On Quantum and Gravity

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Mesoscopic Quantum Gravity

- Quantum & Gravity: **What is to be quantized?** The naïve answer is, we should quantize gravitons (with unknown UV completion). This is the Effective (Quantum) Field Theoretical way, but there are problems: e.g., when energy density is sufficiently high, we e.g., expect that black holes are being formed, which are certainly behind the scope of applicability of EFT.
- **The mesoscopic bottom—up approach** aims at capturing the essence of what it means for geometry to become quantum and how quantum geometry goes beyond effective field theory. It is hoped that the understanding of mesoscopic gravity will reveal the fundamental structures behind the theory of quantum gravity, much as the understanding of Brownian motion paved the way for atomic theory and quantum mechanics.
- Then again **What are the "degrees of freedom" to be quantized?**

The program

- The cosmological constant problem.
- Corner proposal in GR.
- ~~Noise in gravitational interferometers and physics of causal diamonds.~~

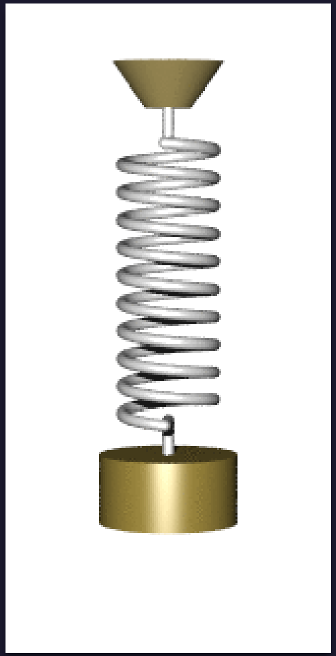
The Cosmological Constant

The cosmological constant problem – what's the problem?

- In quantum mechanics, the ground (vacuum) state of an oscillator of frequency ω has energy $E_0 = 1/2\ \hbar\omega$.
- Field theory describes an infinite number of oscillators (one per momentum), and the total vacuum energy density is infinite.

$$E_0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \omega_p \sim \lim_{\Lambda \to \infty} \frac{1}{16\pi^2} \Lambda^4$$

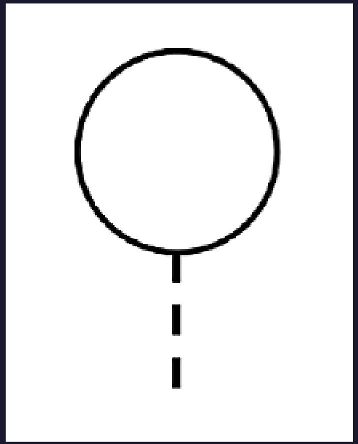
- If gravity is present, due to its universal nature the infinite vacuum energy produces an infinite gravitational field. Or, if there is a natural cutoff, Planck energy say, the cosmological constant is going to be of order of this cutoff, many orders of magnitude larger than what is observed.



Contributions to the cosmological constant from matter loop diagrams *

- In the leading order the matter-linearized gravity (graviton) coupling is

$$S_{int} \sim \int h_{\mu\nu} T^{\mu\nu}$$



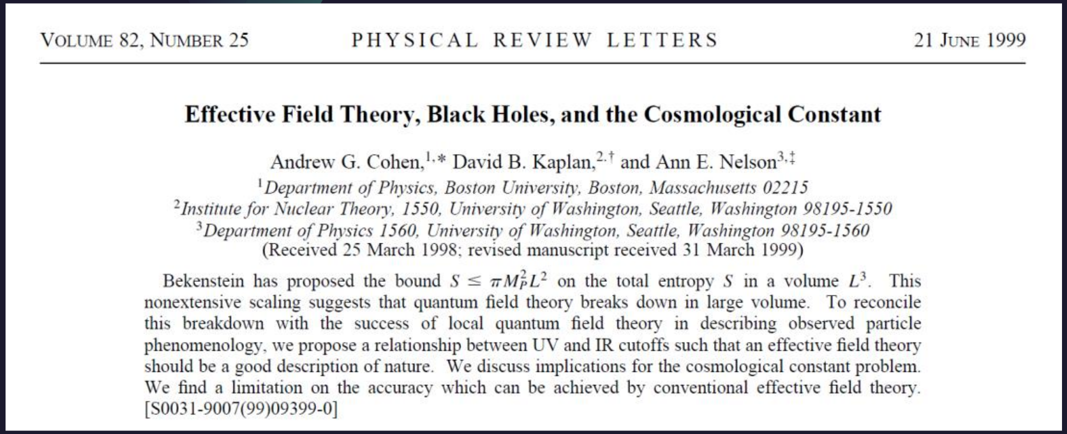
- Computing the tadpole diagram, we get

$$\Delta\mathcal{L} \sim h_{\mu\nu} \times \int \frac{d^4p}{(2\pi)^4} \frac{2p^\mu p^\nu - \eta^{\mu\nu}\left(p^2 - m^2\right)}{p^2 - m^2 + i\epsilon}$$
$$\sim h_{\mu\nu} \times \eta^{\mu\nu} \frac{1}{64\pi^2} \Lambda^4$$

