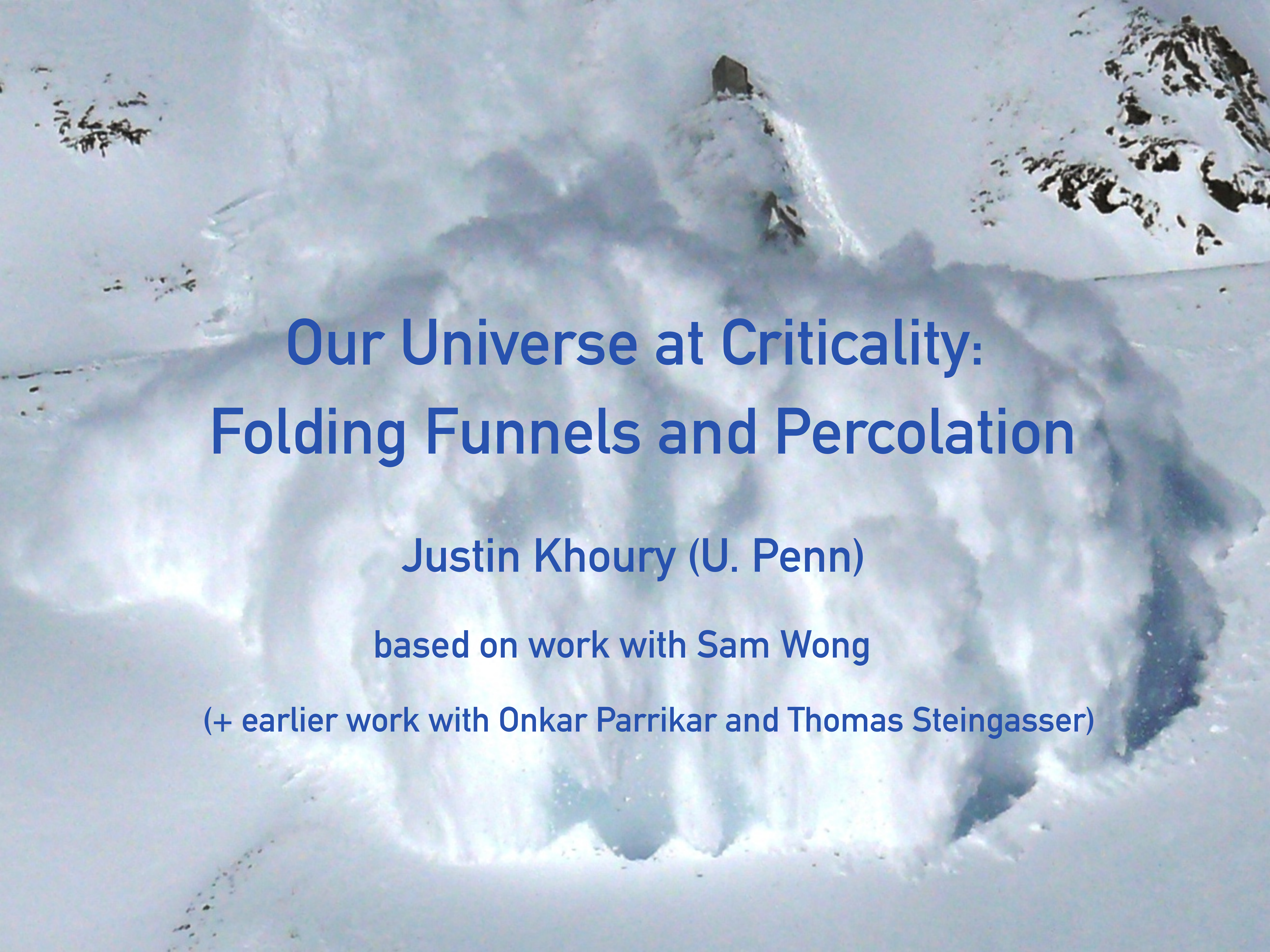


Our Universe at Criticality: Folding Funnels and Percolation

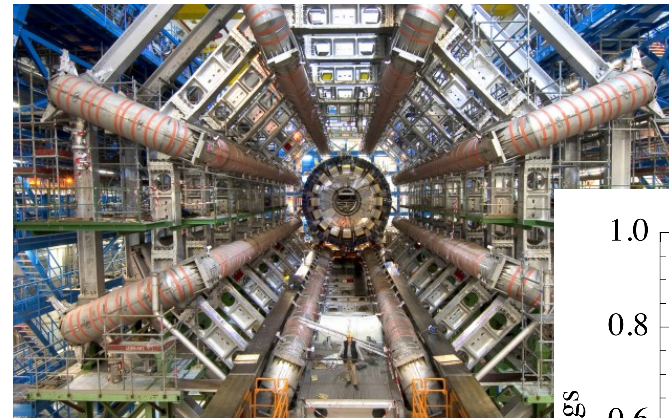
Justin Khoury (U. Penn)

based on work with Sam Wong

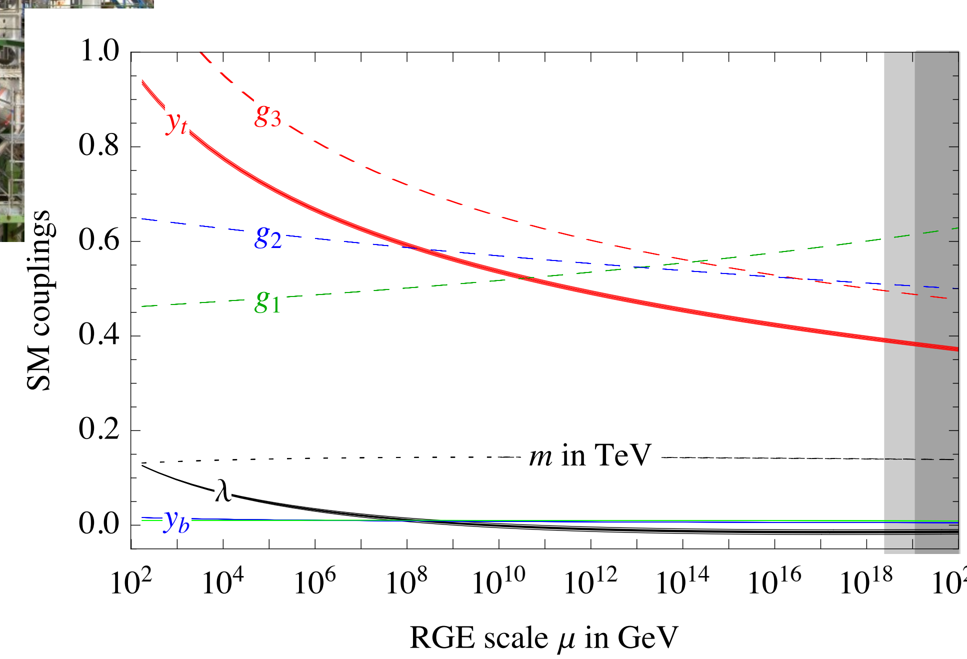
(+ earlier work with Onkar Parrikar and Thomas Steingasser)

WHY IS OUR UNIVERSE SIMPLE/MINIMAL?

Particle physics



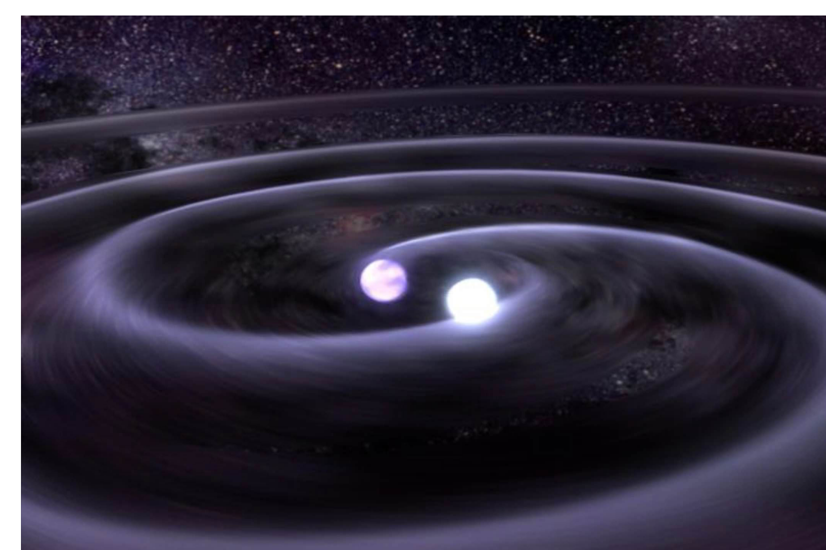
- Discovery of 125 GeV Higgs boson marks successful completion of SM
- No convincing sign of new physics at LHC
- Standard Model perturbative up to Planck scale



$\Longrightarrow$ What explains the weak scale?

Gravity

- GR stands triumphant, now tested in strong-field regime
- Intriguing tensions in cosmology
  (Hubble/S_8 tensions, recent hints of dynamical dark energy)
- Cosmic acceleration appears to be driven by a cosmological constant or vacuum energy



$\Longrightarrow$ What explains the magnitude of the vacuum energy?

Enticing opportunity is that the answer lies in **cosmology.**

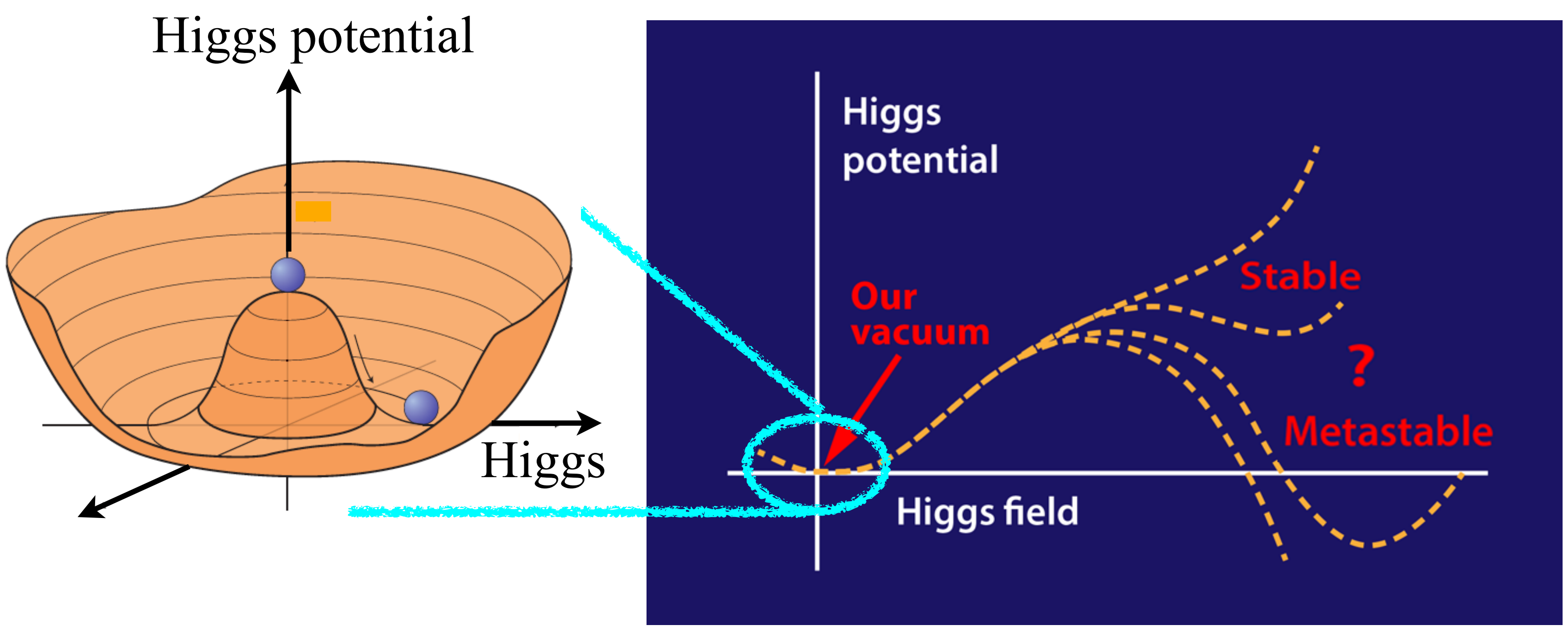
The idea that various physical parameters (Higgs mass, vacuum energy etc.) have reached their current values through **dynamical evolution** has a long and venerable history.

