Scale invariant extension of the Starobinsky inflation model and primordial non-Gaussianity

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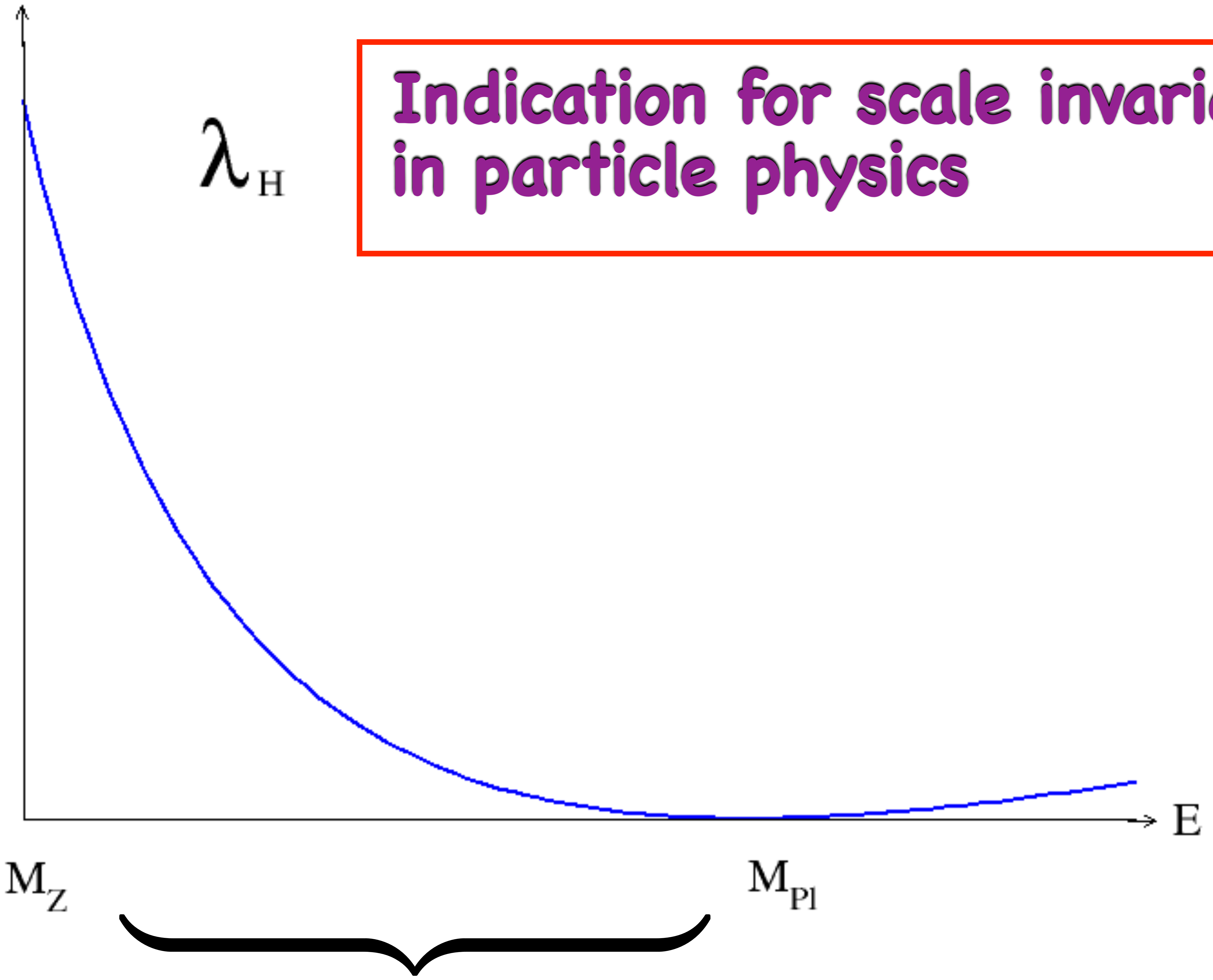
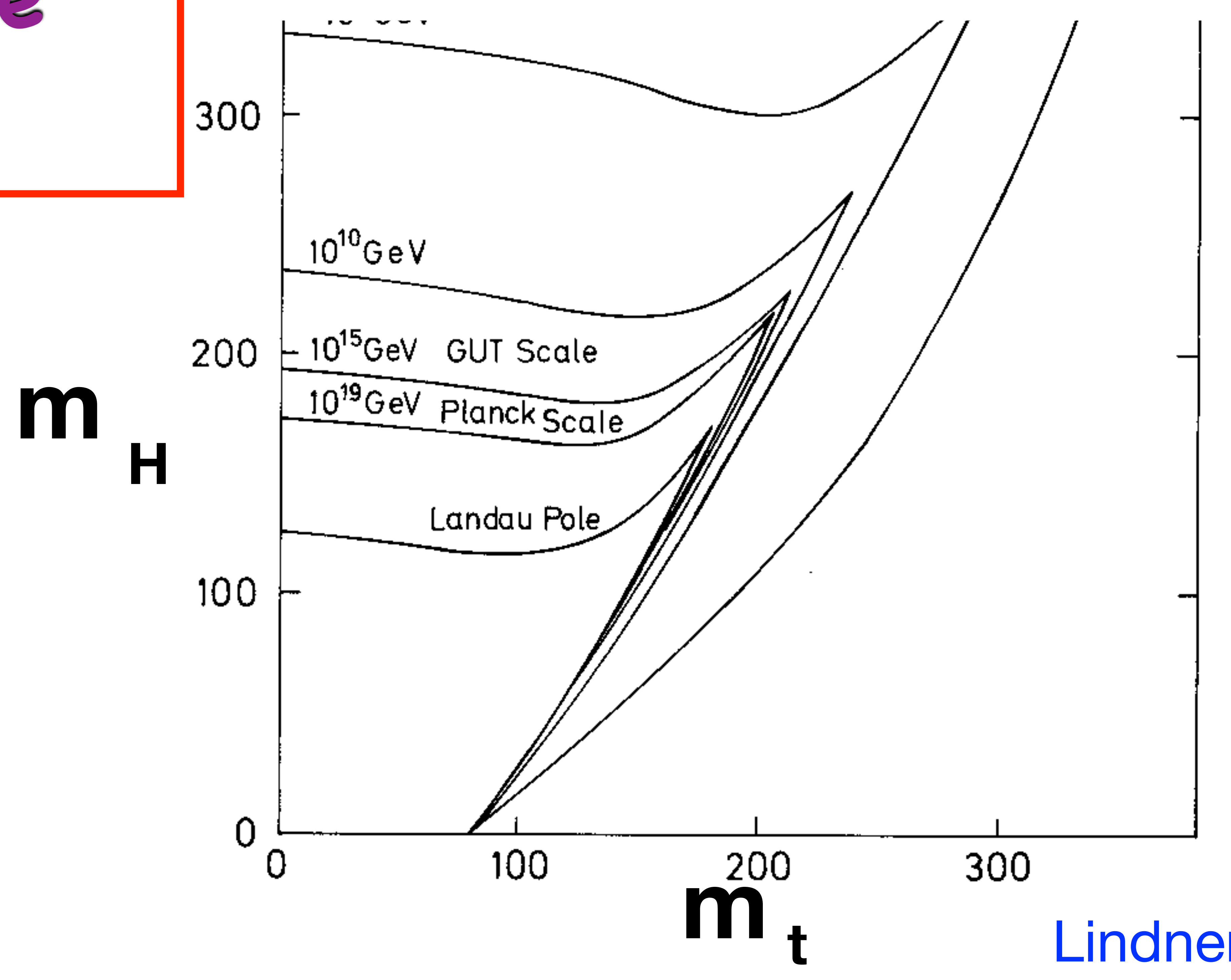
based on Aoki, JK+ Yang, JCAP 05 096 (2401.12442)

Indication for scale invariance in particle physics





Lindner,'86

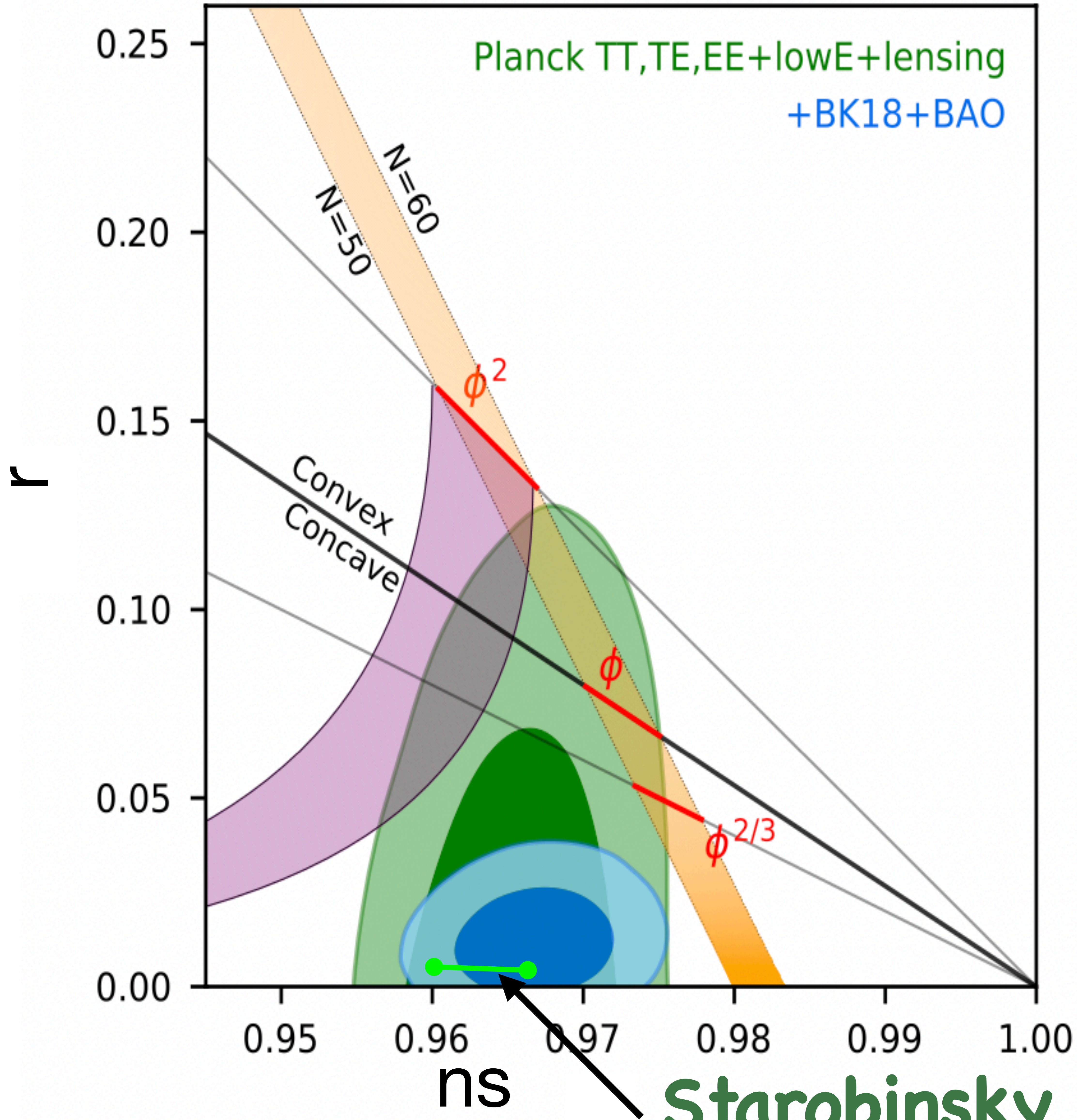
Desert => Scale invariance is broken only by anomaly if μ_H = 0. Bardeen,'95

$\{\lambda_H\ ,\ y_t\ ,\ g_3\}$ - system is asymptotically free.

