# Approximate Minimal SU(5), Several Fundamental Scales, Fluctuating Lattice

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Korfu, September 2024

- The GUT SU(5) only comes about as a symmetry of the plaquette lattice action for a "natural" Standard Model action in the classical approximation. Quantum corrections need an extra factor 3 to agree.
- Show a plot of 6 (?) suggestive fundamental energy scales against the dimensionallity of the related coefficients in the Lagrange density: straight line!

# Two Phenomenological Coincidences, Explained by Fluctuating Lattice

- **Approximate SU(5) for Fine Structure Constants.** We assume a lattice theory for the Standard Model Group with the plaquette actions being traces of an SU(5)-matrix representation of the Standard Model Group plaquette variable (summed over the plaquettes of course). So classically we get just the SU(5) relation between the fine structure constants. But the genuine SU(5) minus the SMG Yang Mills do **not** exist. Quantum corrections **break the only accidental SU(5) relations**. In nature breaking just by **3 times** what true lattice quantum corrections give.
- **A Line of log Energy scales against "Coupling" dimensionality.** "Coupling" means the coefficient to the Lagrangian density for the for the energy scale relevant Lagrangian density, e.g. $1/\kappa$ for $R/\kappa$ or $\frac{-1}{16\pi}$ for $\frac{-1}{16\pi}F_{\mu\nu}^2$, ...

# Elegant Plaquette Action for Standard Model (Group by O'Raifeartaigh)

$$\text{“SM plaquette action”} =$$

$$= \quad ReTr \begin{pmatrix} u_{11}^{(2)} & u_{12}^{(2)} & 0 & 0 & 0 \\ u_{21}^{(2)} & u_{22}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & u_{11}^{(3)} & u_{12}^{(3)} & u_{13}^{(3)} \\ 0 & 0 & u_{21}^{(3)} & u_{22}^{(3)} & u_{23}^{(3)} \\ 0 & 0 & u_{31}^{(3)} & u_{32}^{(3)} & u_{33}^{(3)} \end{pmatrix} \tag{1}$$

Imposing unitarity of the two submatrices (one $2 \times 2$ and one $3 \times 3$) and that the determinant of the $5 \times 5$ matrix be 1, the matrix is restricted to be in the group $S(U(2) \times U(3))$, which is the **group** for the **Standard Model** in a phenomenological sense defined by L. O'Raifeartaigh.

# Standard Model Group $S(U(2)\times U(3))$

$$S(U(2)\times U(3)) =$$

$$= \left\{ \begin{pmatrix} u^{(2)}_{11} & u^{(2)}_{12} & 0 & 0 & 0 \\ u^{(2)}_{21} & u^{(2)}_{22} & 0 & 0 & 0 \\ 0 & 0 & u^{(3)}_{11} & u^{(3)}_{12} & u^{(3)}_{13} \\ 0 & 0 & u^{(3)}_{21} & u^{(3)}_{22} & u^{(3)}_{23} \\ 0 & 0 & u^{(3)}_{31} & u^{(3)}_{32} & u^{(3)}_{33} \end{pmatrix} \middle| \right.$$

$$\left. det \begin{pmatrix} u^{(2)}_{11} & u^{(2)}_{12} & 0 & 0 & 0 \\ u^{(2)}_{21} & u^{(2)}_{22} & 0 & 0 & 0 \\ 0 & 0 & u^{(3)}_{11} & u^{(3)}_{12} & u^{(3)}_{13} \\ 0 & 0 & u^{(3)}_{21} & u^{(3)}_{22} & u^{(3)}_{23} \\ 0 & 0 & u^{(3)}_{31} & u^{(3)}_{32} & u^{(3)}_{33} \end{pmatrix} = 1, \&\text{matrix unitary} \right\}$$

# The lattice action is as if SU(5) classically

The matrix is obviously an $SU(5)$ representation of a Standard Model group element:

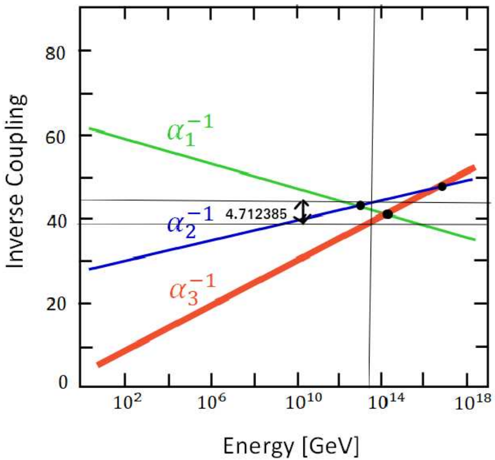
$$\mathrm{U}(\Box) = \begin{pmatrix} u^{(2)}_{11} & u^{(2)}_{12} & 0 & 0 & 0 \\ u^{(2)}_{21} & u^{(2)}_{22} & 0 & 0 & 0 \\ 0 & 0 & u^{(3)}_{11} & u^{(3)}_{12} & u^{(3)}_{13} \\ 0 & 0 & u^{(3)}_{21} & u^{(3)}_{22} & u^{(3)}_{23} \\ 0 & 0 & u^{(3)}_{31} & u^{(3)}_{32} & u^{(3)}_{33} \end{pmatrix} \quad (2)$$

but the genuine SU(5) degrees of freedom are **missing.**

$$\text{“Plaquette Action”} \propto ReTr(U(\Box)) \quad (3)$$

will classically give the **SU(5) relations** between the fine structure constants.

# Our prediction of Deviation from $SU(5)$.



# To compute Quantum Corrections we Taylor expanded the Plaquette Action to Fourth Order in expression $H + h$

Her $h$ is the continuum field $A_\mu(x)$ translated to the lattice matrix, and $H$ the quantum fluctuations.
We Taylor expand the Plaquette action

$$\begin{aligned} ReTr(\exp(i(H+h)) &= ReTr(1) + \frac{1}{2}ReTr((i(H+h))^2) + \\ &+ \frac{1}{24}ReTr((i(H+h))^4), \\ &\text{(odd powers give zero).} \end{aligned} \quad (4)$$

# Calculate Quantum corrections to the Classical SU(5) relation

Dropping but the $h^2$ order terms we get

$$S_{plaquette}|_{h^2-part} = ReTr(U(\Box))|_{h^2-part} \quad (5)$$

$$= \frac{1}{2}ReTr(h^2) + \frac{1}{24} * 6ReTr(h^2H^2) \quad (6)$$

( provided that $h$ and $H$ commute)

$$\text{Otherwise :} \quad = \frac{1}{2}ReTr(h^2) + \frac{1}{6}ReTr(h^2H^2) + \frac{1}{12}ReTr(hHhH).$$

For matrix element row 1 column 2, say, we get

$$< |H_{\text{row 1 column 2}}|^2 > \;=\; < (H^1/2)^2 + (H^2/2)^2 > \quad (7)$$
$$= \; 1/2 * 2\pi\alpha = \pi\alpha. \quad (8)$$

It is such an - most easy off diagonal element we denote by $H_{\text{one component}}$ and its numerical average square is thus for **one layer**

$$< |H_{\text{one component}}|^2 > |_{\text{one layer}} \;=\; \pi\alpha. \quad (9)$$
$$\text{Want } \frac{1}{2} < |H_{\text{one component}}|^2 > |_{\text{one layer}} \;=\; \pi/2 * \alpha. \quad (10)$$

The reason we want this half of the average square of the matrix element in the $5 \times 5$ matrix, is that in the Taylor expansion (76) has a factor 2 deviation between the two terms,which we shall compare.