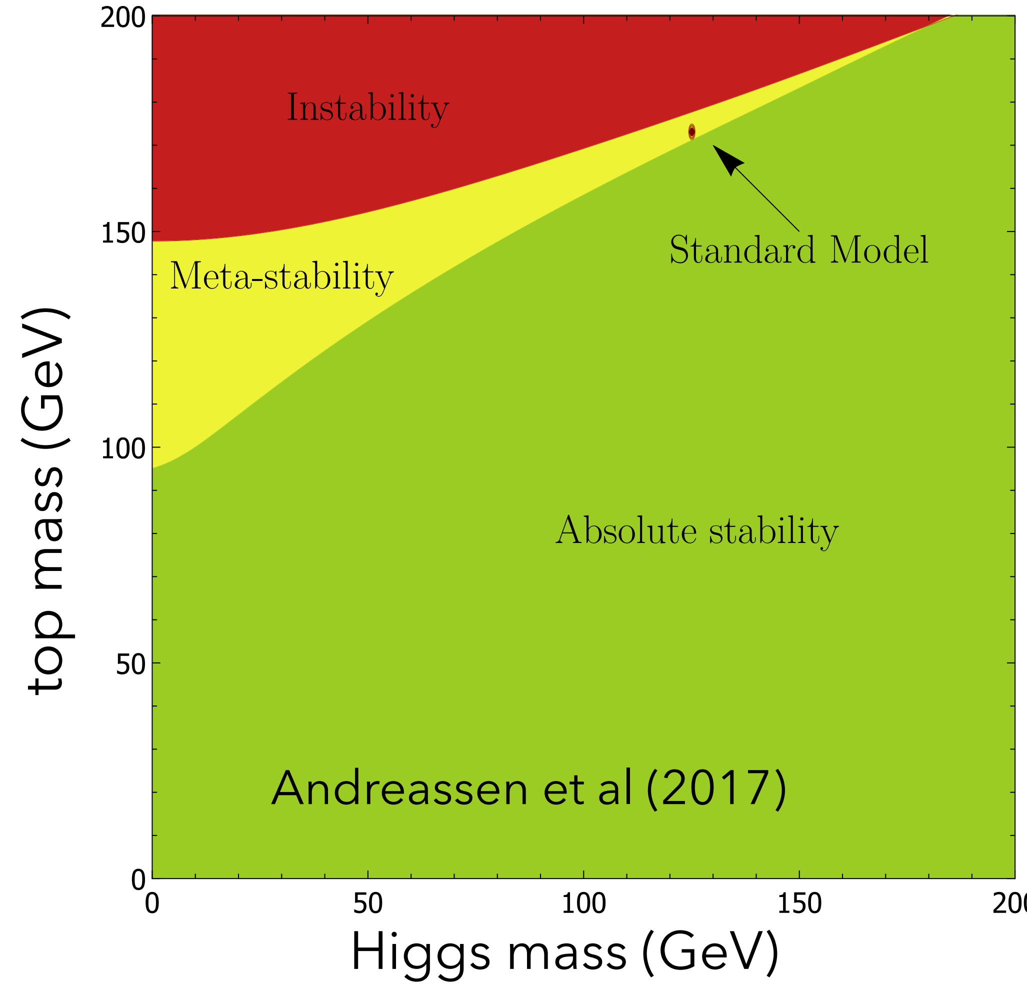
- Abbott's relaxation of the CC (1985)
- Dvali & Vilenkin (2003)
- Damour & Polyakov (1994)
- Graham et al. Relaxion mechanism (2015)
- Giudice, McCullough & You (2021)

I believe there are hints in the data that our universe has a **statistical origin.**

NEAR CRITICALITY OF OUR VACUUM

- Since Standard Model remains perturbative, can e





- Surprising consequence of extrapolation:

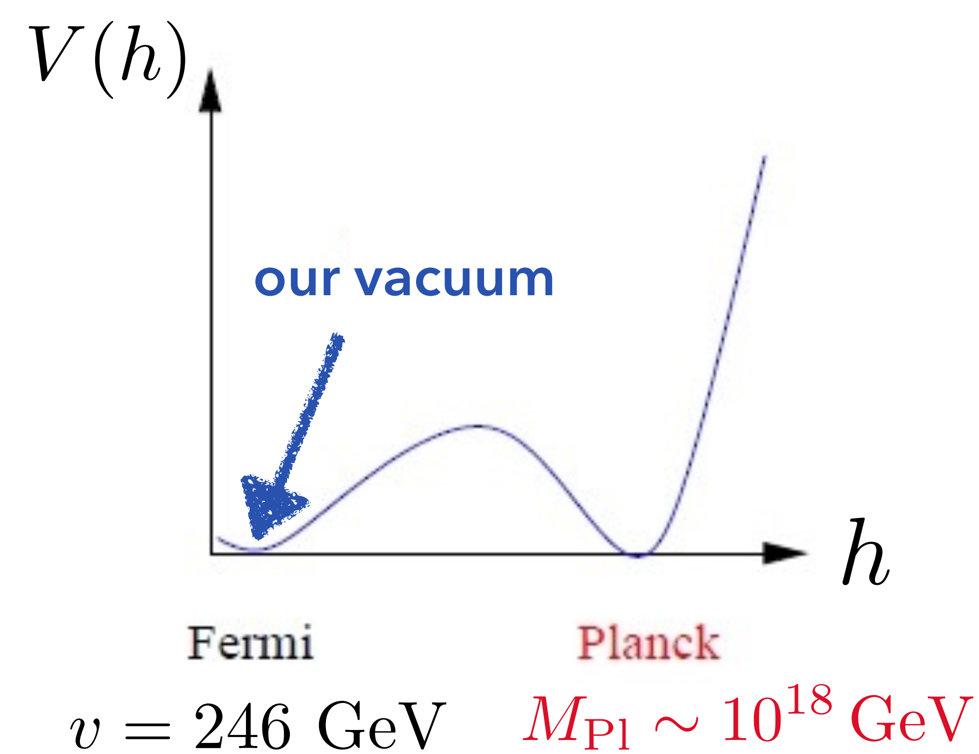
near-criticality of our vacuum

Exquisitely sensitive to **top quark** and **Higgs masses.**

Froggatt & Nielsen (1995)
("Multiple Point Principle")

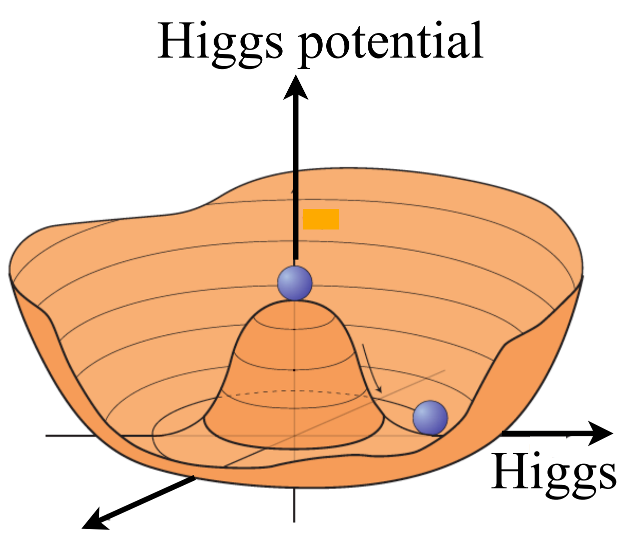
$$m_{\rm h} = 135 \pm 9 \ {\rm GeV}$$
$$m_{\rm t} = 173 \pm 5 \ {\rm GeV}$$

Remarkable that Standard Model (without gravity) "knows" about Planck scale!

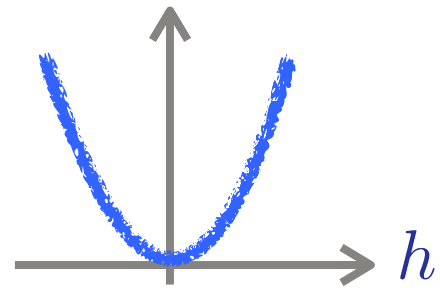


Remarkably, other fine-tuning problems can be thought of as **problems of near criticality.**

- Hierarchy problem Giudice & Rattazzi (2006)

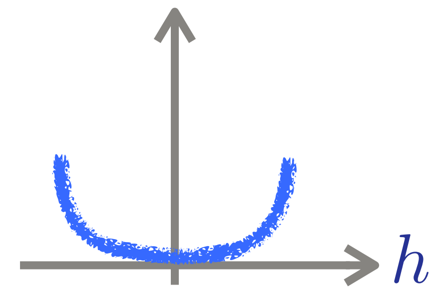


$m_h^2 \simeq M_{\rm Pl}^2$



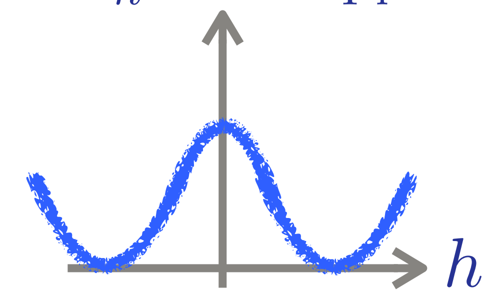
$\langle h \rangle = 0$

$m_h^2 \simeq 0$



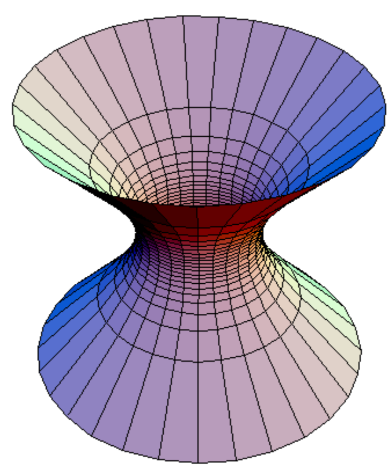
$\langle h \rangle = 246\ \text{GeV}$

$m_h^2 \simeq -M_{\rm Pl}^2$



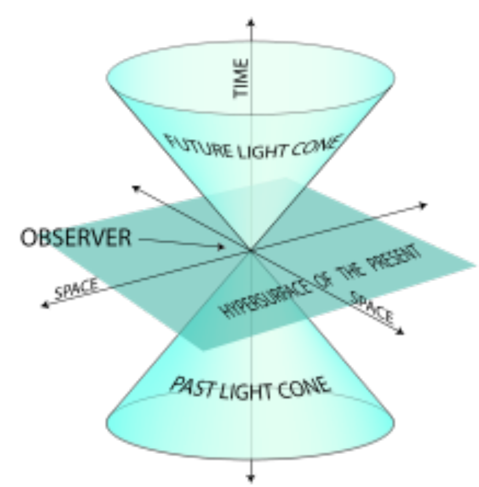
$\langle h \rangle \sim M_{\rm Pl}$

- Cosmological constant



de Sitter

Expands forever



Minkowski

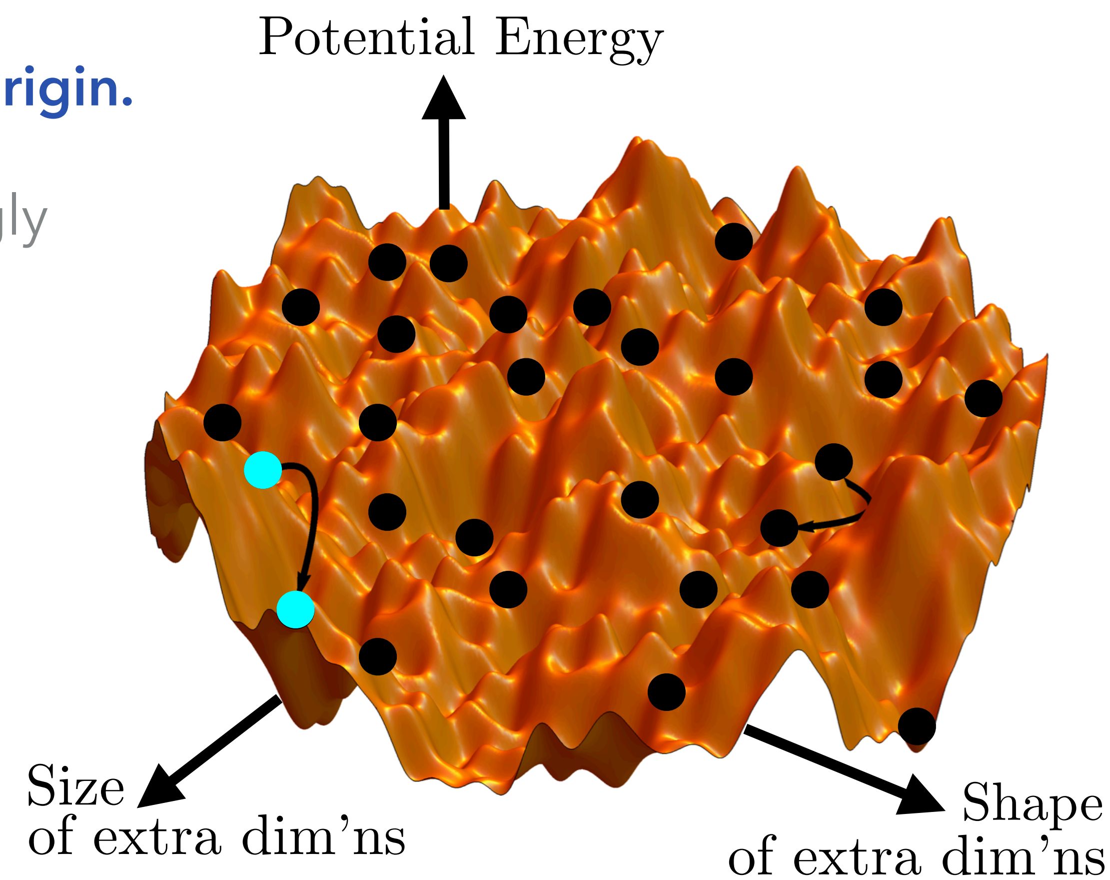
$\Lambda_{\rm obs} \simeq 10^{-122}\, M_{\rm Pl}^4$

Anti de Sitter

Big crunch

Near-criticality of our universe suggests a **statistical physics origin.**

Two major theoretical developments have led to the seemingly inescapable conclusion that we are part of a vast **multiverse.**



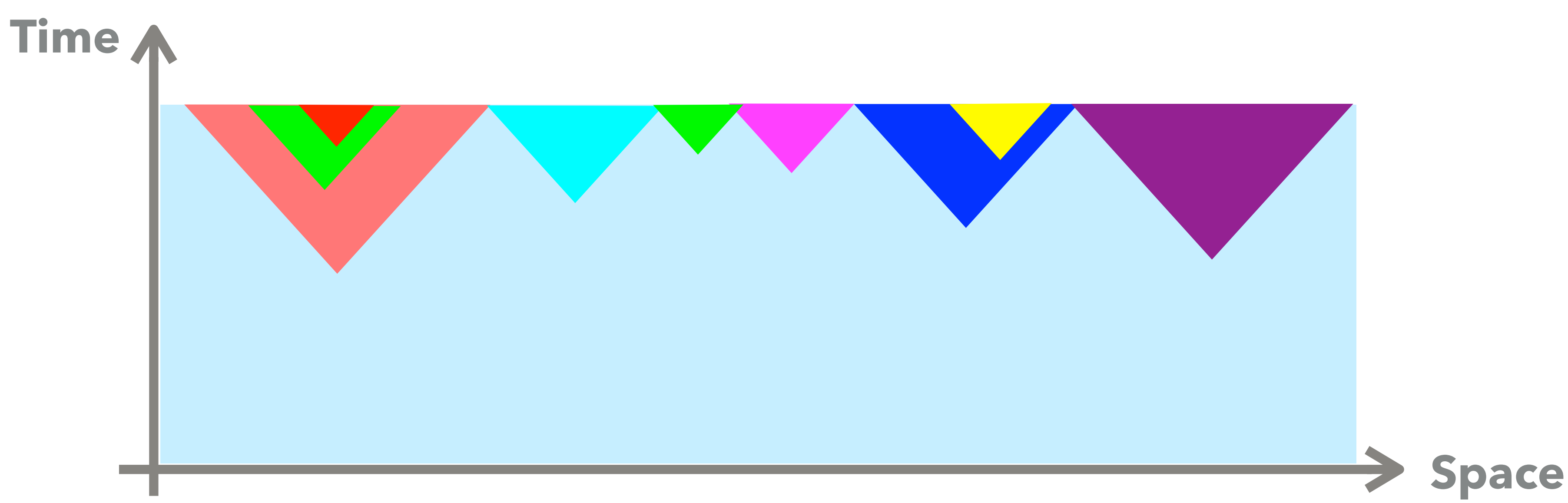
String landscape

- **Myriad metastable states (or "vacua"),** each with different force laws, particle content etc.
- Much remains to be understood about this landscape **("swampland" program).**

We currently inhabit one such state, but which one?

