Asymmetric orbifolds and rank reduction in non-supersymmetric heterotic strings

Ioannis Florakis

University of Ioannina

based on work with C. Angelantonj, G. Leone and D. Perugini

2407.09597 [hep/th]

Quantum Gravity, Strings and the Swampland
Corfu
7/09/2024

Supersymmetry breaking in String Theory

tachyons

background instabilities (tadpoles)

quantum corrections

runaway moduli

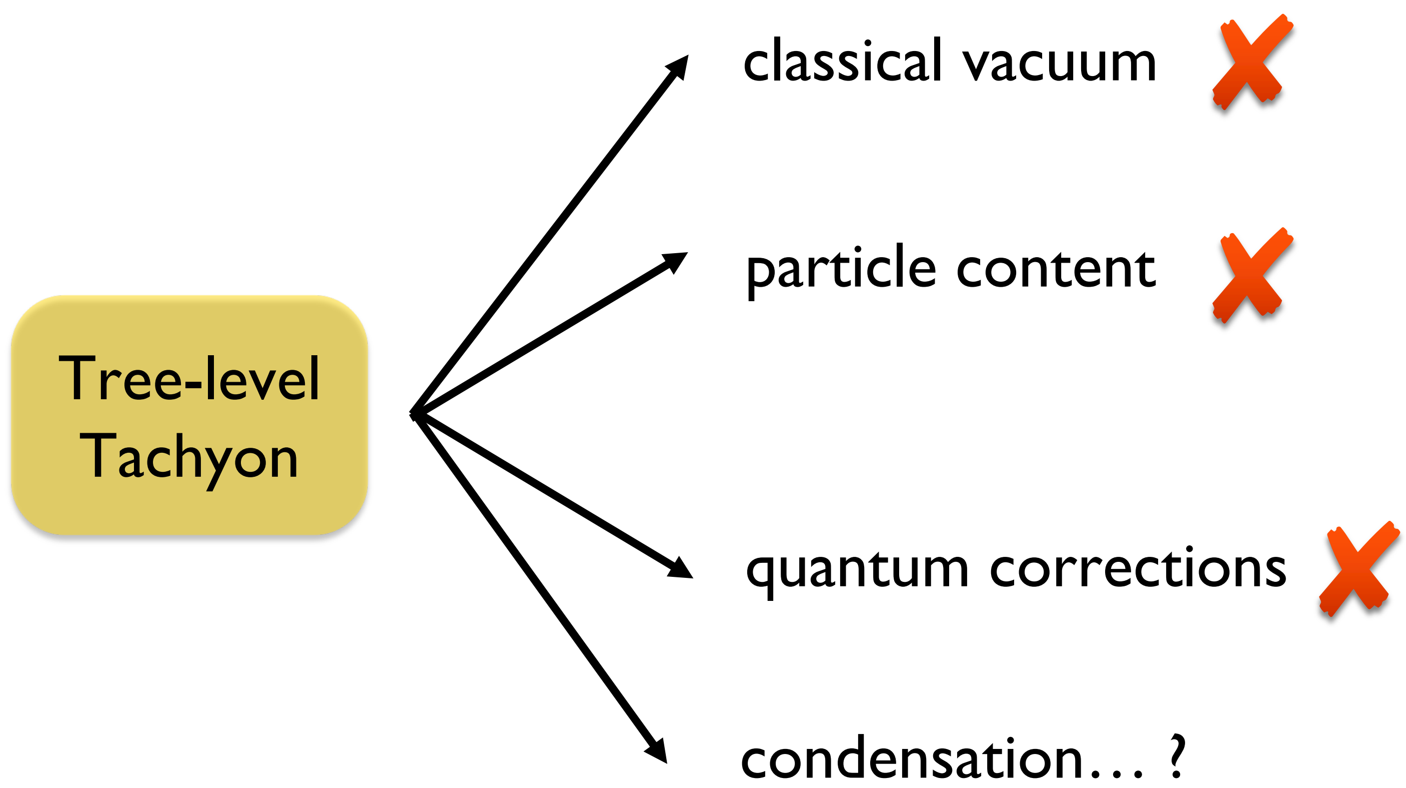
decompactification problem

String Theory is a theory of first quantization.
Tree level instabilities appear as tachyonic states in the spectrum.

independent of moduli	dependent on moduli
e.g. bosonic string	e.g. superstrings with Scherk-Schwarz
$m^2 = -\frac{4}{\alpha'}$	$m^2 = \left(\frac{1}{R} - \frac{R}{\alpha'}\right)^2 - \frac{2}{\alpha'}$

SUSY: stable vacua with no tachyons

No SUSY (or broken): tachyons may and usually do appear;
if that happens, the theory is not predictive.


