Supersymmetric discrete gravity

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1. Introduction and motivations

- Discretization of space-time as regularization in UV divergent field theories.
- Discretization as intrinsic quantum feature.
- Continuum limit may be ill defined, or hard to obtain.
- Difficulties in recovering classical theory, emergent from quantum discrete theory.

Try to define quantities in the discrete theory that mimic typical quantities of the continuum theory.

In the case of discrete gravity: all quantities pertaining to differential calculus, i.e. tangent vectors, vielbein, connection, curvature, torsion etc

Thus: differential calculus on discrete structures.

Differential calculi on Hopf algebras constructed by Woronowicz (1989).

Discrete structures associated to finite groups (finite group “manifolds”) have a canonical differential calculus, due to their Hopf algebra structure.

Can define actions formally identical to continuous actions.

Dimakis, Mueller-Hoissen, Striker (1993); Bresser,Mueller-Hoissen, Dimakis, Sitarz (1996); Bonechi, Giachetti, Maciocco, Sorace, Tarlini (1996); Majid, Raineri (2000); LC (2001); Pagani, LC (2002); Aschieri, Isaev, LC (2003); Chamseddine, Mukhanov (2021); Chamseddine, Khaldieh (2024)

2. Differential calculus on finite groups

G = finite group of order ***n***, generic element ***g*** and unit ***e***

Fun(G) = set of complex functions on *G*

An element *f* of *Fun(G)* is specified by its values $f_g = f(g)$.

- *f* can be written as:

$$f = \sum_{g \in G} f_g x^g, \quad f_g \in \mathbb{C}$$

where the functions x^g are defined by

$$x^g(g') = \delta_{g,g'}$$

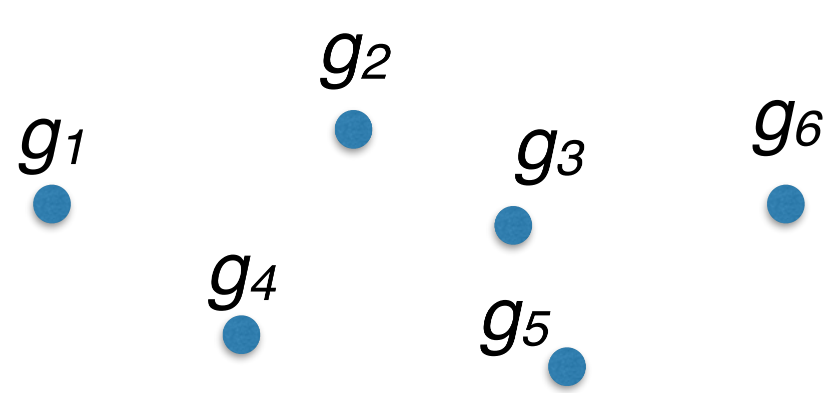
- Thus *Fun(G)* is a *n*-dim vector space, and the functions x^g provide a basis (coordinate functions)

- *Fun(G)* is also a commutative algebra, with the usual pointwise sum and product, and unit *I* defined by *I(g)=1* for all *g*

- In particular

$$x^g x^{g'} = \delta_{g,g'} x^g, \qquad \sum_{g \in G} x^g = I$$

- So far the *G* group manifold is represented by a collection of points:



- The left and right action of *G* on itself :

$$L_g g' = gg' = R_{g'} g$$

induce left and right action of *G* on *Fun(G)* :

$$\mathcal{L}_g f(g') = f(gg') = \mathcal{R}_{g'} f(g)$$

- For ex.

$$\mathcal{L}_{g_1} x^{g_2} = x^{g_1^{-1} g_2} \qquad \mathcal{R}_{g_1} x^{g_2} = x^{g_2 g_1^{-1}}$$