# Our Success with Fitting Deviations from SU(5) with 3 times the Quantum Correction

From the Experimentally determined Fine structure constants at the $Z^0$ mass

$$1/\alpha_{1\ SM} = 98.347 \pm 0.02 \quad (11)$$

$$1/\alpha_{1\ SU(5)} = 59.008 \pm 0.013 \quad (12)$$

$$1/\alpha_2 = 29.569 \pm 0.017 \quad (13)$$

$$1/\alpha_3 = 8.446 \pm 0.05 \quad (14)$$

we determine our

$$\textbf{Deviation parameter } q_{fit} = 4.618 \quad (15)$$

$$\textbf{while our theory } q_{theory} = \frac{3\pi}{2} = 4.7124 \quad (16)$$

# Approximate Minimal SU(5), Several Fundamental Scales, Fluctuating Lattice

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Korfu, September 2024

Several seemingly "fundamental" scales like the Planck scale, the see-saw scale, and the unification scale, are not so equalto each other as we would have expected in a philosophy of their being only **one** "fundamental energy scale".
To cure this fact we bring forward the idea of a truly existing lattice, which has **wildly different sizes**, of say the link length, in different places and/or in different components of a superposition

# Table of “Fundamental ?” Energy Scales

| Name | Energy value | $n$ of $(1/a)^n$ | Coef. dim. | Fit | Lagrangian d. |
|---|---|---|---|---|---|
| Planck scale | $1.22 * 10^{19}$ | 6 | -2 | $2.44 * 10^{18}$GeV | $\frac{1}{2\kappa}R$ |
| reduced Planck | $2.43 * 10^{18}$GeV | 6 | -2 | $2.44 * 10^{18}$GeV | $\frac{1}{2\kappa}R$ |
| Min. $SU(5)$ app. | $5.3 * 10^{13}$GeV | 4 | 0 | $3.91 * 10^{13}$ GeV | $-\frac{1}{16\pi\alpha}F_{\mu\nu}^2$ |
| Susy SU(5) | $10^{16}$ GeV | 4 | 0 | $3.91 * 10^{13}$ GeV | $-\frac{1}{16\alpha} * F_{\mu\nu}^2$ |
| Inflation | $10^{14}$GeV | 4 | 0 | $3.91 * 10^{13}$ GeV | $\lambda\phi^4$ |
| See-saw | $10^{11}$ GeV | 3 | 1 | $1.56 * 10^{11}$GeV | $m_R\bar{\psi}\psi$ |
| Scalars | $\frac{see-saw}{Little-hierachy}$ | 2 | 2 | $6.25 * 10^8$GeV | $m_b^2\|\phi\|^2$ |
| Fermion extrapolate | $10^4$GeV | 0 | 4 | $10^4$ GeV | “1” |

Using $c = \hbar = 1$,

$$\text{“Reduced Planck”} = \frac{1}{\sqrt{G_{Newton} * 8\pi}} = 2.43 * 10^{18} GeV$$

$$\text{“unified (approximate) SU(5)”} = \text{“where lines closest”}$$

$$= (say) 5.3 * 10^{13} GeV$$

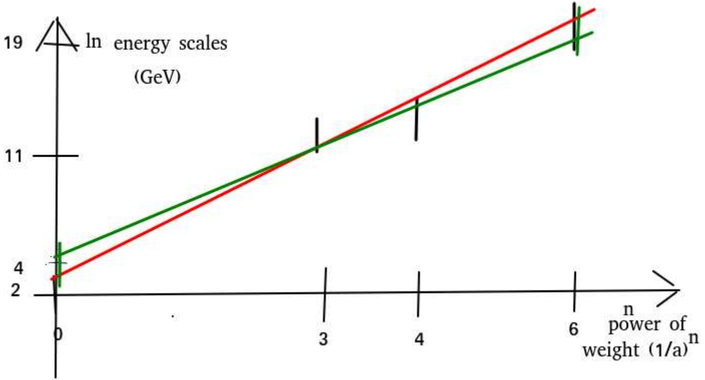
$$\text{“see-saw”} \sim \text{“typical right handed neutrino mass”}$$

The ‘scales” **scalars** and **Fermion extrapolate** are my inventions

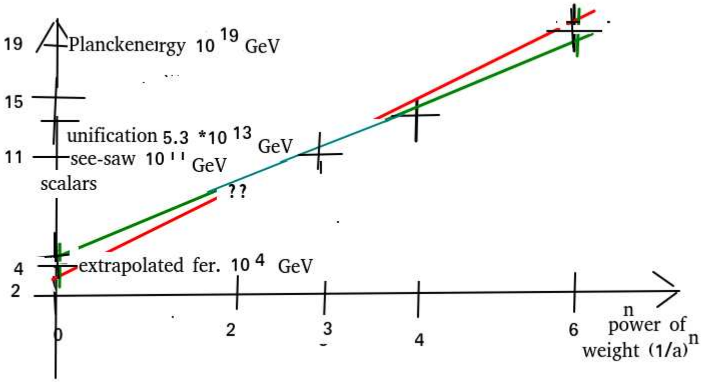
Our fitting of the curve of scales by a linear function as function of the power $n$ may be presented as

$$\begin{aligned} \text{“energy scale”} &= 10^4 GeV * 250^n && (17)\\ \text{or } log_{GeV}(\text{“energy scale”}) &= 4 + 2.40 * n(+logGeV) && (18)\\ \text{or } \ln_{GeV}(\text{“energy scale”}) &= 9.21 + 5.53 * n(+\ln GeV) && (19) \end{aligned}$$

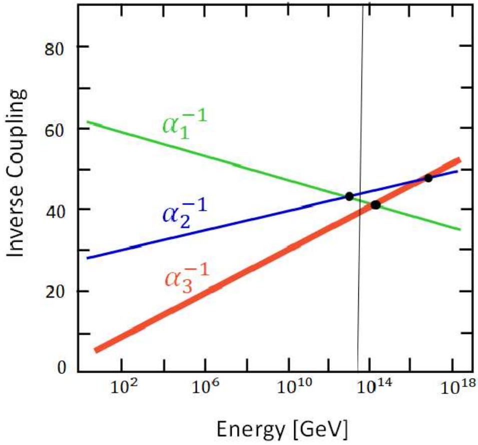
# Different "Energy-scales" versus n = 4 -"Coupling dimension"



# Different Energy-scales versus n = 4 - “Coupling Dimension”



# If I do not believe Susy, just look where lines closest



# My Speculations on GUT SU(5): Approximate

- There is a physically existing lattice, and the Plaquette action happens **classically** to be SU(5) symmetric.
- Quantum corrections break the classical SU(5) symmetry of the lattice action.
- Because the lattice lies in three layers, the quantum correction SU(5) breaking is just a factor 3 bigger than true quantum correction. The factor 3 is the number of families.

