Half-maximal gauged supergravities from 10d heterotic DFT

Based on: F. Hassler, Y. Sakatani, LS, *Generalized Dualities for Heterotic and Type I Strings*, JHEP 08 (2024), 059.

Luca Scala

Summary

Generalised dualities

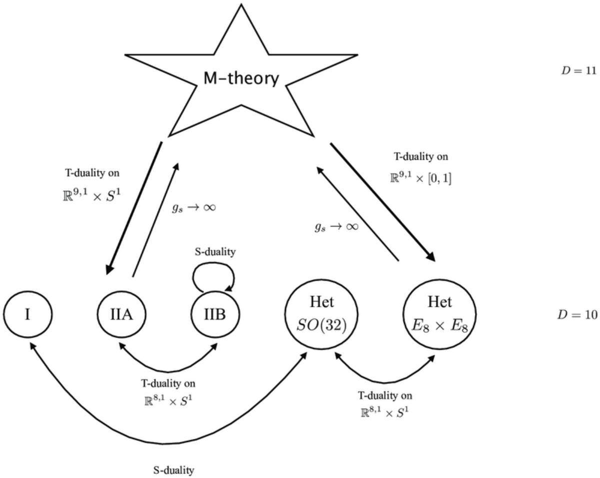

Figure – The web of generalised dualities, from A. Fontanella, *Black Horizons and Integrability in String Theory*, arXiv :1810.05434 [hep-th].

Supergravities

Superstring theories $\xrightarrow[\text{low energy limit}]{}$ SUGRAs.

Maximal [1] (32 supercharges)

- M-theory
- Type IIA
- Type IIB

Half-maximal (16 supercharges)

- Heterotic $E_8 \times E_8$
- Heterotic SO(32)
- Type I.

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10-dim SUGRA $\underset{\text{uplifts}}{\overset{\text{consistent truncations}}{\rightleftarrows}}$ (10-d)-dim (g)SUGRA.

What are there conditions allowing this procedure ?
How do we describe dualities in the lower dimensional theory ?

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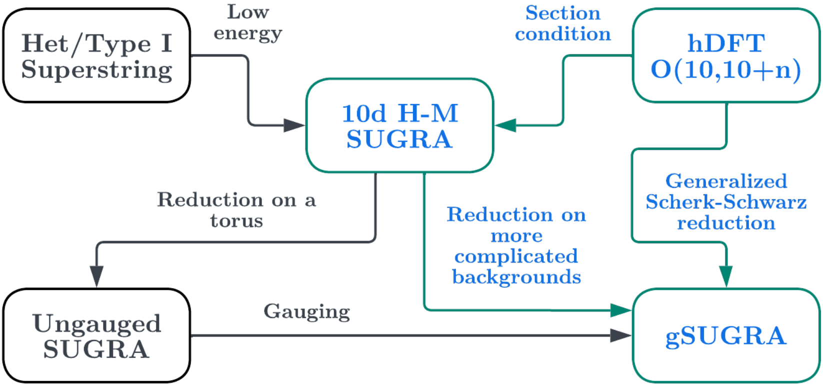
From hDFT to 10d Half-Maximal SUGRA



Heterotic Double Field Theory (hDFT)

10d hDFT : $\mathrm{O}(10, 10+n)$-covariant field theory defined by [2] :

$$\mathcal{L} = \mathrm{e}^{-2d}\Big(-\tfrac{1}{12}\, H^{\mathbb{A}\mathbb{D}}\, H^{\mathbb{B}\mathbb{E}}\, H_{\mathbb{C}\mathbb{F}}\, \hat{F}_{\mathbb{A}\mathbb{B}}{}^{\mathbb{C}}\, \hat{F}_{\mathbb{D}\mathbb{E}}{}^{\mathbb{F}} - \tfrac{1}{4}\, H^{\mathbb{A}\mathbb{B}}\, \hat{F}_{\mathbb{A}\mathbb{D}}{}^{\mathbb{C}}\, \hat{F}_{\mathbb{B}\mathbb{C}}{}^{\mathbb{D}} + \\ + H^{\mathbb{A}\mathbb{B}}\, F_{\mathbb{A}} F_{\mathbb{B}} - \tfrac{1}{6}\, \hat{F}_{\mathbb{A}\mathbb{B}}{}^{\mathbb{C}}\, \hat{F}^{\mathbb{A}\mathbb{B}}{}_{\mathbb{C}} - 2\, D_{\mathbb{A}} F^{\mathbb{A}} + F_{\mathbb{A}}\, F^{\mathbb{A}}\Big)\,, \qquad (1)$$