- Moreover

$$\mathcal{L}_{g_1} \mathcal{L}_{g_2} = \mathcal{L}_{g_2 g_1} \qquad \mathcal{R}_{g_1} \mathcal{R}_{g_2} = \mathcal{R}_{g_1 g_2} \qquad \mathcal{L}_{g_1} \mathcal{R}_{g_2} = \mathcal{R}_{g_2} \mathcal{L}_{g_1}$$

- The *G* group structure induces a Hopf algebra structure on *Fun(G)*, → construction of differential calculi

- Differential calculi on Hopf algebras: general method in Woronowicz 1989

- Defined by a linear map *d* : *Fun(G)* $\longrightarrow \Gamma$ satisfying the Leibniz rule $d(ab) = (da)b + a(db)$

- The space of 1-forms Γ is a bimodule on *Fun(G)*,
→ its elements can be multiplied on the left and on the right by elements of *Fun(G)*

- From $da = d(Ia) = (dI)a + I(da) \Rightarrow \quad dI = 0$

- From $0 = dI = d\sum_{g \in G} x^g = \sum_{g \in G} dx^g$ only *n-1* independent dx^g

- Left and right action of *G* on the space of 1-forms:

$$\mathcal{L}_g(adb) = (\mathcal{L}_g a)d(\mathcal{L}_g b) \qquad \mathcal{R}_g(adb) = (\mathcal{R}_g a)d(\mathcal{R}_g b)$$

$\longrightarrow$ bicovariant calculus

- Left-invariant 1-forms: $\theta^g \equiv \sum_{h \in G} x^h dx^{hg}$

then : $\mathcal{L}_k \theta^g = \theta^g, \quad \mathcal{R}_k \theta^g = \theta^{kgk^{-1}}$

- From $\sum_{g \in G} dx^g = 0$ we have also $\sum_{g \in G} \theta^g = 0$

only *n-1* independent θ

$\longrightarrow$ can take the $\theta^g, \quad g \neq e$ as basis of Γ

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Bicovariant diff. calculi in 1-1 correspondence with unions of conjugation classes of G

- From $\sum_{g \in G} dx^g = 0$ we have also $\sum_{g \in G} \theta^g = 0$

only *n-1* independent θ

$\longrightarrow$ can take the $\theta^g, \quad g \neq e$ as basis of Γ

- commutations: $\theta^g x^h = x^{hg^{-1}} \theta^g$

  imply: $$\boxed{\theta^g f = (\mathcal{R}_g f)\theta^g}$$ $g \neq e$

- Thus functions commute between themselves, but do not commute with the basis of 1-forms θ

- From inversion formula $dx^h = \sum_g x^{hg^{-1}} \theta^g$ one finds the differential of a function:

$$df = \sum_h f_h dx^h = \sum_{g \neq e} (\mathcal{R}_g f - f)\theta^g \equiv \sum_{g \neq e} (t_g f)\theta^g$$

- the finite difference operators $t_g = \mathcal{R}_g - I$ are the analogues of (left-invariant) tangent vectors

- an exterior product is defined as

$$\boxed{\theta^g \wedge \theta^{g'} = \theta^g \otimes \theta^{g'} - (\mathcal{R}_g \theta^{g'}) \otimes \theta^g} \qquad (g, g' \neq e)$$

compatible with left and right action of *G*, i.e. if we define

$\mathcal{L}(\theta^i \otimes \theta^j) = \mathcal{L}\theta^i \otimes \mathcal{L}\theta^j$ and $\mathcal{R}(\theta^i \otimes \theta^j) = \mathcal{R}\theta^i \otimes \mathcal{R}\theta^j$

we find $\mathcal{L}(\theta^i \wedge \theta^j) = \mathcal{L}\theta^i \wedge \mathcal{L}\theta^j$ and $\mathcal{R}(\theta^i \wedge \theta^j) = \mathcal{R}\theta^i \wedge \mathcal{R}\theta^j$

- can be generalized to *k*-forms

$$\theta^{i_1} \wedge \ldots \wedge \theta^{i_k} \equiv A^{i_1 \cdots i_k}_{j_1 \cdots j_k} \theta^{j_1} \otimes \ldots \otimes \theta^{j_k}$$