The max. value of $y_t^2/\lambda_H \quad \rightarrow \quad m_t^2/m_H^2 \simeq 1.9 \left([m_t^2/m_H^2]_{\rm exp} \simeq 2.0 \right)$

JK, Sibold, Zimmermann, '85;'88

Indication for scale invariance in cosmology



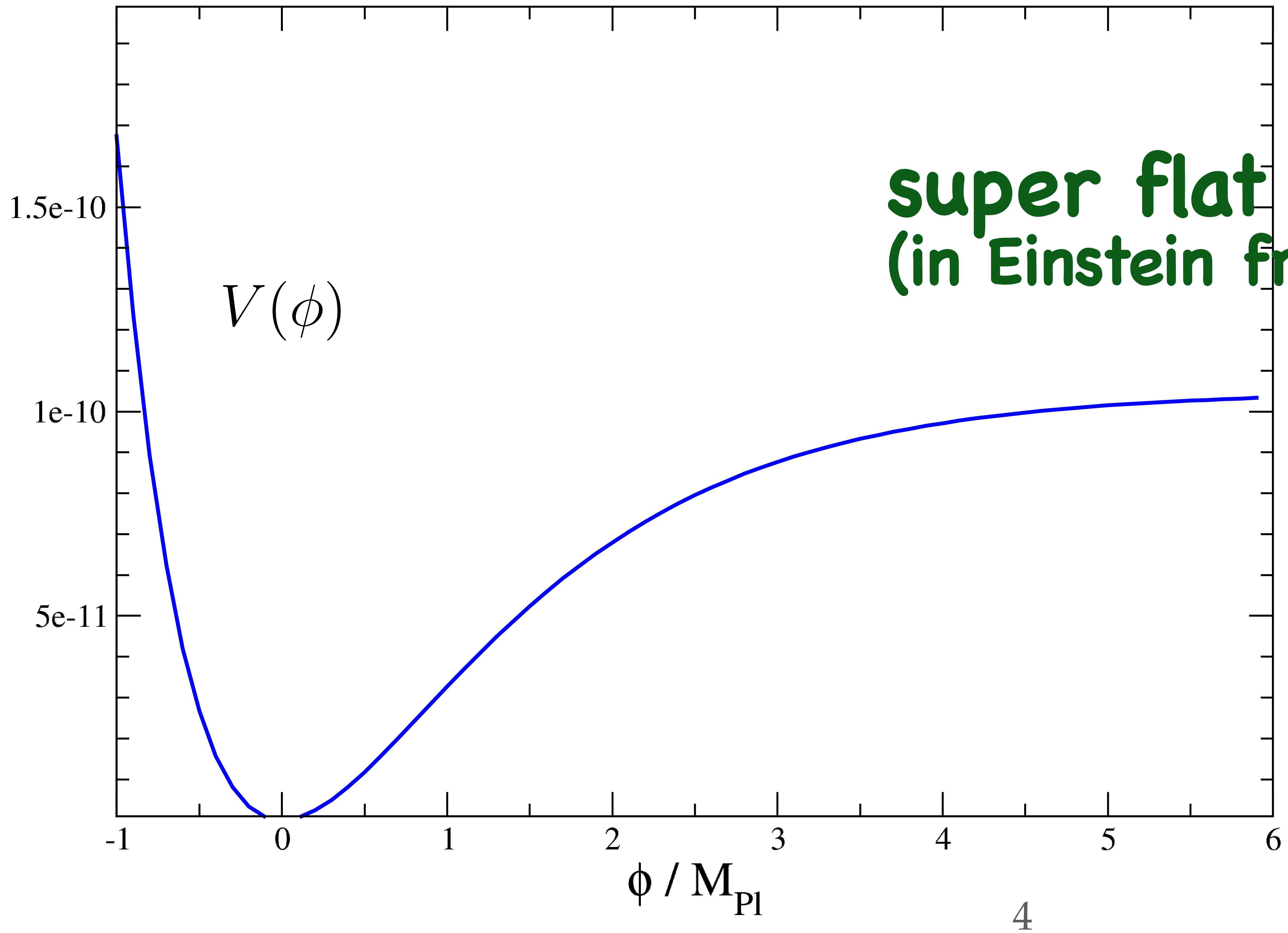
Planck-BICEP/keck

Starobinsky (R^2) inflation

scale invariant

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_{\rm Pl}^2}{2} R + \begin{cases} \gamma R^2 \quad (\gamma \sim 10^9) \\ \beta |H|^2 R - \lambda_H |H|^4 \quad (\beta \sim 10^4) \end{cases}$$

$$\text{for} \begin{cases} R^2 \text{ inflation, Starobinsky, '80} \\ \text{Higgs inflation, Bezrukov and Shaposhnikov, '08.} \end{cases}$$



super flat potential (plateau)
(in Einstein frame)

$$r \propto \left(\frac{V'(\phi)}{V(\phi)} \right)^2$$

Scale invariance is hardly broken
by scale anomaly.

Callan, '70; Symanzik,'70

However, scale anomaly
can not directly generate mass gap.

To generate a mass gap, scale invariance
has to be spontaneously broken.

Spontaneous generation (SG) of M_{Pl}
= SG of Einstein-Hilbert theory

Induced gravity

with scalars

*Fujii '74
*Minkowski, '77
*Englert, Gunzig, Truffin+Windey,'75
*Minkowski, '77
*Chundnovsky,'78
*Fradkin+Vilkovisky,'78
*Zee,'79
*Smolin,'79
*Terazawa,'81
*Nieh, '82
....

without scalars

*Akama, Chikashige+ Matsuki,'78
*Adler,'80 ;
*Zee,'81
....

An simple way to generate M_{Pl} :

$$\frac{\xi_S}{2} S^2 R \to \frac{\xi_S}{2} \langle S \rangle^2 R \to \frac{M_{\rm Pl}^2}{2} R$$

$$\boxed{M_{\rm Pl} = \sqrt{\xi_S} \langle S \rangle}$$

Recent papers:

Salvio+Strumia,'14; Kannike et al,'15; Rinaldi et al,'14; Farzinnia+Kouwn,'15;

Ghilenea +H.M.Lee,'18; Karam+Pappas+Tamvakis,'18;JK,Lindner,Schmitz+Yamada,'18;……………

see e.g. Aoki+JK+Yang, '24 and Cecchini et al, '24 for more references

Extension of Starobinsky inflation based on Renormalizable Quadratic Gravity (QG):

$$\frac{\mathcal{L}_{\rm QG}}{\sqrt{-g}} = -\frac{1}{2}\xi_{ab}\phi_a\phi_b R + \gamma\, R^2 - \kappa W^2 + \frac{\mathcal{L}_{\rm Matter}(\phi,\cdots)}{\sqrt{-g}}$$

Weyl tensor^2

(classically) scale invariant

renormalizable Stelle, `77

unitary during inflation
(if the inflation scale < $M_{\text{spin-2-ghost}}$)

JK+Kuntz, `23; JK+Kugo, `23;`24

$$\frac{\mathcal{L}_{\rm QG}}{\sqrt{-g}} \ni -\frac{1}{2}\xi_{ab}\phi_a\phi_b R + \gamma\, R^2 - \kappa W^2$$

γ , ξ_{ab} , κ control n_s , r , f_{NL} :

γ for R^2 Inflation

Starobinsky,'80

ξ_{ab} for Higgs (-like) inflation

Bezrukov+Shaposhnikov,'07

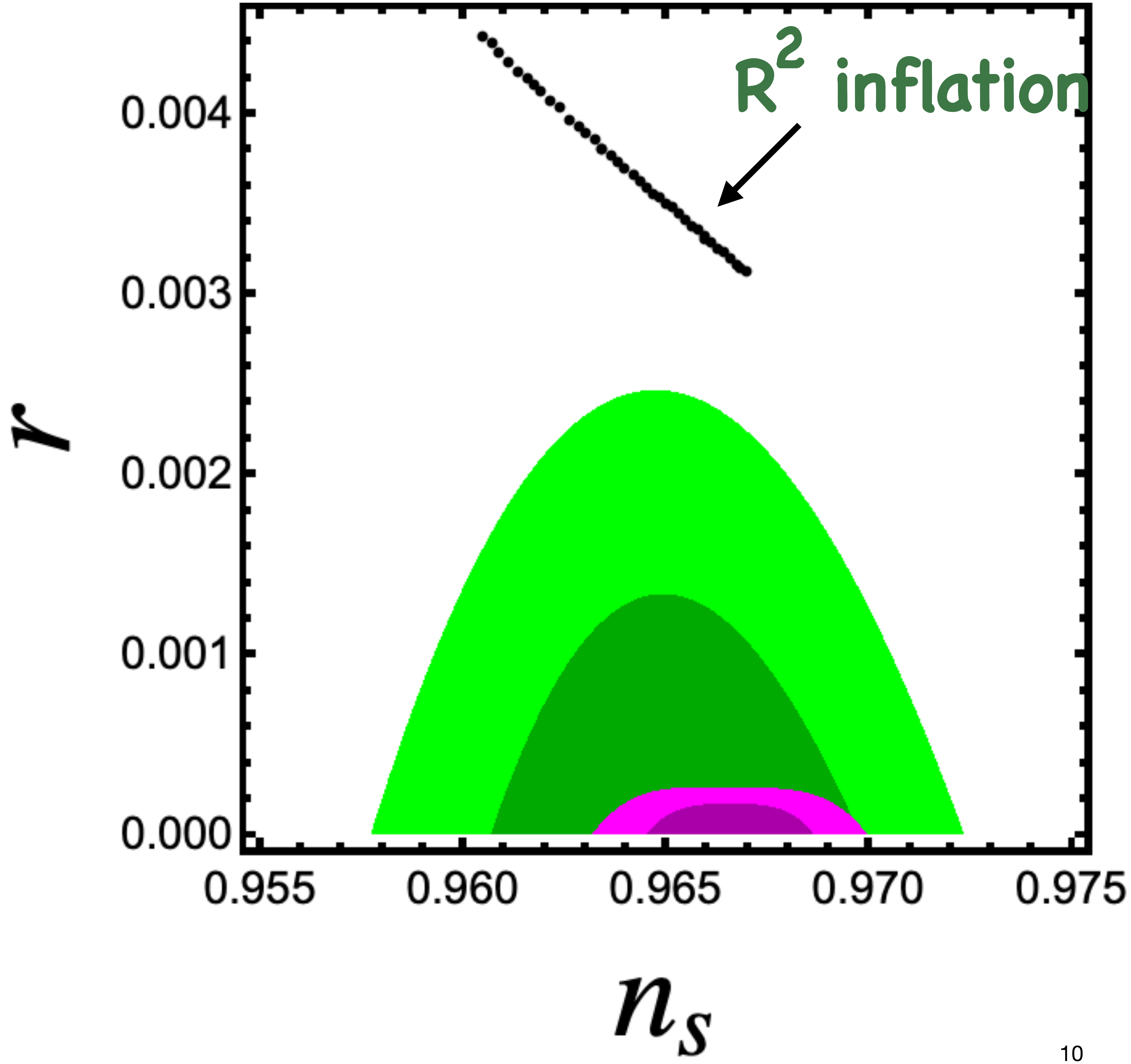
κ contributes to r

Clunan+Sasaki,'09; Baumann et al,'15; Salvio,'17; Anselmi et al, '20

triggers SSB of scale invariance

JK,Kuntz,Rezacek+Saake,'22

Future experiments for n_s and r



LiteBird/Planck (95% CL)

PICO (95%CL)

in the case of null detection

Further:

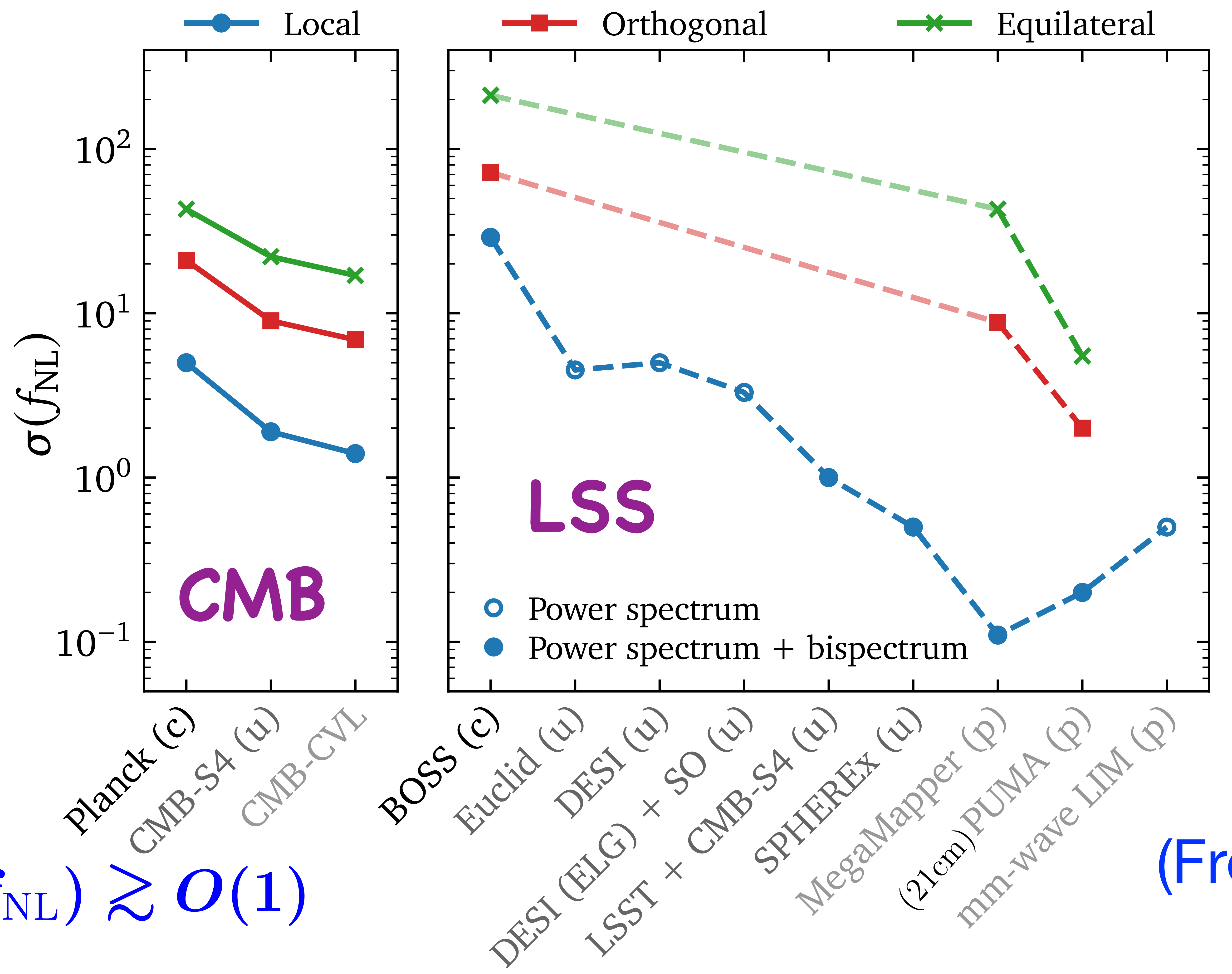
Simons, CMB-S4, ……

(see e.g. Snowmass 2021)

$$f_{NL}^{local} = -0.9 \pm 5.1 \;\; (f_{NL}^{equil} = -26 \pm 47\ ,\ \ f_{NL}^{equil} = -38 \pm 24)$$

