T-duality and Riemann^4

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Motivation

The low-energy effective action for the massless fields of the closed string is an expansion

$$S_{eff} = \sum_{k,l=0}^{\infty} \alpha'^{k} g_s^{2(l-1)} S^{(k,l)}$$

in

- α' – inverse string tension
- g_s – string coupling

Usually we work only with $S^{(0,0)}$, but in some situations the higher corrections become important, e.g. in the study of black holes

Here we will ignore the g_s corrections, i.e. restrict to the tree-level string effective action

$$S_{tree} = \int d^{10}x\sqrt{-g}e^{-2\Phi}\left(L_0 + \alpha' L_1 + \alpha'^2 L_2 + \dots\right)$$

We further confine ourselves to the NSNS sector fields: g, B and Φ

At lowest order we have $(H = dB)$

$$L_0 = R + 4(\nabla\Phi)^2 - \frac{1}{12}H^2$$

We are interested in the higher derivative corrections L_1, L_2, L_3, ...

They can be found from tree-level string scattering amplitudes

This involves two steps: (1) compute the relevant amplitudes and (2) make an ansatz for the effective action and match the amplitudes

The first correction in the type II case is [Gross, Witten '86]

$$L_3 = \zeta(3)\mathrm{Riem}^4 + \dots$$

It is actually present also for the heterotic and bosonic string.

To find the full result for L_3 would require computing amplitudes up to 8 points and matching to an ansatz for the effective action with $\mathcal{O}(1000)$ terms!

However, the form of the correction is highly constrained by:

(1) Supersymmetry

(2) Duality symmetries

In fact, in a series of papers M. Garousi ('18-'20) has shown that T-duality completely fixes L_3 in the NSNS sector (up to the overall coefficient)!

He started from an ansatz in 10 dimensions, reduced on S^1 and required the result to be invariant under T-duality.

The starting ansatz for L_3 has $\mathcal{O}(1000)$ terms and the answer hundreds of terms.

Unfortunately, it is very hard to recognize any structure in the result.

I will describe a simplified approach, which also makes it is easier to recognize what structures should appear in the answer.

Outline

1. $O(d,d)$ symmetry
2. Necessary condition for $O(d,d)$
3. The α'^3 correction
4. Why T-duality/$O(d,d)$ cannot be made manifest

$O(d, d)$ symmetry

When the closed string effective action is reduced from 10 to $10 - d$ dimensions, i.e. taking all fields independent of d coordinates $y^{m'}$ ($m' = 1, \ldots, d$), an $O(d, d)$ symmetry appears [Meissner, Veneziano '91]

The non-trivial part of this symmetry is

$$O(d) \times O(d) \subset O(d, d)$$

It arises because the left/right-moving modes of $y^{m'}$ become effectively independent and each can be rotated by an $O(d)$. This symmetry persists to all orders in α' (though it is not an exact symmetry). [Sen '91]

This can be used to constrain the form of the (tree-level) α' corrections to the action.

We first need the action of $O(d,d)$ on the (NSNS) fields.

In the dimensional reduction we get

$$\begin{aligned} \underline{g}_{\underline{mn}} &\to g_{mn}\,, \quad \underline{g}_{m'n} = A^{(1)}_{m'n}\,, \quad \underline{g}_{m'n'} \\ \underline{B}_{\underline{mn}} &\to B_{mn}\,, \quad \underline{B}_{m'n} = A^{(2)}_{m'n}\,, \quad \underline{B}_{m'n'} \\ \underline{\Phi} &\to \Phi \end{aligned}$$

Besides g, B and Φ of the reduced theory we get $2d$ vectors and d^2 scalars, which we group into

$$\mathcal{A}_M = \begin{pmatrix} A^{(1)m'} \\ A^{(2)}_{m'} \end{pmatrix} \qquad \mathcal{H}^{MN} = \begin{pmatrix} (g - Bg^{-1}B)_{m'n'} & (Bg^{-1})_{m'}{}^{n'} \\ -(g^{-1}B)^{m'}{}_{n'} & g^{m'n'} \end{pmatrix}$$

transforming as a vector and a symmetric 2-index tensor (a.k.a. generalized metric) under $O(d,d)$

$O(d,d)$ invariants

We will consider only the terms in the reduced action quadratic in the KK vectors.