- exterior derivative *d*, satisfying (graded) Leibniz rule:

$$d(\rho \wedge \rho') = d\rho \wedge \rho' + (-1)^k \rho \wedge d\rho'$$

with ρ *k*-form

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$\longrightarrow$

$\theta^g \wedge \theta^g = 0$

$\theta^g \wedge \theta^{g'} = -\theta^{g'} \wedge \theta^g$ if $[g, g'] = 0$

- can be generalized to *k*-forms

$$\theta^{i_1} \wedge \ldots \wedge \theta^{i_k} \equiv A^{i_1 \cdots i_k}_{j_1 \cdots j_k} \theta^{j_1} \otimes \ldots \otimes \theta^{j_k}$$

- exterior derivative *d*, satisfying (graded) Leibniz rule:

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with ρ *k*-form

- in general for a diff. calculus with *m* independent θ there is an integer *p* $\geq$ *m* such that the linear space of left-invariant *p*-forms is 1-dimensional, and (*p+1*)-forms vanish identically.

- then every product of *p* 1-forms is proportional to one of these products, that can be chosen as volume form

$$\theta^{i_1} \wedge ... \wedge \theta^{i_p} = \varepsilon^{i_1 \cdots i_p} \; vol$$

- integration of a *p*-form ρ :

$$\int \rho = \int \rho_{i_1 \ldots i_p} \theta^{i_1} \wedge ... \wedge \theta^{i_p} = \int \rho_{i_1 \ldots i_p} \varepsilon^{i_1 \cdots i_p} \; vol = \sum_G \rho_{i_1 \ldots i_p}(g) \varepsilon^{i_1 \cdots i_p}$$

↑ $\rho_{i_1 \ldots i_p}$ $\in \; Fun(G)$

- picture of a finite group and its diff. calculus:

a collection of points corresponding to the group elements with links associated to tangent vectors $t_h = \mathcal{R}_h - I$, or equivalently to the right actions $\mathcal{R}_h$, h belonging to union of conjugacy classes characterizing the diff. calculus

- link is oriented from x^g to $x^{g'}$ if $x^{g'} = \mathcal{R}_h x^g$
i.e. if $g' = gh^{-1}$. (NB unoriented if $h = h^{-1}$)

- Two examples follow: Z_N and S_3

Bresser, Mueller-Hoissen, Dimakis, Sitarz (1996)

3. Differential calculus on Z_n

- Elements: $\{e, u, u^2, ... u^{n-1}\}$
- Basis of dual functions: $\{x^e, x^u, x^{u^2}, ..., x^{u^{n-1}}\}$

  Left and right actions coincide, since the group is abelian:

$$\mathcal{L}_{u^i} x^{u^j} = x^{u^{j-i}} = \mathcal{R}_{u^i} x^{u^j}$$

- Conjugation classes: $\{e\}, \{u\}, \{u^2\}, ... \{u^{n-1}\}$

  here we use the diff. calculus corresponding to $\{u\}$

  all the left-invariant 1-forms θ^{u^i} are set to zero except

$$\theta^u = \sum_{j=0}^{n-1} x^{u^j} dx^{u^{j+1}}$$

- Commutations: $\theta^u f = (\mathcal{R}_u f)\theta^u$

- Tangent vector: $t_u = \mathcal{R}_u - I$
- Differential: $df = (t_u f)\theta^u$, where the partial derivative