- The loop contribution to cosmological constant is proportional to the regularized volume of momentum space. It is implicitly assumed that the process takes place in the fixed infinite Minkowski space (hidden IR divergence).

*J. F. Donoghue, Phys. Rev. D **104**, 045005

The cosmological constant problem



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Effective Field Theory, Black Holes, and the Cosmological Constant

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Bekenstein has proposed the bound $S \leq \pi M_P^2 L^2$ on the total entropy S in a volume L^3. This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, we propose a relationship between UV and IR cutoffs such that an effective field theory should be a good description of nature. We discuss implications for the cosmological constant problem. We find a limitation on the accuracy which can be achieved by conventional effective field theory. [S0031-9007(99)09399-0]

- 25 years ago, in a renowned paper Cohen, Kaplan, and Nelson postulated that to solve the cosmological constant problem one should append EFT with a constraint of the QG origin relating IR cutoff (the size of the spacetime region *L*) and UV cutoff (the size of momentum space Λ)

$$L^3\Lambda^4 \lesssim L M_P^2$$

- If L is identified with the Hubble size H^{-1}, the UV cutoff is bounded by 10^{-3} eV, which agrees with the current value of the cosmological constant.

- Not surprisingly the CKN bound can be understood as a dramatic depletion of the number of states in UV.

- Could we obtain a bound like that from the first principles? Which "degrees of freedom" should be employed?

An idea

- The crucial observation is that the problem is not only about UV, but also about IR.
- To see this, we revisit the standard computation from slightly different perspective*. Instead of computing the loop diagram with no external legs, following Polchinski (String Theory, Ch. 7) the start point is the circle amplitude of a particle moving on a circle S^1

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \mathrm{Tr}\, e^{i\hat{\mathscr{H}}\tau} \sim \rho V_4$$

- It turns out that this integral is over the phase space.

* L. Freidel, JKL, R. Leigh, D. Minic, *Phys.Rev.D* 107 (2023) 12, 126016, [2212.00901](2212.00901) and *Int.J.Mod.Phys.D* 32 (2023) 14, 2342004 [2303.17495](2303.17495)

Regularization

- Not surprisingly, the loop integral is divergent. We regularize it by choosing a finite region in the phase space. To this end, we split the phase space into cells of minimal size

$$\boxed{\varepsilon\lambda = 2\pi\hbar}$$

- It should be emphasized that, except for this constraint, the scales ε and λ are arbitrary; nothing forces us to identify them with the Planck mass/length. These scales can be chosen contextually, vis a vis the specific physical problem at hand.

- The regularized region of the phase space has size $M = N_p\,\varepsilon$ in momentum direction and $L = N_p\,\lambda$ in spacetime direction. Also, the total number of cells = total number of degrees of freedom = dimension of the Hilbert space is

$$\boxed{N = (N_q N_p)^4 = \left(\frac{LM}{2\pi\hbar}\right)^4}$$

Vacuum energy

- Massaging the loop formula, we eventually leads to the bound

$$\rho \leq \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar}\right]^4 = \hbar \left[\frac{M}{2\pi\hbar}\right]^4$$

- If M is identified with a large mass scale such as Planck mass, then the usual conundrum pertains. ***But there is nothing here that makes it necessary/natural.***

- We can rewrite

$$\rho \leq \hbar \frac{N}{V_q}, \quad V_q = L^4$$

- It relates the product of vacuum energy density and space-time volume, ρ and V_q, to N. But (in the limit of small $\hbar$) N is the dimension of the Hilbert space of the system, and therefore, basically, its entropy.

$$\boxed{\rho \leq \hbar \frac{S}{L^4}}$$

Entropy and N

- The gravitational entropy scales as an area

$$S_{grav} \sim \ell_{Pl}^{-2}\,\text{Area} \sim \left(\frac{\ell}{\ell_{Pl}}\right)^2$$