Eternal inflation

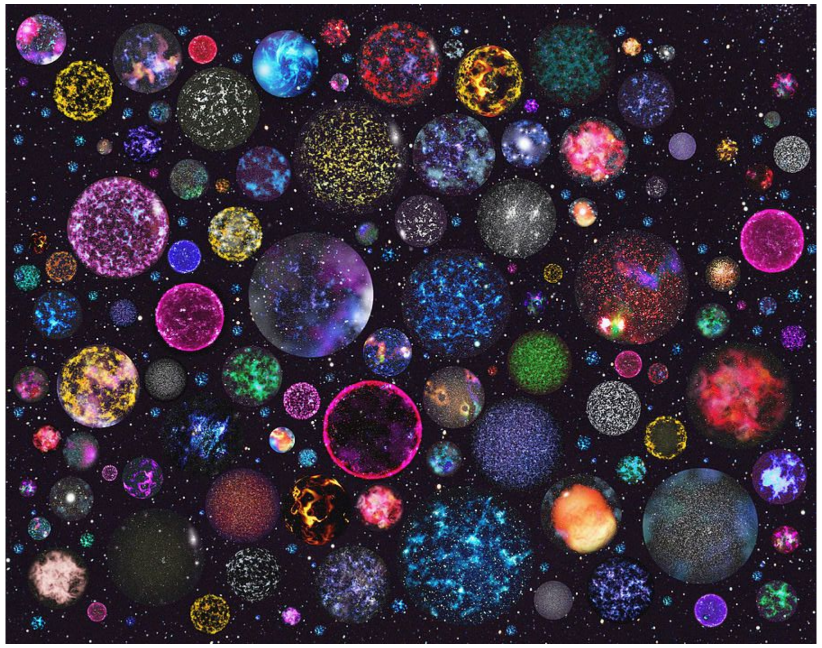
- Dynamical mechanism to instantiate in space-time the different metastable states



How should we reason probabilistically as inhabitant of multiverse?

A CONTROVERSIAL SUBJECT

"Occam's razor": "Multiverse conjectures myriad other universes, and is unnecessarily complicated."



- Energy landscape
- Eternal inflation
- Probability distribution over states

- Random mutations
- Heredity
- Natural selection (fitness function)

Judge a theory by parsimony of its ingredients, not by the richness of its outcomes.

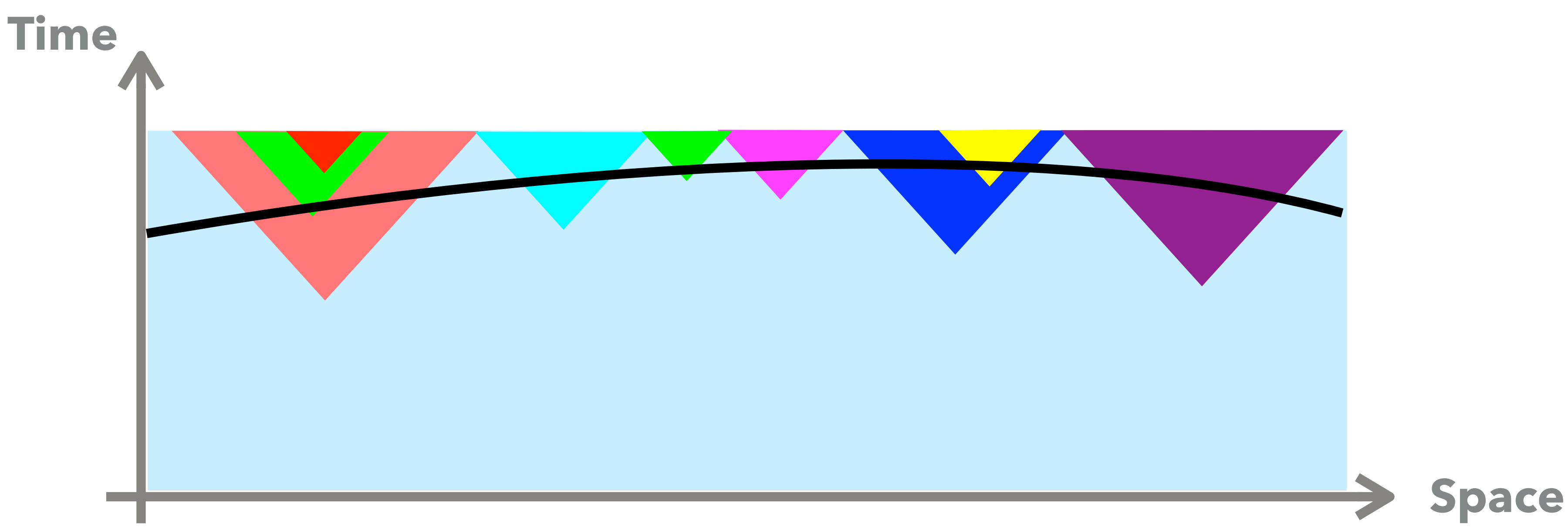
Falsifiability: "Not experimentally testable, because existence of other universes can never be proven."

Most theories make predictions that can never be tested (e.g. interior of black holes in GR).

The important point is to make **some predictions** that are **testable.**

The onus is on multiverse proponents to make predictions for our own universe.

Traditionally, probabilities in eternal inflation are defined in terms of **frequencies (e.g. counting bubbles)**

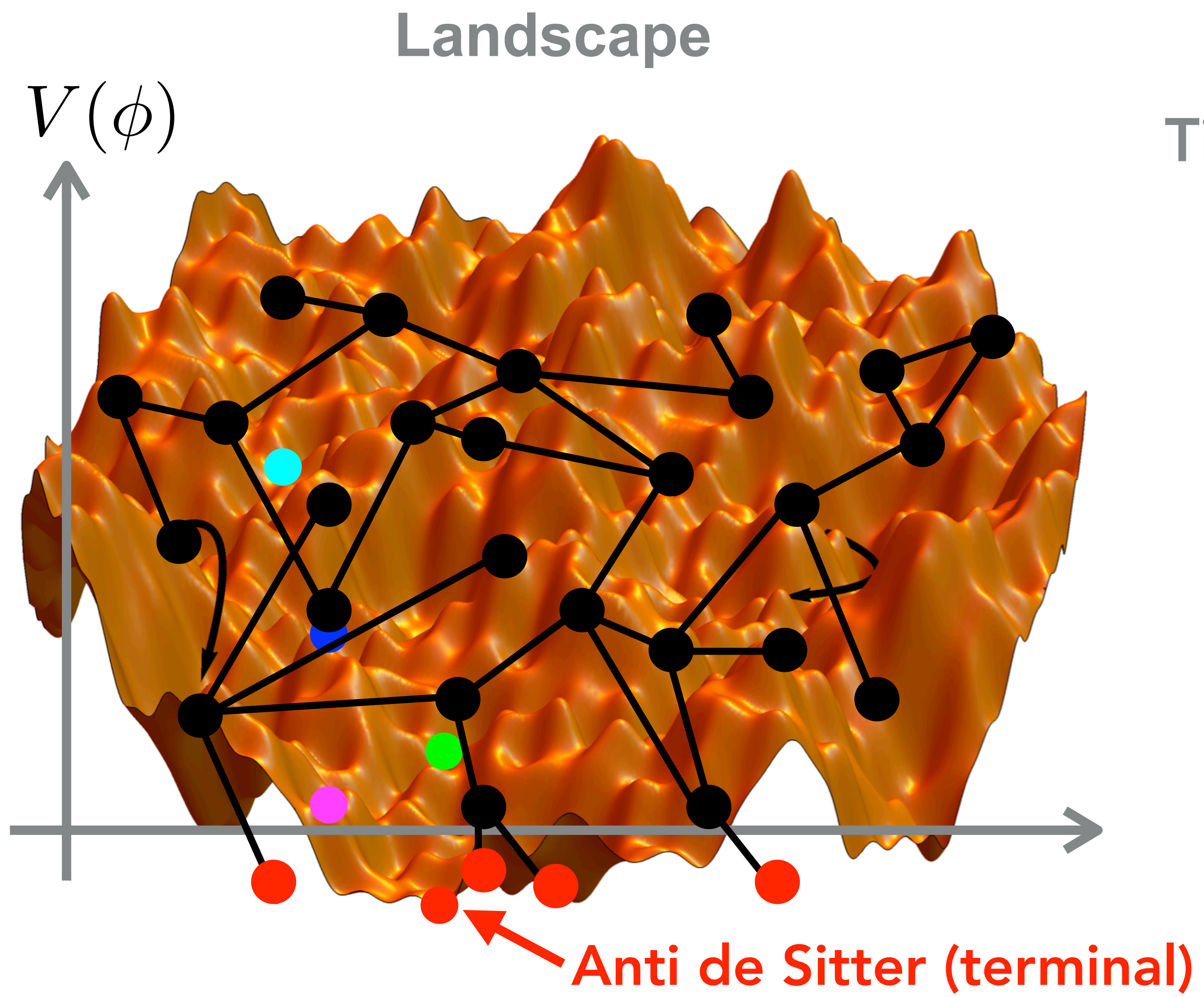


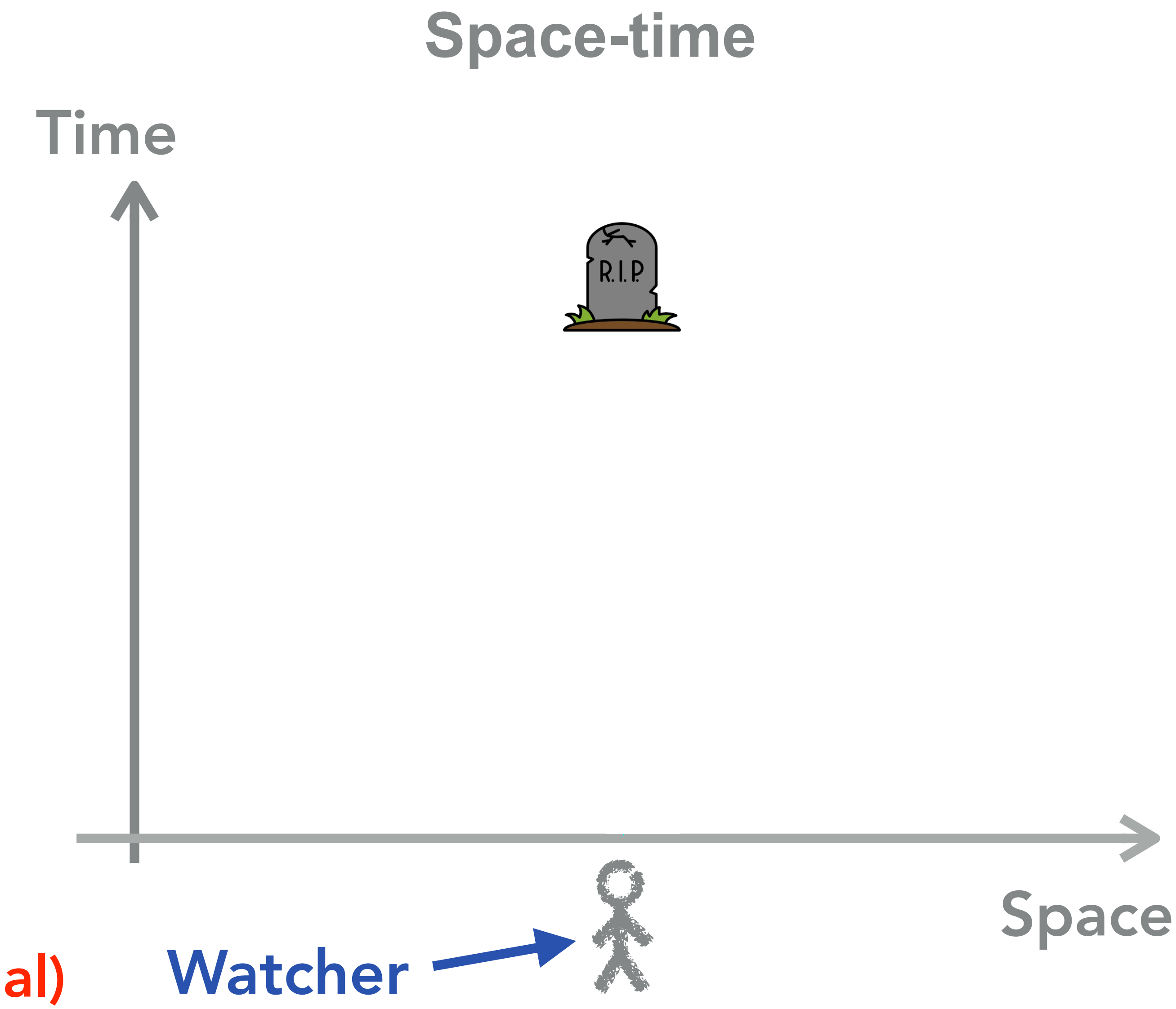
But this is **rife with ambiguities** $\Longrightarrow$ **Measure problem**

Sam Wong and I recently proposed a systematic approach, based on **Bayesian reasoning.**

JK & Wong, 2205.11524

A random walk in the landscape





Along world-line, probability $f_I(t)$ to occupy vacuum I as function of time satisfies a **master equation**

Landscape dynamics = Random walk on network of states

$$\frac{\mathrm{d}f_I}{\mathrm{d}t} = \sum_J \kappa_{IJ} f_J - \sum_K \kappa_{KI} f_I$$

Garriga & Vilenkin (1998)