$$(t_u f)(u^i) = (\mathcal{R}_u f)(u^i) - f(u^i) = f(u^{i+1}) - f(u^i)$$

is just a finite difference operator between two neighbour sites

- Integration: the volume form is θ^u, the integral of a 1-form ρ is

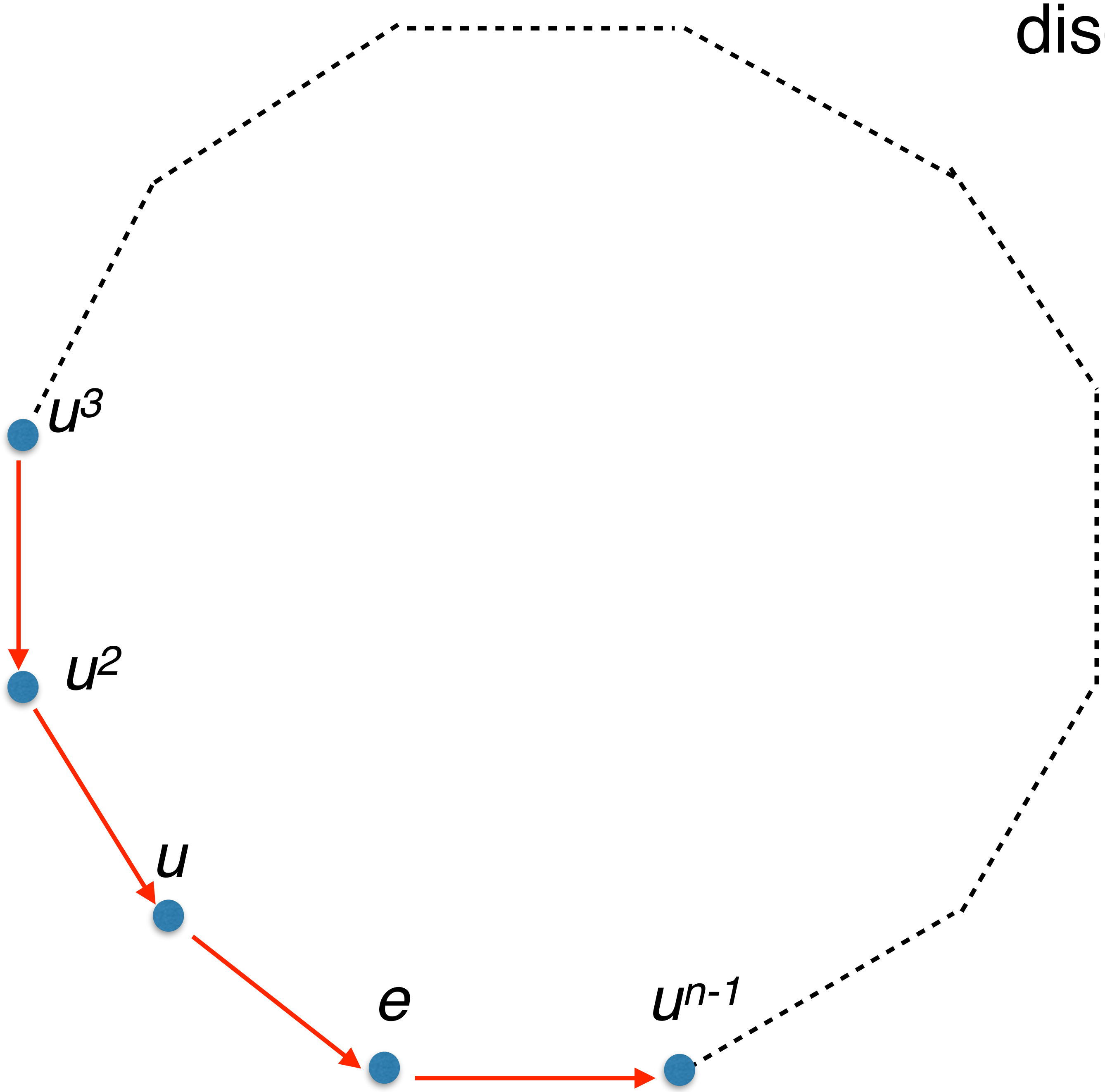
$$\int \rho = \int \rho_u \theta^u = \int \rho_u \; vol = \sum_{g \in \mathbb{Z}_n} \rho_u(g)$$