- The holographic principle states that matter entropy N cannot exceed de Sitter gravitational entropy which gives the vacuum energy bound

$$\rho \leq \hbar\,\frac{N}{V_q} \lesssim \frac{\hbar}{\ell^2 \ell_{Pl}^2}\,, \qquad V_q = \ell^4$$

- which gives the value of the vacuum energy contribution to cosmological constant

$$\boxed{\Lambda_{cc} \sim \rho G \sim \frac{1}{\ell^2}}$$

The lesson

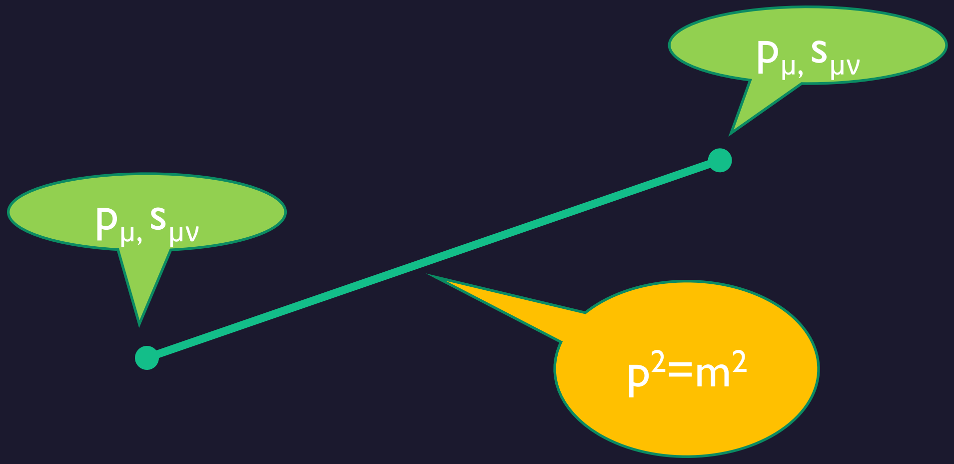
- The cosmological constant is small because the universe is large. Why the universe is large? It is large, because it is stable against fluctuations. For N degrees of freedom, the statistical fluctuations are of order of $N^{1/2}$ and are (relatively) small if N is large.
- The calculation suggests that it is the phase space that serves as the arena for quantum gravity, with phase space cells being the elementary quanta. (Born geometry, relative locality, …)
- Can this calculation be generalized? E.g., can we undertand the Higgs mass?

The Corner Proposal

Intuition: Quantum Field Theory (perturbative)

- You can think of (perturbative) quantum field theory as a set of Feynman diagrams. The building block of Feynman diagram is a line with Hamiltonian constraint (mass-shell condition) in the bulk and conserved charges (momentum, angular momentum) at the boundaries.

- The algebra of symmetries of boundary charges is, universally, the Poincarè algebra. Then we can join segments to obtain more complex diagrams. It is being said that QFT is about irreducible representations of the Poincarè group.



Intuition: gravity

Charges on the boundary

Wheeler—De Witt in the bulk

- Perhaps we can similarly think of gravity with regions surrounded by boundaries and a symmetry of boundary charges that can be an analog of Poincarè algebra and that can organize quantization.
- Instead of solving WDW equation, we can build spacetime by combining the regions (Feynman diagrams of bubbles).
- The boundary is central, ~~the bulk is not that interesting~~.

The corner proposal

- The corner C is the codimension 2 boundary of a codimension 1 region. There are non-vanishing Noether charges carried by the corner, associated with diffeomorphisms. Remarkably, in metric theory the (Poisson) algebra the charges is universal and called Extended Corner Symmetry algebra

$$\mathfrak{ecs}_D = \mathfrak{diff}(S_{D-2}) \ltimes \left(\mathfrak{sl}(2,\mathbb{R}) \ltimes \mathbb{R}^2\right)$$

- The proposal: Quantum gravity is a representation theory of ECS in the same way that QFT is a representation theory of the Poincare group.

- ***Forget about the origin of ECS and consider it as a starting point of the construction***.

Representations of ECS_2 *

- In D=4 the representation theory of ECS is not known, but the problem becomes manageable in D=2, when the corner becomes a point

$$\mathfrak{ecs}_2 = \mathfrak{sl}(2,\mathbb{R}) \ltimes \mathbb{R}^2$$

* L. Ciambelli, JKG, and L. Varrin, arXiv:2406.07101 [hep-th]

Representations of ECS_2 *

- In quantum theory we are interested in projective representations of symmetries. Bargmann&Mackey theorem says that the projective representations are equivalent to the ordinary unitary representations of its maximal central extension. To obtain the physical representations of a symmetry group we should therefore consider its central extensions.