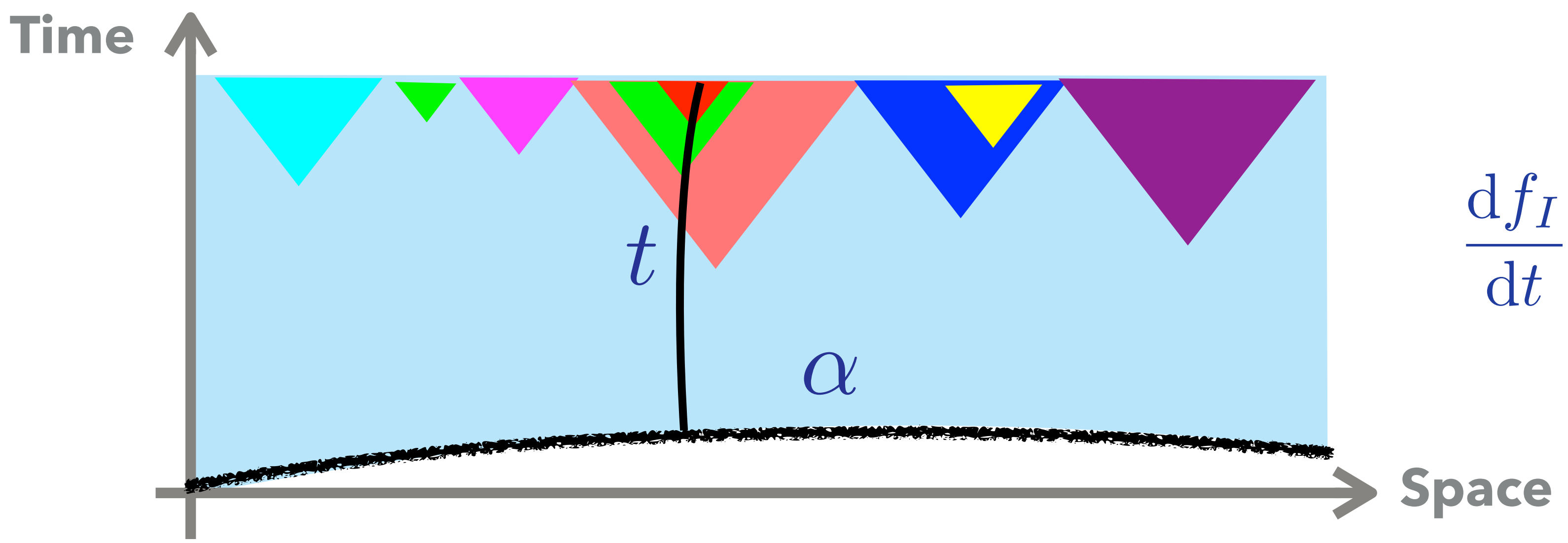
$J \to I$ transition rate

- de Sitter (positive energy) states = transient nodes
- Anti de Sitter (negative energy) states = terminal/absorbing nodes **(Big crunch)**

Thus landscape dynamics reduce to a **linear Markov process.**

JK & Wong, 2205.11524

A unique solution requires **2 pieces of prior information**, both pertaining to **initial conditions**.



$$\frac{\mathrm{d} f_I}{\mathrm{d}t} = \sum_J \kappa_{IJ} f_J - \sum_K \kappa_{KI} f_I$$

$J \rightarrow I$ transition rate

- Eternal inflation is not eternal in the past — Borde, Guth & Vilenkin (2003)

  $\Longrightarrow$ started a **finite time** t in our past, but **we don't know how long ago.**

- Along our past world-line, eternal inflation started out in some **primordial vacuum** α, but **we don't know which one.**

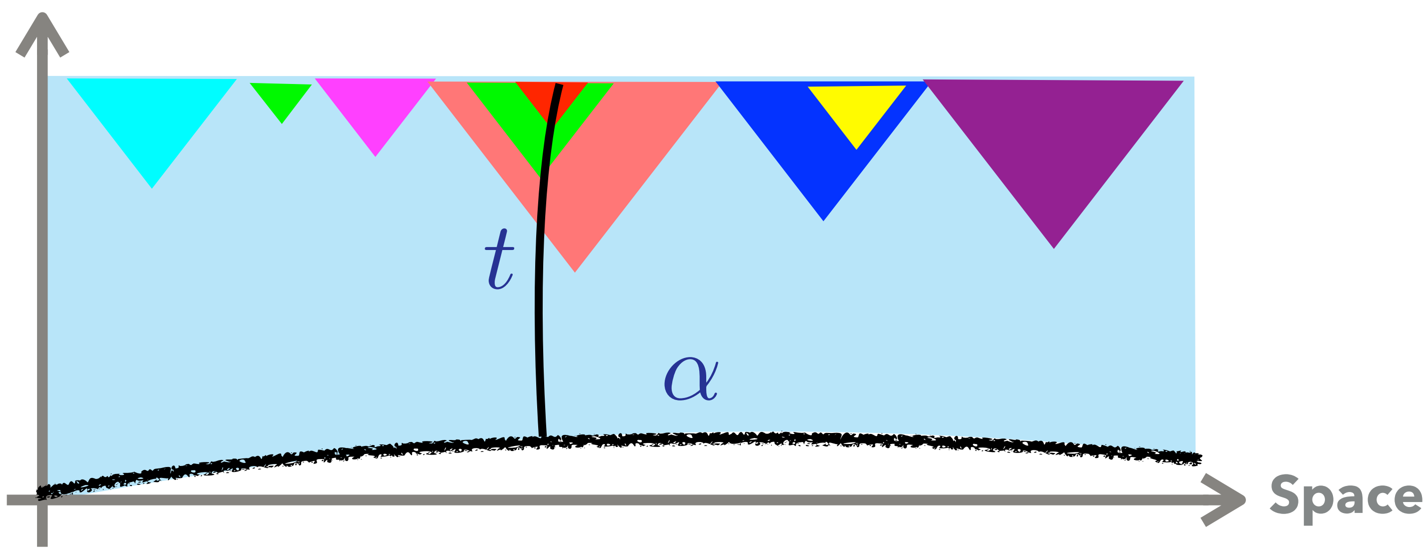
Two model parameters:

1. **time of existence** t $\Longrightarrow$ **prior density** $\rho(t)$
2. **primordial vacuum** α $\Longrightarrow$ **prior** p_α **over initial dS**

Different probability distributions (measures) correspond to different priors.

IGNORANCE IS BLISS

Priors should reflect all information at hand, but otherwise **minimally informative**.



E.T. Jaynes

Primordial vacuum:

$$p_\alpha = \frac{1}{N_{\mathrm{dS}}}$$

Laplace's principle of indifference

Time of existence: Master equation $\frac{\mathrm{d}f_I}{\mathrm{d}t} = \sum_J \kappa_{IJ} f_J - \sum_K \kappa_{KI} f_I$ is **time-translation invariant.**

(To be precise, invariant under translations in **proper time** or **e-folding time.**)

$$\Longrightarrow \quad \rho(t) = \mathrm{const.}$$

Uniform prior

These priors are minimally informative and consistently reflect our ignorance.

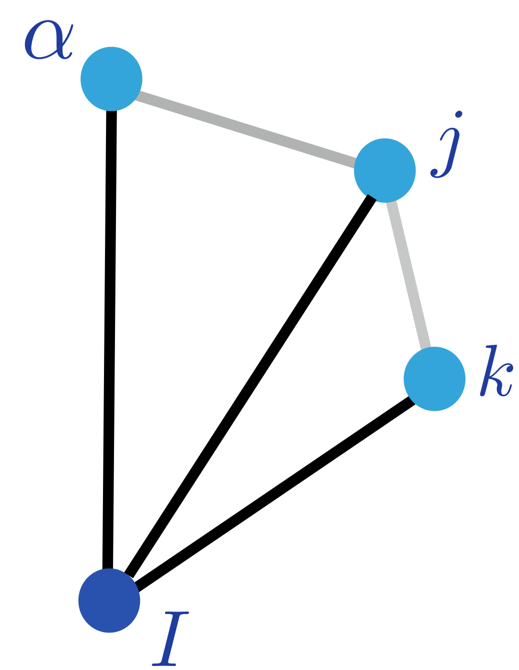
Marginalizing over parameters gives the desired **probability distribution over vacua:**

$$P(I) \sim \sum_{\alpha} \left(T_{I\alpha} + \sum_{j} T_{Ij} T_{j\alpha} + \sum_{j,k} T_{Ik} T_{kj} T_{j\alpha} + \ldots \right)$$

where $T_{Ij} = \dfrac{\kappa_{Ij}}{\sum_K \kappa_{Kj}}$

branching ratio

Probability to occupy I, irrespective of time of existence t and primordial vacuum α.



$P(I) =$ **Total branching probability, summed over all walks that reach I.**

Can be summed up succinctly:

$$P(I) \sim \sum_{\alpha,j} T_{Ij} \underbrace{(\mathbb{1} - T)^{-1}_{j\alpha}}_{\text{fundamental matrix}}$$

JK & Wong, 2205.11524

Probabilities favor vacua that are well-connected to very large number of ancestors.

Folding funnels

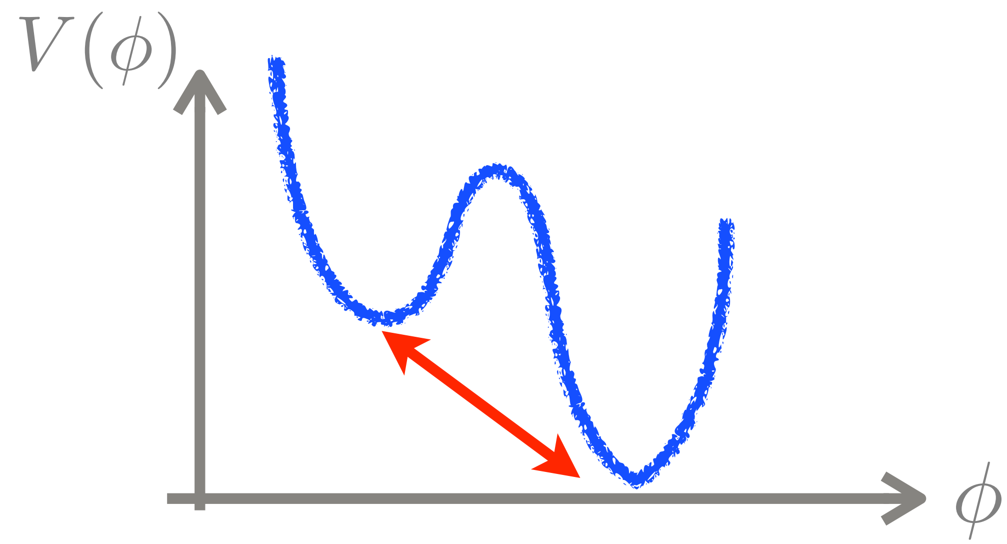
DOWNWARD APPROXIMATION

- **Transitions rates** κ_{IJ} between vacua are mediated by **instantons,** e.g. Coleman-De Luccia, Hawking-Moss, Brown-Teitelboim.

- Importantly, these instantons between dS vacua satisfy **"detailed balance":**

$$\frac{\kappa_{\text{up}}}{\kappa_{\text{down}}} \sim \mathrm{e}^{\underbrace{-24\pi^2 M_{\text{Pl}}^4 \left(\frac{1}{V_{\text{low}}} - \frac{1}{V_{\text{high}}}\right)}_{\text{difference in dS entropy}}}$$

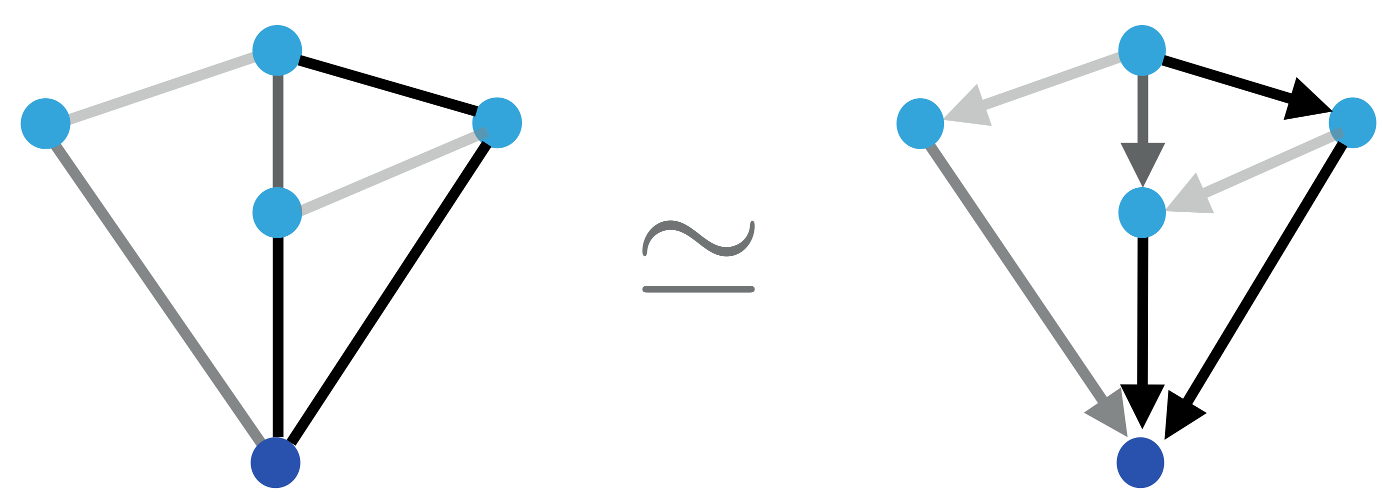
Lee & Weinberg (1987)



$\Longrightarrow$ Can **ignore upward jumps** to leading order **(downward approximation)**

Schwartz-Perlov & Vilenkin ('06); Olum & Schwartz-Perlov ('07)

Graphs become directed:



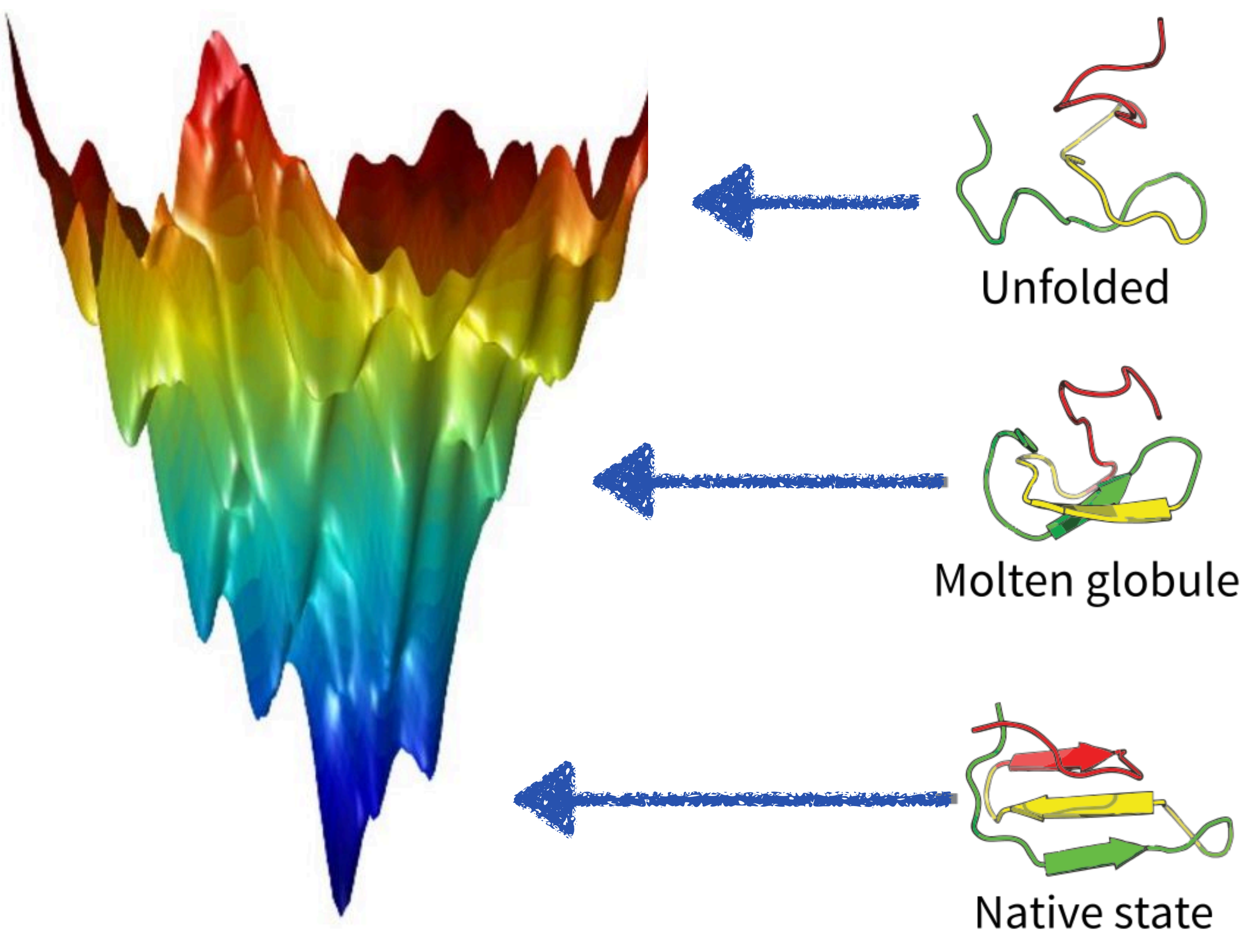
JK & Parrikar ('19); JK ('19); JK & Wong ('21, '22)

$$P(I) \sim \sum_{\alpha,j} T_{Ij} \left(\mathbb{1} - T\right)^{-1}_{j\alpha}$$

Probabilities maximized for vacua that can be accessed from many other vacua by **sequence of downward transitions.**

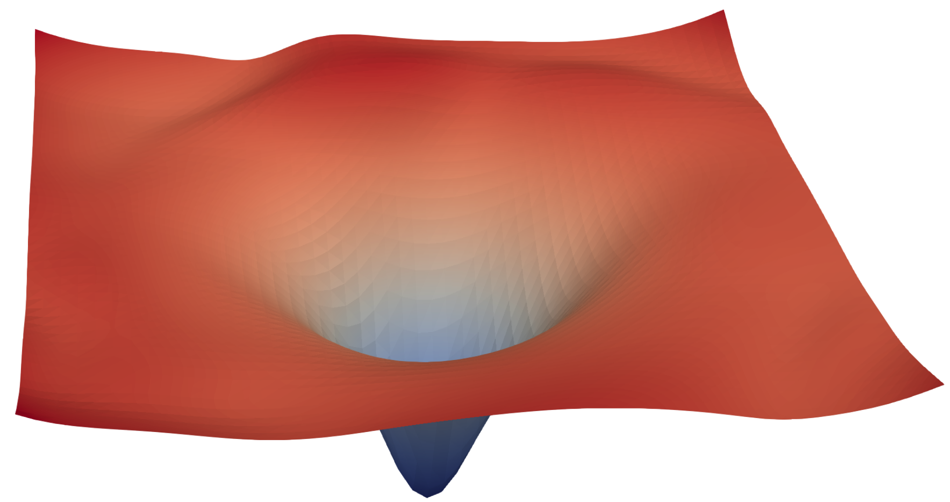
Such vacua have a surrounding landscape topography of a **deep valley or funnel.**

Folding funnels of proteins Bryngelson & Wolynes (1990)



Funnels are ubiquitous to search optimization on complex energy landscapes.

- **Deep learning:** "Good" minima of the loss function have **large basins of attraction**



(b) with skip connections

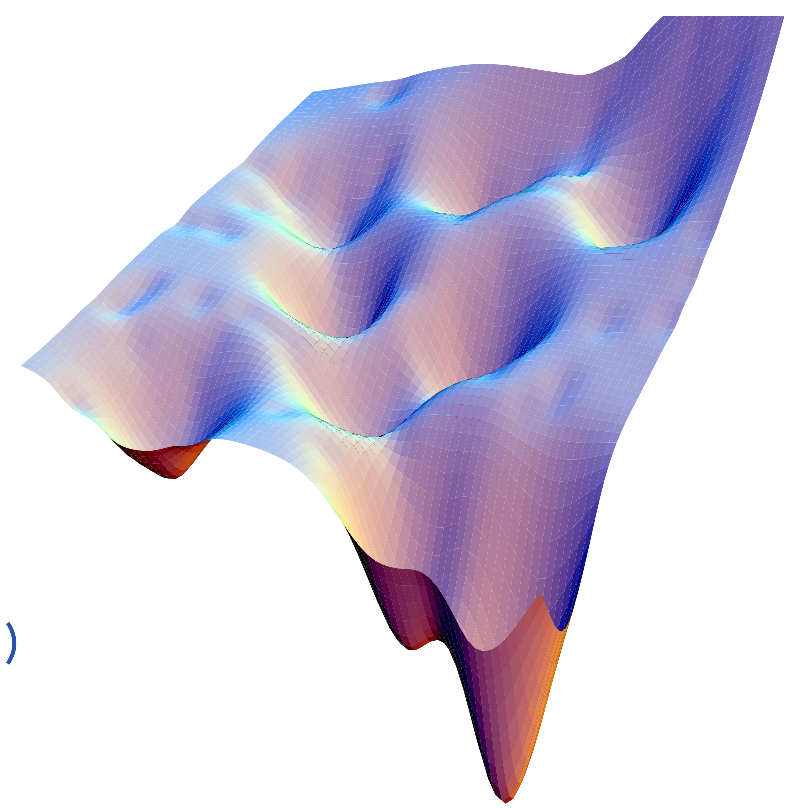
L. Wu, Z. Zhu and W. E (2017)

- **Atomic clusters** (Lennard-Jones potentials)
  Doye (2002); Massen & Doye (2005, 2006)

- **Combinatorial optimization** (e.g. Traveling Salesman Problem)

"Big Valley hypothesis"

Boese et al. (1994)
Ochoa & Veerapen (2016)



Directed percolation

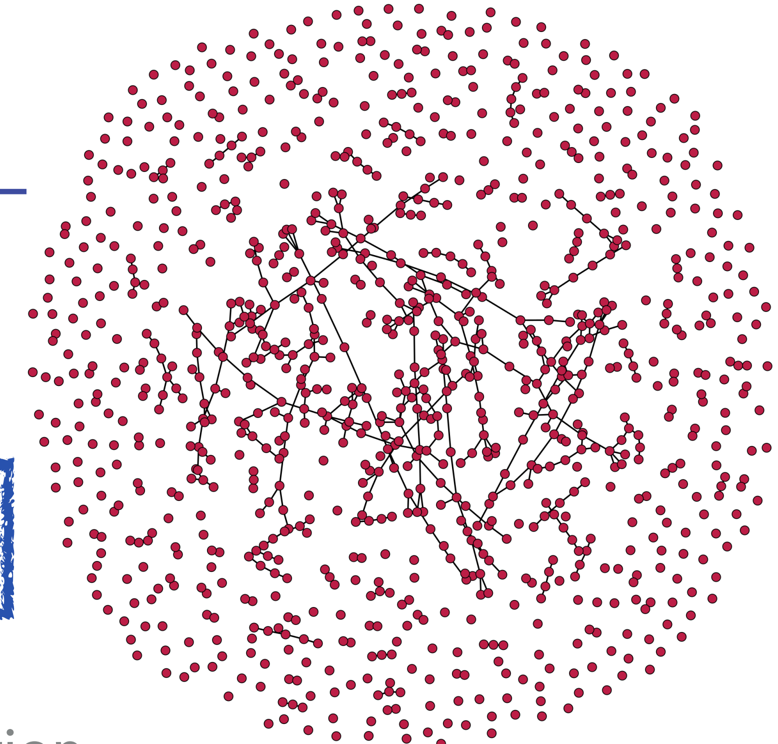
We inhabit a funnel, but can we be more quantitative?

Yes, because our problem can be naturally mapped to a problem of **percolation.**

JK & Wong, 2308.09736

Percolation: Fluid flow through porous media.

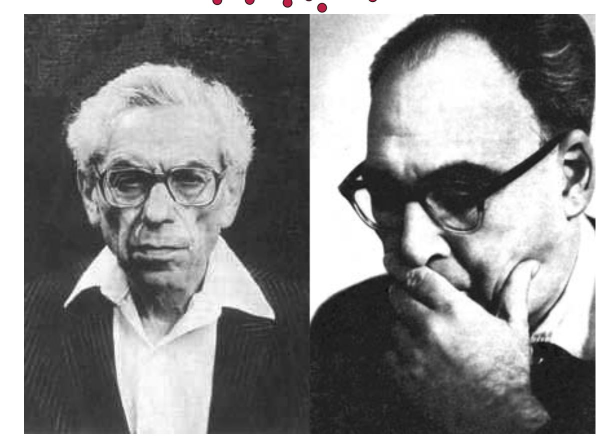
PERCOLATION ON RANDOM NETWORKS



Consider a **random graph**: Set of N nodes with randomly-assigned links.

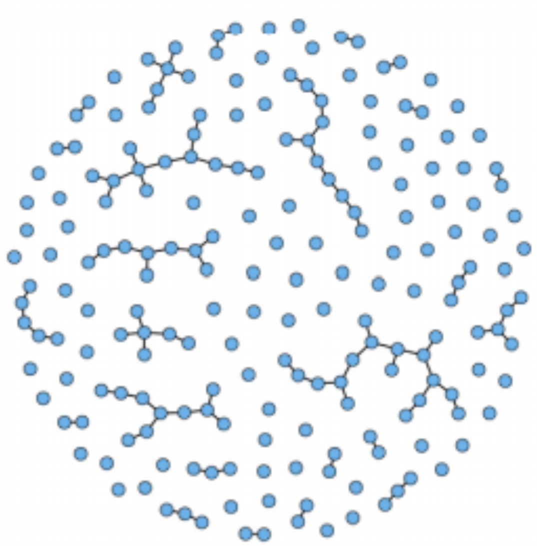
$p =$ probability that any pair of nodes is connected by link

Average degree (# of connections) of each node is $z = p(N-1)$

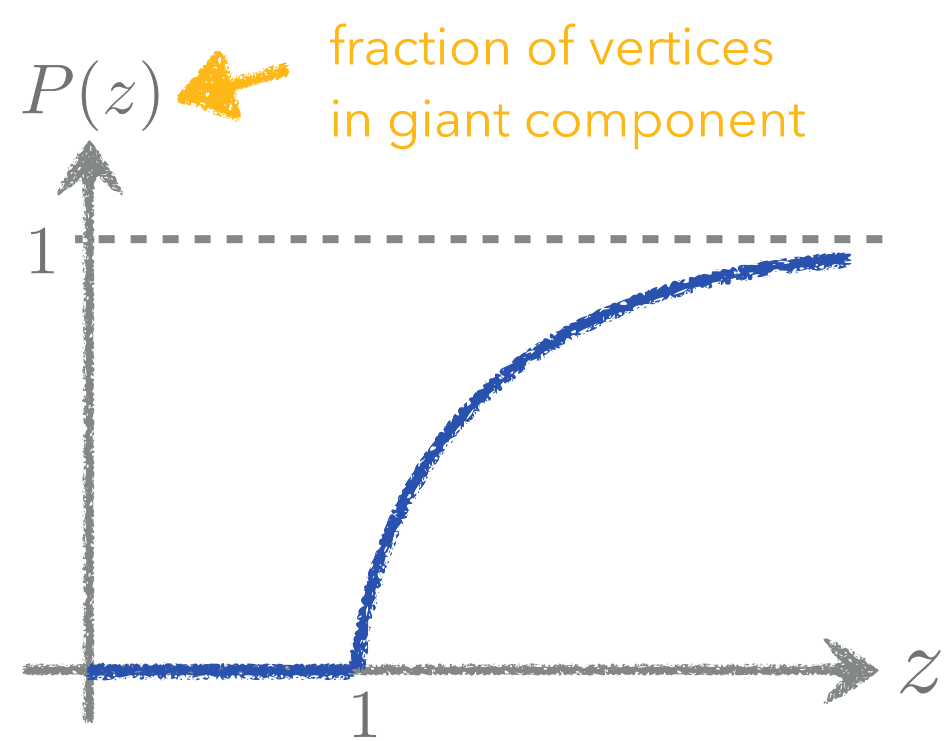


Erdös & Rényi (1959): Ensemble of such graphs exhibits a phase transition as function of z

- $z < 1$: disconnected components of size $\mathcal{O}(\log N)$.
- $z_c = 1$: **giant component** of size $\mathcal{O}(N^{2/3})$ emerges.
- $z > 1$: giant component include $\mathcal{O}(1)$ fraction of all nodes.