Integration by parts holds since

$$\int df = \int (t_u f)\theta^u = \int (\mathcal{R}_u f - f) \; vol = \sum_{g \in \mathbb{Z}_n} (\mathcal{R}_u f - f)(g) = 0$$

Z_n

discrete approximation of S^1

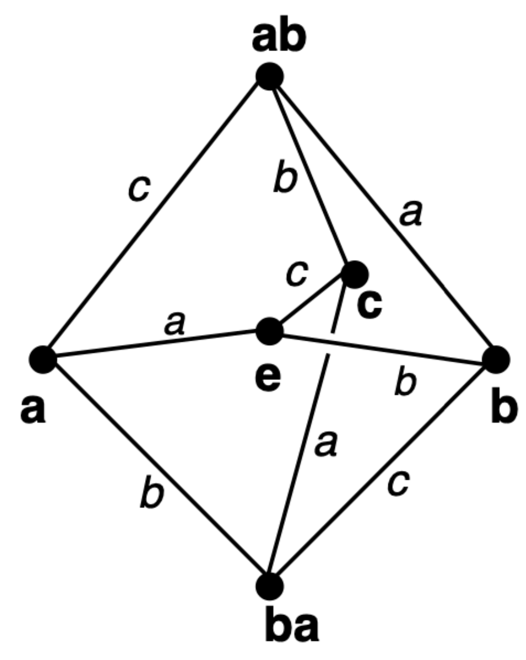


4. Differential calculi on S_3

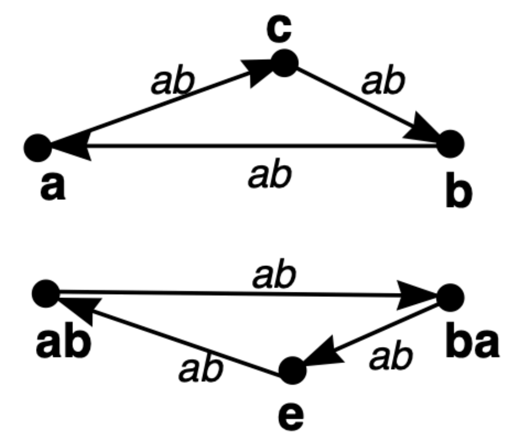
Elements: $a = (12)$, $b = (23)$, $c = (13)$, $ab = (132)$, $ba = (123)$, e.

Multiplication table:

	e	a	b	c	ab	ba
e	e	a	b	c	ab	ba
a	a	e	ab	ba	b	c
b	b	ba	e	ab	c	a
c	c	ab	ba	e	a	b
ab	ab	c	a	b	ba	e
ba	ba	b	c	a	e	ab



S_3 manifold (BC_I)



S_3 manifold (BC_{II})

Nontrivial conjugation classes: $I = [a, b, c]$, $II = [ab, ba]$.

There are 3 bicovariant calculi BC_I, BC_{II}, BC_{I+II} corresponding to the possible unions of the conjugation classes. They have respectively dimension 3, 2 and 5.

volume form: $\theta^a \wedge \theta^b \wedge \theta^a \wedge \theta^c$

In Catenacci, Debernardi, Pagani, LC (2003), diff calculi for all finite G of order ≤ 8

5. Finite group discretization of gravity (coupled to fermions)

- Classical gravity + fermions, summary

index-free notation:

$$S = \int Tr\left(i\ R \wedge V \wedge V \gamma_5 - [(D\psi)\bar{\psi} - \psi(D\bar{\psi})] \wedge V \wedge V \wedge V \gamma_5\right)$$

basic fields: $V = V^a_\mu \gamma_a dx^\mu$, $\Omega = \Omega^{ab}_\mu \gamma_{ab} dx^\mu$, ψ

curvature: $R = d\Omega - \Omega \wedge \Omega$

$$\longrightarrow \quad R = \frac{1}{4} R^{ab} \gamma_{ab} = \frac{1}{4} R^{ab}_{\mu\nu} dx^\mu \wedge dx^\nu \gamma_{ab}$$

$$\longrightarrow \quad R^{ab} = d\omega^{ab} - \omega^a{}_c \wedge \omega^{cb}$$

covariant exterior derivative: $D\psi \equiv d\psi - \Omega\psi$

(used in Aschieri, LC (2009) for Drinfeld twist $\star$ deformation)