Planck 2018

Future experiments for f_{NL}



$\sigma(f_{NL}) \gtrsim O(1)$

(From Snowmass 2021)

Two models

I: $$\frac{\mathcal{L}_{\rm I}}{\sqrt{-g}} = \frac{1}{2} g^{\mu\nu}\partial_\mu S \partial_\nu S - \frac{\lambda_S}{4} S^4 - \frac{1}{2}\beta S^2 R + \gamma R^2 - \kappa W^2 + \cdots$$

($\beta = \xi$, not β -function)

Coleman-Weinberg:

Fluctuation of the massive spin-two ghost $\rightarrow \langle S \rangle$ JK,Kuntz,Rezacek+ Saake,'23

II: $$\frac{\mathcal{L}_{\rm II}}{\sqrt{-g}} = \frac{\mathcal{L}_{\rm I}(\kappa \to 0)}{\sqrt{-g}} - \frac{1}{2}{\rm tr} F^2 + \underbrace{{\rm tr}\bar{\psi}(i\gamma^\mu \not{D}_\mu}_{\text{Hidden QCD-like sector}} - yS)\psi + \cdots$$

Aoki, JK, Yang, '22,'24

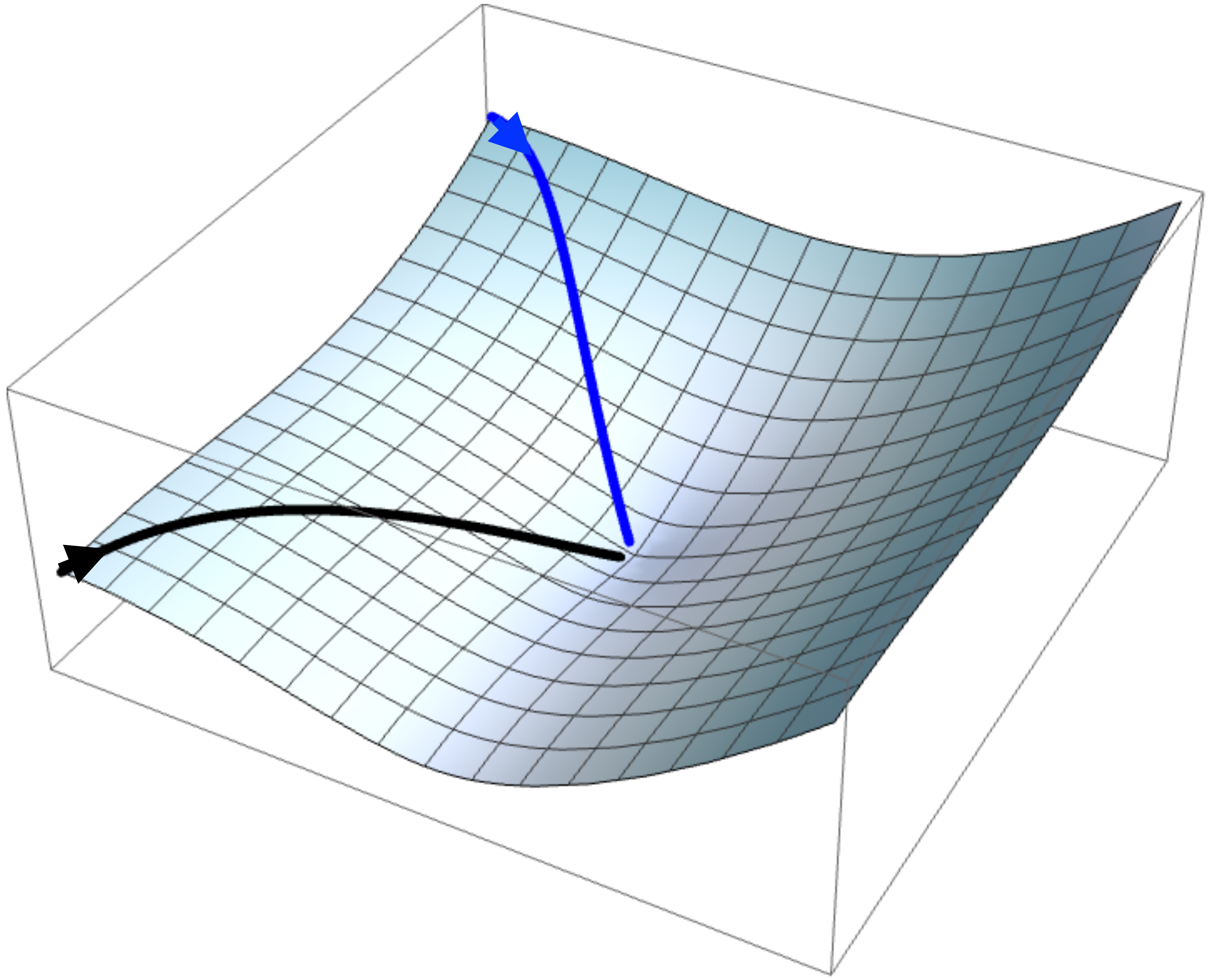
Chiral symmetry breaking: $\langle \bar{\psi}\psi \rangle \rightarrow \langle S \rangle$

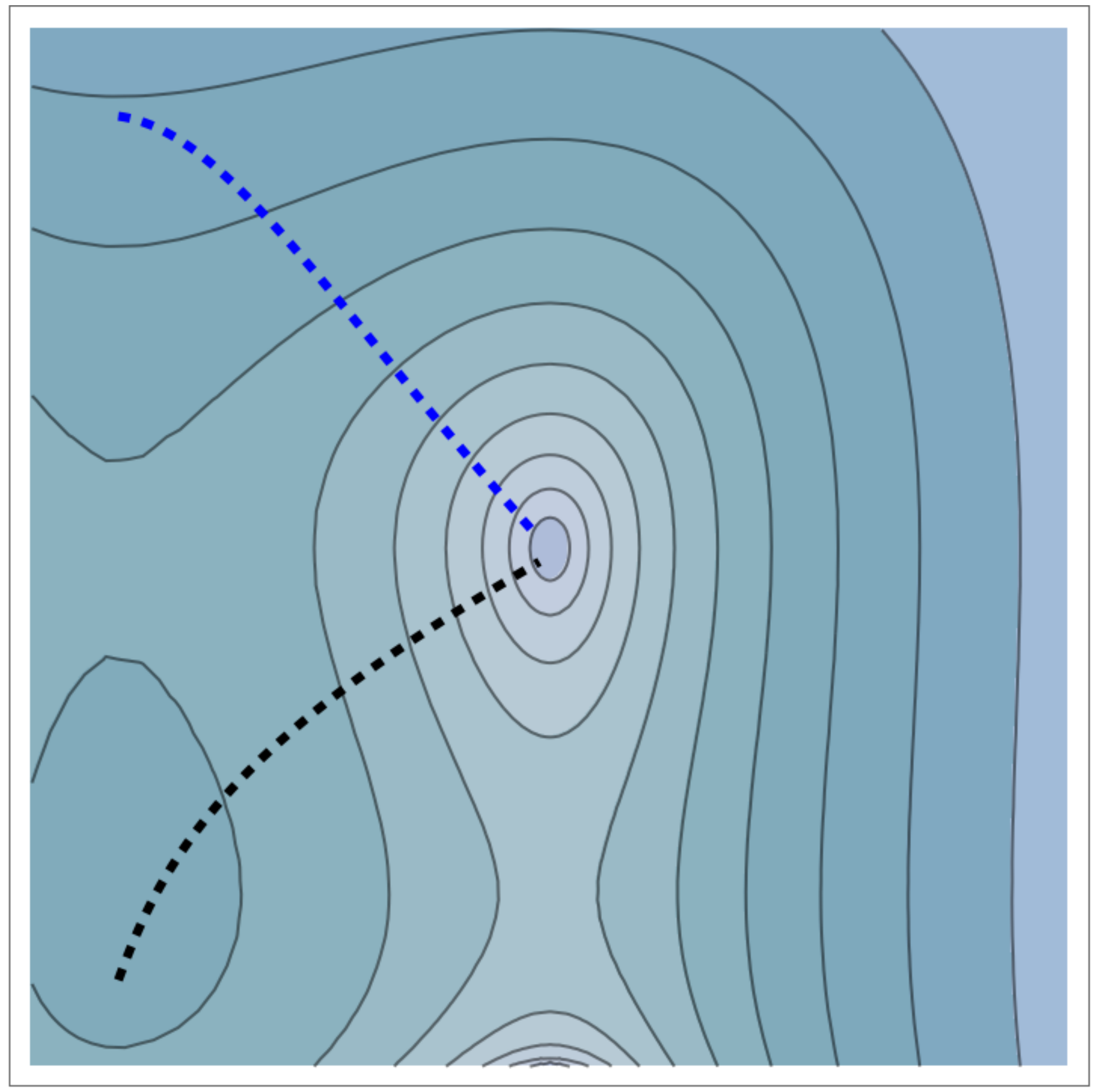
$$\left(\begin{array}{c} M^2_{\rm ghost} = \frac{\beta}{4\kappa}\langle S \rangle^2 \\ m^2_\varphi = \frac{\beta}{12\gamma}\langle S \rangle^2 \end{array} \right)$$