Tree-level Tachyon
classical vacuum ✗
particle content ✗
quantum corrections ✗
condensation… ?

Obviously, we want to construct stable configurations (at least classically)

but this is a **notoriously hard** problem!

Naively, not so difficult : specific points in the classical moduli space

However, massless scalars corresponding to marginal deformations of the 2d sigma model are typically present

eventually we end up back into tachyonic regions

“Truly” Tachyon-Free models : cannot be continuously deformed into tachyonic ones (classically)

In 10d heterotic strings, the answer is well-known : O(16) x O(16) string

Lower dimensions? Until very recently, no truly stable examples were known

[Baykara, Tarazi, Vafa ’24]
[Angelantonj, I.F., Leone, Perugini ’24]

Consider $E_8 \times E_8$ heterotic string in 10d

Left-movers (RNS) | Right-movers (bosonic)

$$X_L^\mu(z), \psi^\mu(z)$$

$$|0\rangle_{\rm NS} \to \Delta = -\tfrac{1}{2}$$

$$|0\rangle_{\rm R} \to \Delta = 0$$

$$\otimes$$

$$X_R^\mu(\bar z), \tilde\lambda^a(\bar z)$$

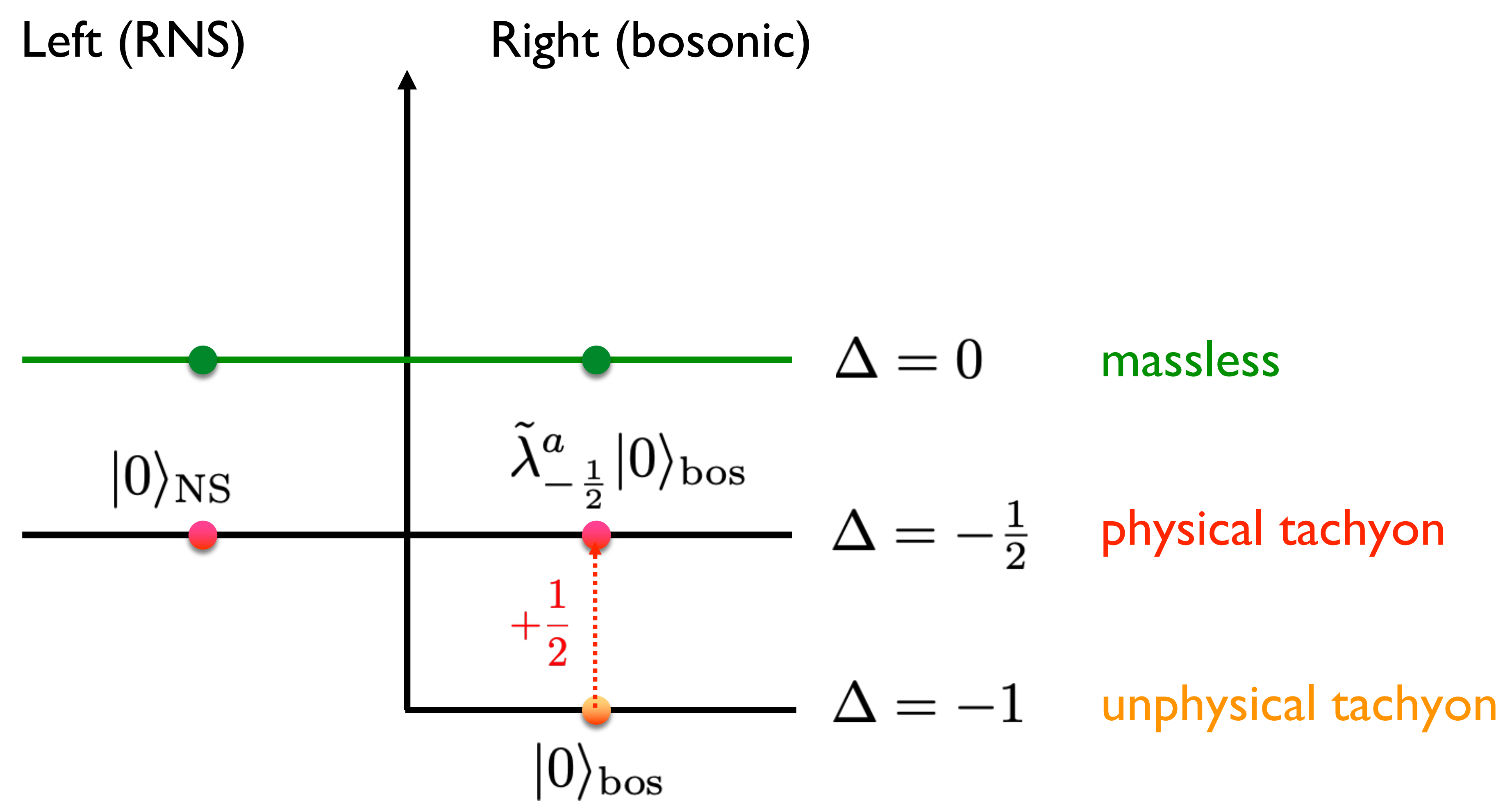
$$|0\rangle_{\rm bos} \to \bar\Delta = -1$$