- Carrying out the Tr on spinor indices:

$$S = \int Tr\left(i\; R \wedge V \wedge V\gamma_5 - [(D\psi)\bar{\psi} - \psi(D\bar{\psi})] \wedge V \wedge V \wedge V\gamma_5\right) \longrightarrow$$

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} + i[\bar{\psi}\gamma^a D\psi - (D\bar{\psi})\gamma^a\psi] \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}$$

- Symmetries
  - Lorentz

$$\delta_\varepsilon V = [\varepsilon, V] \qquad \delta_\varepsilon \Omega = d\Omega + [\varepsilon, \Omega] \qquad \begin{array}{l} \delta_\varepsilon \psi = \varepsilon\psi \\ \delta_\varepsilon \bar{\psi} = -\bar{\psi}\varepsilon \end{array}$$

with $\varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab}$

Then $\delta_\varepsilon R = [\varepsilon, R]$, $\qquad \delta_\varepsilon (D\psi)\bar{\psi} = [\varepsilon, (D\psi)\bar{\psi}]$

algebra: $\quad [\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] = -\delta_{[\varepsilon_1, \varepsilon_2]}$

  - Infinitesimal diff.s

S invariant under Lie derivative $\quad \ell_v = i_v d + d i_v$

- Discrete gravity + fermions

  - Formally the same action:

$$S = \int Tr \left(i \; R \wedge V \wedge V \gamma_5 - [(D\psi)\bar{\psi} - \psi(D\bar{\psi})] \wedge V \wedge V \wedge V \gamma_5\right)$$

where now the 1-forms V and Ω are expanded on the basis of left-invariant 1-forms θ^i and on the Dirac basis of gamma matrices.

  - The gamma expansion must now include new contributions

$$V = (V_i^a \gamma_a + \tilde{V}_i^a \gamma_a \gamma_5)\theta^i \qquad \Omega = (\frac{1}{4}\omega_i^{ab}\gamma_{ab} + i\omega_i 1 + \tilde{\omega}_i \gamma_5)\theta^i$$

since the gauge variations $\delta_\varepsilon V = [\varepsilon, V]$ and $\delta_\varepsilon \Omega = d\Omega + [\varepsilon, \Omega]$ contain also anticommutators of gamma matrices.

The gauge parameter ε , however, has the same expansion as in the classical case, because functions on G commute
The gauge group is still Lorentz.

The extra gamma contributions in the connection produce extra contributions in the curvature:

$$R = (\frac{1}{4} R^{ab}_{ij} \gamma_{ab} + i r_{ij} 1 + \tilde{r}_{ij} \gamma_5)\ \theta^i \wedge \theta^j$$

- Invariances

- The action is invariant under Lorentz variations provided that the volume form commutes with functions. This is the case for $G=S_3$, but not for the 4-dim calculus of $(Z_n)^4$

- When $\int d\rho = 0$, S is invariant under Lie derivative

with a caveat: modified Leibniz rule

- Gauge variations (Lorentz)

$$\delta_\epsilon V^a = \frac{1}{2}(\varepsilon^a{}_b \star V^b + V^b \star \varepsilon^a{}_b) + \frac{i}{4}\varepsilon^a{}_{bcd}(\tilde{V}^b \star \varepsilon^{cd} - \varepsilon^{cd} \star \tilde{V}^b)$$
$$+ \varepsilon \star V^a - V^a \star \varepsilon - \tilde{\varepsilon} \star \tilde{V}^a - \tilde{V}^a \star \tilde{\varepsilon}$$