Multi-field system for inflation

A. Basin-like potential

Slow-roll trajectory



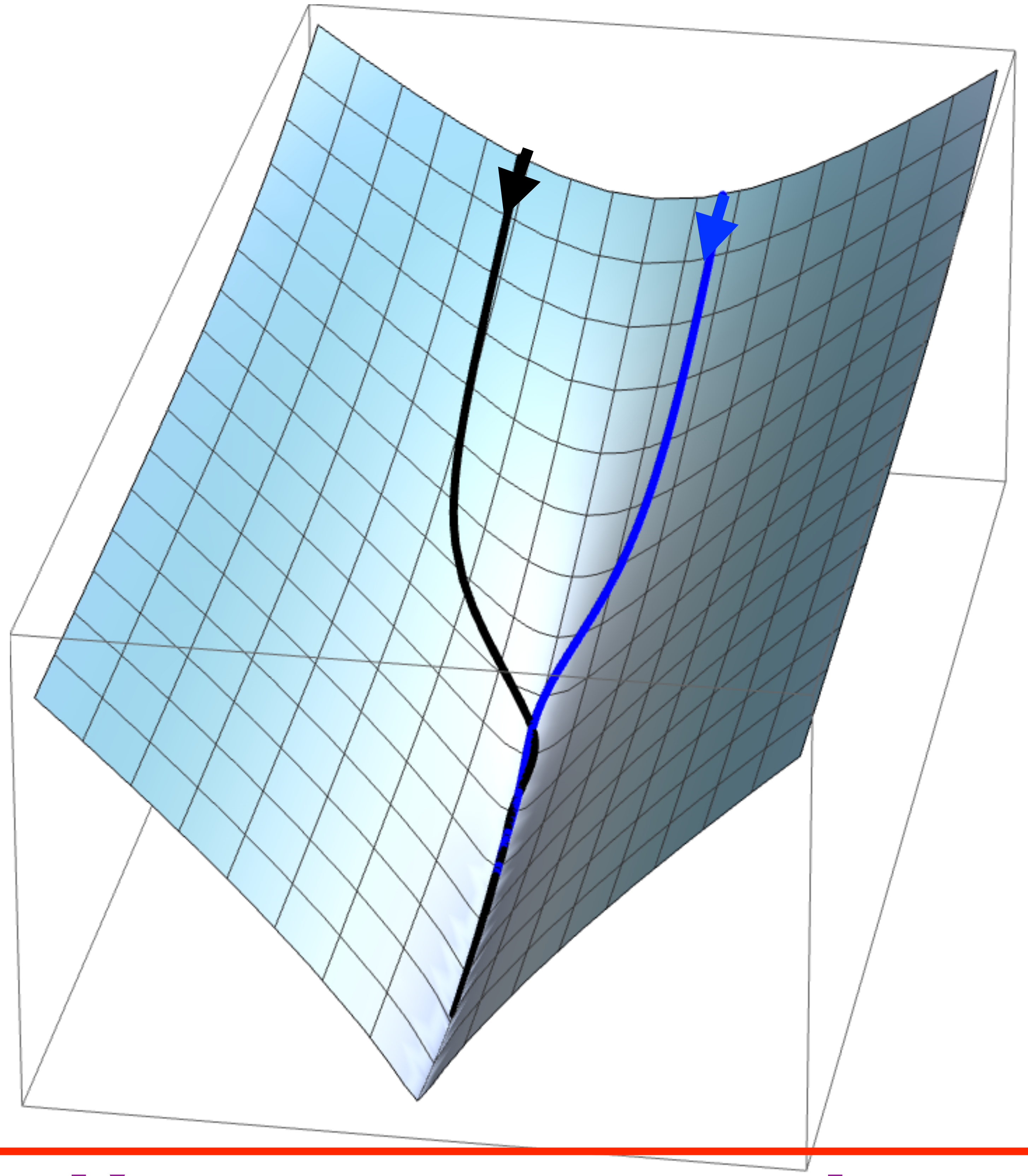


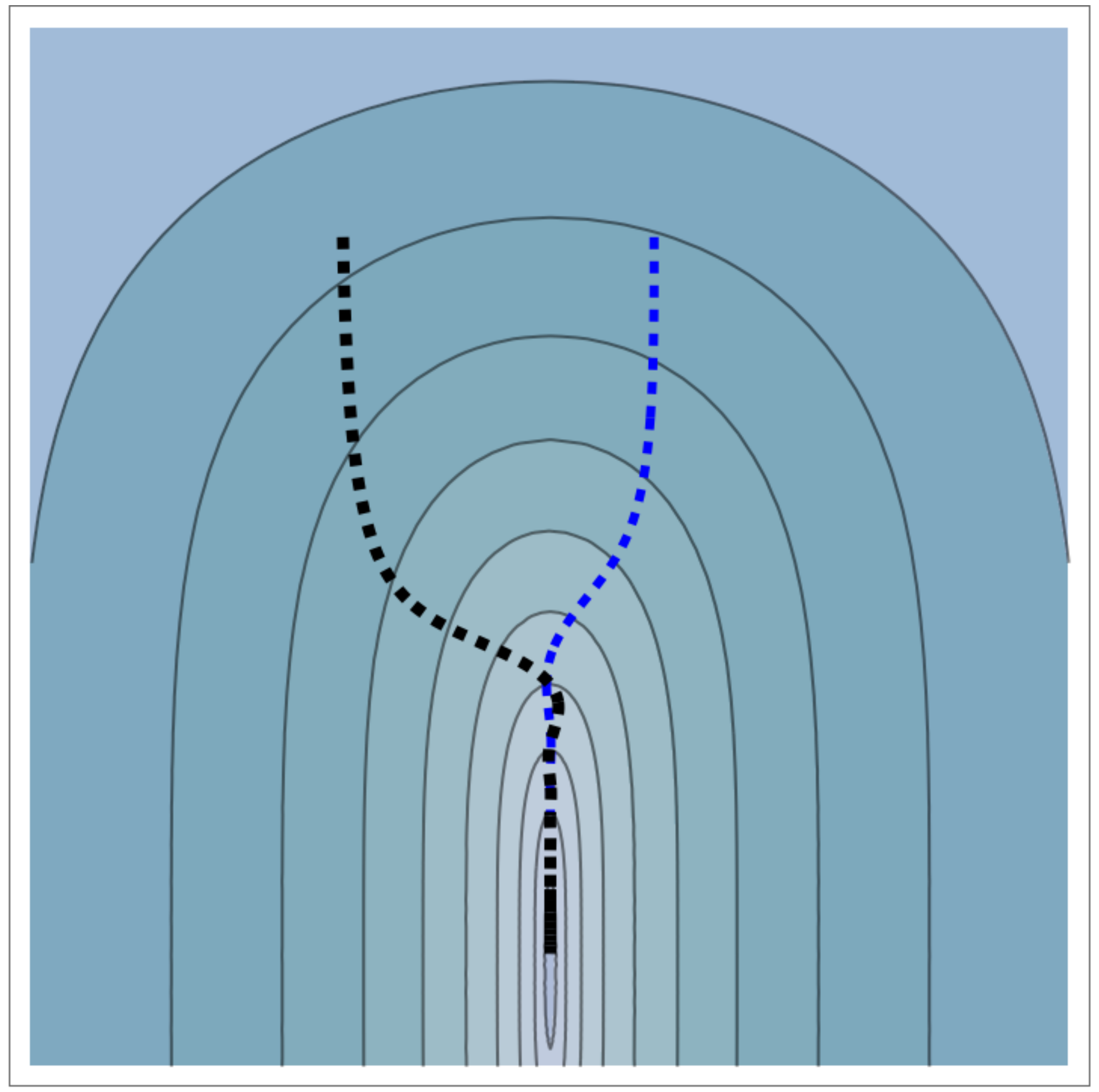
Contour

Inflationary parameters depend on the starting point very much.

B. Vally-like potential

Slow-roll trajectory

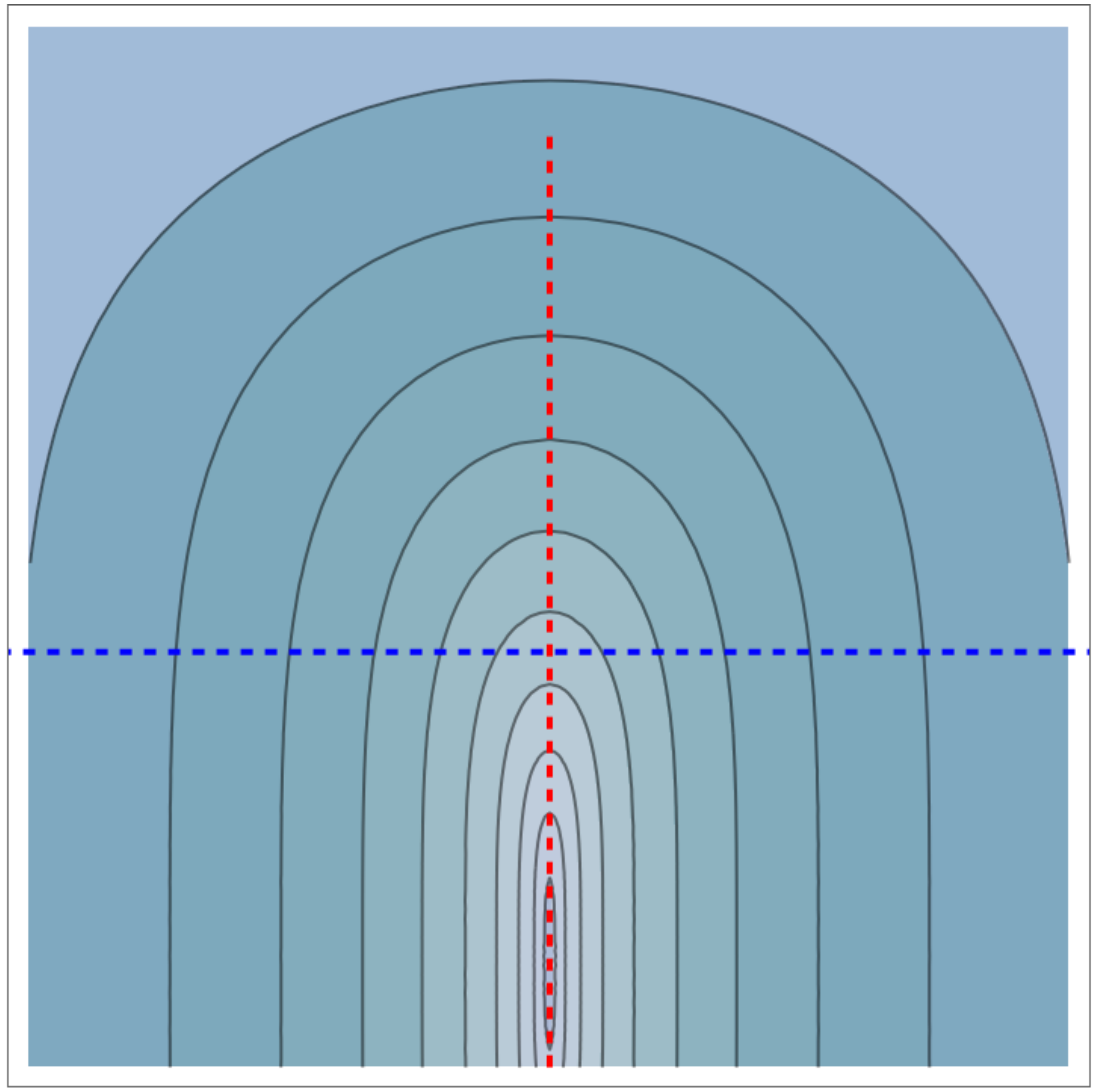




Contour

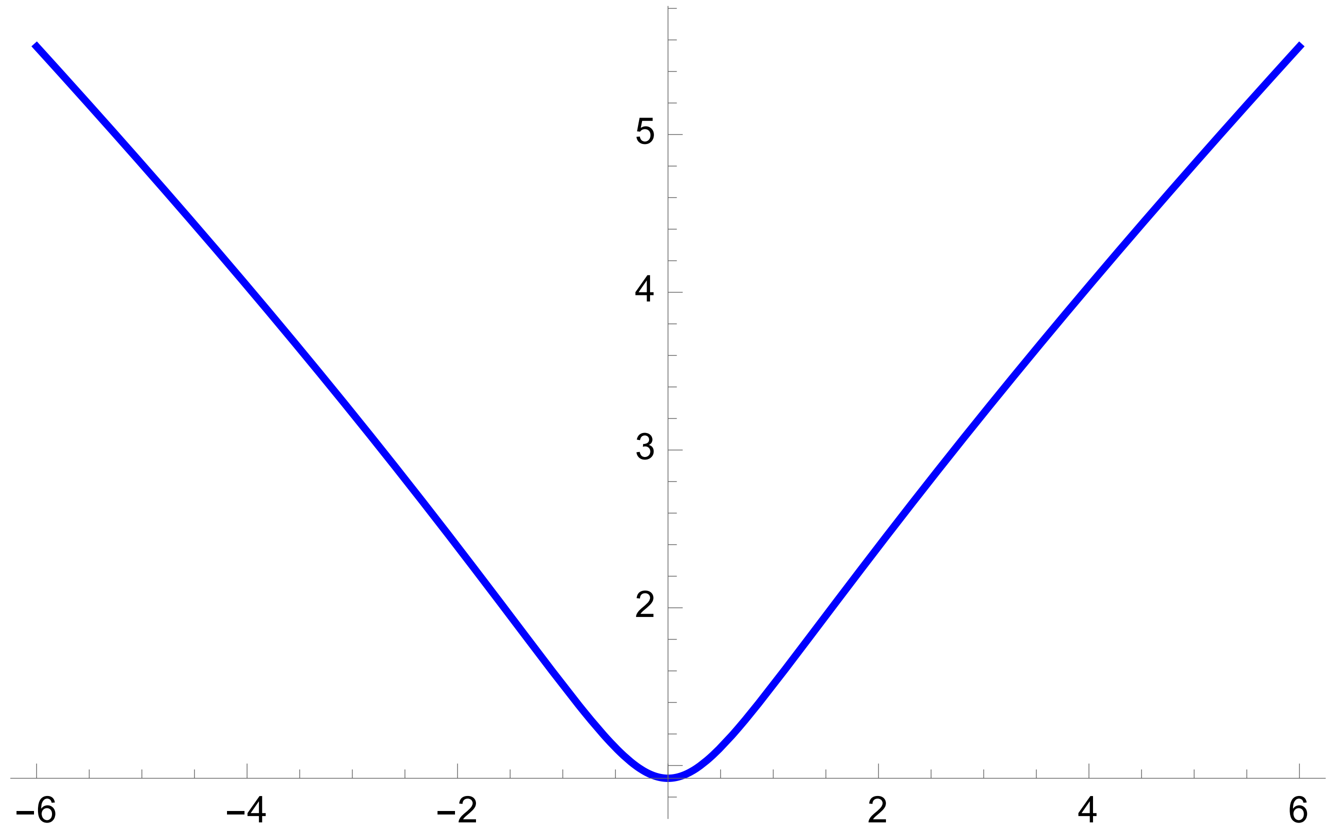
Inflationary parameters do not depend on the starting point very much; effectively a single-field system.