Modular invariance : Kac-Moody fermions realize $E_8 \times E_8$ or SO(32) lattice

bosons : periodic
fermions : periodic or anti-periodic

⟶ oscillators raise/lower conformal weight by integral or half-integral units

$$|\text{state}\rangle \to (\Delta, \bar\Delta)$$ plus constraints (level matching, GSO, …)



Level matching : with half- or integral moding, only possible tachyon is of the form

$$|\text{tachyon}\rangle = |0\rangle_{\rm NS} \otimes \tilde\lambda^a_{-\frac12}|0\rangle_{\rm bos}$$

$(-)$ $(++)$

the left/right GSO projections eliminate it

$$|\text{state}\rangle = |\text{left}\rangle \otimes |\text{right}\rangle$$

$(-)$ $(++)$

survives the left/right GSO projections

Consider the Z2 orbifold with element $\gamma = (-1)^F \, e^{2\pi i v \cdot Q}$

F : spacetime fermion number (SUSY breaking)

v : shift vector on E8xE8 maximal torus

Q : U(1) charges in Cartan subalgebra (lattice momenta)

Partition function : (un)twisted sectors, and project onto invariant states for each

$$Z = \sum_{h=0,1} \frac{1}{2} \sum_{g=0,1} \mathrm{Tr}_{\mathcal{H}_h} \left[\gamma^g \, q^{L_0} \, \bar{q}^{\bar{L}_0} \right]$$

$$= \sum_{\Delta, \bar{\Delta}} d(\Delta, \bar{\Delta}) \, q^{\Delta} \, \bar{q}^{\bar{\Delta}}$$

$q = e^{2\pi i \tau}$

complex structure $\tau = \tau_1 + i\tau_2 \in \mathbb{H}$

E.g. take $\gamma = (-1)^{F+F_1}$

Twisted sector: inverts GSO on RNS fermions and for fermions realising the first E8

$$|\mathrm{tachyon}\rangle = |0\rangle_{\mathrm{NS}} \otimes \tilde{\lambda}^a_{-\frac{1}{2}} |0\rangle_{\mathrm{bos}} \qquad a = 1, \ldots, 8$$

$(+) \quad (-+)$

✔ survives GSO

10d heterotic SO(16) x E8 theory (tachyonic!)