- It is well known that the Poincare group does not have central extension, but Galileo group does, and in this case the central extension is identified with mass.

- Surprisingly also ECS_2 allows for central extension; the central element c replaces the translational algebra $\mathbb{R}^2$ with the Heisenberg algebra $\mathbb{H}_2$. In this way we obtain the quantum corner algebra QCS_2, whose irreducible, unitary representations we must investigate

$$\mathfrak{qcs}_2 = \mathfrak{sl}(2,\mathbb{R}) \ltimes \mathbb{H}_2$$

* The representation theory of QCS_2 is discussed in L. Varrin, 2409.10624

Metaplectic representation of QCS_2

- There is a irreducible, unitary representation of QCS_2

$$\mathbb{H}_2 : \quad P_- = \sqrt{c}a, \quad P_+ = \sqrt{c}a^\dagger, \quad C = c,$$

$$\mathfrak{sl}(2,\mathbb{R}) : \quad L_+ = \frac{1}{2}a^\dagger a^\dagger, \quad L_- = \frac{1}{2}aa, \quad L_0 = \frac{1}{2}a^\dagger a + \frac{1}{4}$$

- with two Casimirs, one being the central element C, and a cubic one

$$\mathscr{C}^{(3)}_{\mathrm{QCS}} = C\left(L_0(2L_0+3) - 2L_-L_+\right) + L_-P_+^2 + L_+P_-^2 - 2L_0P_-P_+$$