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where

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$F_{\mathbb{A}} := 2\, D_{\mathbb{A}} d - \partial_{\mathbb{J}} E_{\mathbb{A}}{}^{\mathbb{J}}$, $\hat{F}_{\mathbb{A}\mathbb{B}}{}^{\mathbb{C}} := F_{\mathbb{A}\mathbb{B}}{}^{\mathbb{C}} + E_{\mathbb{A}}{}^{\mathbb{I}} E_{\mathbb{B}}{}^{\mathbb{J}} E_{\mathbb{K}}{}^{\mathbb{C}} \Sigma_{\mathbb{I}\mathbb{J}}{}^{\mathbb{K}}$,

$F_{\mathbb{A}\mathbb{B}\mathbb{C}} := -3\, D_{[\mathbb{A}} E_{\mathbb{B}}{}^{\mathbb{I}} E_{|\mathbb{I}|\mathbb{C}]}$ (generalized fluxes),

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&F_{\mathbb{A}\mathbb{B}\mathbb{C}} := -3\, D_{[\mathbb{A}} E_{\mathbb{B}}{}^{\mathbb{I}}\, E_{|\mathbb{I}|\mathbb{C}]} \qquad \text{(generalized fluxes)}, \qquad (2)\\
&E_{\mathbb{A}}{}^{\mathbb{I}} \in \mathrm{O}(10, 10+n) \qquad \text{(generalized frame)},\\
&D_{\mathbb{A}} := E_{\mathbb{A}}{}^{\mathbb{I}}\, \partial_{\mathbb{I}} \qquad \text{(flat derivative)},\\
&\Sigma_{\mathbb{I}\mathbb{J}}{}^{\mathbb{K}} \qquad \text{constant torsion term.}
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From 10-dim hDFT to 10-dim SUGRA (1)

To be well defined we have to impose the constraint

$$\partial^{\mathbb{I}} \cdot \partial_{\mathbb{I}} \cdot = 0, \tag{3}$$

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To obtain 10-dim half-maximal SUGRA we branch indices :

$$\mathrm{O}(10, 10+n) \to \mathrm{GL}(10) \times \mathcal{G}. \tag{4}$$

The constraints are solved by the canonical section

$$\partial_{\mathbb{I}} = \begin{pmatrix} \partial_m & \partial_{\mathcal{I}} & \partial^m \end{pmatrix} = \begin{pmatrix} \partial_m & 0 & 0 \end{pmatrix} , \tag{5}$$

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$$\Sigma_{\mathbb{IJ}}{}^{\mathbb{K}} = \begin{cases} f_{\mathcal{I}\mathcal{J}}{}^{\mathcal{K}} & \text{structure constants of } Lie(\mathcal{G}), \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

From 10-dim hDFT to 10-dim SUGRA (2)

Standard parametrization

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\begin{aligned}
E_{\mathbb{A}}{}^{\mathbb{I}} &= E_{\mathbb{A}}{}^{\mathbb{I}}(e, A, B_2)\,, \\
\mathrm{e}^{-2d} &:= \mathrm{e}^{-2\Phi}\sqrt{-g}\,,
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we find the 10-dim half-maximal SUGRA action

$$
S = \int \mathrm{d}^{10}x\ \sqrt{-g}\,\mathrm{e}^{-2\Phi}\Big(R + 4\,\partial_m\Phi\,\partial^m\Phi - \tfrac{1}{12}\,\hat{H}_{mnp}\,\hat{H}^{mnp} - \tfrac{1}{4}\,\kappa_{\mathcal{I}\mathcal{J}}\,F_{mn}{}^{\mathcal{I}}\,F^{mn\mathcal{J}}\Big)\,, \tag{8}
$$