Similarly, one gets the other 10d non-SUSY heterotic theories

$\mathrm{SO}(32)$	$\mathrm{SO}(8) \times \mathrm{SO}(24)$
$\mathrm{SO}(16) \times \mathrm{E}_8$	$\mathrm{U}(16)$
$(\mathrm{SU}(2) \times \mathrm{E}_7)^2$	E_8

tachyonic

and one celebrated exception :

$$\mathrm{SO}(16) \times \mathrm{SO}(16)$$

$$\gamma = (-1)^{F+F_1+F_2}$$

tachyon free!

$$|\text{tachyon}\rangle = |0\rangle_{\mathrm{NS}} \otimes \tilde{\lambda}^a_{-\frac{1}{2}} |0\rangle_{\mathrm{bos}} \quad ✗$$

$(+)$ $(--)$

eliminated by right GSO

What about lower dimensions? e.g. compactify on torus T^d :

classical moduli space $\quad G_{IJ}, B_{IJ}, Y_I^a \in \dfrac{\mathrm{SO}(d, d+16)}{\mathrm{SO}(d) \times \mathrm{SO}(d+16)}$

tuning Wilson lines continuously connects the theory to the tachyonic regime!

For example, start with SO(16) x SO(16) in 10d and compactify on a circle $S^1(R)$

$$Z_{\rm 9d}=\frac{1}{(\sqrt{\tau_2}\,\eta\bar\eta)^7}\,\frac{1}{2}\sum_{h,g\in\mathbb{Z}_2}\frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\,(-1)^{ga+bh+hg}\,\frac{\vartheta\left[{a \atop b}\right]^4}{\eta^4}\,\frac{\Gamma_{1,17}\left[{h \atop g}\right](R,Y)}{\eta\bar\eta^{17}}$$

non-compact bosons — standard GSO — RNS fermions — Narain lattice

Hunting tachyons : $a=0, h=1$

level matching : $\Delta-\bar\Delta = mn - \frac{\vec Q^2+\vec Q'^2}{2} = -\frac{1}{2}$

right GSO :

$$\sum_i Q_i = 1 \,(\mathrm{mod}\ 2)$$

$$\sum_i Q'_i = 1 \,(\mathrm{mod}\ 2)$$

$$(Q,Q')\in(\mathbb{Z},\mathbb{Z}+\tfrac{1}{2}) \quad \text{or} \quad (Q,Q')\in(\mathbb{Z}+\tfrac{1}{2},\mathbb{Z})$$

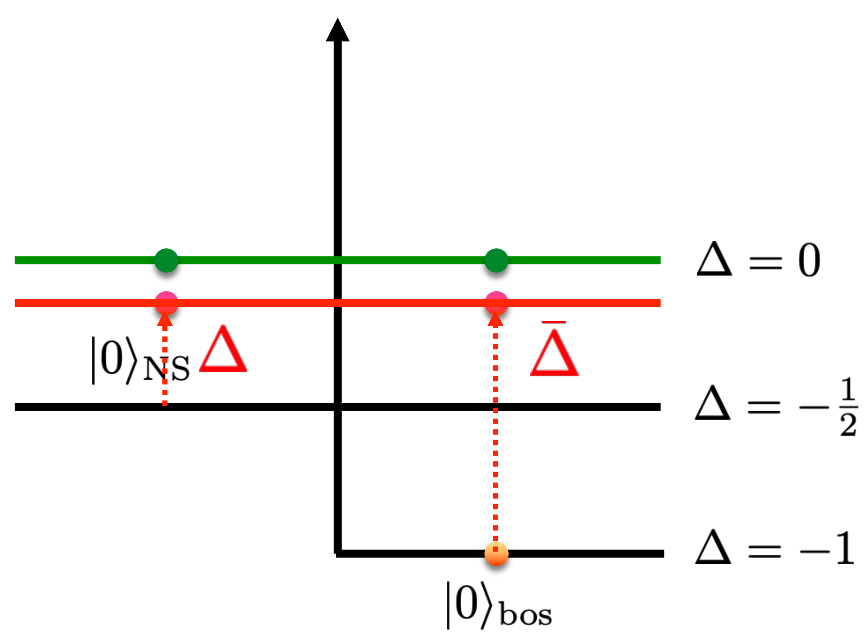
Consider the Wilson line $\mathbf{Y} = (Y, 0^7; Y', 0^7)$ along the circle

e.g. pick the state $Q = (+\frac{1}{2}, -\frac{1}{2}, (+\frac{1}{2})^6)$, $Q' = (+1, 0^7)$, $m = n = -1$

pick the radius that minimises Δ : $R^2 = \frac{3}{8} + \frac{(Y - \frac{1}{2})^2 + (Y' - 1)^2}{2}$

$$\Delta = \frac{1}{2} P_L^2 = R^2 = \frac{3}{8} + \frac{(Y - \frac{1}{2})^2 + (Y' - 1)^2}{2} \rightarrow \boxed{\frac{3}{8} < \frac{1}{2}}$$

physical tachyons!