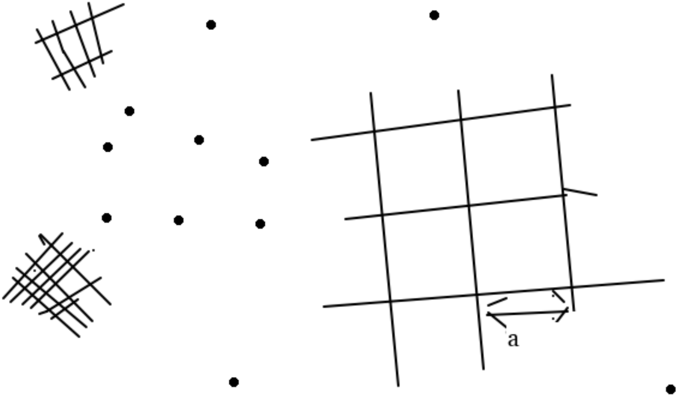
# Take at First that our Energy Scales (4 of them) are Observed Phenomenologically on Logarithmic Plot as function of the Coupling Constant Dimension $[GeV^{dim}]$ lie on Straight line

Since the power $n$ to which the inverse link length $a$ comes into the action $S$ for the Lagrangian densities for the different sort of physics related to the different (fundamental?) energy scales, $(1/a)^n$, is linearly related to the dimension of the coefficient [“coefficient”]

$$n = 4 - Dim(\text{“}Coefficient''), \qquad (20)$$

linearity - i.e. straight line - of the logarithm of the energy scales as function of $n$ means also, that we **observed straihgt line for the relation of Dim(“coefficient”) to logarithm(“energy scale”)**

# "Fluctuating lattice" (in superposition of) being dense somewhere and rough somewhere and often deformed



# Densities of, say, Sites in Fluctuating Lattice with several Layers

For the distribution of different densities of links or of sites in a fluctuating lattice with shall distinguish:

$$\#layers = (say)\ \#sites \text{ per } a^4 \quad (21)$$

$$density_{/m^4} = \#sites \text{ per } m^4 \quad (22)$$

$$density_{/m^4} = \#layers * a^{-4} \quad (23)$$

Ansatz:

Probability: (24)

$$P(\ln(1/a))d\ln(1/a)d^4x \propto$$

$$\propto \exp\left(-\frac{(\ln(1/a) - \ln(10^4 GeV))^2}{2\sigma}\right) \quad d\ln(1/a)d^4x \quad (25)$$

where $10^4 GeV$ is our "fitted" value; $\sigma$ is a spreading to be fitted.

# Distribution of Contribution for one of the to Scales associated actions

One of the actions associated with the candidates for fundamental scales as e.g. the Einstein-Hilbert-action $\frac{1}{2\kappa}R\sqrt{-g}d^4x$ with $(1/a)^n$ proportional contribtuion get of the form:

$$
\begin{aligned}
S = & \int \mathcal{L}(x) d^4x \propto \\
\propto & \int \exp\left(\frac{-(\ln(1/a)) - \ln(10^4 GeV))^2}{2\sigma}\right)(1/a)^n d\ln(1/a) \\
= & \exp\left(\frac{-(\ln(1/a) - \ln(10^4 GeV))^2 + n * 2\sigma * \ln(1/a)}{2\sigma}\right) d\ln(1/a)
\end{aligned}
$$

# An Action depends on the spread $\sigma$ like:

$$
\begin{aligned}
S &= \int \mathcal{L}(x) d^4x \propto \\
&\propto \int \exp(-\frac{(\ln(1/a)) - \ln(10^4 GeV) - n*\sigma)^2 + (n*\sigma)^2}{2\sigma}) d\ln(1/a) \\
&= \int \exp(-\frac{(\ln(1/a)) - \ln(10^4 GeV))^2 + (n*\sigma)^2}{2\sigma}) d\ln(1/a) \\
&= \int \exp(-\frac{(\ln(1/a)) - \ln(10^4 GeV))^2}{2\sigma}) d\ln(1/a) * \exp(n^2\sigma/2)
\end{aligned}
$$

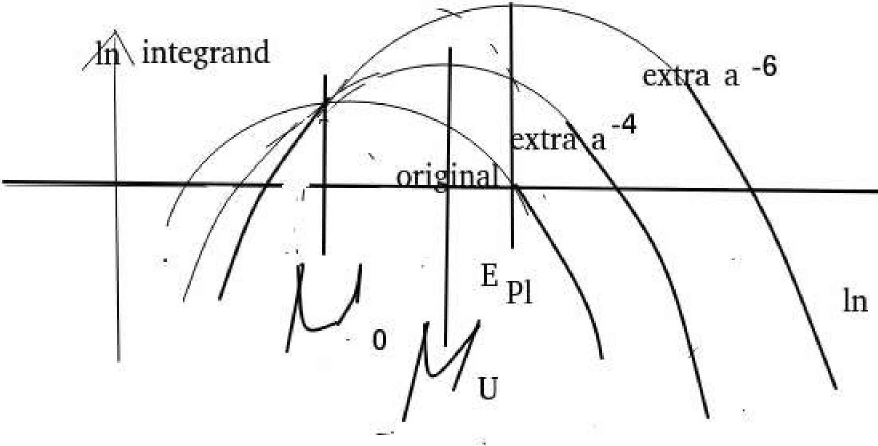
where only the last factor $\exp(n^2 * \sigma/2)$ depends on $n$. This was foran action $S \propto (1/a)^n$.

Interpreting the factor $\exp(n^2\sigma/2)$ as correcting the $n$ factors $(1/a)$ we get $(1/a)_{eff} = (1/a) * \exp(n\sigma/2)$.

# The Effect of the in $\ln(1/a)$ Broadened Distribution is $1/a \to (1/a)_{eff} = \exp(n\sigma/2) * (1/a)$

We shall interprete correction to the effective $1/a$ (= the inverse of the link size) as a correction of the "energy scale". So the effect of the spreading with Gauss distribution in the logarithm $\ln(1/a)$ with a width given by $\sigma$ as

$$\begin{aligned} \text{Replace "energy scale"} &\to \text{"energy scale"} * \exp(n * \sigma/2) \quad (26) \\ \text{So } 250 &= \exp(\sigma/2)(\text{where 250 from our empirical fit}) \\ \text{and thus } \sigma &= 11.0. \quad (27) \end{aligned}$$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# Explaing a bit more Scales

- “Inflation” Inflation stands for the parameter of dimension mass orenergy extractable from the potential $V(\phi)$ for the inflaton fields ( whether this be a special inflaton field or e.g. the Higgs field). It is about $10^{-5} * M_{Pl}$. Determined via relative fluctuations in CMB.
- Scalars An energy scale we predict to have many scalar particle masses and scalar particle field expectation values. Many symmetries broken here. Determined as giving “little heirachy scales” relative to the see-saw scale. So our model: “Little hierarchy scale” $\sim$ the 250.