$z < 1$



A simplifying fact: **Transition rates between states are exponentially staggered.**

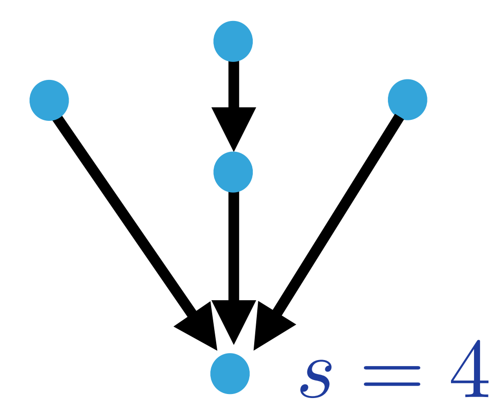
$$\kappa_{ij} \sim \mathrm{e}^{-S_{\mathrm{E}}}$$

depends on shape + height of potential barrier

Branching ratios $T_{Ij} = \frac{\kappa_{Ij}}{\sum_K \kappa_{Kj}}$ tend to be dominated by **single, dominant decay channel:**

In this approximation, probability distribution simply counts the **number s_I of ancestors:**

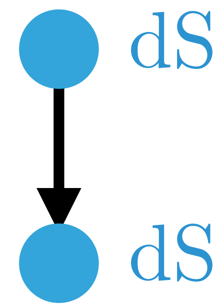
$$\boxed{P(I) \sim s_I}$$



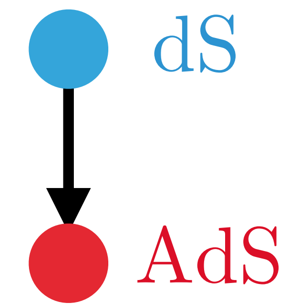
$\therefore$ **High probability corresponds to having many ancestors.**

To see how this maps to a problem of **(directed) percolation,** note there are 2 types of links:

Transient → transient:

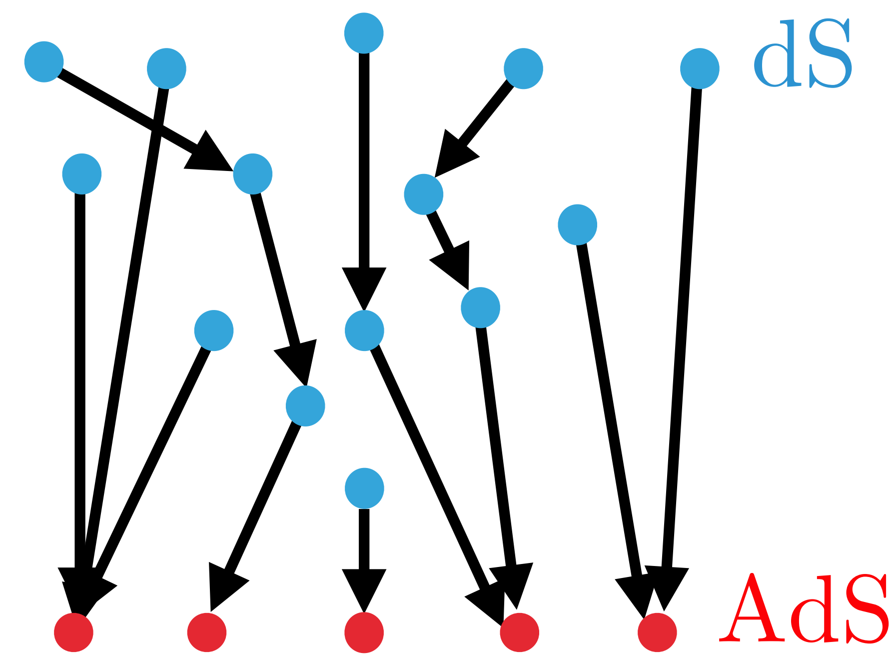


Transient → **terminal:**



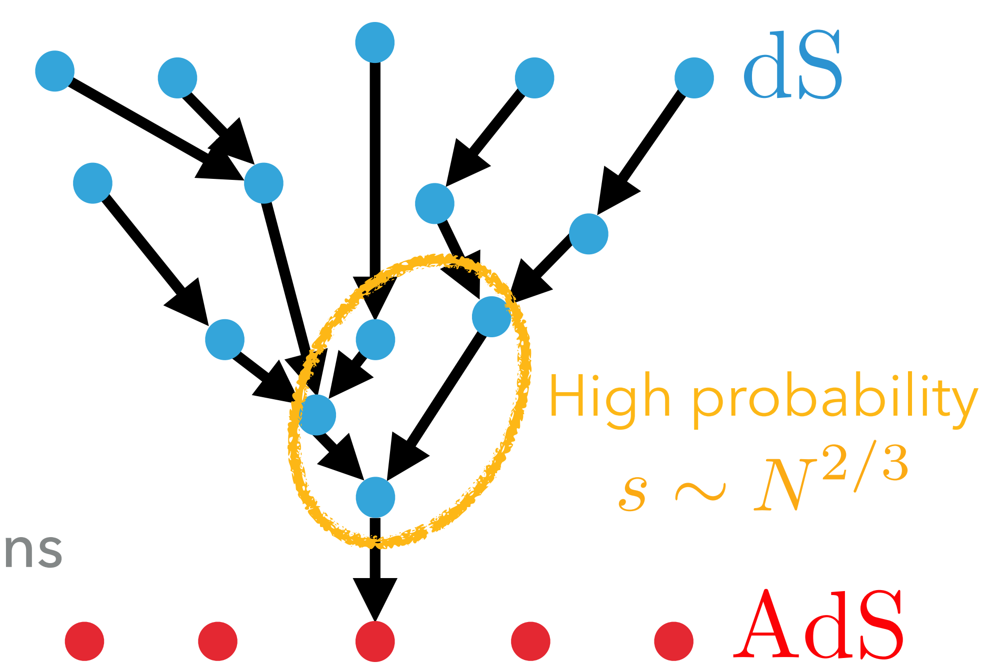
- If **transients** decay mainly to **terminals**, then many **small disconnected components**

$$\implies \quad s_I \sim \mathcal{O}(\text{few})$$ **(Subcritical)**



- If **transients** decay mainly to other **transients**, then **giant connected component** emerges

$$\implies \quad s_I \sim N^{2/3}$$ **(Critical)**



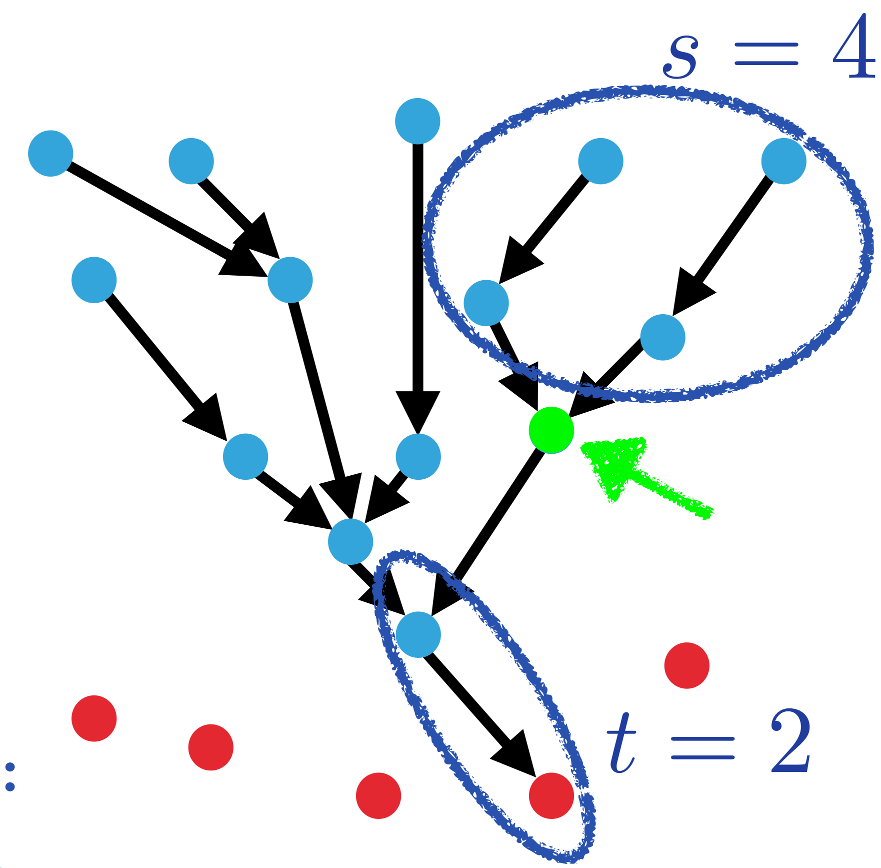
∴ Our probabilities $P(I) \sim s_I$ favor landscape regions nearly tuned at **directed percolation criticality.**

JK & Wong, 2308.09736

- Predictive power of criticality lies in **universality.**
- **Scale-invariant** observables, characterized by **critical exponents.**

$s \equiv$ # of ancestors of a randomly-selected node

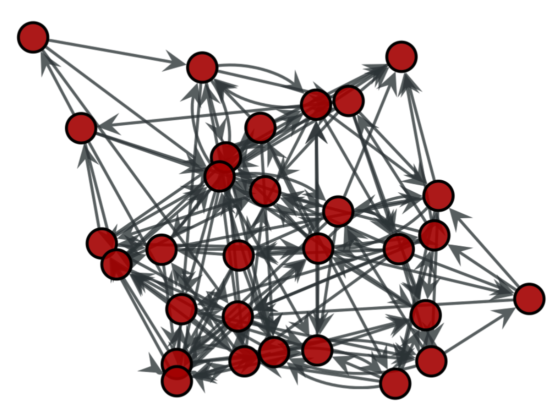
$t \equiv$ # of descendants of randomly-selected node



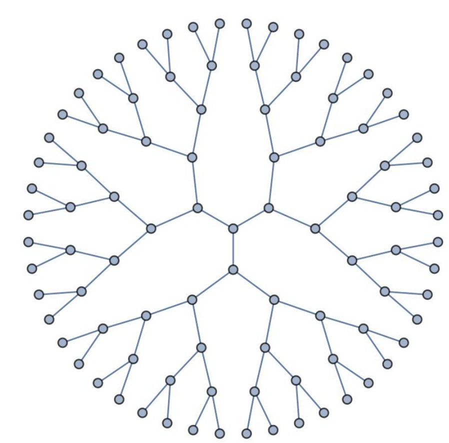
At criticality, their distributions have universal **power-law tails:**

$$P_{\text{in}}(s) \sim \frac{1}{s^{3/2}}; \quad P_{\text{out}}(t) \sim \frac{1}{t^{3/2}} \qquad (s, t \gg 1)$$

Very broad universality class:

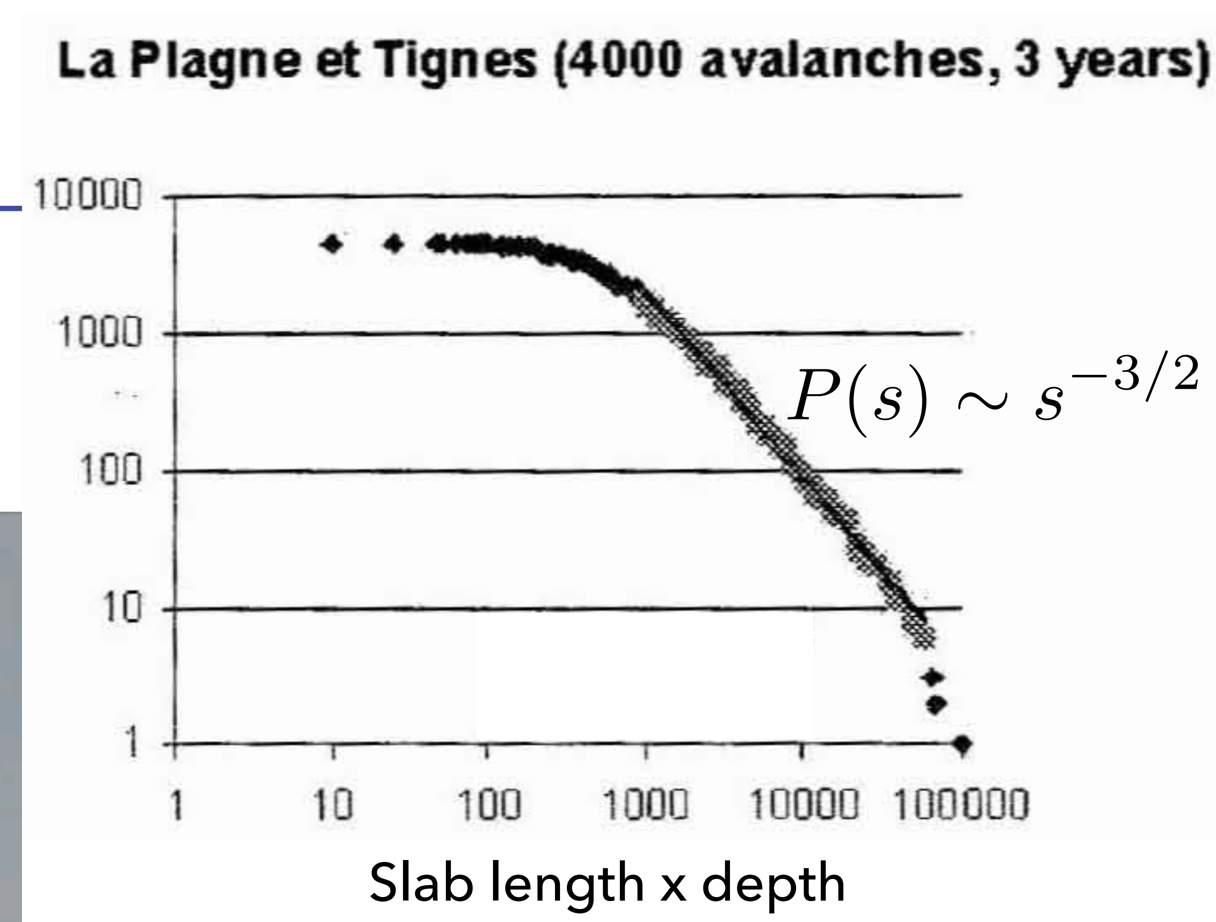


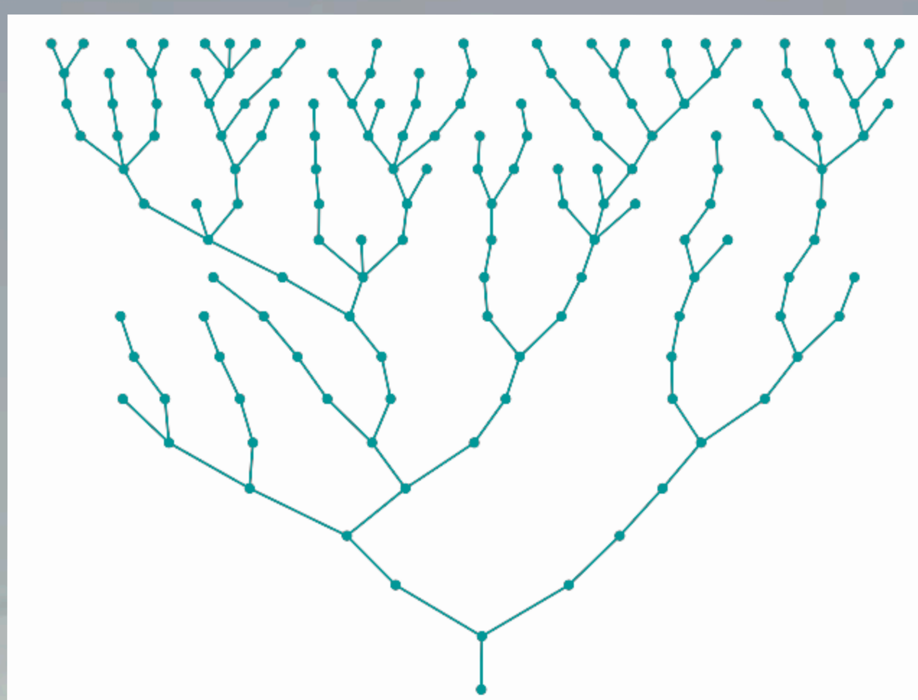
Random graphs
Erdös & Rényi (1959)



