle F| - \mathbb{1}$$

there are two outcomes F, $\neg F$. The last includes the other flavor or decays products. Choosing $F = K^0$, with $O_0 = F$, $O_1 = \neg F$, and $O_2 = \neg F$ and K_0 produced at t_0, we find there is only a non trivial NSIT condition[13]

$$\mathcal{N}(t) \ = \ 0$$

with

$$\mathcal{N}(t) := P_{F\to F}(2t) - P^2_{F\to F}(t) - P_{F\to \bar{F}}(t) P_{\bar{F}\to F}(t)$$

[13] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Phys. Rev. A **109**, 062209 (2024)

NSIT for meson oscillations

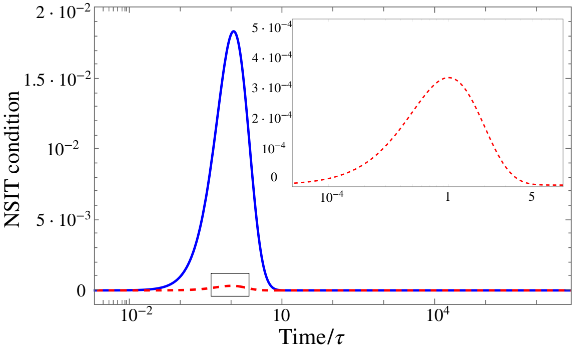


Figure 4: Function $\mathcal{N}(t)$ of the neutral kaon (blue solid curve) and of the strange B meson (red dashed curve) as functions of time scaled by the proper mean lifetime $\tau = 8.954 \cdot 10^{-11}$ s for a neutral kaon and $\tau = 1.470 \cdot 10^{-12}$ s for a strange B meson. We assume that the kaon is produced in flavor $F = K^0$, and the parameters $\Gamma = 5.5939 \times 10^9$ s^{-1}, $\Delta\Gamma = 1.1149 \times 10^{10}$ s^{-1}, and $\Delta m = 0.5293 \times 10^{10} \hbar$ s^{-1} for neutral kaon system are chosen in accordance with the corresponding experimental values provided by the Particle Data Group. For the strange B meson, the particle is produced in the flavor $F = B_s$ and the parameters are $\Gamma = 6.615 \times 10^{11}$ s^{-1}, $\Delta\Gamma = 9.14 \times 10^{10}$ s^{-1}, and $\Delta m = 1.776 \times 10^{13} \hbar$ s^{-1}. All the quantities that appear in the plot are dimensionless.

LGIs for Kaon oscillations

LGIS:

$$\begin{aligned}
\mathcal{L}_1(t) &= P_{F\to F}(2t) + P^2_{F\to F}(t) \\
&- P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \geqslant 0 \\
\mathcal{L}_2(t) &= P_{F\to F}(2t) - P^2_{F\to F}(t) \\
&+ P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \geqslant 0 \\
\mathcal{L}_3(t) &= -P_{F\to F}(2t) - P^2_{F\to F}(t) + 2P_{F\to F}(t) \\
&+ P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \geqslant 0, \\
\mathcal{L}_4(t) &= -P_{F\to F}(2t) + P^2_{F\to F}(t) + 2\Big(1 - P_{F\to F}(t)\Big) \\
&- P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \geqslant 0.
\end{aligned}$$