But another problem

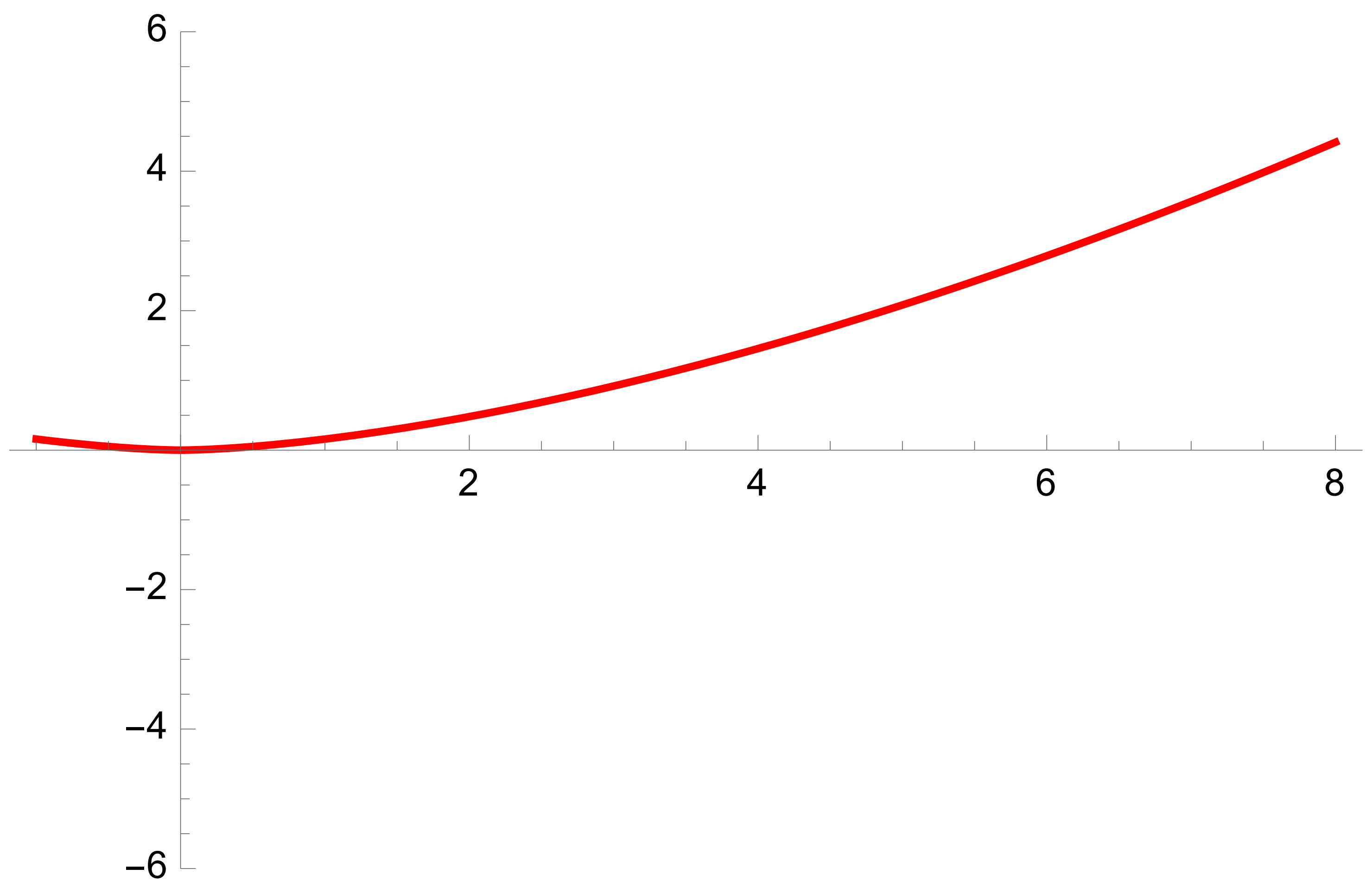


Contour

Potential



Potential

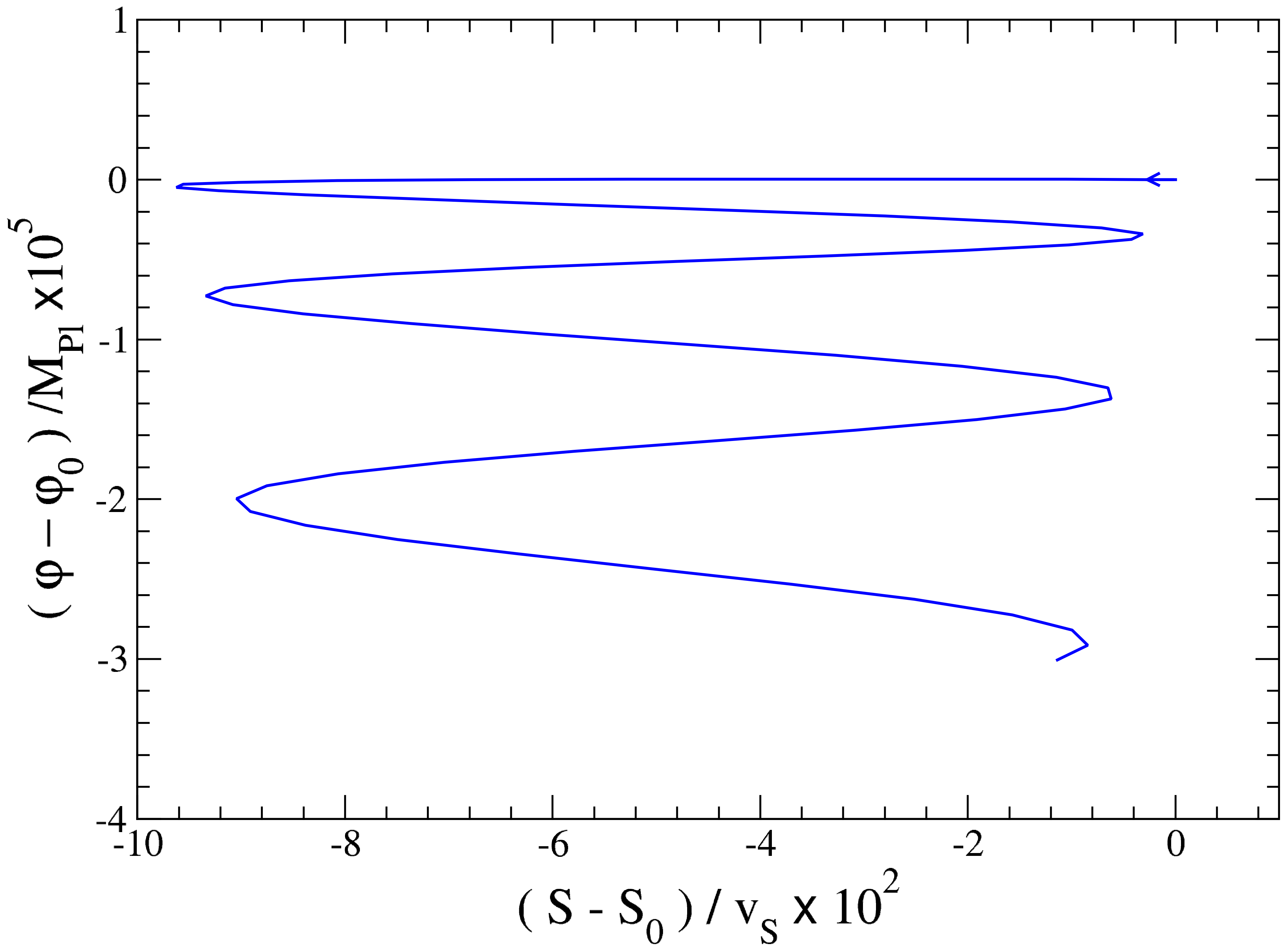


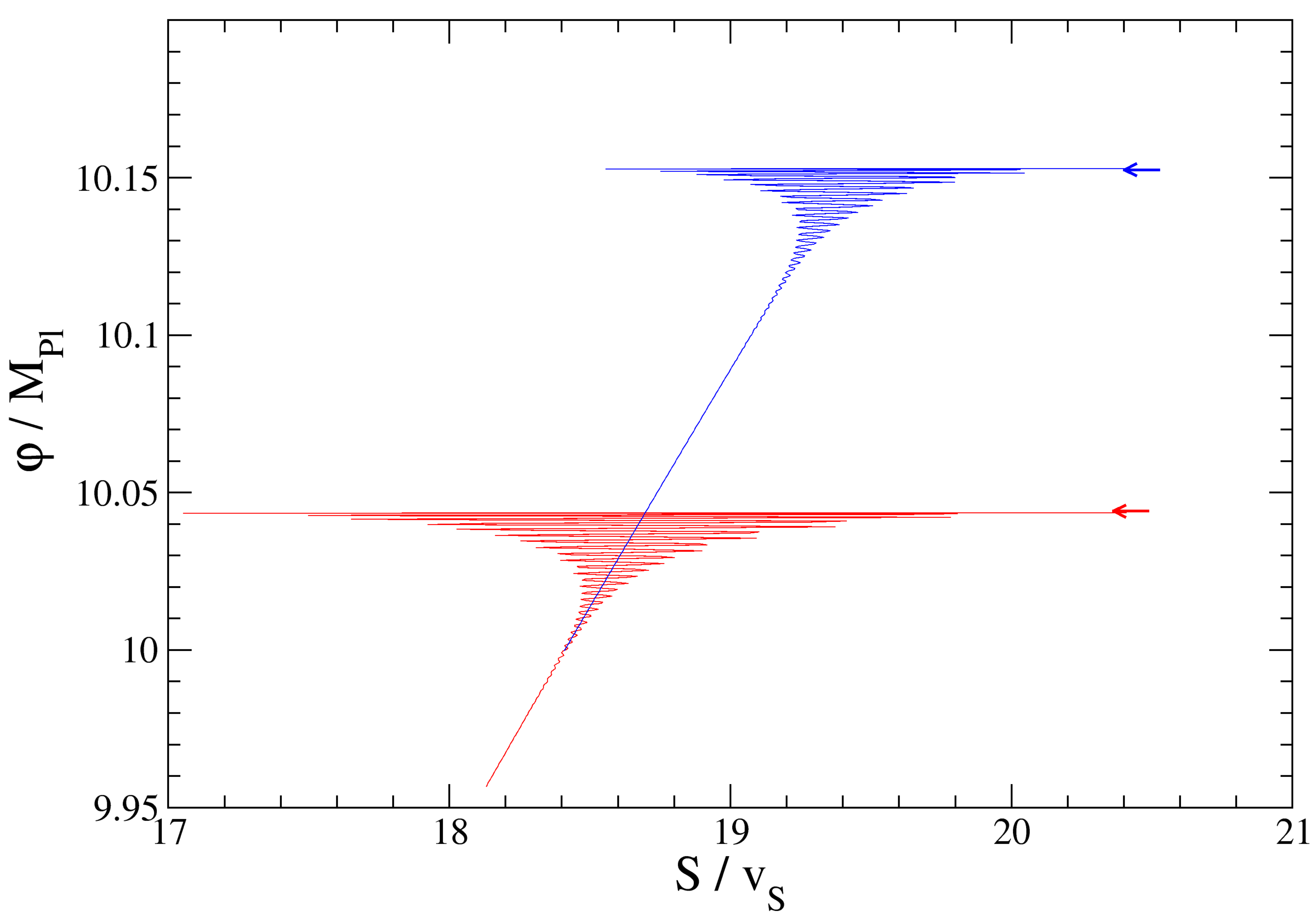
Slow-roll condition

X

○

Zoomed near the starting point (I in Einstein frame)

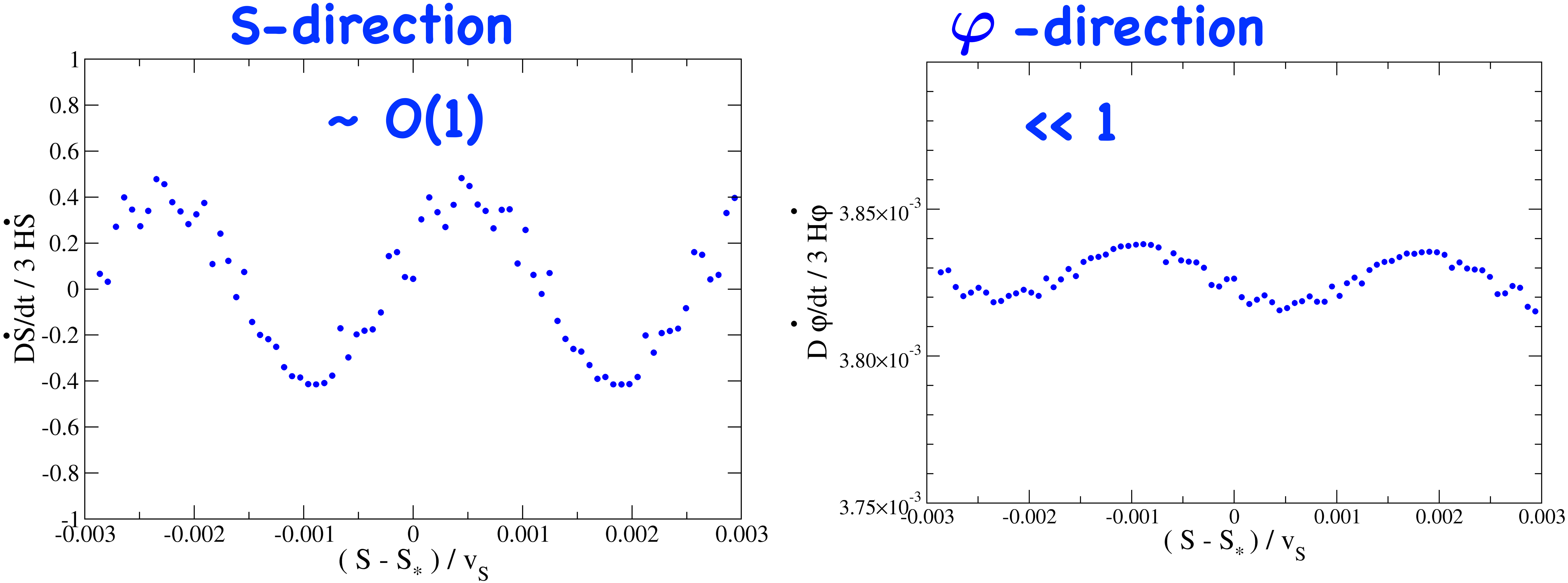




Oscillating just after the start, but conversing fast to a „fixed point" trajectory

Initial value dependence is suppressed.