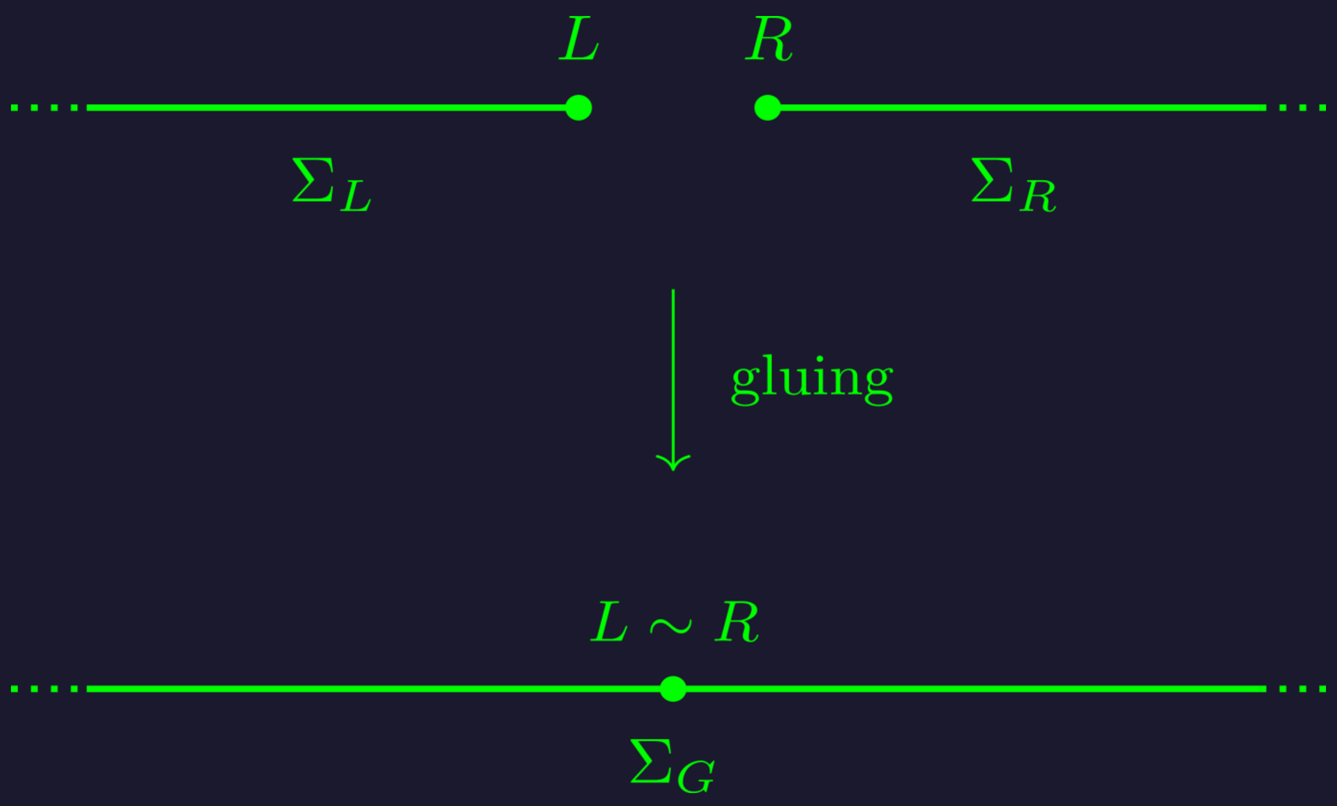
Gluing the segments

- Having two segments we can glue them into one.

Gluing the segments

- Roughly, gluing is possible only when the charges of the L and R corners are the same, so that they cancel each other and the net charge of the connection point is zero.
- The entangled product

$$\mathscr{H}_G = \mathscr{H}_L \sqcup_{\text{QCS}} \mathscr{H}_R \subset \mathscr{H}_L \otimes \mathscr{H}_R = \tilde{\mathscr{H}}_G,$$

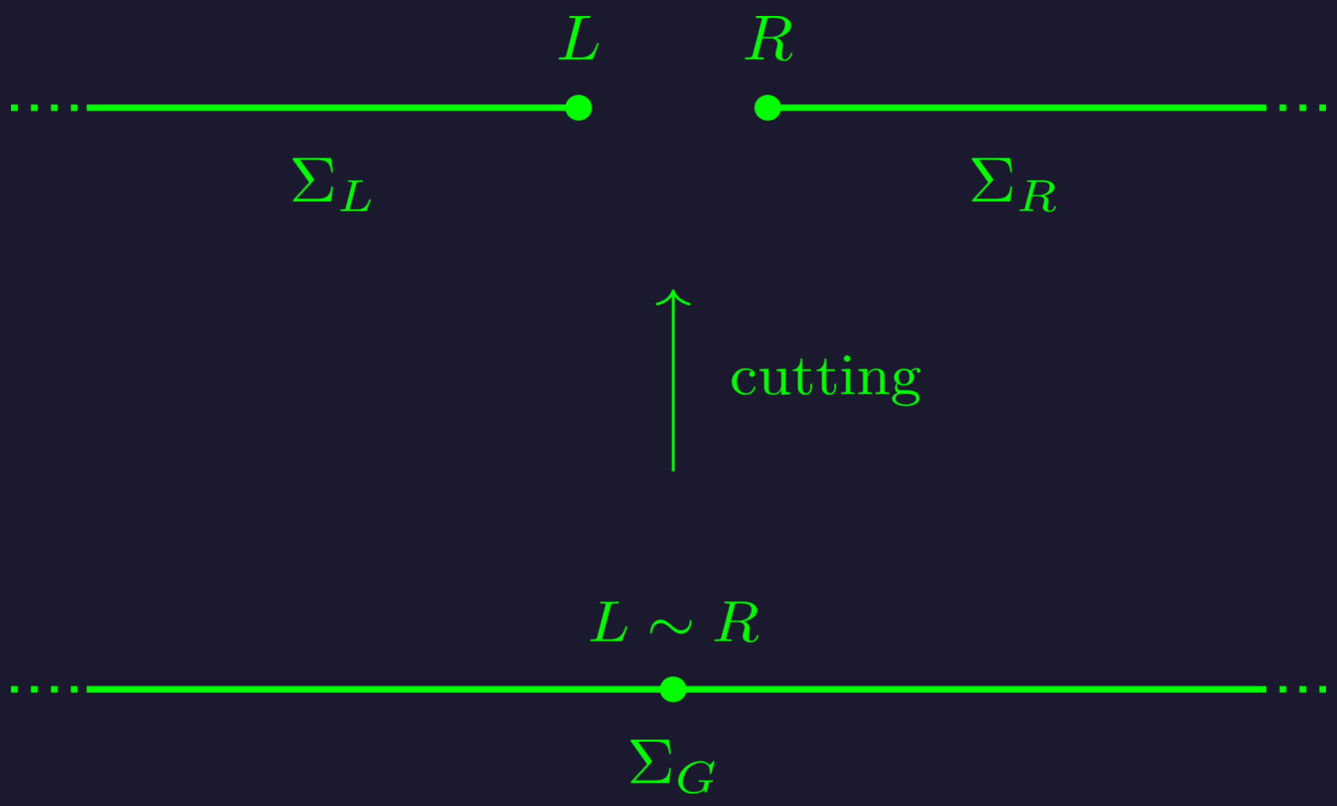


Cutting the segments

- Roughly, gluing is possible only when the charges of the L and R corners are the same, so that they cancel each other and the net charge of the connection point is zero.
- The entangled product

$$\mathscr{H}_G = \mathscr{H}_L \sqcup_{\mathrm{QCS}} \mathscr{H}_R \subset \mathscr{H}_L \otimes \mathscr{H}_R = \tilde{\mathscr{H}}_G,$$

- Knowing how to glue, we can also cut back.

