3d Gravity and Ensembles of CFTs

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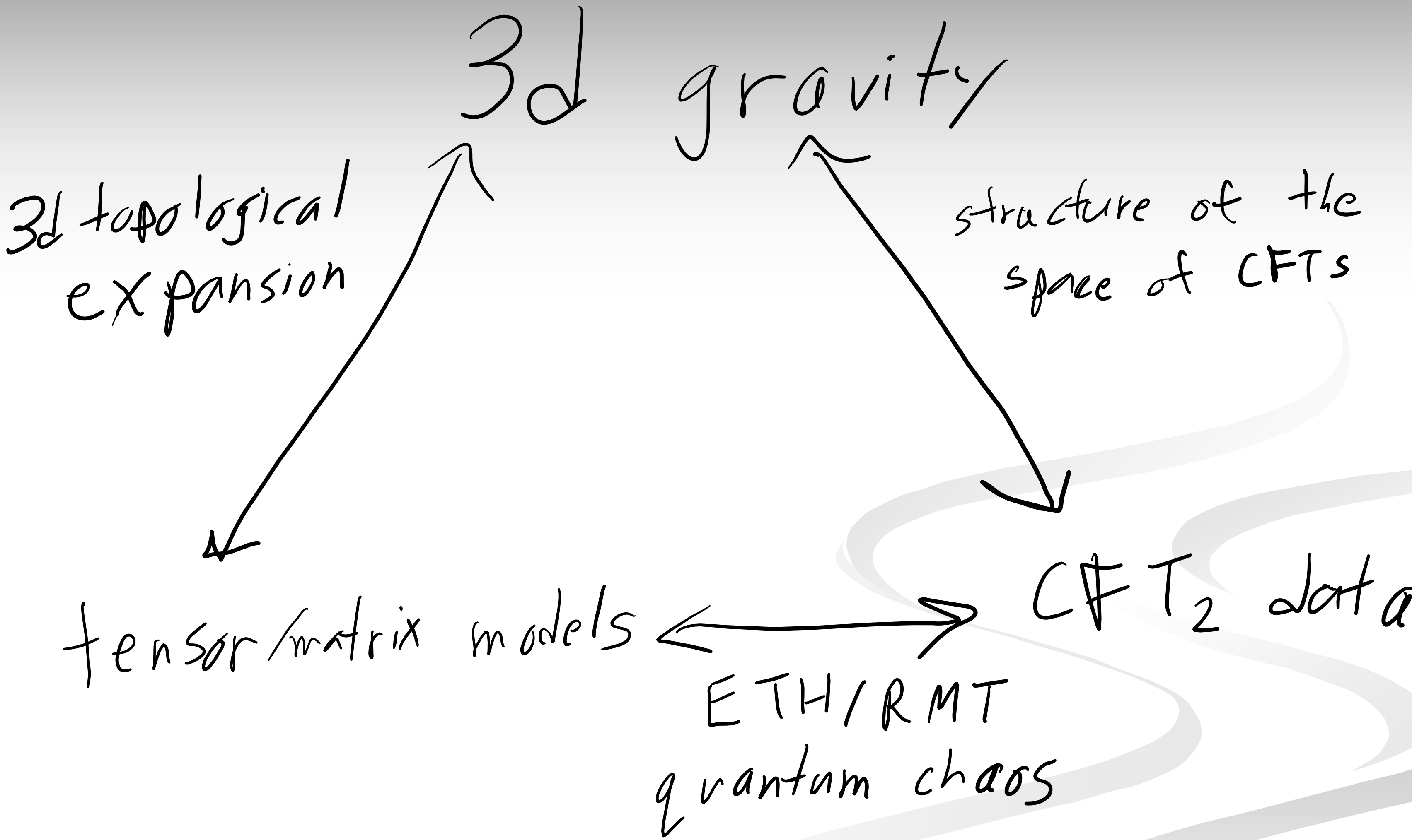
Based on work with Liza Rozenberg, Gabriel Wong, and with
Alex Belin, Jan de Boer, Pranjal Nayak, Julian Sonner, and in progress

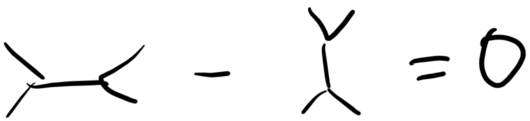

3d gravity
3d topological expansion
structure of the space of CFTs
tensor/matrix models
CFT$_2$ data
ETH/RMT quantum chaos