$$\delta_\epsilon \tilde{V}^a = \frac{1}{2}(\varepsilon^a{}_b \star \tilde{V}^b + \tilde{V}^b \star \varepsilon^a{}_b) + \frac{i}{4}\varepsilon^a{}_{bcd}(V^b \star \varepsilon^{cd} - \varepsilon^{cd} \star V^b)$$
$$+ \varepsilon \star \tilde{V}^a - \tilde{V}^a \star \varepsilon - \tilde{\varepsilon} \star V^a - V^a \star \tilde{\varepsilon}$$

$$\delta_\epsilon \omega^{ab} = \frac{1}{2}(\varepsilon^a{}_c \star \omega^{cb} - \varepsilon^b{}_c \star \omega^{ca} + \omega^{cb} \star \varepsilon^a{}_c - \omega^{ca} \star \varepsilon^b{}_c)$$
$$+ \frac{1}{4}(\varepsilon^{ab} \star \omega - \omega \star \varepsilon^{ab}) + \frac{i}{8}\varepsilon^{ab}{}_{cd}(\varepsilon^{cd} \star \tilde{\omega} - \tilde{\omega} \star \varepsilon^{cd})$$
$$+ \frac{1}{4}(\varepsilon \star \omega^{ab} - \omega^{ab} \star \varepsilon) + \frac{i}{8}\varepsilon^{ab}{}_{cd}(\tilde{\varepsilon} \star \omega^{cd} - \omega^{cd} \star \tilde{\varepsilon})$$

$$\delta_\epsilon \omega = \frac{1}{8}(\omega^{ab} \star \varepsilon_{ab} - \varepsilon_{ab} \star \omega^{ab}) + \varepsilon \star \omega - \omega \star \varepsilon + \tilde{\varepsilon} \star \tilde{\omega} - \tilde{\omega} \star \tilde{\varepsilon}$$

$$\delta_\epsilon \tilde{\omega} = \frac{i}{16}\varepsilon_{abcd}(\omega^{ab} \star \varepsilon^{cd} - \varepsilon^{cd} \star \omega^{ab}) + \varepsilon \star \tilde{\omega} - \tilde{\omega} \star \varepsilon + \tilde{\varepsilon} \star \omega - \omega \star \tilde{\varepsilon}$$

- When a classical limit can be defined (for ex. in the Z_n case) do the extra fields disappear in this limit ?

In the $(Z_n)^4$ a lattice spacing *a* can be introduced, and extra fields appear always multiplied by (powers of) *a*

6. Finite group discretization of Osp(1|4) supergravity

- Classical action (Mac Dowell-Mansouri)

$$S = 2i \int Tr(R \wedge R\gamma_5 + 2\Sigma \wedge \bar{\Sigma}\gamma_5)$$

OSp(1|4) connection: 5 x 5 supermatrix

$$\mathbf{\Omega} = \begin{pmatrix} \Omega & \psi \\ \bar{\psi} & 0 \end{pmatrix} \qquad \Omega = \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{i}{2}V^a\gamma_a$$

OSp(1|4) curvature

$$\mathbf{R} = d\mathbf{\Omega} - \mathbf{\Omega} \wedge \mathbf{\Omega} = \begin{pmatrix} R & \Sigma \\ \bar{\Sigma} & 0 \end{pmatrix}$$

$$R = \frac{1}{4}R^{ab}\gamma_{ab} - \frac{i}{2}R^a\gamma_a \quad \begin{cases} R^{ab} = d\omega^{ab} - \omega^a{}_c\omega^{cb} + V^aV^b + \frac{1}{2}\bar{\psi}\gamma^{ab}\psi \\ R^a = dV^a - \omega^a{}_cV^c - \frac{i}{2}\bar{\psi}\gamma^a\psi \end{cases}$$

$$\Sigma = d\psi - \frac{1}{4}\omega^{ab}\psi + \frac{i}{2}V^a\psi$$

- Action (explicit): N=1, D=4 anti de Sitter SG

$$S = \int \mathcal{R}^{ab} V^c V^d \epsilon_{abcd} + 4\bar{\rho}\gamma_a\gamma_5\psi V^a + \frac{1}{2}(V^a V^b V^c V^d + 2\bar{\psi}\gamma^{ab}\psi V^c V^d)\epsilon_{abcd}$$