LGIs for Kaon oscillations: Plots

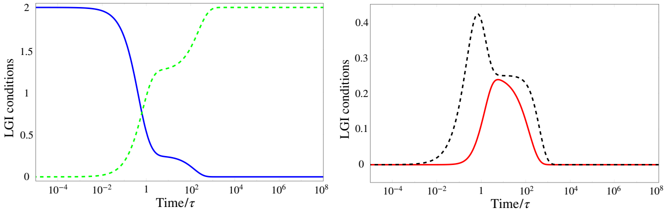


Figure 5: Functions $\mathcal{L}_1(t)$ (blue solid curve, first panel), $\mathcal{L}_2(t)$ (red solid curve, second panel), $\mathcal{L}_3(t)$ (black dashed curve, second panel), and $\mathcal{L}_4(t)$ (green dashed curve, first panel) as functions of time scaled by the proper mean lifetime $\tau = 8.954 \cdot 10^{-11}$ s of a neutral kaon. We assume that the kaon is produced in flavor $F = K^0$, and the parameters $\Gamma = 5.5939 \times 10^9$ s^{-1}, $\Delta\Gamma = 1.1149 \times 10^{10}$ s^{-1}, and $\Delta m = 0.5293 \times 10^{10} \hbar$ s^{-1} for neutral kaon system are chosen in accordance with the corresponding experimental values provided by the Particle Data Group. All the quantities that appear in the plots are dimensionless.

WLGIs for Kaon oscillations

WLGIs:

$$
\begin{aligned}
\mathcal{W}_1(t) &= P_{F\to F}(2t) - P_{F\to F}(t) \\
&- P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \leqslant 0 \\
\mathcal{W}_2(t) &= P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) - P_{F\to F}(2t) \leqslant 0 \\
\mathcal{W}_3(t) &= P_{F\to F}(2t) - P_{F\to F}(t) \\
&- P_{F\to\bar{F}}(t)P_{\bar{F}\to F}(t) \leqslant 0
\end{aligned}
$$

WLGIs for Kaon oscillations: Plots

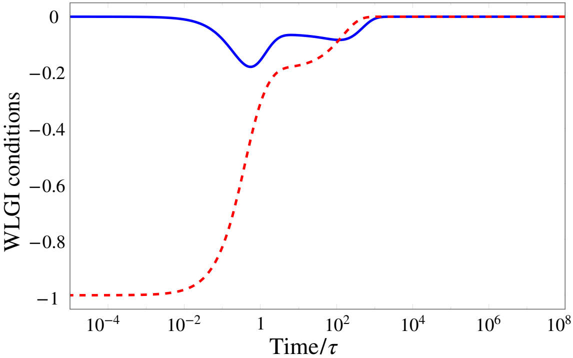


Figure 6: Functions $\mathcal{W}_1(t)$ (blue solid curve) and $\mathcal{W}_2(t)$ (red dashed curve) as functions of time scaled by the proper mean lifetime $\tau = 8.954 \cdot 10^{-11}$ s of a neutral kaon. We assume that the kaon is produced in flavor $F = K^0$, and the parameters $\Gamma = 5.5939 \times 10^9$ s^{-1}, $\Delta\Gamma = 1.1149 \times 10^{10}$ s^{-1}, and $\Delta m = 0.5293 \times 10^{10} \hbar$ s^{-1} for neutral kaon system are chosen in accordance with the corresponding experimental values provided by the Particle Data Group in [?]. All the quantities that appear in the plot are dimensionless.

Both LGIs and WLGIs are never violated in the present case!

Conclusions and Perspectives

Conclusions

- Violations of Macrorealism can be tested through (W)LGIs
- (W)LGIs do not fully carachterize macrorealism
- A set of equalities NSIT/AoT provide a necessary and sufficient condition
- NSIT/AoT can be used to test macrorealism in flavor oscillations
- Violations of macrorealism are observed when (W)LGIs are not violated[14][15]

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[14] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Eur. Phys. J. C **83** , 688 (2023)

[15] M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov and L.S.. Phys. Rev. A **109**, 062209 (2024)

Perspectives

- Violations of LGIs were tested in neutrino oscillations experiments[16]: similar tests could be repeated for NSIT/AoT conditions
- The present analysis could be repeated in cases including CP violations (e.g. in Kaon oscillations or three-flavor neutrino oscillations). CPT violations also worth to be studied
- Effects of gravity could be included: gravitational decoherence

[16] J. A. Formaggio, D. I. Kaiser, M. M. Murskyjj and T. E. Weiss, Phys. Rev. Lett. **117**, 050402 (2016)

Thank you for the attention!