# Conclusion on the SU(5) Deviations of Fine structure constants

We extracted a parameter $q$ for deviation of the three (inverse) Fine staructure constants in the Standard Model to meet as they should by SU(5), and it fits remakable well to just **three times** the value it should have had, if the approximate SU(5) meet ing of the running couplings constants were due a lattice action making SU(5) exact classically, so that the deviation was just a quantum correction.

$$q_{fit} = 4.618 \quad ; \quad q_{theory} = 4.7124 \tag{28}$$

# Conclusion on the Straight line fitting of Scales

- We presented an emprical straight line fit to three wellknown energy scales, valid to crude order of magnitude accuracy,

$$\text{“energy scale”} = 10^4\, GeV * 250^n \quad (29)$$

$$\text{or “energy scale”} = 10^4\, GeV * 250^{4-dim(coefficient)} \quad (30)$$

(where $dim(coefficient)$ is the dimension in energy units of the coefficient multiplying in the Lagrangian density the field (product).)

- We explain this empirical fit with a speculated “fluctuating lattice” with a fluctuation distribution being a Gauss distribultion in the logarithm of the statistically fluctuating link length $a$, i.e. a Gauss distribution in $\ln(1/a)$:

$$P \propto \exp\left(-\frac{(\ln(1/a) - \ln(10^4\, GeV))^2}{2 * 11.0}\right). \quad (31)$$

# Old Slide Talk: Approximate SU(5)

# Approximate $SU(5)$, Fine Structure Constants

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" Bled", 9. of July 2024

**Fit the three Fine Structure Constants in the Standard Model with three Parameters, Derivable in Our Theory.**
Shall fit with

- $q =$ "number of families" $* \pi/2$.
- $\alpha_{5\ uncorrected} = \alpha_{5\ critical}$ (unifying coupling (ours)).
- We shall put our replacement for unification energy scale $\mu_U$ into aline of four different energy scales, fitting a line in the logarithm of the energy scale versus dimension of related couplings.

# Abstract for First part: "Approximate $SU(5)$, Fine structure constants"


## Abstract

We suggest a model with a physical lattice for the gauge groups in the Standard Model with link variables taking values in the according to O´Raifeartaigh Standard Model**group**, $S(U(2) \times U(3))$, and it is so similar to $SU(5)$, that in what we can call the classical approximation, it gives the same ratios between the three fine structure constants. But including quantum fluctuations we get deviation from the GUT prediction, becuase there is not true $SU(5)$ and thus the true $SU(5)$ quantum fluctuations are lacking, unless they belong to the StandardModelgroup. The remarkable thing is, that apart from just a factor 3 the deviations caused by these quantum fluctuations reproduce within uncertainties in the very accurately measured finestructure constants fit the data.

# Abstract for Second Part: Several "Fundamental" Scales, Their logarithms Fitted on a Line as function of Dimension of the Coefficient in Lagrangian Term Related

Since our replacement for the unification coupling scale is even more deviating from the Planck scale than more popular unifications with susy, we give up that the various " fundamental scales found, see saw, unification (or approximate unification) and Planck scale, should be at the same energy. Rather we allow them to vary in a systematic way with the dimensionality of the related coefficients in the Lagrangian in the quantum field theory.

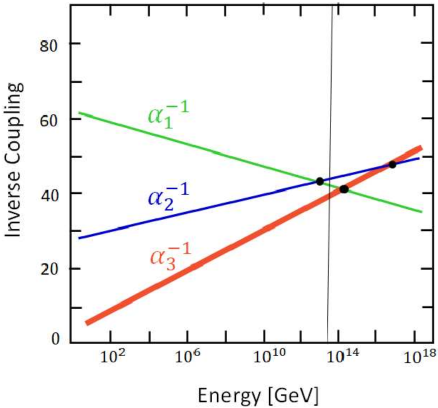
# Abstract for part II continued: Several "Fundamental" Scales, Their logarithms Fitted on a Line as function of Dimension of the Coefficient in Lagrangian Term Related

We interpret this fitting with a model of a truly existing lattice (probably irregular) which is fluctuating both in size and local shape, in a way corresponding to a fluctuation in the reparametrization gauge of general relativity. We though assume that it is somehow cut off so that the distribution of the link length say fluctuate on a logarithmic scale much like a Gaussian distribution in the logarithm.

# Yet moreabstract forpart II:

When one asks for different powers of the link-length for different purposes or different types of interactions, one gets the dominant link length to be somewhat different. This gives different scales for different purposes or lagrange terms: Planck scale, Unification scale, See saw scale,and then a scale related to the fermion masses (to be explained).

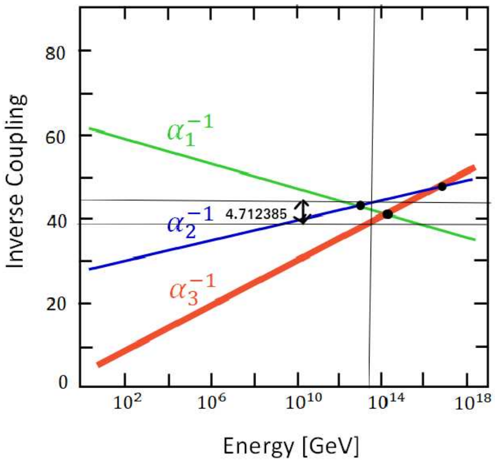
# Crossing in one point of Minimal $SU(5)$ Running (inverse) Fine structue constants not perfect.



# Caption of Crossing Running Fine structure constants figure

This is the usual graph representing the three Standard Model inverse fine structure constants with the $\alpha_1^{-1}$ being in the notation suitable for $SU(5)$, meaning it is 3/5 times the natural normalization, $\alpha_{1\ SU(5)}^{-1}$ $=3/5 * \alpha_{1\ SM}^{-1}$ $=3/5 * \alpha_{EM}^{-1} cos^2\Theta_W$. The vertical thin line at the energy scale $\mu_U = 5 * 10^{13} GeV$ points out "our unified scale", which is as can be seen not really unifying the couplings, but rather is the scale where the ratio of the two independent differences, $\alpha_2^{-1} - \alpha_{1\ SU(5)}^{-1}$ and $\alpha_{1\ SU(5)}^{-1} - \alpha_3^{-1}$ have just the ratio 2/3 as our model predicts at the " our unification scale". One may note, that this "our unified scale" is actually very close to, where the three inverse couplings are nearest to each other, and in that sense an "approximate" unification scale.

# Our prediction of Deviation from $SU(5)$.



## Caption for figure 2: Our prediction of deviation form minimal $SU(5)$

Same as figure 1, but now with our prediction inserted, marked as the number $4.712385 = 3 * \pi/2$, which is predicted to be at the "our unified scale" the difference $1/\alpha_2 - 1/\alpha_3$. Our prediction is, that just at horizontal thin black line, at $5 * 10^{13} GeV$, corresponding to the scale $\mu_U$, given by our fitted to the green line crossing point dividing the region between the blue and the red in the ratio 2 to 3, we shall have the difference in ordinate between the red and the blue crossing points with the vertical black being $3\pi/2$.