with field strengths

$$
\begin{aligned}
\hat{H}_3 &:= \mathrm{d}B_2 - \tfrac{1}{2}\,\kappa_{\mathcal{I}\mathcal{J}}\,A^{\mathcal{I}} \wedge \mathrm{d}A^{\mathcal{J}} - \tfrac{1}{3!}\,f_{\mathcal{I}\mathcal{J}\mathcal{K}}\,A^{\mathcal{I}} \wedge A^{\mathcal{J}} \wedge A^{\mathcal{K}}\,, \\
F_2^{\mathcal{I}} &:= \mathrm{d}A^{\mathcal{I}} + \tfrac{1}{2}\,f_{\mathcal{J}\mathcal{K}}{}^{\mathcal{I}}\,A^{\mathcal{J}} \wedge A^{\mathcal{K}}\,.
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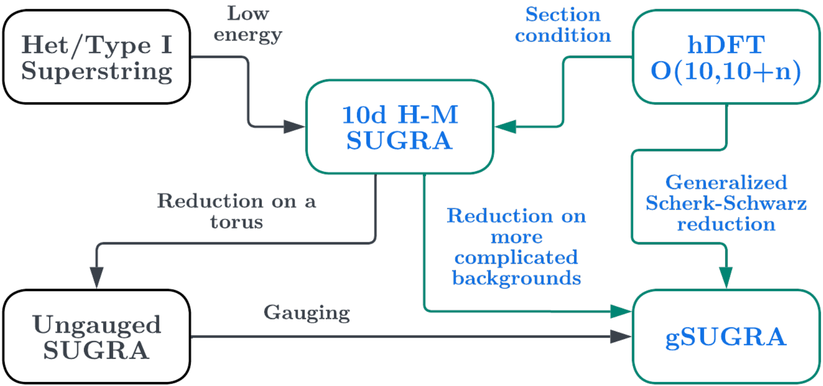
Reduction to lower dimensional gSUGRAs



Preparing the lower dimensional reduction

We want to reduce on an internal d-dimensional space. We perform the following splitting of indices[3]

$$x^{\mathbb{I}} = \begin{pmatrix} x^{\mu} & x^{I} & x_{\mu} \end{pmatrix}, \qquad \text{with} \qquad x^{I} := \begin{pmatrix} x^{i} & x^{\mathcal{I}} & x_{i} \end{pmatrix}. \tag{10}$$

The decomposition reads

$$\begin{aligned} \mathrm{O}(10, 10+n) &\to \mathrm{GL}(10-d) \times \mathrm{O}(d, d+n) & (11) \\ &\to \mathrm{GL}(10-d) \times \mathrm{GL}(d) \times \mathcal{G}\,. & (12) \end{aligned}$$

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Standard parametrization for $E_{\mathbb{A}}{}^{\mathbb{I}}$ and e^{-2d}.

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Embedding tensor

Doing dimensional reduction, a subgroup G of the global duality group

$$\mathrm{G}_D = \tilde{\mathrm{G}}_D(d) \times \mathrm{O}(d, d+n) \qquad \text{with} \qquad \tilde{\mathrm{G}}(d) = \begin{cases} \mathbb{R}^+ & 0 < d \leq 5 \\ \mathrm{SL}(2) & d = 6 \end{cases} \tag{13}$$

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Half-maximal gSUGRAs in $10 - d$ dimensions are classified by the embedding of their gauge group G into G_D [4] $\rightarrow$ embedding tensor $X_{\hat{A}\hat{B}}{}^{\hat{C}}$.

$X_{\hat{A}\hat{B}}{}^{\hat{C}}$ coming from reduction of the 10-dim theory $\rightarrow$ geometric gaugings.

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$$X_{\hat{A}\hat{B}}{}^{\hat{C}} \xrightarrow[\text{IRREP decomp.}]{} \begin{cases} \{F_{ABC},\, \xi_A\} & d \leq 4\,, \\ \{F_{ABC},\, \xi_A,\, \underline{\boldsymbol{\vartheta}_A,\, \xi_{AB},\, \boldsymbol{\vartheta}_*}\} & d = 5\,, \\ \{F_{\underline{\alpha}ABC},\, \xi_{\underline{\alpha}A},\, \underline{\boldsymbol{\vartheta}_{\alpha A}}\} & d = 6\,. \end{cases} \tag{14}$$

Latin indices are in $\mathrm{O}(d, d+n)$, $*$ in $\mathbb{R}^+$ and $\alpha = +, -$ in $\mathrm{SL}(2)$.