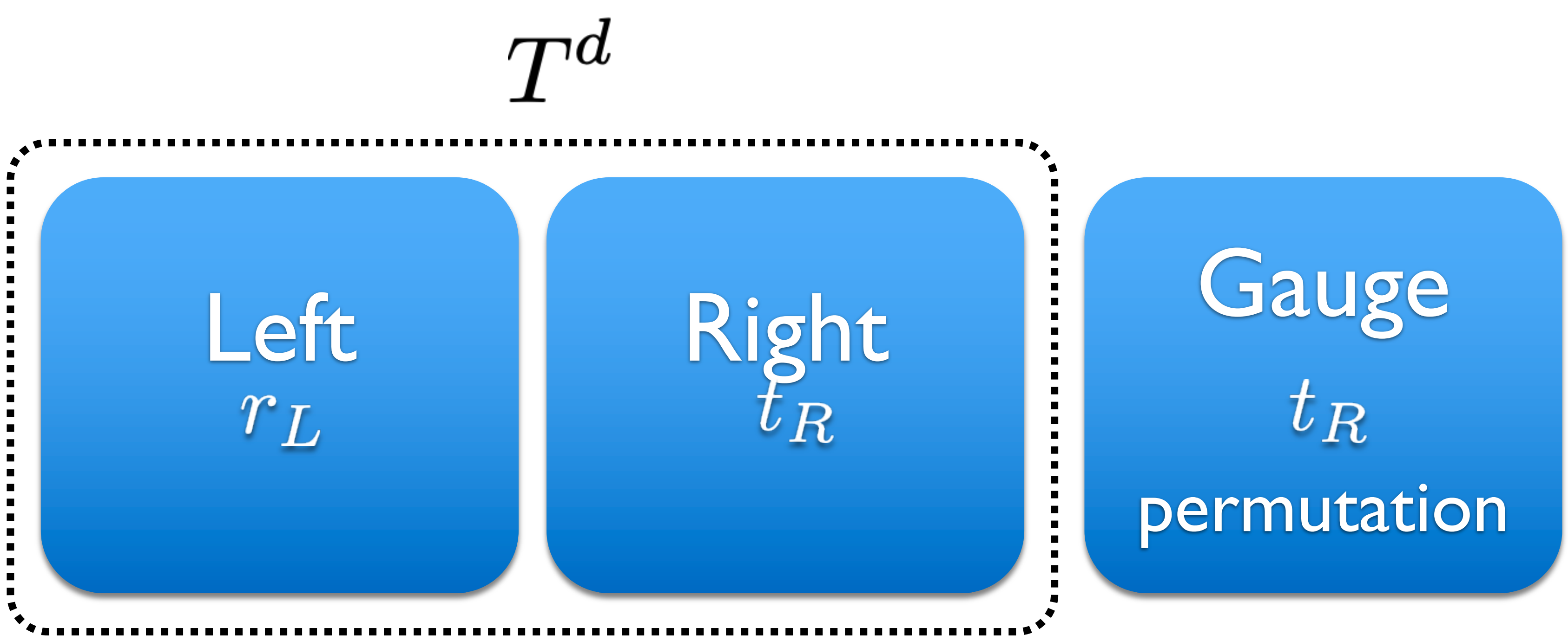
The 9d circle compactification of SO(16) x SO(16) is unstable (no big news here)

[Angelantonj, I.F., Leone, Perugini ’24]

To obtain something stable, we need to remove the moduli

Compactify the stable 10d SO(16) x SO(16) theory on asymmetric orbifolds

couple purely left-moving rotations to
CHL-like permutations of the two SO(16) dof’s
+suitable shifts on the right-movers