# Our formulas to be fitted:

The three standard model fine structure constants (inverted):

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 11/5 * q \qquad (32)$$

$$\frac{1}{\alpha_2}(\mu_U) = \frac{1}{\alpha_{5\ uncor.}} - 9/5 * q \qquad (33)$$

$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 14/5 * q, \qquad (34)$$

where the one parameter $\frac{1}{\alpha_{5\ uncor.}}$, is essentially the unified coupling, although we do not have unification proper.

# We work with two related " unified" couplings, $\alpha_{5\ uncor.}$ and $\alpha_{5\ cor.}$

$$\frac{1}{\alpha_{5\ cor.}} = \frac{1}{\alpha_{5\ uncor.}} - 24/5 * q. \qquad (35)$$

The other parameter $q$ we believe to have calculated in our model with its 3 families of fermions and in a Wilson lattice in a lowest order approximation:

$$q = \text{``}\#families'' * \pi/2 = 3 * \pi/2 = 4.712385. \qquad (36)$$

# Our formulas in "corrected form":

Using this notation we could equally well use the formulation

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 13/5 * q \quad (37)$$

$$\frac{1}{\alpha_2(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 3 * q \quad (38)$$

$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 2 * q. \quad (39)$$

# Table of Fitting the Three parameters

| Parameter | Formula | From $\alpha$'s | Theory | Deviation | Section |
|---|---|---|---|---|---|
| q | q=$1/\alpha_2(\mu_U) - 1/\alpha_3(\mu_U)$ | 4.618 | 4.712385 | -0.094$\pm$0.05 | 16, 1 |
| $1/\alpha_{5\ uncor.}(\mu_U)$ | see above | 51.705 | 45.927 | 5.778$\pm$ 3.5 | 22 |
| $\ln(\frac{\mu_U}{M_Z})$ | $\ln(\frac{\mu_U}{m_t}) = \frac{2}{3} * \ln(\frac{E_{Pl\ red}}{M_t})$ | 27.04 | 24.76 | 2.28$\pm$ 1 or 0.02 | 24 |

In the third line we now replace the top mass $m_t$ with a mass value gotten by extrapolating from the whole spectrum of quarks and leptons, which is about $10\,TeV$ and the agreement got very good indeed.

# We assume ANTI-GUT: Diagonal subgroup breaking

$$G_{full} = SMG \times SMG \times SMG \quad (40)$$

$$\text{where } SMG = S(U(1) \times U(3)) \quad (41)$$

$$= (R \times SU(2) \times SU(3))/Z_{app} \quad (42)$$

where

$$Z_{app} = \{(r, U_2, U_3)| \exists n \in Z[(r, U_2, U_3) = (2\pi, -1, \exp(i2\pi/3)1)^n]\}$$

$$SMG_{as\ observed} = \{(g, g, g)|g \in SMG\} \subset G_{full} \quad (43)$$

# Action: Trace of in $5 \times 5$ imbedding of SMG

It is our crucialassumption that we have a lattice theory with plaquette-action given proportional to the trace of the representative of the plaquette group element $U_{Pl}(\Box)$ in the/a "smallest" representation - taken here as the representation in the five-plet $SU(5)$ representation 5:

$$\rho(U_{pl}(\Box) \quad : \quad 5 \to 5 \tag{44}$$

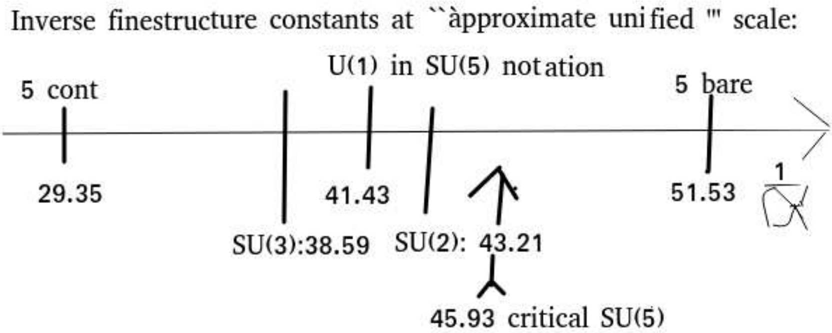
$$\text{or } \rho(U_{pl}(\Box)) \quad \in \quad UnitaryMarix(5 \times 5) \tag{45}$$

We have once pointed out that the very standard model group *SMG* is selected as the one having with appropriate definition the smallest relative to the group faithful representation.

# We predict two differences between $1/\alpha$s in Absolute number

Taking our $q = 3\pi/2$ as just given in our model, and we predict the differences from a to be fitted "unified" inverted coupling $1/\alpha_{5\ uncor.}$ at a to be fitted "unification scale" $\mu_U$, we really only provide **one predicted parameter at first**. Really we predict the two independent differences, say $1/\alpha_2 - 1/\alpha_3 = q$ and $1/\alpha_2 - 1/\alpha_{1\ SU(5)} = 2/5 * q$ at the "unification scale".
E.g. select the scale by having the ratio of the two differences the predicted one; then the absolute size is a true prediction.
We got $q = 4.6$ by the fine structure constant data and the $3\pi/2 = 4.712$.

# Inverse Fine structure constants at the $\mu_U$-scale



# Figure Caption about Critical Coupling

Explaing figure:The axis is the axis of inverse fine structure constants; The group names U(1), SU(2) and Su(3) are the by renormalization group tothe replacement of unification scale $\mu_U$ extrapolated expeimental inverse fine structure constants for these groups respectively. The two $SU(5)$ inverse fine structure constants are repectively with and without the quantum fluctuation contribution.

# Helping Approximations to justify Critical Coupling

To justify that the above figure implies that the unified coupling represented by the inverse fine structure constant has indeed the critical value (for a phase transition, presumably between confinement or not) we make use of the following three approximations/assumptions:(see next slide)

# The approximations or assumtions

- The critical couplings for a true $SU(5)$ lattice theory and for the Standard Modelgroup deviate only little, because the standard model group can be considered an attenuation of the SU(5) one.
- We can trust a rather simple formula for the critical couplings for the SU(N) groups,

$$\frac{1}{\alpha_N} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha^{-1}_{U(1)crit} \quad (46)$$

$$\text{where } \alpha^{-1}_{U(1)crit} = 0.2 \pm 0.015 \quad (47)$$

found in an article with Laperashvili and others[21].
- The critical coupling for the Standard Model group $S(U(3) \times U(2))$ should be compared to couplings with equally many quantum fluctuation contributions as it has itself.