There are three possible combinations

$$A_m^{(1)} \cdot A_n^{(1)}\,, \qquad A_m^{(2)} \cdot A_n^{(2)} \qquad \text{and} \qquad A_m^{(1)} \cdot A_n^{(2)}$$

However, there are only two $O(d,d)$ invariants

$$\mathcal{A}_{mM}\eta^{MN}\mathcal{A}_{nN} \qquad \text{and} \qquad \mathcal{A}_{mM}\mathcal{H}^{MN}\mathcal{A}_{nN}$$

where

$$\eta^{MN} = \begin{pmatrix} 0 & \delta_{m'}^{n'} \\ \delta_{n'}^{m'} & 0 \end{pmatrix}$$

is the $O(d,d)$ invariant metric

A necessary condition

Defining

$$A = -\frac{1}{2}(A^{(1)} + A^{(2)}) , \qquad \hat{A} = \frac{1}{2}(A^{(1)} - A^{(2)})$$

the invariants are

$$A_m \cdot A_n \qquad \text{and} \qquad \hat{A}_m \cdot \hat{A}_n ,$$

while

$$A_m \cdot \hat{A}_n$$

explicitly violates $O(d, d)$

A necessary condition for $O(d, d)$ symmetry is therefore that all terms in the reduced action involving $A_m \cdot \hat{A}_n$ cancel out

This condition is very strong (it fixes everything except terms involving only derivatives of the dilaton)

Order α'^3

To fifth order in fields one finds $\mathcal{R}^4$, $H^2\mathcal{R}^3$ and $H^4\mathcal{R}$ terms:

$$\begin{aligned}
L_3 = {} & \frac{1}{4!}t_8t_8\mathcal{R}^4 + \frac{1}{4(4!)}(\varepsilon_8\varepsilon_8)'\mathcal{R}^4 - \frac{1}{2(3!)}t_8t_8H^2\mathcal{R}^3 + \frac{1}{2(3!)^3}(\varepsilon_9\varepsilon_9)'H^2\mathcal{R}^3 \\
& - \frac{1}{2(3!)^2}H^2(\varepsilon_6\varepsilon_6)'\mathcal{R}^3 + \frac{1}{2(3!)^3}(\varepsilon_9\varepsilon_9)'[H^2]\mathcal{R}^3 + 2\varepsilon_4H^2\varepsilon_4\mathcal{R}^2\mathcal{R} \\
& + \frac{1}{2}\tilde{t}_8\tilde{t}_8H^2(\nabla H)^2\mathcal{R} - \frac{1}{2(3!)^2}(\varepsilon_9\varepsilon_9)'[H^2](\nabla H)^2\mathcal{R} - 4\varepsilon_4H^2\varepsilon_4(\nabla H)^2\mathcal{R} \\
& - 4!(\nabla H)^{abef}(\nabla H)^{cd}{}_{ef}H_{k[ab}\nabla^2H^k{}_{cd]} + 4!\nabla^eH^{fab}\nabla_eH_f{}^{cd}H_{k[ab}\nabla^2H^k{}_{cd]} \\
& - 2(4!)H^2_{[abcd]}(\nabla H)^{abef}(\nabla H)_{ghef}\mathcal{R}_S^{cdgh} - 2(4!)H^2_{[abcd]}\nabla^eH^{fag}\nabla_eH_f{}^{bh}\mathcal{R}_S{}^{cd}{}_{gh} \\
& + 8\nabla_eH^{kcd}\nabla^eH_{fcd}H_{abk}\nabla^2H^{abf} - 8\nabla_eH_{kcd}H^{fcd}\nabla^eH_{abk}\nabla^2H^{abf} \\
& + 19 \text{ more } H^2(\nabla H)^2\mathcal{R}\text{-terms}
\end{aligned}$$

where

$$\mathcal{R}_{abcd} = R_{abcd} - (\nabla H)_{abcd} + \frac{1}{2}H^2_{a[cd]b}$$

with $(\nabla H)_{abcd} = \nabla_{[a}H_{b]cd}$ and $H^2_{abcd} = H_{abe}H^e{}_{cd}$.

T-duality cannot be made manifest

The $O(d,d)$ violating terms cancel only up to total derivatives and terms proportional to the lowest order e.o.m.. The latter can be removed by field redefinitions.

Interesting question: Can we find a choice of Lagrangian in 10d so that no field redefinitions are needed to achieve $O(d,d)$ upon dimensional reduction?

The answer turns out to be no. One can prove that some required field redefinitions cannot be lifted to 10d. [Work in progress]

This gives a clear explanation why approaches like Gen. Geometry/DFT/ExFT fail to capture this correction.

[Hronek, Wulff '20]

Conclusions and outlook

- Cancellation of $O(d, d)$ violating terms quadratic in the KK vectors can be used to find α'-corrections to string eff. actions
- Leads to simpler expressions than previously available in the literature [Liu, Minasian '19; Garousi '20; Gholian, Garousi '23]
- $O(d, d)$ symmetry appears only after field redefinitions which cannot be lifted to 10d $\rightarrow$ Extended symmetry approaches fail at this order (in 10d)

- The structure of the corrections remains to be better understood
- At α'^3 it remains to complete the correction with terms of the form $H^4\mathcal{R}^2$, $H^6\mathcal{R}$ and H^8 (and RR fields)