Bethe lattices

AVALANCHES AND BRANCHING PROCESS





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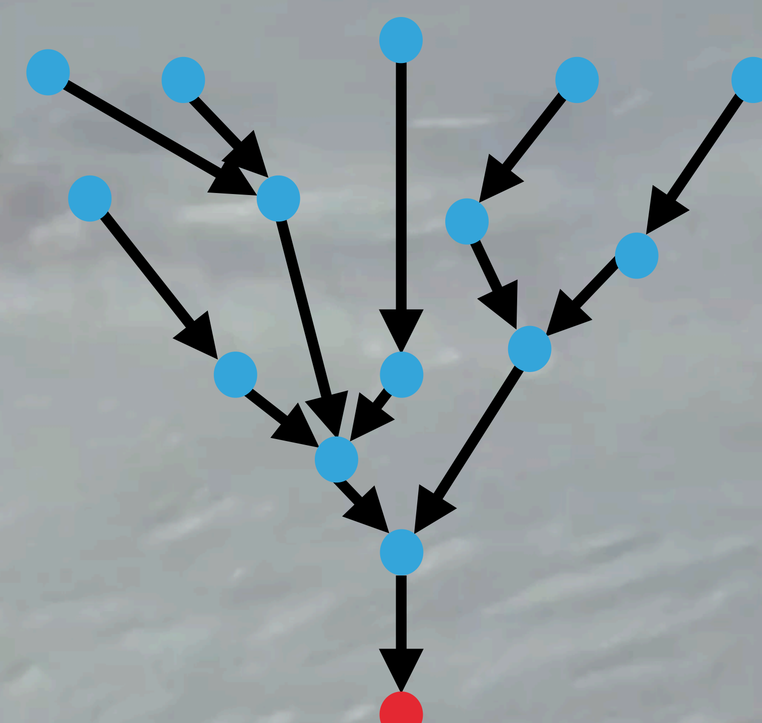
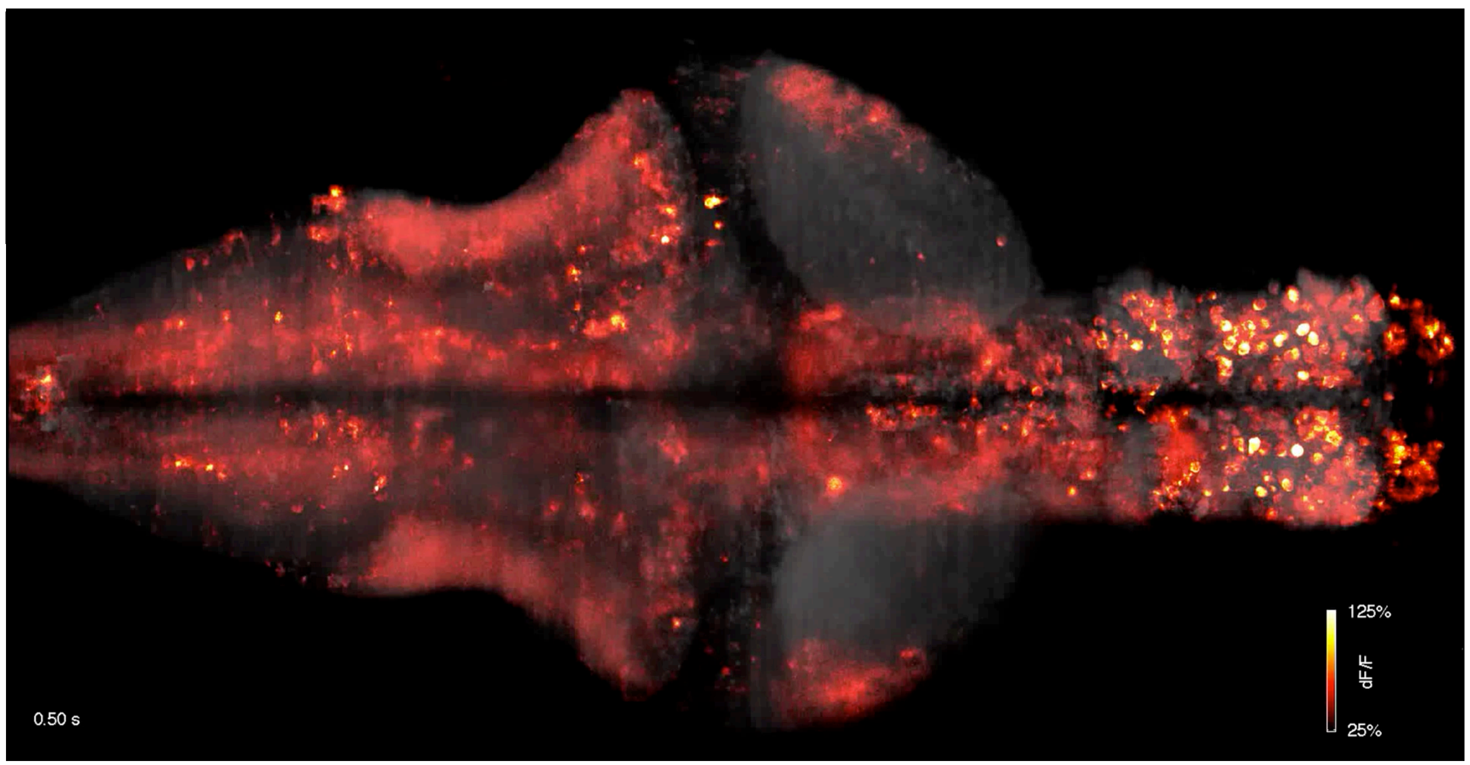


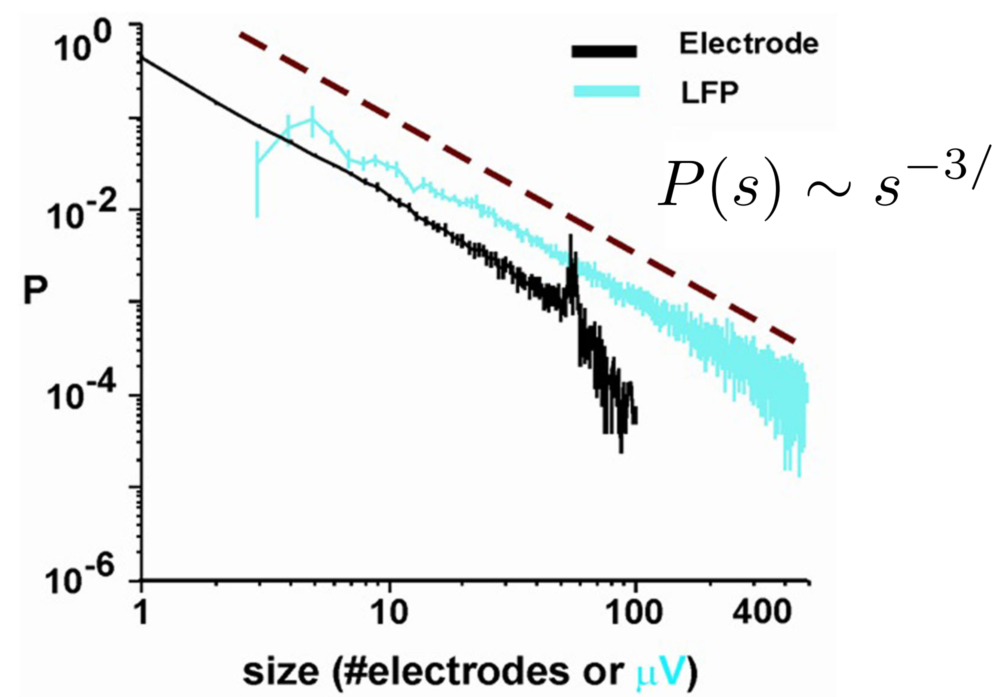
Critical (Galton-Watson) branching process

Directed percolation on graphs

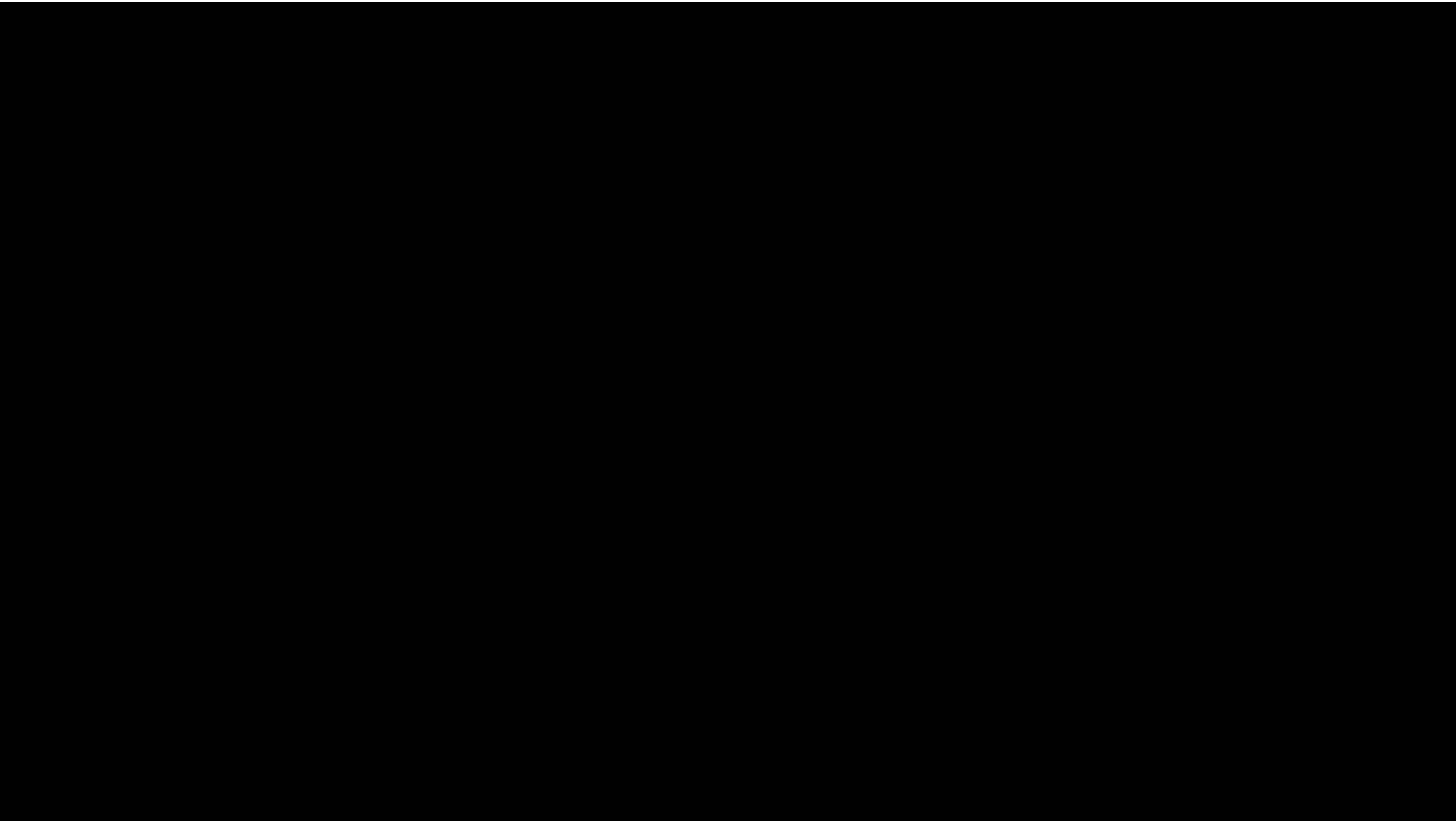
THE BRAIN AT CRITICALITY

Critical boundary between **stable** and **unstable** dynamics (**"edge of chaos"**) maximize computational capabilities





Systems at criticality achieve **optimal compromise** between **robustness** and **adaptability.**

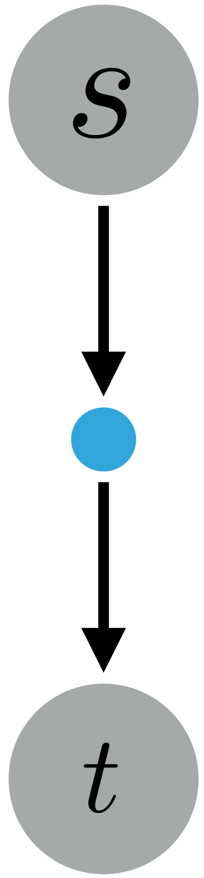


CRITICALITY AND THE VACUUM ENERGY

JK & Wong, 2308.09736

In downward approximation, **ancestors** have **higher vacuum energy**, while **descendants** have **lower vacuum energy.**

$\Longrightarrow$ **vacuum energy distribution follows ordered statistics**



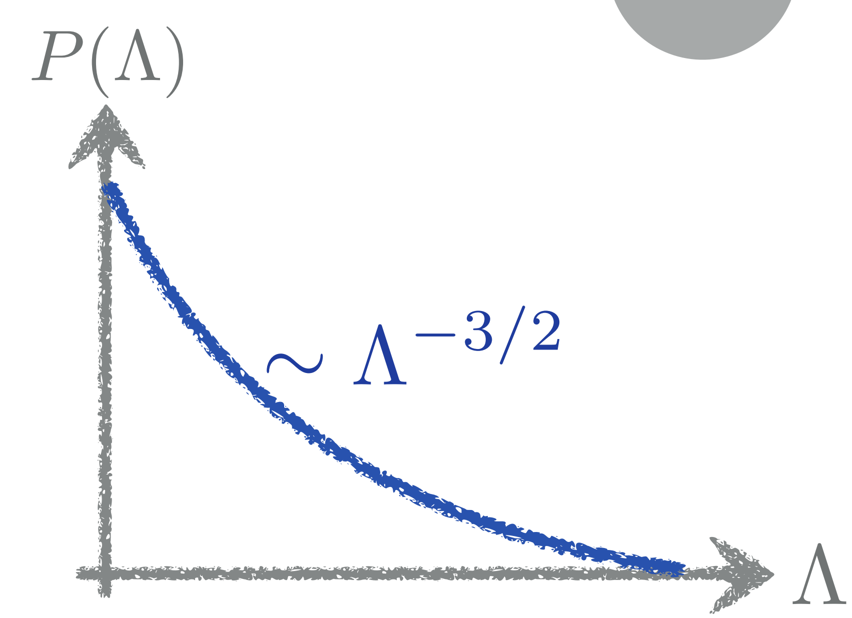
Probability of having s **ancestors** and t **descendants** translates to **probability distribution for** Λ :

$$P_{\rm in}(s) \sim \frac{1}{s^{3/2}} \;\; ; \;\; P_{\rm out}(t) \sim \frac{1}{t^{3/2}} \Longrightarrow \boxed{P(\Lambda) \sim \frac{1}{\Lambda^{3/2}}}$$

Favors small, positive vacuum energy:

$$\Lambda \lesssim \mathcal{O}\left(N^{-2/3}\right) \qquad \text{(95\% C.L.)}$$

$N =$ **# of de Sitter vacua in our local funnel**



Broader points: - **Universality** of this (non-anthropic) vacuum energy distribution.