# A reference to larisa et al.

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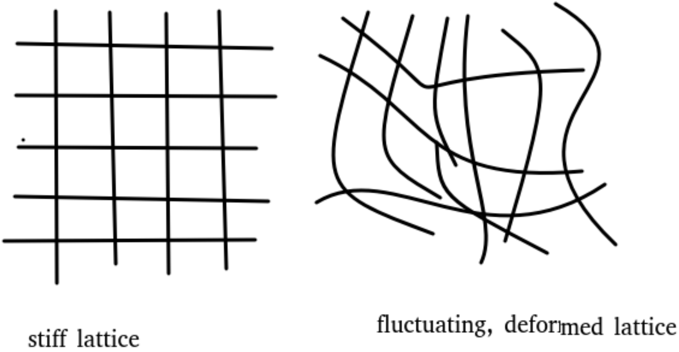
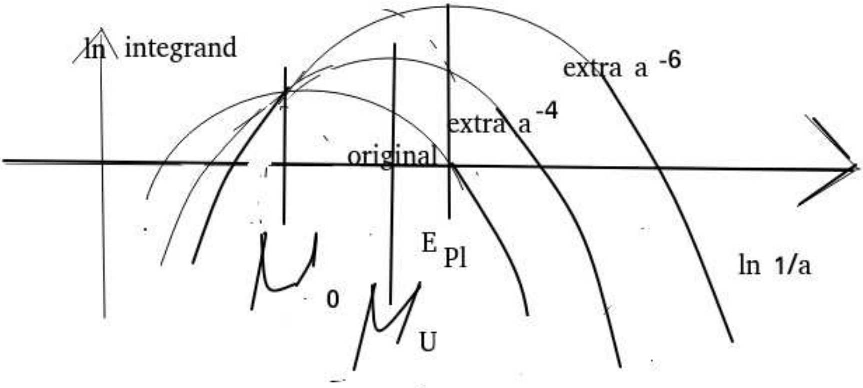
# Part II on the scales in fluctuating lattice model

Really I believe that gauge symmetries could be due to very huge fluctuations in thosedegreesof freedom which arethe guage-monters.
If a lattice were connected to a coordinate system in general relativity, but the guage not fixed but allowed to fluctuate, we should get a lattice fluctuating relative to what we would consider the fixed geometry.

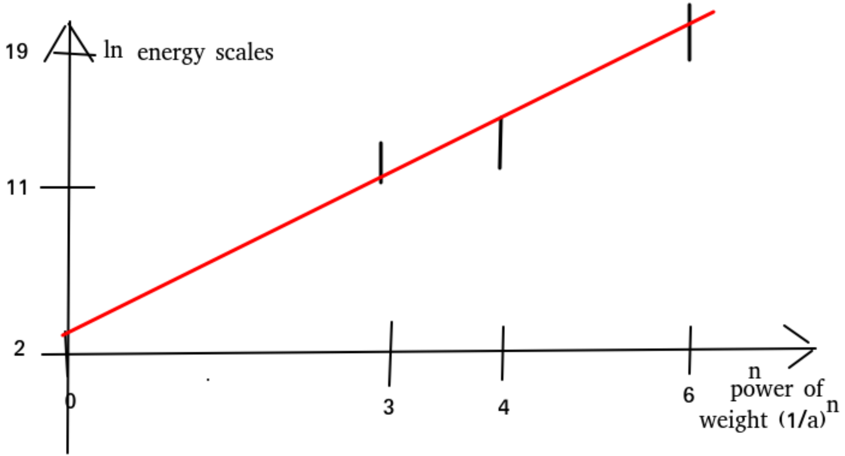
# Fluctuating Lattice Imposed by General relativity



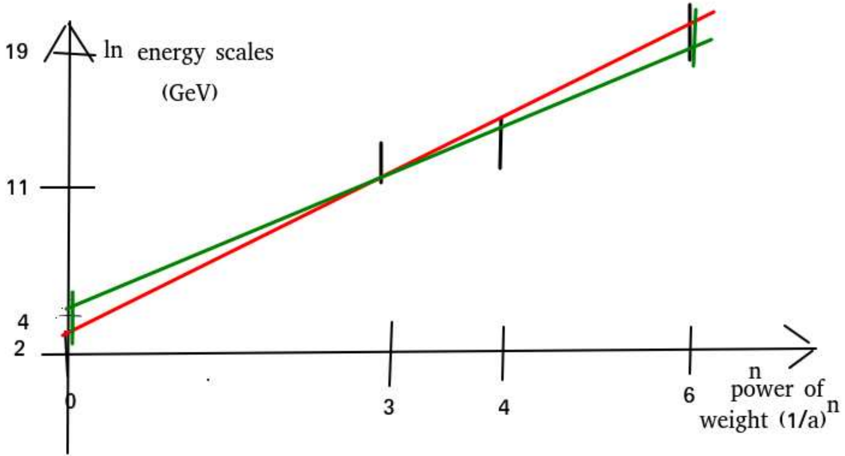
# Contributions as function of ln scale in fluctuating lattice



# Shifted Scales Depending on Weighting with $(1/a)^n$ wiegth factor



# Shifted Scales Depending on Weighting with $(1/a)^n$ wieght factor

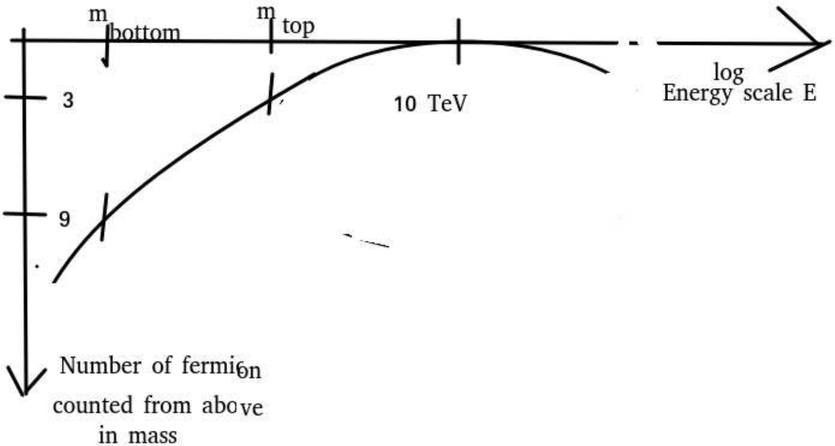


# The plot of scales versus weighting power $n$

We present 4 energy scales of physical interest together with lattice link size $a$ dependent factor comming into the expression in the action or Lagrangian relevant for the scale in question. It is essentially the dimension of the termin the Lagrangian density without counting the coefficient (so it is rather trivially related to this coefficient). We took:

- **0** $(1/a)^0$ This scale is the scale of maximal number of "active"/effectively massless families. (Needs more explanation below.). Below extrapolation $\sim 10\,TeV$.
- **3** $(1/a)^3$ The see-saw neutrino mass scale
- **4** $(1/a)^4$ The our "unification scale",at which the Yang Mills theories are supposed to be given by the truly existing fluctuating lattice.
- **6** $(1/a)^6$ The Planck scale, related to the Einstein-Hilbert-action.

# Fitting Weyl Fermion masses by number after mass by Philosophy of Fluctuating Lattice



# Quark for $m_{mnl} = 10^4 GeV$

$m_{mnl}$ = "maximum number of layers" is the energy scale at which density of the distributuion of inverse lattice sizes $1/a$ is the biggest.