CFT data

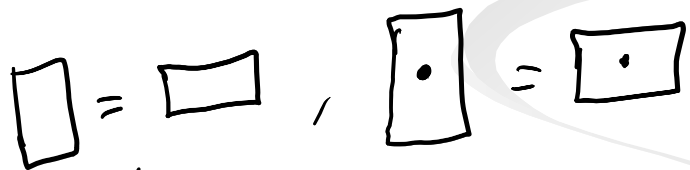
Operator dimensions, Δ, at each spin $s \in \mathbb{Z}$.

Id operator + Virasoro descendants with central charge c.
$(T, \bar{T}, \ldots)$

Locality $\langle 1 | [\mathcal{O}_2, \mathcal{O}_3]_{\text{space-like}} | 4 \rangle = 0 \Rightarrow$ crossing equations



Modular invariance



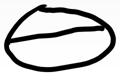
$\tau \to -\frac{1}{\tau}$ $\langle \mathcal{O} \rangle_{\tau^2}$

These imply all higher genus modular invariance.

Tensor integral

$$\mathcal{N} = \int \prod_s d\Delta_s \prod dC_{ijk} \, e^{-\sum_s \mathrm{Tr}\, V^{(s)}_{\text{Cardy}}(\Delta_s)} \exp\left[-\frac{1}{\hbar}\left(\left(4\,\text{pt}\right)^2 + \left(1\,\text{pt}, \tau^2\right)^2 + \left(S\right)^2\right)\right]$$

$$\underline{\text{Ex.}} \quad \langle Z_{\Sigma_2}(\vec{q}) \rangle = \frac{1}{\mathcal{N}} \int \cdots \quad C_{ijk} C_{ijk} \left(q_1^{\Delta_i} q_2^{\Delta_j} q_3^{\Delta_k} + \text{descendants} \right)$$



genus 2 block

$\hbar$ and e^{-c} expansions

'$\hbar$ Hooft expansion of Δ + appears in tensor functions

$\rho(\Delta) \sim e^{c}$

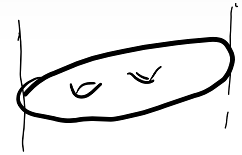
needs to be "resummed" since $C_{ijk} = 0$ is at the top of the potential (Id needs to be compensated by $C_{ijk} \neq 0$)

AdS$_3$ gravity

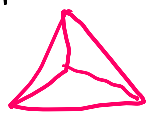
$$S = \int \sqrt{g}\,(R - \Lambda)$$

Locally, first order formalism leads to $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ Chern-Simons theory $A_\pm = \omega \pm e$.

More precisely, one gets Virasoro TQFT



$\mathcal{H}_\Sigma = \{\text{Virasoro blocks}\}$

 = 6j symbol

Even more precisely, one has additional quotients by large diffeomorphisms ("mapping class group"), which changes answers for Seifert manifolds ( $\times S^1$ ...)

Important for Δ matrix integral vandermonde $\prod_{i<j} (\Delta_i - \Delta_j)$.

Tensor integral = 3d gravity!

- It appears that every diagram evaluates exactly to an associated pure 3d gravity partition function!

$$\langle Z(\Sigma, q_i)\rangle = \sum_{M_3} Z_{\mathrm{3dG}}(M_3; c, q_i) f(M_3, \hbar)$$

- For the tensor diagrams, we will see this by matching the basic building blocks and using the logic of VTQFT.
- The matrix model leading "disk" is designed to match the BTZ (solid torus) spectral density, and the "annulus" matches the torus wormhole $T^2 \times I$ 3d gravity result of Cotler-Jensen.

- Conjecture that f $\rightarrow$1 as crossing is imposed exactly.

What is the space of consistent CFTs?

- Equivalent to the question of what are the consistent quantum gravity theories in AdS. Is there an exact CFT whose light operators agree with a given EFTs correlation functions in AdS?

- Around the AdS vacuum, perturbative solutions to the bootstrap exist for any EFT. There are some sharp "naturalness" bounds on the Wilson coefficients from dispersion relations.

  [Heemskerk Penedones Polchinski Sully; Caron-Huot Mazac Rastelli Simmons-Duffin]

- For black hole backgrounds, corresponding to thermal correlators, there are consistent ensembles for any EFT.