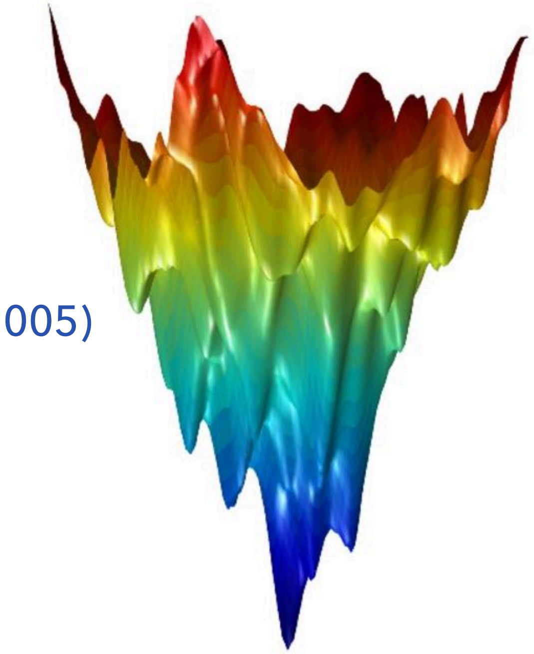
- **Insensitive to detailed understanding** of string landscape (which is lacking)

Outlook: some implications

General expectation: There should be **many other dS nearby**.

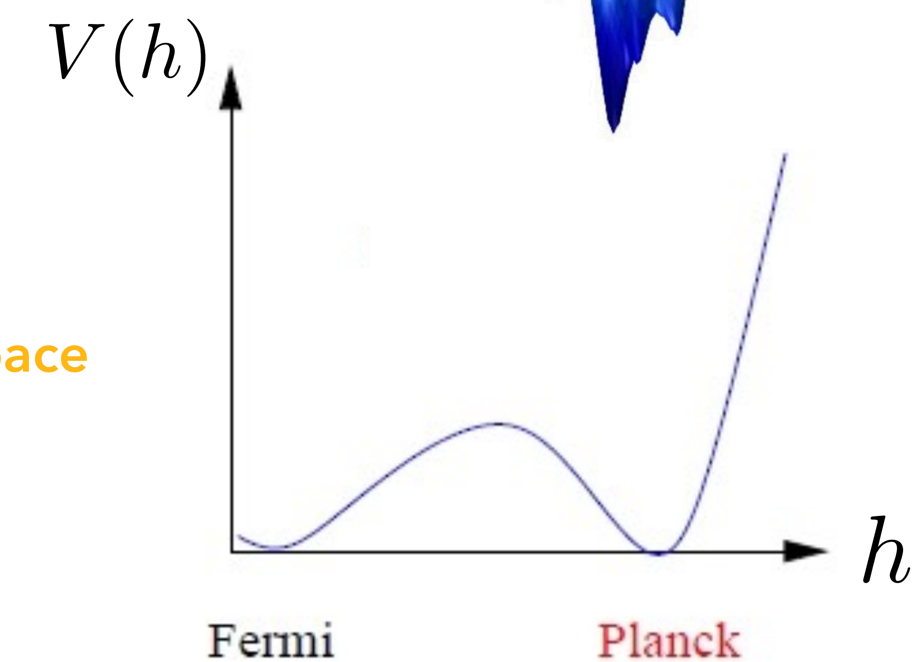
- Using Ashok-Denef-Douglas density of SUSY flux vacua, can estimate **average distance between minima:**

Ashok & Douglas (2004); Denef & Douglas (2004, 2005)



$$\text{distance} \sim \frac{\sqrt{8\pi \mathrm{e}}}{D} M_{\mathrm{Pl}}$$

$D =$ dimensionality of moduli space



Existence of 2nd minimum is sensitive to **top quark mass:**

$$m_{\mathrm{t}} = 171.36 \pm 0.46 \ \mathrm{GeV}$$

Latest experimental value of top pole mass (CMS 2022):

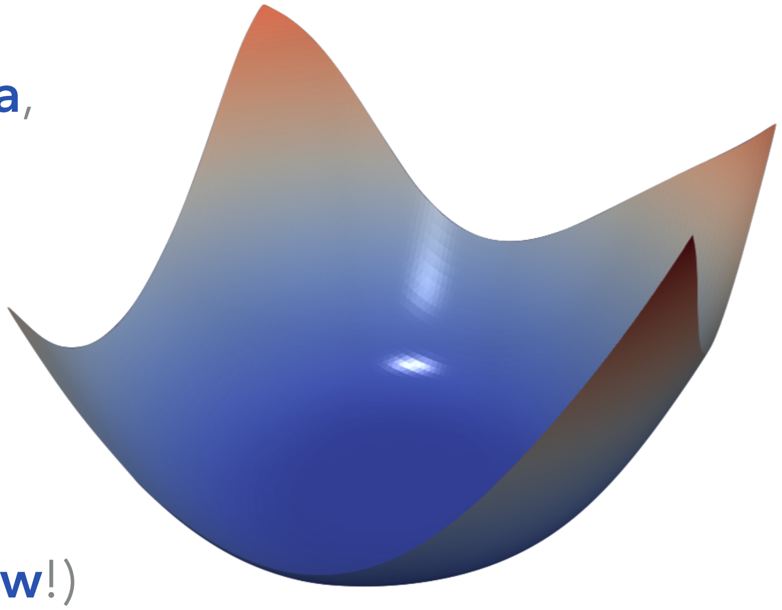
$$m_{\mathrm{t}}^{\mathrm{exp}} = 172.94 \pm 1.37 \ \mathrm{GeV}$$

Can be viewed as prediction, albeit sensitive to new particles at intermediate energy scale.

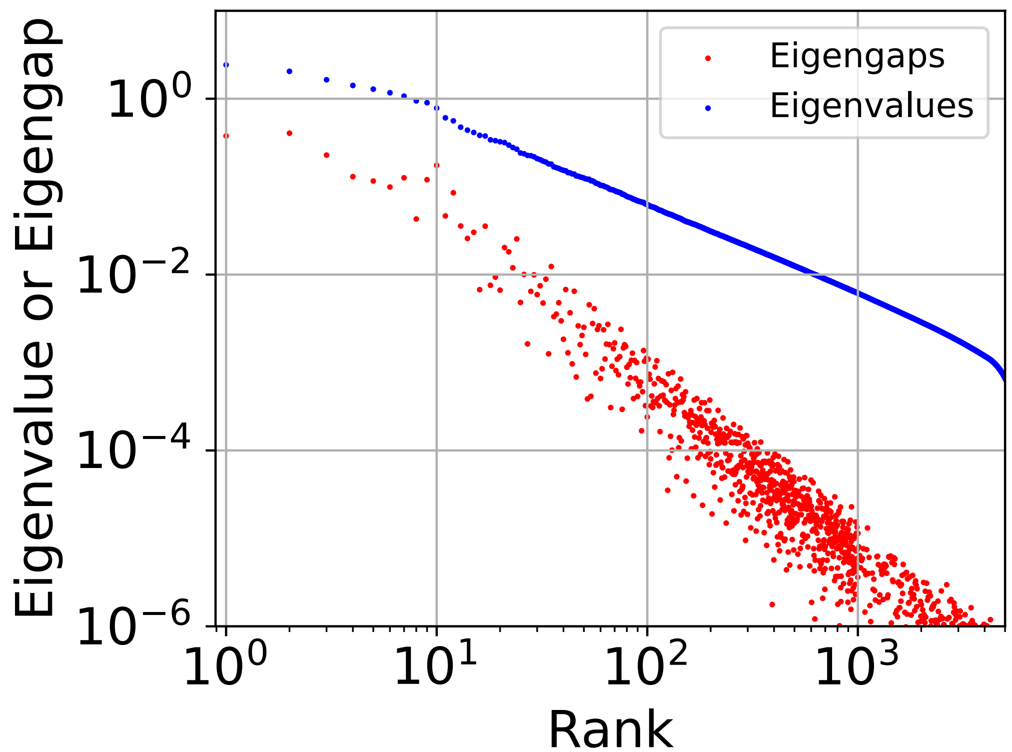
Hochreiter & Schmidhuber (1997); Sagun, Bottou & LeCun (2016); Xie et al. (2022)

- **Deep learning:** Loss function of neural networks feature **wide, flat minima**, that generalize well and are easily accessible.

Large basin volume ⟷ **Low-curvature minima**



- Hessian eigenvalues are **power-law** distributed (consistent with **Zipf's law**!)



$$\rho(\lambda) \sim \frac{1}{\lambda^\alpha}$$

Xie et al. (2022)

Can be derived on general grounds using the **principle of maximal entropy** Visser (2012)

Implications for the weak hierarchy problem?

SCALE-FREE NETWORKS/LANDSCAPES

Georgios Gounaris

Eleni Katifori

- Degree distribution is **scale free**

Rao & Caflisch (2004)

$$P(k) \sim \frac{1}{k^2}$$

$k =$ degree (# of links to given nodes)

Folded states are **"hubs"** of **very high connectivity**

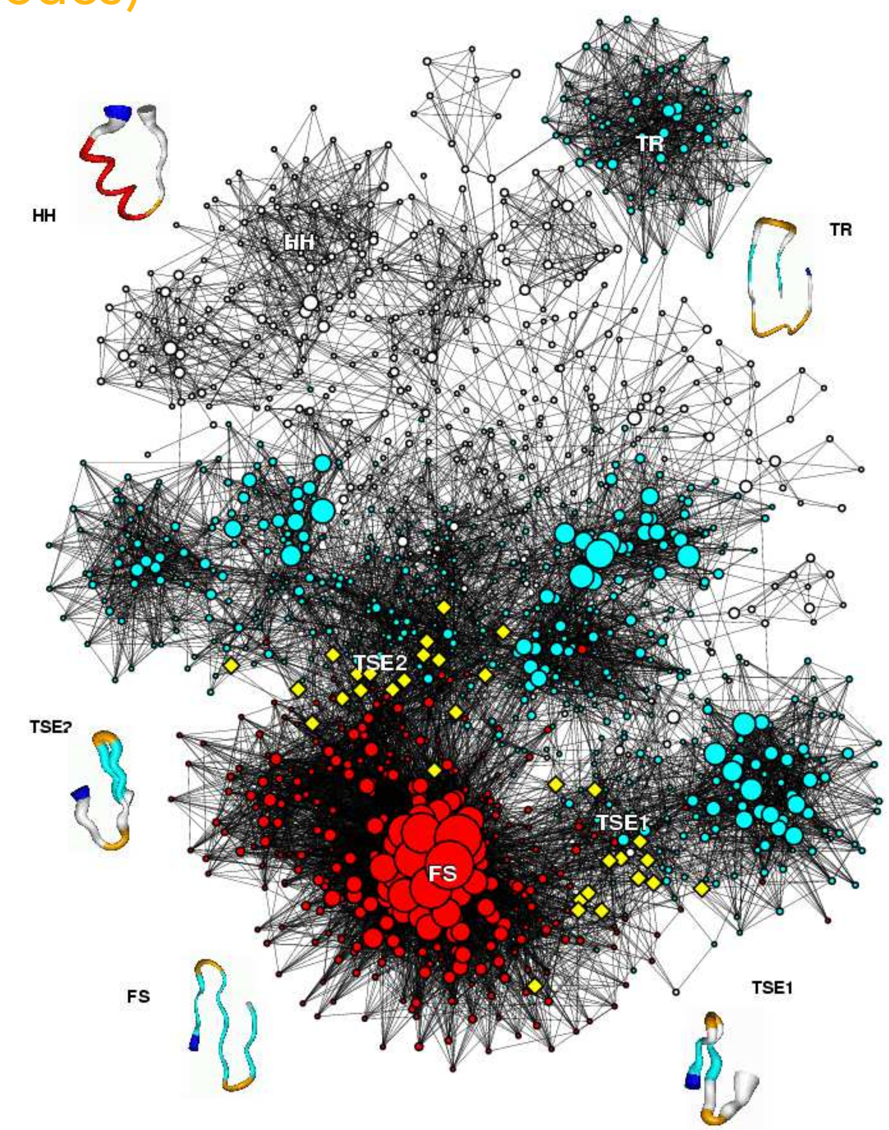
Different percolation universality class!

- "Small-world" property
- Hierarchical: **funnels nested within larger funnels**

These properties are generic of **complex "real-world" networks:**

- World wide web
- Social networks
- Academic citations

Implications for the energy scale of slow-roll inflation?



Beta3s protein