After rescaling $V^a \to \lambda V^a$ $\psi \to \sqrt{\lambda}\psi$ and dividing S by λ^2

the $\lambda \to 0$ limit reproduces usual Minkowski SG

(contraction of OSp(1|4) to superPoincaré)

- Action in terms of OSp(1|4) curvature supermatrix

$$S = 4\int STr[\mathbf{R}(\mathbf{1} + \frac{\mathbf{\Gamma}^2}{2})\mathbf{R}\mathbf{\Gamma}]$$

$$\mathbf{\Gamma} = \begin{pmatrix} i\gamma_5 & 0 \\ 0 & 0 \end{pmatrix}$$

NOT invariant under OSp(1|4) gauge variations

$$\delta_\epsilon \mathbf{\Omega} = d\epsilon - \mathbf{\Omega}\,\epsilon + \epsilon\,\mathbf{\Omega} \qquad \epsilon = \begin{pmatrix} \frac{1}{4}\gamma^{ab}\varepsilon_{ab} - \frac{i}{2}\gamma^a\varepsilon_a & \epsilon \\ \bar{\epsilon} & 0 \end{pmatrix}$$

because $[\mathbf{\Gamma}, \epsilon] \neq 0$, but Lorentz inv. and supersymmetry ok

- OSp(1|4) supergravity:

$$S = 4 \int STr[\mathbf{R}(\mathbf{1} + \frac{\mathbf{\Phi}^2}{2})\mathbf{R}\mathbf{\Phi}]$$

where the auxiliary field supermatrix

$$\mathbf{\Phi} = \begin{pmatrix} \frac{1}{4}\pi + i\phi\gamma_5 + \phi^a\gamma_a\gamma_5 & \zeta \\ -\bar{\zeta} & \pi \end{pmatrix}$$

transforms as

$$\delta_\epsilon \mathbf{\Phi} = -\mathbf{\Phi}\ \epsilon + \epsilon\ \mathbf{\Phi}$$

Then the action is OSp(1|4) gauge invariant

- OSp(1|4) variations:

$$\delta\omega^{ab} = d\varepsilon^{ab} - \omega^{ac}\varepsilon^{cb} + \omega^{bc}\varepsilon^{ca} - \varepsilon^a V^b + \varepsilon^b V^a - \bar{\epsilon}\gamma^{ab}\psi$$

$$\delta V^a = d\varepsilon^a - \omega^{ab}\varepsilon^b + \varepsilon^{ab}V^b + i\bar{\epsilon}\gamma^a\psi$$

$$\delta\psi = d\epsilon - \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon + \frac{i}{2}V^a\gamma_a\epsilon + \frac{1}{4}\varepsilon^{ab}\gamma_{ab}\psi - \frac{i}{2}\varepsilon^a\gamma_a\psi$$

Twisted OSp(1|4) supergravity: LC (2013)

$$S = 4 \int STr[\mathbf{R} \star (\mathbf{1} + \frac{\mathbf{\Phi} \star \mathbf{\Phi}}{2}) \wedge_\star \mathbf{R} \star \mathbf{\Phi}]$$

- On finite group spaces: invariant under OSp(1|4) gauge group (*not* enhanced to U(1,3|1))

7. Conclusions and outlook

- so far only algebraic analysis. Need to understand physical implications
- for ex: problems of introducing fermions in lattice gauge theories.
- study field equations, solutions.
- study continuum limit
- action is a finite (discrete) sum: numerical simulations ?

Thank you !