First instances of non-SUSY, stable, CHL-like, constructions in 4d and 6d

4d: Compactify SO(16)xSO(16) on T^6/Γ

Permutation action P of the two SO(16)s : construct Γ out of Z_2 factors

To rotate all 6 compact left-movers, pick $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$

$$\gamma_1 = (-,+,-)_L \times (\text{shift})_R \times P$$
$$\gamma_2 = (+,-,-)_L \times (\text{shift})_R \times P$$
$$\gamma_3 = (-,-,+)_L \times (\text{shift})_R$$

right-moving shifts
to lift twisted sector moduli

Fermionize all 6 internal coordinates at the "fermionic" point

$$y^I \to \sin X_L^I \quad , \quad \omega^I \to \cos X_L^I \quad , \quad \partial X^I = i y^I \omega^I$$
$$\tilde{y}^I \to \sin X_R^I \quad , \quad \tilde{\omega}^I \to \cos X_R^I \quad , \quad \bar{\partial} X^I = i \tilde{y}^I \tilde{\omega}^I$$

rotations and translations of X's get mapped into sign flips of the free fermions

$$X_L^I \to -X_L^I \quad \leftrightarrow \quad (y^I, \omega^I) \to (-y^I, +\omega^I)$$
$$X_R^I \to X_R^I + \pi \quad \leftrightarrow \quad (\tilde{y}^I, \tilde{\omega}^I) \to (-\tilde{y}^I, -\tilde{\omega}^I)$$

The choice of shifts is constrained by modular invariance

Fermionic point: $R_I = \sqrt{2\alpha'}$

$$\mathrm{SO}(12) \to \mathrm{SO}(4) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$$

$$D_6 \to D_2 \oplus D_2 \oplus D_2$$

$$G_{IJ} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad , \quad B_{IJ} = 0$$

Choose

$$\gamma_1 = (-,+,-)_L \times (1,\delta,\delta)_R \times P$$
$$\gamma_2 = (+,-,-)_L \times (\delta,1,\delta)_R \times P \quad \longrightarrow \quad SO(16)_2 \times [SO(4)_1]^3$$
$$\gamma_3 = (-,-,+)_L \times (\delta,\delta,1)_R$$

gauge group

	$\bar\chi^{1,2}$	$\bar\chi^{3,4}$	$\bar\chi^{5,6}$	$y^{1,2},\omega^{1,2}$	$y^{3,4},\omega^{3,4}$	$y^{5,6},\omega^{5,6}$	$\bar y^{1,2}$	$\bar\omega^{1,2}$	$\bar y^{3,4}$	$\bar\omega^{3,4}$	$\bar y^{5,6}$	$\bar\omega^{5,6}$
γ_1	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	✓	✗
γ_2	✗	✓	✓	✓	✗	✓	✗	✗	✓	✗	✓	✗
$\gamma_1\gamma_2$	✓	✓	✗	✓	✓	✗	✓	✗	✓	✗	✗	✗

(6,6) lattice representation in terms of Jacobi theta functions

$$\Gamma_{6,6}\begin{bmatrix} h_1 \ , \ h_2 \\ g_1 \ , \ g_2 \end{bmatrix} = \frac{1}{2}\sum_{\gamma,\delta=0,1} \vartheta\begin{bmatrix}\gamma\\ \delta\end{bmatrix} \vartheta\begin{bmatrix}\gamma+h_1\\ \delta+g_1\end{bmatrix} \times \bar\vartheta\begin{bmatrix}\gamma+h_2\\ \delta+g_2\end{bmatrix}^2 \qquad I=1,2$$

$$\vartheta\begin{bmatrix}\gamma\\ \delta\end{bmatrix} \vartheta\begin{bmatrix}\gamma+h_2\\ \delta+g_2\end{bmatrix} \times \bar\vartheta\begin{bmatrix}\gamma+h_1\\ \delta+g_1\end{bmatrix}^2 \qquad I=3,4$$

$$\vartheta\begin{bmatrix}\gamma\\ \delta\end{bmatrix} \vartheta\begin{bmatrix}\gamma-h_1-h_2\\ \delta-g_1-g_2\end{bmatrix} \times \bar\vartheta\begin{bmatrix}\gamma-h_1-h_2\\ \delta-g_1-g_2\end{bmatrix}^2 \qquad I=5,6$$

Only compatible with rigid lattice (G,B,Y fixed)