(η) slow-roll condition for I



If η condition were satisfied,

$$\mathcal{D}\dot{\phi}^I + 3H\,\dot{\phi}^I + \mathcal{G}^{IJ}\,V_{,J} = 0$$

δN formalism to compute n_s, r, f_{NL}

Curvature perturbation

Sasaki+Stewart,'95,…….

$$\zeta = \delta N = N(\delta\phi_*) - \bar{N}$$
$$= (\partial N / \partial \bar{\phi}_I)\, \delta\phi_I|_{t=t_*} + \cdots$$

on the uniform energy density hypersurface

$$\bar{\phi}^I(t_*) = \bar{\phi}^I_* \quad \rightarrow \bar{\rho}(t) \rightarrow \bar{\rho}(\bar{t}_{\rm end})$$
$$\|$$
$$\phi^I(t_*, x) = \bar{\phi}^I_* + \delta\phi^I_*(x) \rightarrow \rho(t) \rightarrow \bar{\rho}(t_{\rm end})$$

δN formalism works

if eigenvalues of ε_{IJ} << 1, where

Sasaki+Stewart,'95,.......

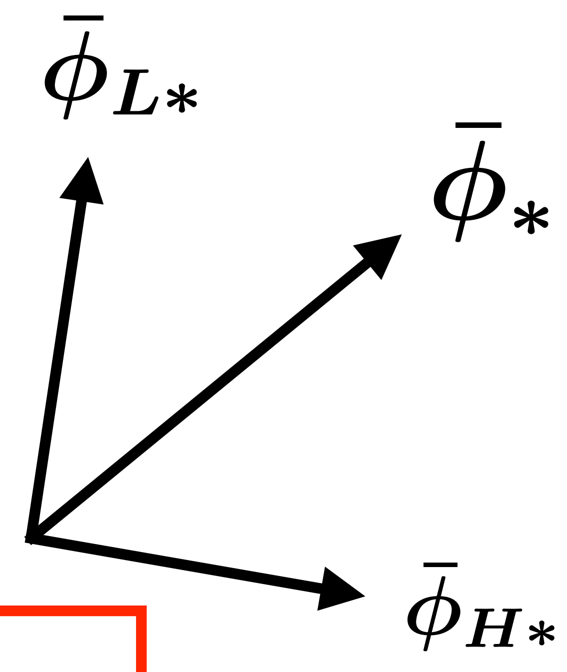
$$\epsilon_{IJ} = \cdots\cdots - \frac{\mathcal{D}_I \mathcal{D}_J V}{3H^2}$$

η_v **slow-roll parameter**

Valley structure -> ε_L << 1 , ε_H >> 1 !!

Moreover, fluctuation of heavy modes are exponentially suppressed in super horizon ! Pilo et al,'14

$$\tilde{\epsilon} = R^T \epsilon \, R = \begin{pmatrix} \tilde{\epsilon}_L & 0 \\ 0 & \tilde{\epsilon}_H \end{pmatrix} \quad \sim \ : \text{ in diagonal basis}$$



„Excite" only the light modes at t$_*$.

$$\delta\phi_* = R \begin{pmatrix} \delta\phi_L \\ \delta\phi_H = 0 \end{pmatrix} \text{ at } t = t_*$$

Aoki, JK, Yang, '24

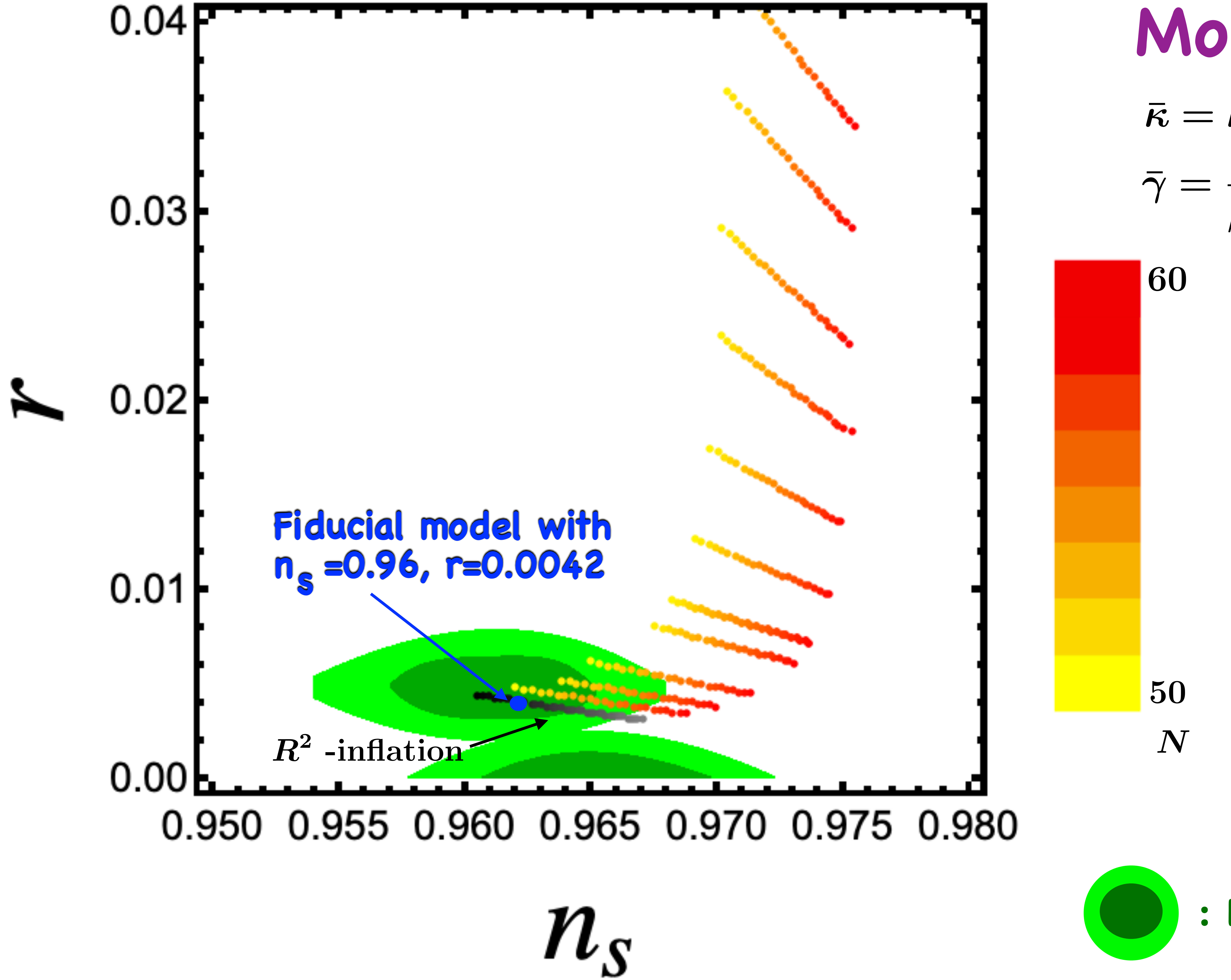
$$n_s = \cdots \, , \ r = \cdots , \quad f_{NL} = \frac{5}{6} \frac{(\mathcal{D}_i \mathcal{D}_j N) N^i N^j}{(N_k N^k)^2}$$

$$\text{with } \ N_i = \frac{\partial N}{\partial \bar{\phi}_L^i}$$

Model I

$$\bar{\kappa} = \kappa/\beta = 1.26$$

$$\bar{\gamma} = \frac{\gamma}{\beta^2} = 5406 \sim 21.6$$



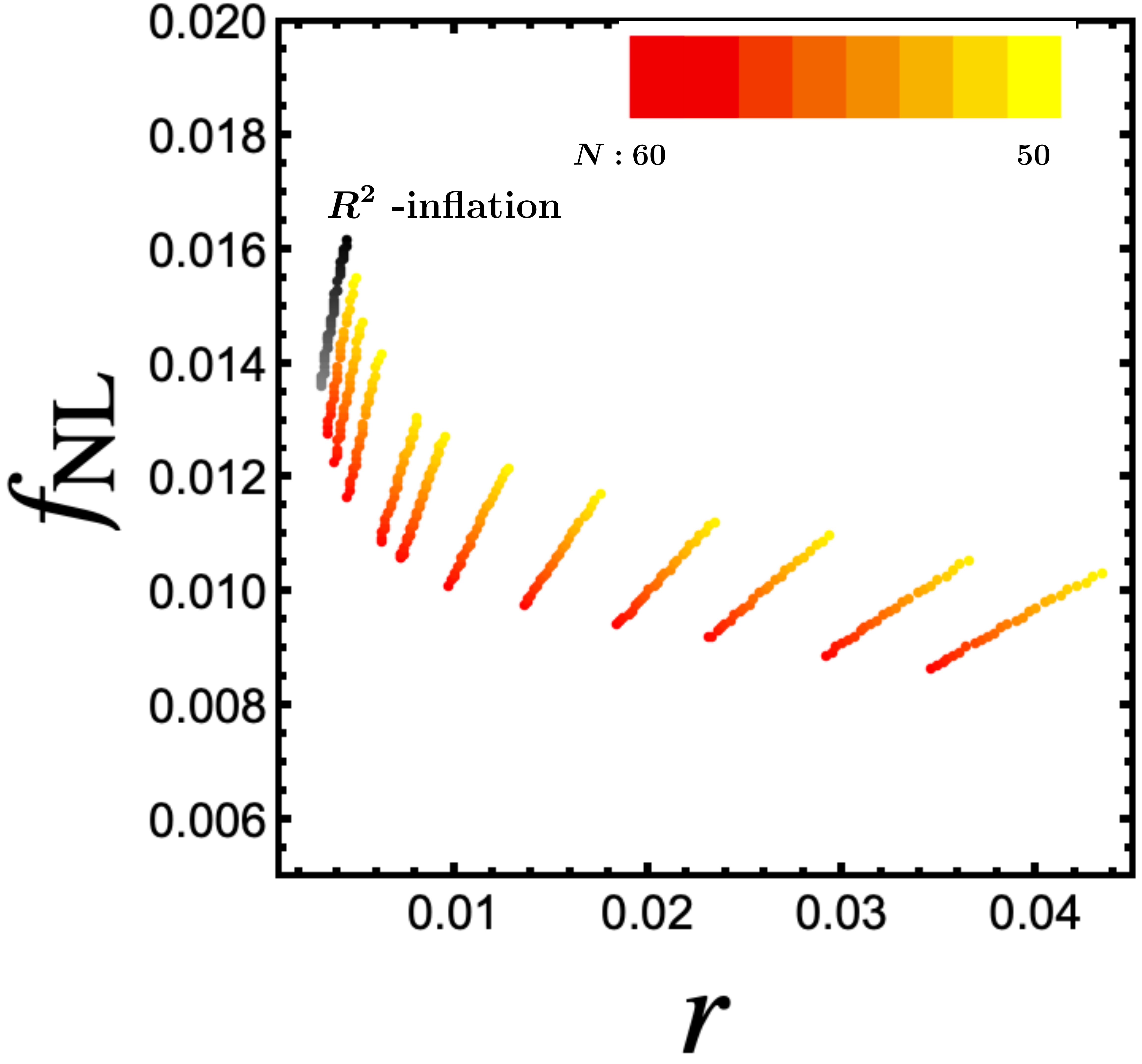
$$M_{\rm ghost} = (1.4 - 2.6) \times 10^{-2} M_{\rm Pl}$$