We fit to this density being proportional to the number of Weyl fermions relative to the scale being light/massless. Since the last column $diff^2/n$ fits a constant 1.12 to about 0.1 we have good fit for the 10 TeV.

| Name | number $n$ | Mass $m$ | $log_{10\ GeV} m$ | $diff = 4 - logm$ | $diff^2$ | $diff^2/n$ |
|---|---|---|---|---|---|---|
| top | 3± 1 | 172.76 ± 0.3 GeV | 2.2374± 0.0008 | 1.7626 | 3.1066± 0.003 | 1.0355 ± 0.001 ±0.4 |
| bottom | 9± 0.3 | 4.18± 0.0079GeV | 0.6212±0.001 | 3.3788 | 11.416± 0.01 | 1.268± 0.001±0.03 |
| charm | 17 or 15 | 1.27± 0.02 | 0.10382± 0.009 | 3.8962 | 15.180± 0.07 | 0.893 ± 0.004 ± 0.06 |
| strange | 25 or 23 | 0.095±0.006 GeV | -1.0223±0.003 | 5.0223 | 25.223 ±0.03 | 1.009±0.001±0.1 |
| down | 31 | 4.79± 0.16 MeV | -2.3197±0.01 | 6.3197 | 39.939± 0.06 | 1.288 ± 0.002 |
| up | 37 | 2.01± 0.14 MeV | -2.6968±0.03 | 6.6968 | 44.847± 0.4 | 1.212± 0.01 |

# Leptons for $m_{mnl} = 10^4\,GeV$

$m_{mnl}$ = "maximum number of layers" is the energy scale at which density of the distributuion of inverse lattice sizes $1/a$ is the biggest.
We fit to this density being proportional to the number of Weyl fermions relative to the scale being light/massless. Since the last column $diff^2/n$ fits a constant 1.19 to about 0.1 we have good fit for the 10 TeV.

| Name | number $n$ | Mass $m$ | $log_{10\ GeV}\, m$ | $diff = 4 - logm$ | $diff^2$ | $diff^2/n$ |
|---|---|---|---|---|---|---|
| $\tau$ | 13 or 19 | 1.77686±0.00012 | 0.2496± 0.00003 | 3.7503 | 14.065± 0.0003 | 1.082 ± 0.00002 ±0.4 |
| mu | 21 or 27 | 105.6583745± $2.4 * 10^{-6}$MeV | −0.9761...± $10^{-8}$ | 4.9761 | 24.761± $10^{-7}$ | 1.179± $4 * 10^{-9}$ ± 0.3 |
| electron | 41 | 0.51099895069±$1.6 * 10^{-10}$ | -3.2915± $4 * 10^{-10}$ | 7.2916 | 53.167± $10^{-8}$ | 1.297± $10^{-11}$ |

# Explaining the tables fitting Fermion Masses to Fluctuating Lattice

In the two foregoing tables - one for quarks, the second one for the charged leptons - youhave in first column the name of the fermion, then its number in the series of fermions counted as Weyl fermions and after mass, the heaviest first then the lighter and lighter ones. A quark flavour corresponds to two Weyl per particle and it has three colors,so there is under each flavour 6 Weyl and werepresent a flavour by the midle one of these 6. So the top quark gets the representative number $n = 3$ ( the midlebetween 0 and 6. We use logarithmic scale and care for the logarithm - we use log of basis 10 for slightly easier calculation - of the ratio of the fermion mass to the scale we test with as $m_{mld} =$ " maximum layerdensity point on the energy scale".

# Table Explanation continued

Because our best fit $m_{mld} = 10^4 GeV = 10 TeV$ the log of it is just the 4 in the column $4 - log_{10} m$ (= diff).Since we want to fit the number of layers as a square function of the logarithm of the masses, we shall square what in the table is called *diff* and which is just the log of theratio mentioned.

If the Fermion masses were indeed arranged so as to make the number of (Weyl)fermions with mass under a given scale be proportional to the a quadratic function in the log dropping down from a maximum as we go more and more below $m_{mnl}$ point, then the ratio in the last column $diff^2/n$ should be constant.

If we would have liked to fit with a Gaussian of the logarithm of the masses, weshould intead of the number $n$ have used $\log \frac{45-n}{45}$, which for the first small $n$ is approximately proportional to $n$ itself.(45 isthe number of Weyl particles in SM).

# What to take for the Seesaw neutrino scale?

| Name | Seesaw-scale | Comments |
|---|---|---|
| Steven King | $3.9 * 10^{10} GeV$ | lowest mass;susy |
| Grimus and Lavoura | $10^{11} GeV$ | |
| Davidson and Ibarra | $\geq 10^{9} GeV$ | |
| "statisic" (my own) | $1.4 * 10^{9} GeV$ | |
| Mohapatra | $10^{14}$ to $10^{15}$ GeV | very crude guess |
| Modernized Takanishi and me(own) | $1.2 * 10^{15} GeV$ | |
| Average of most trustable | $10^{11} GeV$ | |

# Conclusion

We had a successful agreement with the values of the fine structure constants in a minimal (i.e. no susy!) approximate GUT SU(5) “unification” at the scale $\mu_U = 5.13 * 10^{13} GeV$ (compared susy-models a very lowenergy scale, but not far at all from the scale needed for see-saw neutrinoes to fit the neutrino oscillations).

# Conclusion (continued): Three parameters.

We have three parameters **predicted by our theory**:

- $q = 3 * \pi/2$ is a parameter going into the **deviation** from full GUT-SU(5).
- The replacement for the unification coupling $\alpha_{5\ uncor.}$ or $\alpha_{5\ cor.}$,whichever one of them we want to think of, or rather the thirds of one of them, should correpond to the critical value,in the sense that it should be at borderline of two phases of the vacuum.
- The replacement for the unification scale $\mu_U$ goes into a series of "fundamentalscales" fittedon a line.(see next slide).

# Conclusion (yet continued); Several "Fundamental scales"

- We related four **different** physical/"fundamental" scales by a line relating the energy scale to their dimention of the related Lagrange density term.
  **2.5 orders of magnitude per dimension of the Lagrange term coefficient**.
  The four scales are:
  - A scale relatedto the fermion mass distribution of formal dimension of coupling $[GeV^4]$.
  - The See saw scale, coupling dimension $[GeV]$,
  - Approximate Grand unification scale, coupling dimension [1],
  - Planck i.e. gravity scale, coupling dimension $[GeV^{-2}]$

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# The PaperItself

# Approximate $SU(5)$, Fine Structure Constants

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“ Bled”, 9. of July 2024

## Abstract

We fit the three finestructure constants of the Standard Model with three, in first approximation theoretically estimable parameters, 1) a "unified scale",turning out *not* equal to the Planck scale and thus only estimable by a very speculative story, 2) a "number of layers" being a priori the number of families, and 3) a unified coupling related to a critical coupling on a lattice. So formally we postdict the three fine structure constants!
In the philosophy of our model there is a physical lattice theory with link variables taking values in a (or in the various) "small" representations of the Standard Model **Group**. We argue for that these representations function in first approximation as were the theory a genuine $SU(5)$ theory. Next we take into account fluctuation of the gauge fields in the lattice and obtain a correction to the a priori $SU(5)$ approximation, because of course the link fluctuations not corresponding any Standard model Lie algebra, but only to the SU(5), do not exist