  [DLJ Kolchmeyer Mukhametzhanov Sonner]

- Example: are there large c 2d CFTs with no operators below the black hole threshold beside the identity Virasoro module?

Higher d has more constraints

- An important point is that unlike QM, where any Hamiltonian is allowed, CFTs have many consistency constraints. 4 point crossing encodes locality, the vanishing of operator commutators outside the lightcone. We are imposing that on the black hole microstates, in the limit $\hbar$ goes to 0.

- We do not expect there to exist ensembles with e^S parameters over exact CFT data, in contrast to QM Hamiltonians. No disordered exact duals are possible for higher dimensional gravity.

Swampland is doubly nonperturbative

- UV completability is the question of whether, given some light operator data, one can adjust the heavy operators so it obeys crossing. We encapsulated that as finding the minimum of a particular function (sum over crossing^2).

- If we expand the tensor model around the Cardy density, we are asking whether pure gravity has a UV completion. Imposing CFT consistency tells us to do the topological expansion.

- It's an asymptotic series in e^{-c}. So this is a $e^{-e^{c}}$ question of whether minimizing kicks many eigenvalues out of the cut.

Adding matter

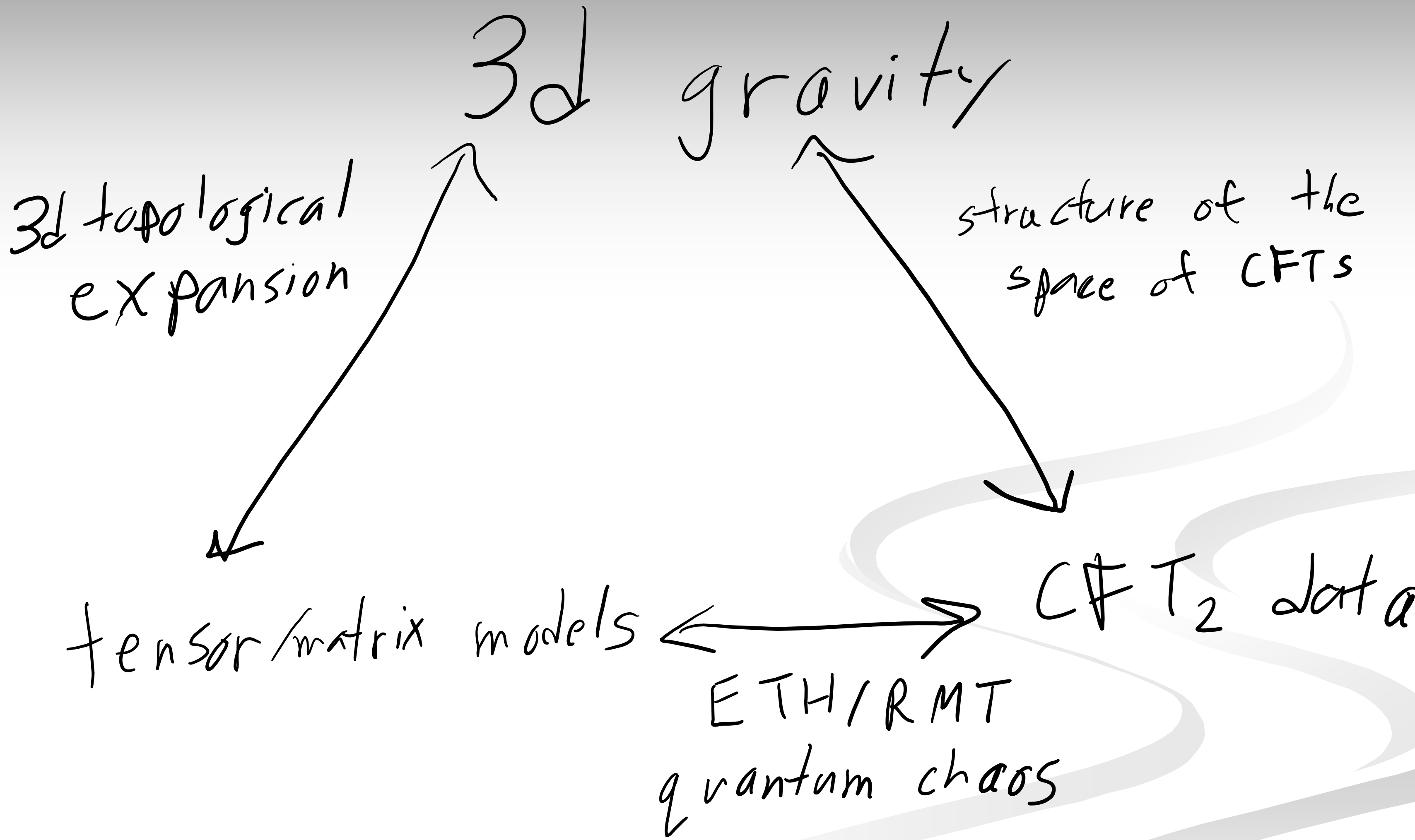
- Working on a simple addition to the matrix/tensor model that introduces a light operator, in such a way that all of the multi-products automatically appear. The expansion is the same 3d gravity topological sum, but now one sums over a matter Wilson line that can go anywhere. This is equivalent to a bulk field.