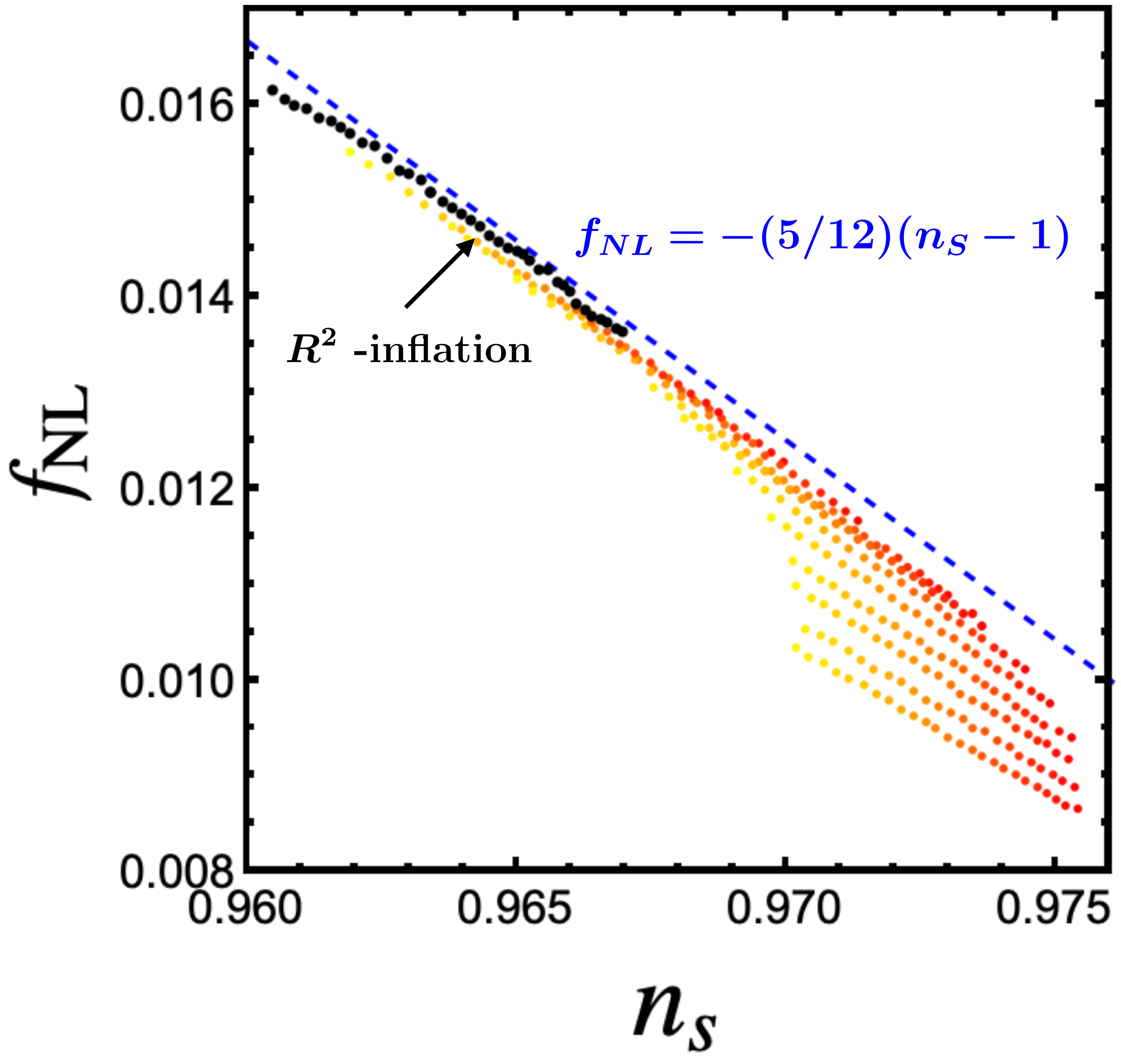
$$H = (0.31 - 3.5) \times 10^{-4} M_{\rm Pl}$$

$r > r_{R\hat{}2}$

: LiteBird/Planck constraint

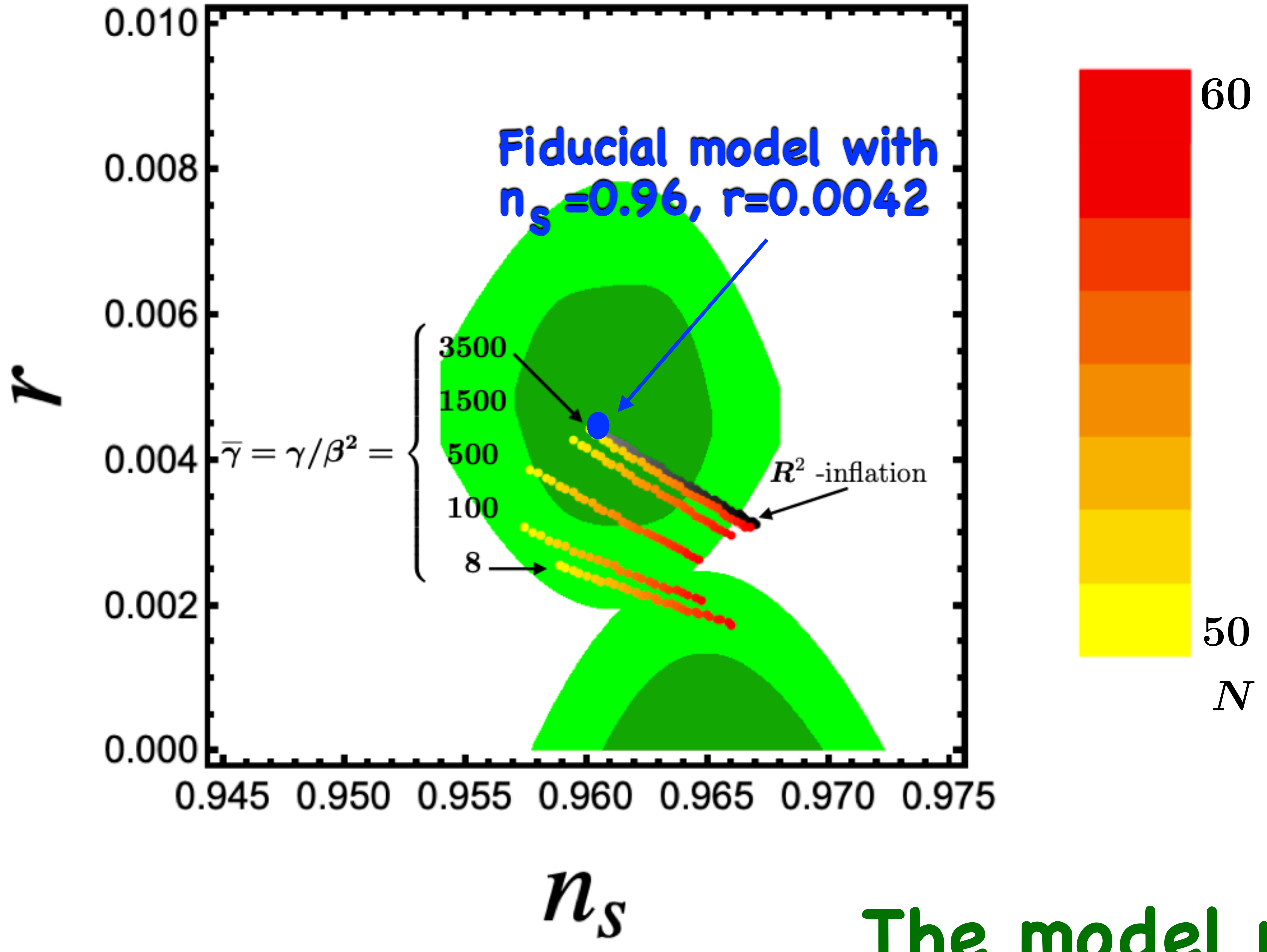
Non-Gaussianity (I)





The model behaves similar to a single-field model, except for $n_s \gtrsim 0.97$.