Independent orbit vanishes : no discrete torsion

Write as 3 Z2's

$$\begin{array}{rcl} \mathbb{Z}_2(h,g) & : & \mathrm{E}_8\times\mathrm{E}_8/(-1)^{F+F_1+F_2}\to \mathrm{SO}(16)\times\mathrm{SO}(16) \\ \mathbb{Z}_2(h_1,g_1) & : & T^6/(-,+,-)_L\times(1,\delta,\delta)_R\times P \\ \mathbb{Z}_2'(h_2,g_2) & : & T^6/(+,-,-)_L\times(\delta,1,\delta)_R\times P \end{array}$$

$$Z_{4\mathrm{d}} = \frac{1}{(\sqrt{\tau_2}\,\eta\bar\eta)^2}\,\frac{1}{2}\sum_{h,g\in\mathbb{Z}_2}\frac{1}{2}\sum_{h_1,g_1\in\mathbb{Z}_2}\frac{1}{2}\sum_{h_2,g_2\in\mathbb{Z}_2}\frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\,(-1)^{ga+bh+hg}$$
$$\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h_1\\b+g_1\end{bmatrix}\vartheta\begin{bmatrix}a+h_2\\b+g_2\end{bmatrix}\vartheta\begin{bmatrix}a-h_1-h_2\\b-g_1-g_2\end{bmatrix}}{\eta^4}\,\frac{\Gamma_{6,6}\begin{bmatrix}h_1\ ,\ h_2\\g_1\ ,\ g_2\end{bmatrix}}{\eta^6\bar\eta^6}\,\Xi\left[\begin{array}{c|c} h & h_1+h_2\\ g & g_1+g_2\end{array}\right]$$

$$\Xi\left[\begin{array}{c|c} h & 0\\ g & 0\end{array}\right] = \sum_{k,\rho}(-1)^{g(k+\rho)}\,\bar\chi_h^k\bar\chi_h^\rho(\bar\tau)$$

$$\Xi\left[\begin{array}{c|c} h & 0\\ g & 1\end{array}\right] = \sum_{k}\bar\chi_h^k(2\bar\tau)$$

$$\Xi\left[\begin{array}{c|c} h & 1\\ g & 0\end{array}\right] = \sum_{k}(-1)^{gk}\,\bar\chi_{2h}^k(\bar\tau/2)$$

$$\Xi\left[\begin{array}{c|c} h & 1\\ g & 1\end{array}\right] = e^{2\pi i/3}\sum_{k}(-1)^{(g-h)k}\,\bar\chi_{2h}^k\left(\tfrac{\bar\tau+1}{2}\right)$$

in terms of SO(16) characters

$$\bar\chi_0^0=\bar O_{16}$$
$$\bar\chi_1^0=\bar V_{16}$$
$$\bar\chi_0^1=\bar S_{16}$$
$$\bar\chi_1^1=\bar C_{16}$$

Light spectrum:

Sector	fields	$SO(16)_2 \times SO(4)_1 \times SO(4)_1 \times SO(4)_1$
Untwisted	$g_{\mu\nu}, B_{\mu\nu}, \phi$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$
	A_μ	$(\mathbf{120},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$
	ψ_{D}	$2\,(\mathbf{128},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{136},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{120},\mathbf{1},\mathbf{1},\mathbf{1})$
	2ϕ	$(\mathbf{120},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{4},\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{4})$
γ_1 twisted	$2\phi+\psi_{\mathrm{D}}$	$(\mathbf{1},\mathbf{1},\mathbf{2}_s+\mathbf{2}_c,\mathbf{2}_s+\mathbf{2}_c)$
γ_2 twisted	$2\phi+\psi_{\mathrm{D}}$	$(\mathbf{1},\mathbf{2}_s+\mathbf{2}_c,\mathbf{1},\mathbf{2}_s+\mathbf{2}_c)$
$\gamma_1\gamma_2$ twisted	2ϕ	$(\mathbf{1},\mathbf{2}_s+\mathbf{2}_c,\mathbf{2}_s+\mathbf{2}_c,\mathbf{4})$

$$\Lambda = -\frac{1}{(2\pi\sqrt{\alpha'})^4}\int_{\mathcal{F}}\frac{d^2\tau}{2\tau_2^3}Z \quad \simeq \; 3.9\times 10^{-2}\,\alpha'^{-2}$$

- no tachyons
- no neutral scalars /deformation moduli, except dilaton
- reduced rank 14
- non-chiral

Thank you