- Non-renormalizability of gravity+matter is traded for the asymptotic-ness of the sum over Wilson links, now in an e^{-m} expansion.

Relation to emergence conjecture

- No higher curvature terms ever appear directly in the 3d gravity expansion of the tensor models – they can only be generated in the Wilsonian sense by integrating out states. Even the Chern-Simons level can be understood as generated by integrating out the boundary gravitons. In a sense, the UV is pure group theory.

- Given a UV completable EFT, we can write an associated tensor model that will impose crossing on the heavy operators, and expand it. So one might be able to argue that the above applies to all consistent theories.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Statistics of pseudo-random microdata

- Features of high energy microstates of chaotic quantum systems are pseudo-random. Randomizing over the choice of microstate is equivalent to what ensemble of theories?

- "Generalized RMT/ETH" is that this is the maximum entropy ensemble consistent with EFT data and micro-consistency. Applies to all time scales up to those which resolve the fine-grained microstates.

3d topological expansion

- A very special choice for the square of the crossing equation has amazing integrability properties. Diagrams fall into classes labelled by 3d-manifolds, and are equal in each class up to overall factors of $\hbar$.

- What is interesting is to study the theory in the 't Hooft matrix + triple line tensor Feynman expansion. The expansion parameter is e^{-c}. Find a finite $\hbar \to 0$ limit term by term.

- The ultimate fate of the strict limit at fixed c is a "doubly non-perturbative" $e^{-e^{c}}$ question - sum over all topologies.

“Local” version of crossing

- The Ponsot-Teschner crossing kernel F is exactly the object which transforms s-channel to t-channel blocks.

$$\mathcal{F}_{ijmn}\left(O_k|x\right) = \int \mathrm{d}[O_l]\, \mathbf{F}_{kl}\begin{bmatrix} n & j \\ m & i \end{bmatrix} \mathcal{F}_{imjn}\left(O_l|1-x\right)$$

- Expand the crossing equation in s-channel principal series

$$\sum_q \left(C_{i_1 i_2 q} C_{i_3 i_4 q} \delta^{(2)}\left(P_s - P_q\right) - C_{i_1 i_4 q} C_{i_2 i_3 q} \left| \mathbf{F}_{P_q P_s} \begin{bmatrix} P_3 & P_4 \\ P_2 & P_1 \end{bmatrix} \right|^2 \right) = 0\,,$$

- For Virasoro, the principal series are the above threshold physical weights. For other operators (including Id), the δ involves contour manipulation. The equation should be understood in a limiting procedure in which the δ is smeared.

$$P = \sqrt{h - (c-1)/24}$$

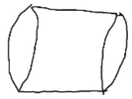
Tensor model

- Instead of specifying moments, write an explicit ensemble:

$$\mathcal{Z} = \int D[L_0, \bar{L}_0]\, D[C]\; e^{-\frac{1}{\hbar} V[L_0, \bar{L}_0, C]}$$

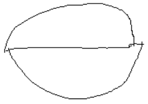
- The maximum ignorance ensemble consistent with crossing has V given by the sum of squares of the constraints.

$$V_4 = 2 \sum_{i_1 \cdots i_4}{}' \sum_{p,q} \left(\frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_1 i_2 q} C_{i_3 i_4 q}}{|\rho_0(p) C_0(12p) C_0(34p)|^2} \delta^{(2)} \left(P_p - P_q\right) - \frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_4 i_1 q} C_{i_2 i_3 q}}{|C_0(12p) C_0(34p) C_0(41q) C_0(23q)|^2} \left\{ \begin{matrix} q & 4 & 1 \\ p & 2 & 3 \end{matrix} \right\} \right)$$