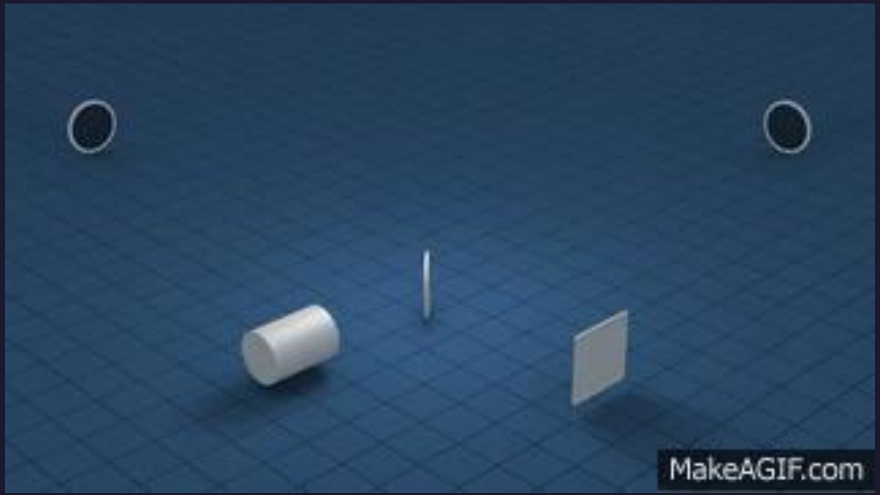


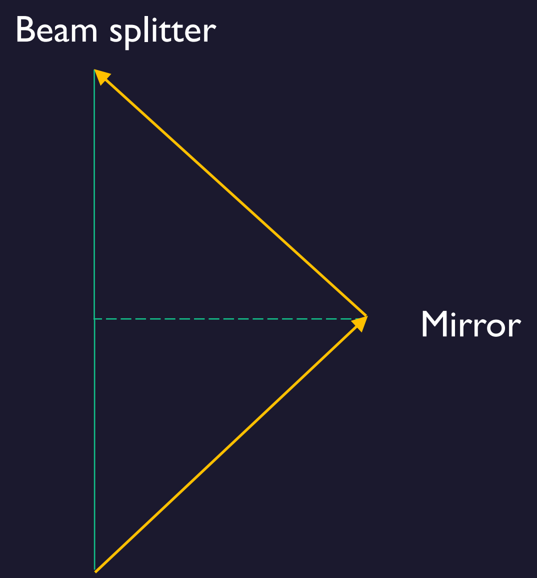
Outlook

- Entanglement entropy in terms of representation theory,
- So far, we considered only the bivalent vertex. It would be certainly possible to generalize the construction to the case of 3-, 4-, … valent vertices.
- Generalize to the higher dimensional case where Diff part of ECS starts being relevant.
- Corners, there and back again: is it possible to reconstruct the (classical/quantum) spacetime from ECS/QCS representation theory? There are some curious hints coming from relations between $\mathfrak{sl}(2,\mathbb{R})$ and symmetries of causal diamonds *.

M. Arzano (et. al) *JHEP* 05 (2020) 072, [2002.01836](2002.01836); *JHEP* 10 (2023) 165, [2306.12291](2306.12291)

On causal diamond





- Each photon traces the boundary of a causal diamond.
- Verlinde&Zurek proposal *:
  - Suppose that the QG DEGREES OF FREEDOM have the dynamics governed by the modular Hamiltonian K associated with the causal diamond fluctuate. Let

$$\boxed{\langle \Delta K^2 \rangle \sim \langle K \rangle = A}$$

  - Then such fluctuations can be in principle observed in present or near future interferometer.

E.Verlinde & K. Zurek, *Phys.Lett.B* 822 (2021) 136663, 1902.08207 and *Phys.Rev.D* 106 (2022) 10, 106011, 2208.01059

Thank you