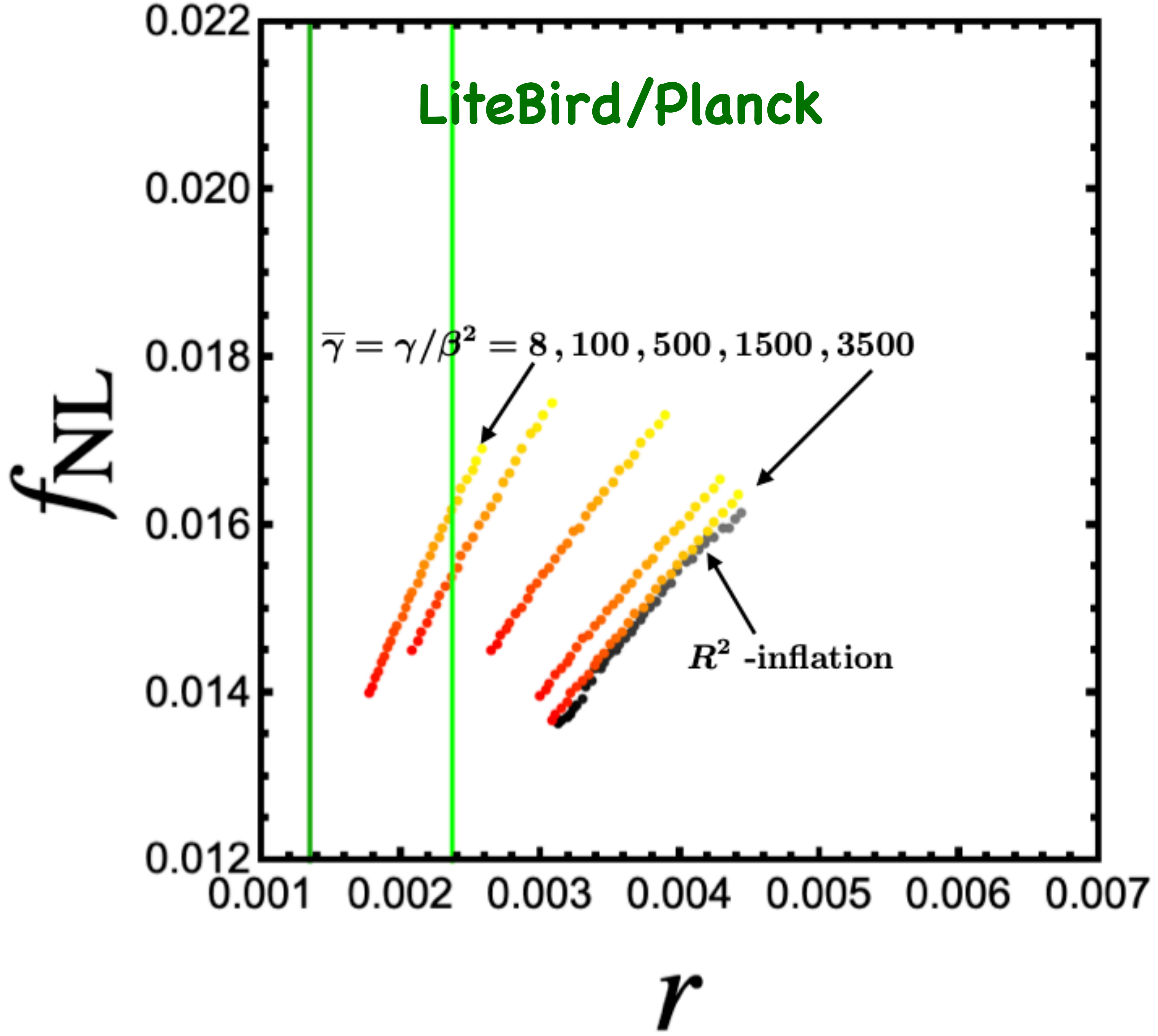
Model II

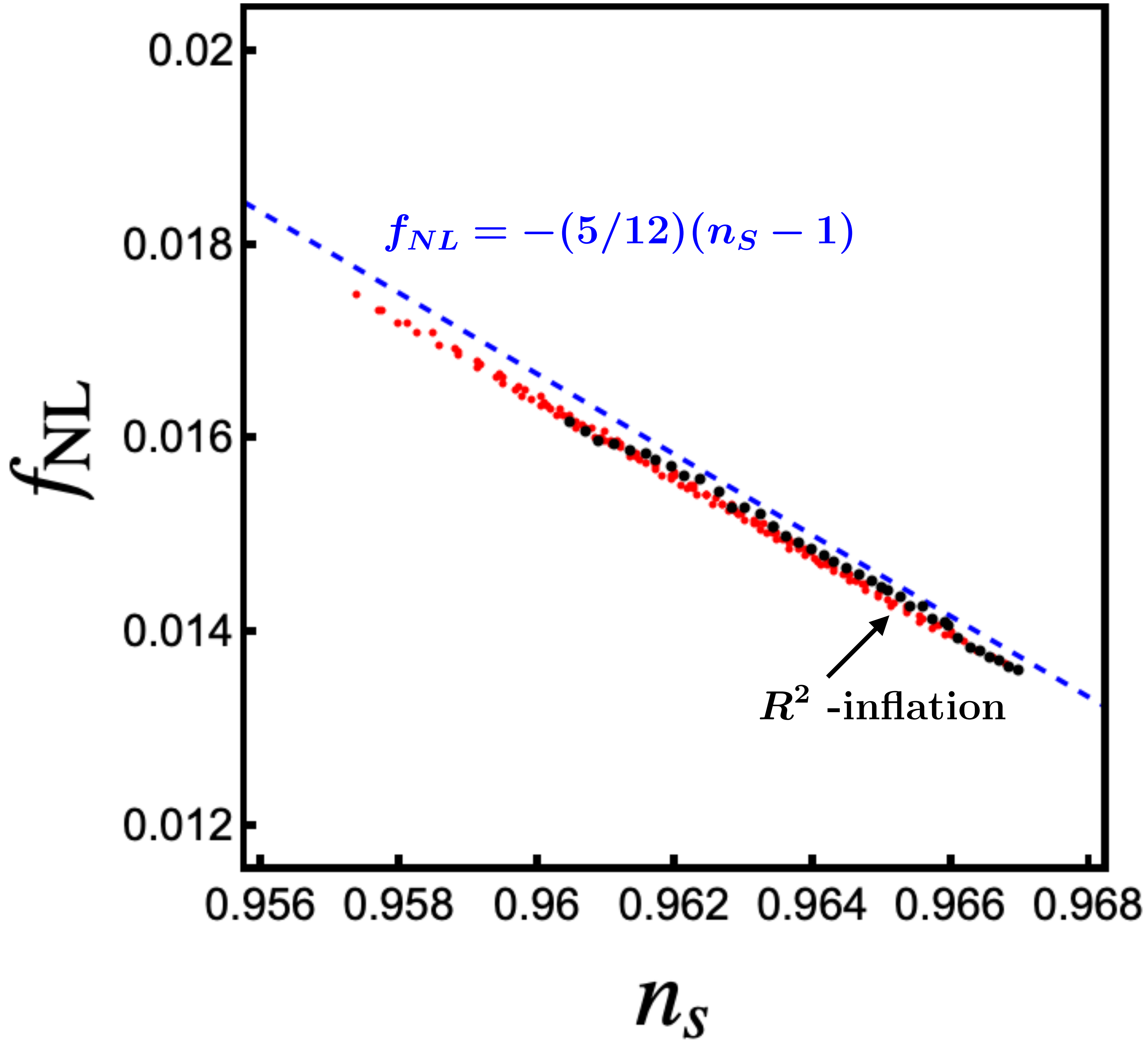


$r < r_{R\hat{}2}$

The model model will be consistent with null detection at LiteBIRD.

Non-Gaussianity (II)





The model behaves like a single-field model (as expected).

Summary

1. If the origin of all energy scales is known, a new route toward a solution of the hierarchy problem might open.

2. Various indications in particle physics + cosmology that the underlying theory is scale invariant.

3. Extension of the R^2 model
   $\Rightarrow$ More than two scalar fields involved in inflation
   $\Rightarrow$ Multi-field system for inflation.

4. Valley-like potential $\Rightarrow$ Initial value dependence is suppressed, but δN formalism has to be accordingly adjusted.

5. r of our models can be measured at future experiments, but not f_{NL}.

Ευχαριστώ

L_H: QCD-like sector

χSB (chiral symmetry breaking)

= Origin of all scales

$<\bar{\psi}\psi> \neq 0$ & $<S> \neq 0$

L_G: $Mpl^2 = \beta_S <S>^2$ and inflation

L_N: $m = y_M <S> \sim 10^7$ GeV (ν option)

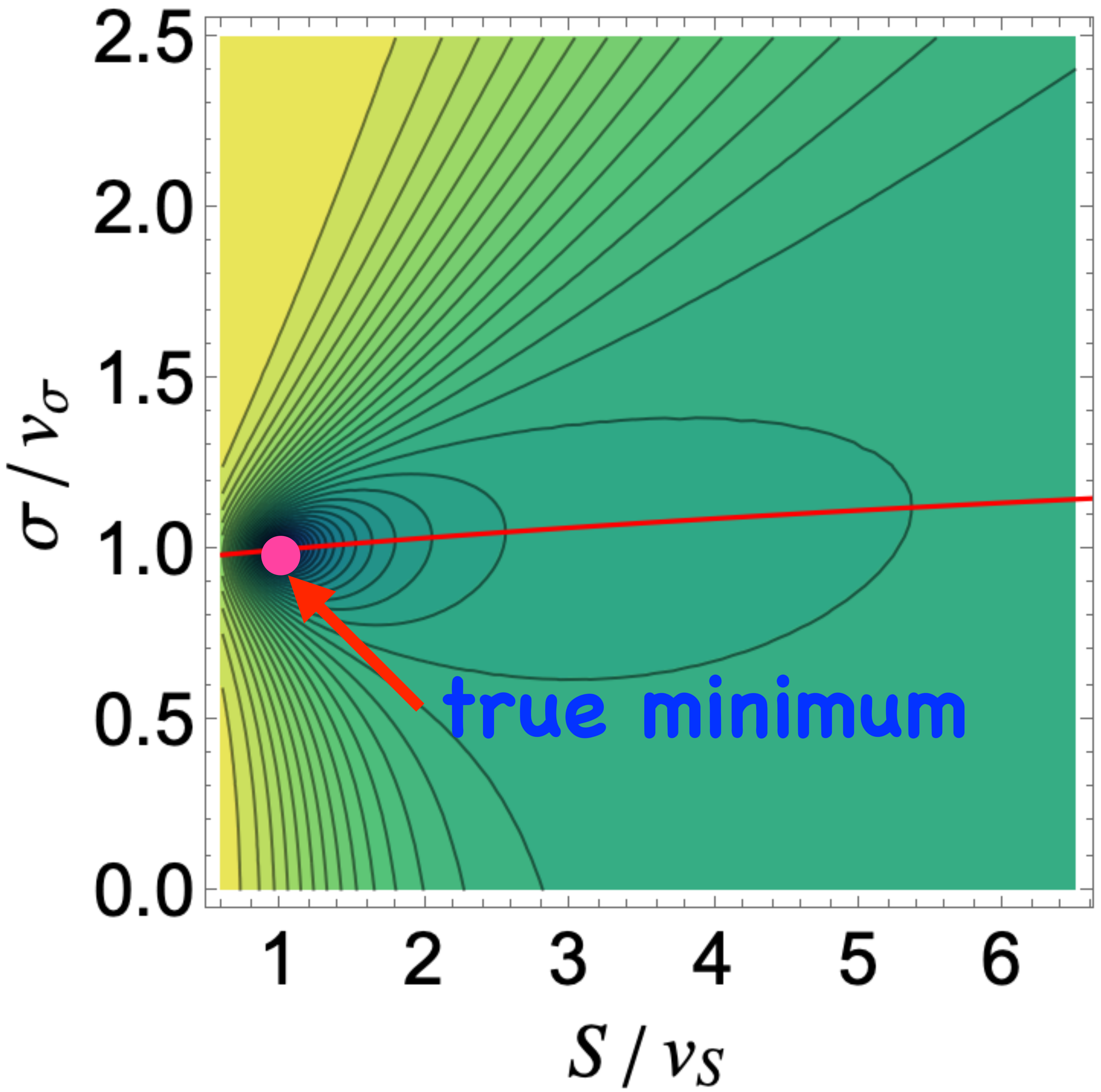
Brivio+Trott,'17,…….

The full potential is: $V_T(S, \sigma, \varphi)$ (φ = scalaron)
in the Einstein frame

A three-field system of cosmic inflation

But

Contour of $V_T(S, \sigma, \varphi_v(S, \sigma))$



local minimum of φ for given S and σ

σ ≈ constant = v_σ

Effectively two-field system in the Einstein frame described by

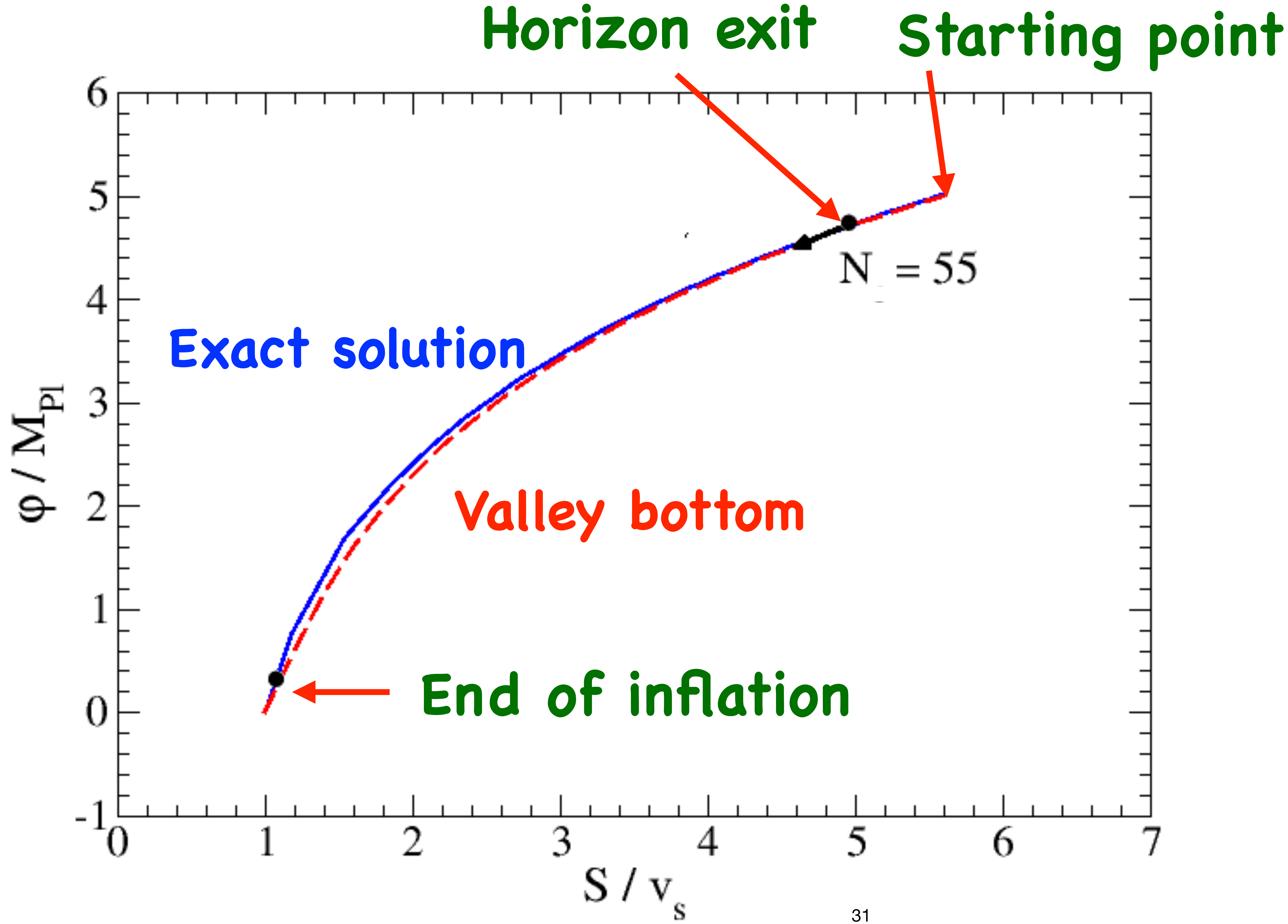
$$\ddot{\varphi} + \left(1/\sqrt{6}M_{\rm Pl}\right) e^{-(2/3)^{1/2}\varphi/M_{\rm Pl}}\dot{S}^2 + 3H\dot{\varphi} + \frac{\partial V(\varphi,S)}{\partial\varphi} = 0\,,$$

$$\ddot{S} - 2\left(1/\sqrt{6}M_{\rm Pl}\right)\dot{\varphi}\dot{S} + 3H\dot{S} + e^{(2/3)^{1/2}\varphi/M_{\rm Pl}}\frac{\partial V(\varphi,S)}{\partial S} = 0\,,$$

$$\left(\mathcal{D}\dot{\phi}^I + 3H\,\dot{\phi}^I + \mathcal{G}^{IJ}\,V_{,J} = 0\right)$$

where $\varphi = \varphi(t)$, $S = S(t)$ and H is the Hubble parameter

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm Pl}}\left(\frac{1}{2}(\dot{\varphi}^2 + e^{-(2/3)^{1/2}\varphi/M_{\rm Pl}}\dot{S}^2) + V(\varphi,S)\right)$$


Horizon exit
Starting point
N = 55
Exact solution
Valley bottom
End of inflation
φ / M_Pl
S / v_s
6
5
4
3
2
1
0
-1
0
1
2
3
4
5
6
7