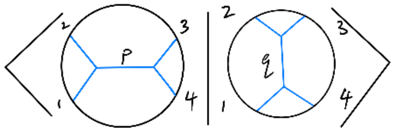
- p=Id, 1=2, 3=4 gives the propagator.

$$V_2 = -\sum_{ijk} \frac{C_{ijk}^2}{|C_0(ijk)|^2}$$



Verlinde measure

- Alternatively, one can understand the potential as the square of the crossing equation in the Verlinde measure on the space of holomorphic conformal blocks.



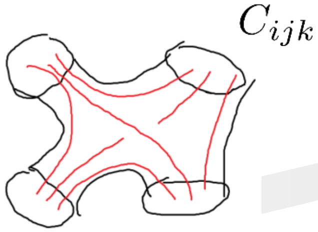
$$= \frac{1}{C_0(12p)C_0(34p)C_0(41q)C_0(23q)} \begin{Bmatrix} q & 4 & 1 \\ p & 2 & 3 \end{Bmatrix}$$

- This directly relates to the associated VTQFT amplitude on that Wilson line network in S^3 with the vertices excised as thrice punctured spheres.

3d gravity

- Conjecture: in the case with no fixed primaries aside from Id (thus a gap to the black hole threshold), then in e^{-c} expansion, the triple scaled limit of the tensor model is exactly pure 3d gravity, including the sum over all hyperbolic 3-manifolds!

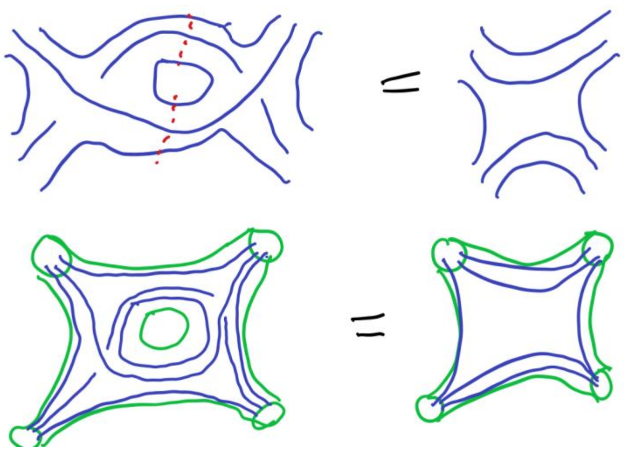
- Building block of the tensor part is the 4 boundary wormhole associated to the 6j vertex. The index loops are filled in with 't Hooft diagrams of the matrices – leading disk corresponds to BTZ filling of solid torus.

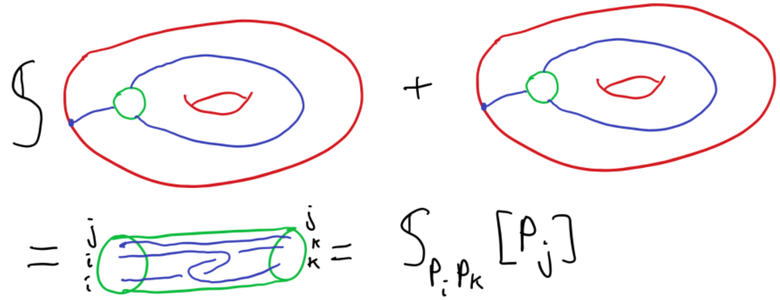


[2 * (Collier Eberhardt Zhang)]

Relation to 3d gravity

- Examples of tensor diagrams evaluating to 3d gravity partition functions, dressed with Wilson lines. Closed Wilson lines are integrated over the BTZ spectrum, corresponding to S transform toroidal surgery.





$$\int dP \rho_0(P) \frac{\begin{Bmatrix} \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_3 \\ \mathcal{O}_4 & \mathcal{O}_5 & \mathcal{O}_p \end{Bmatrix} \begin{Bmatrix} \mathcal{O}_1 & \mathcal{O}_4 & \mathcal{O}_p \\ \mathcal{O}_5 & \mathcal{O}_2 & \mathcal{O}_6 \end{Bmatrix}}{C_0(14p)C_0(25p)} = \frac{C_0(123)C_0(345)}{\rho_0(3)}\delta(P_3-P_6)$$

All manifolds are produced

- Gluing the basic blocks produces rather simple manifolds with only S^2 handles, threaded by Wilson lines. All lines are closed (some through the external observable).