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Geometric gaugings

$d = 5$: we have to supplement to the section condition [5]

$$\partial_* = 0 \longrightarrow \xi_{AB} = \boldsymbol{\vartheta}_* = 0. \tag{15}$$

$d = 6$: we have to supplement to the section condition

$$\partial_{-I} = 0 \longrightarrow F_{-ABC} = 0. \tag{16}$$

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Then from G_D frames (gen. paralleliz.)

$$\mathcal{L}_{E_{\hat{A}}} E_{\hat{B}}{}^{\hat{I}} = -X_{\hat{A}\hat{B}}{}^{\hat{C}}\, E_{\hat{C}}{}^{\hat{I}}, \tag{17}$$

we obtain $\mathrm{O}(d, d+n)$ frames

$$\mathcal{L}_{E_A} E_B{}^I = -X_{AB}{}^C\, E_C{}^I. \tag{18}$$

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Truncating of the 10d action

Then Scherk-Schwarz reduction to truncate.

$$F_A \propto \left(2\, D_A \Phi - \xi_A - \partial_I E_A{}^I\right), \qquad F_{ABC} = X_{[ABC]}\,. \tag{19}$$

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If the trombone gauging ($\boldsymbol{\vartheta}$'s) vanish : computing the action it splits into

$$S = \int_{ext} \mathrm{d}^{10-d}x\, \mathcal{L}_{\text{ext}} \int_{int} \mathrm{d}^d x\, v\,, \tag{20}$$

v : scalar density $\longrightarrow$ left-invariant integration measure.

We can, then, truncate the internal part.

Frames and generalised dualities

From the geometric gaugings we define the geometric (Leibniz) algebra

$$T_{\hat{A}} \circ T_{\hat{B}} = X_{\hat{A}\hat{B}}{}^{\hat{C}}\, T_{\hat{C}}\,, \qquad (T_{\hat{A}})_{\hat{B}}{}^{\hat{C}} = -X_{\hat{A}\hat{B}}{}^{\hat{C}}\,. \tag{21}$$

This algebra contains the Lie subalgebra $Lie(\mathrm{G})$ (remember : embedding tensor tells how G is embedded in G_D).

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We choose a subgroup H of G, and construct frames in G/H, starting from the parametrization

$$E_{\hat{A}}{}^{\hat{I}} = M_{\hat{A}}{}^{\hat{B}}\, V_{\hat{B}}{}^{\hat{J}}\, N_{\hat{J}}{}^{\hat{I}}\,, \qquad (M^{-1})_{\hat{A}}{}^{\hat{C}}\, \mathrm{d}M_{\hat{C}}{}^{\hat{B}} = -v^{\hat{C}}\, X_{\hat{C}\hat{A}}{}^{\hat{B}}\,, \tag{22}$$

$$N_{\hat{I}}{}^{\hat{J}} := \left[\exp\!\left(-\tfrac{1}{2!}\,\mathfrak{b}_{ij}\, R^{ij}\right)\, \exp\!\left(-\mathfrak{a}_k^{\mathcal{I}}\, R_{\mathcal{I}}^k\right)\right]_{\hat{I}}{}^{\hat{J}}\,. \tag{23}$$

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From the geometric gaugings it is possible to construct all these quantities. Choosing a different H results in the same physics : generalized dualities.

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4. we found the frames corresponding to the dimensional-reduced gSUGRAs and analysed generalised dualities coming from G/H construction ;
5. (In the paper : explicit examples : constructing frames from random gaugings and an explicit example of a generalised duality. Analysed obstructions coming from the trombone gauging.)

Thanks for your attention !

Bibliography :

1 F. Hassler, Y. Sakatani, LS, *Generalized Dualities for Heterotic and Type I Strings*, JHEP 08 (2024), 059 ; 2312.16283 [hep-th].

2 F. Hassler, Y. Sakatani, *All maximal gauged supergravities with uplift*, PTEP 2023 (2023) 8, 083B07.

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5 J. Schon, M. Weidner, *Gauged N=4 supergravities*, JHEP 05 (2006).

6 E. Malek, *Half-Maximal Supersymmetry from Exceptional Field Theory*, Fortsch.Phys. 65 (2017) 10-11, 1700061.

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