- The Cardy density corresponds to toroidal surgery on the internal Wilson loops. Wilson lines ending at external insertions are excised, giving annulus components of the boundary.

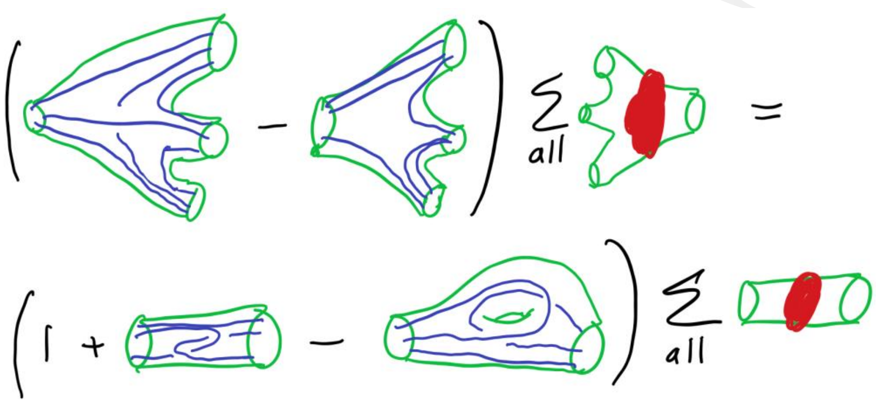
- Using inner and outer lines to remove all S^2 handles results in a general link of the middle triple line, giving all 3-manifolds.

$$\langle Z(\Sigma, q_i) \rangle = \sum_{M_3} Z_{\mathrm{3dG}}(M_3; c, q_i) f(M_3, \hbar)$$

Summing the series in h

- Conjecture is that the coefficients of each topology becomes 1 (up to symmetry factors) as h → 0, so one obtains exactly the 3d gravity partition function. It's clear that all 3-manifolds are produced since one can obtain surgeries on arbitrary links.

- If true, this implies infinitely many Schwinger-Dyson equations that are topological/combinatoric 3d analogues of 2d topological recursion.



Connection to simplicial gravity

- Also involved tensor models with 6j symbols. Didn't work, in the sense that in the double scaled limit, the diagrams didn't correspond to dense tetrahedral decompositions of smooth manifolds. Instead, melonic dominance.

- Here, we also have the "pillow" vertex, so there is a delicate balance. Moreover, the basic BTZ saddle comes from the matrix part of the theory.

- Mathematical results (used often in Chern-Simons theory) relate toroidal surgeries on similar building blocks to simplicial decomposition, but we're still working out the exact story here.

Doubly non-perturbative completion

- The e^{-c} expansion is an asymptotic series, as the number of manifolds grows faster than exponentially in the volume (on-shell 3d gravity action). There are also accumulation points, but these always correspond to hyperbolic cusps in an SL(2,Z) sum over solid torii in some piece of the manifold, which are usually regulated by zeta functions.

- However, since the action is the square of a constraint, one expects a very different completion than for 2d gravity – a specific CFT_2 (probably with some (many?) operators pushed off the above-threshold contour).

Membrane worldvolume

- Deep connection between matrix models and 2d worldsheets. The SSS model giving 2d JT gravity is a special case.

- This tensor model should be a special case of a more general relation to membrane theories.

- An intriguing fact is that the reduction of the 2 M5 brane theory on S^3 in the context of $\mathcal{N}=2$ sphere partition functions is exactly the integration cycle for SL(2,C) Chern-Simons theory that gives TTQFT on the 3-manifold.

[Dimofte Gaiotto Gukov; Cordova DLJ; Mikhaylov]

Summary

- Described a tensor and matrix model that is completely determined by conformal symmetries – maximum ignorance ensemble. The topological expansion of the tensor diagrams is given exactly by pure 3d gravity, as computed by VTQFT.

- The sum over all of the hyperbolic 3-manifolds is successively imposing exact CFT crossing!

- The ultimate existence of a specific CFT is a doubly non-perturbative question about the asymptotic series over 3-topologies.