- long time ago - worked on fitting the fine structure constants - especially the non-abelian ones - in a model based on the main assumptions:

- **Critical Couplings at Fundamental Scale** Preferably the gauge couplings should be at some multiple critical point for a lattice theory at the "fundamental scale" . And it was in the spirit of that model, that there indeed would exist a lattice theory in Nature.
- **AntiGUT** The gauge group was at the "fundamental scale" the Cartesian product $G \times G \times ... \times G$ of the same group $G$ with itself, one time for every family of fermions.

but mainly the Abelian coupling of $U(1)$ was not so well predicted contrary to the non-abelian ones (the attempt by Don Bennet and myself [14] got good numbers, but the theory is a bit complicated). Further Laperashvili and Das and Ryzhikh [16, 21, 5] have even united this type of model with grand unification with $SU(5)$

[16, 21]. They used also supersymmetry in their picture.
Now it is the point of the present article to also make such a combination of $SU(5)$ GUT[1, 3] and the A(nti)GUT theory(AGUT= "anti grand unification theory" meaning the type of theory with a cross product of several copies of the standard model group, e.g. one cross product factor for each family of fermions) just mentioned, but **without SUSY**. Our model would be an example of the type [45] by Michael Zantedeschi and Goran Senjanovic in as far as we shall have higher order effective Lagrangian at about $10^{14} GeV$. We shall here seek an $SU(5)$-like "unification" **without taking the** $SU(5)$ **theory as really true**, but rather taking the $SU(5)$ as an **approximate symmetry** appearing, because of the link variables have a form reminding of $SU(5)$. In fact one possible argumentation is to assume, that the link variables are constructed as matrices (with dynamical matrix element with somewhat restricted movability) for a most simple

and smallest faithfull representation (a sort of principle [26, 27, 28] of smallest link-representation). Another similar argumentation is to use our earlier work [26, 27, 28] telling, that one can define a concept of "small representation" so that the standard model **group**[29][1]gets selectedas the combination of a group and representation as the smallest representation (in a lightly arbitrary way) compared to the group. This would, taken seriously, tell, that it is important, that the group chosen by Nature should have small representation, and that makes it natural, that the link degrees of freedom corresponds to a "small" faithfull representation of the standard model group. Then it turns out, that a typical such small representation is the one obtained by starting from the 5-plet of $SU(5)$ and restrict to the Standard Model Group as contained in $SU(5)$. Really the standard model group $S(U(2) \times U(3))$ is, even in the notation as used here, an obvious subgroup of just $SU(5)$, the notation of which- the $5 \times 5$ matrices - is used to write it.

In the game we proposed[26, 27, 28] to specify the Standard model group as a **group**, it turns out, that a cross product of several isomorphic groups gets the same "points"(in the game of our reference [26, 27, 28]) so that the AGUT model believed in this article is on a shared first place with the single Standard Model **group**) as the group itself, so a group $G_{SMG} \times G_{SMG} \times ... G_{SMG}$ would be equally favoured by the our game.

. In any case the idea is, that the link variables are in terms of the fundamental physics, that is imagined to be behind, represented by variables like in some "small" representation [26] of the standard model group, and that then this representation happens to be / naturally is effectively an $SU(5)$-representation. This means, that the link variables can formally be interpreted as $SU(5)$ variables; but **in reality they are not**. (i.e. there is *no* $SU(5)$ symmetry for turning around the matrix elements in link **5**-plet, **only under the Standard Model subgroup**.) There is **no true** $SU(5)$

**symmetry/theory in our model**! But we can describe the model in terminology of an $SU(5)$-theory, which is broken fundamentally. It is not broken by Higgs mechanism as in the usual SU(5)-theories (a priori at least), but other gauge fields than the ones in the standard model subgroup do not exist (in the first place). There are only gauge fields corresponding to the degrees of freedom in the standard model groups - one set for each family, however, -. (So you must imagine either, that we really have the gauge group $G_{SMG} \times G_{SMG} \times \cdots \times G_{SMG}$ with as many standrad model group factors as there are families of fermions, 3, or you imagine there to be three layers of a usual lattice, so that we have three links, where you usually have only one.)
In the very crudest approximaton for a lattice action - linear in the trace of the representation matrix, the similarity to the $SU(5)$ matrix theory is so great, that the coupling constant ratios at the fundamental lattice theory in the first approximation become just

as in the GUT $SU(5)$ unification scale. However, when it now comes to perturbative corrections due to the fluctuation of the lattice theory degrees of freedom, it becomes important that the degrees of freedom present in $SU(5)$ theory, but not in the Standard model, are missing, and therefore cannot fluctuate. So the quantum corrections from the fluctuation of these - in standard model not present - degrees of freedom are lacking, and thus makes the effective couplings observed in the continuum limit get different values from what they would have gotten in a true $SU(5)$ theory. Being quantum corrections one would usually treat them perturbatively and expect them to be small. If this is indeed the case, then the usual $SU(5)$ predictions will be **approximate**! We can say that it is the main point of present article to calculate this deviation from the exact $SU(5)$ predictions to the usual picture of unifying gauge couplings. Thus the Standard Model (inverse) fine structure constants do not truly unify (at a unification scale, but

we shall talk in the present paper about an "our unified scale", which is the scale at which there is unification except for our (quantum) corrections, and that scale we call $\mu_U$), but we calculate the degree of lack of unification, and even make **prediction of the numerical value of the deviation from GUT $SU(5)$.**
The main point of the present article is really to predict the deviation from exact $SU(5)$ GUT at a certain scale $\mu_U$, at which we calculate the corrections to the exact $SU(5)$ inverse fine structure constants in the standard model as due to quantum fluctuations in the lattice theory assumed to be really physically existent at this scale $< mu_U$. Since we predict the absolute values of the differences between the inverse finestructure constants at the scale, we have at this scale two numerical predictions, and thus we can afford to use one of these predictions at the fundamental scale to fit the scale, and we shall still have predition of one parameter left. For instance we can use the prediction at the scale,

at which the ratio of the difference $1/\alpha_2(\mu_U) - 1/\alpha_{1\ SU(5)}(\mu_U)$ to the other difference $1/\alpha_{1\ SU(5)}(\mu_U) - 1/\alpha_3(\mu_U)$ shall be 2 to 3 (as our calculation implies). This is illustrated on figure 1, and one shall remark, that the three crossings of the inverse fine structure constant with the vertical black line on the figure at the scale about $5.1 * 10^{13} GeV$ has been fitted, so that the three crossings lie with the ratio 2 : 3 of the two intervals. The $U(1)$ inverse fine structure constant passes in between the $SU(2)$, above it with a piece that is proportional to 2, and the $SU(3)$ line, then below it with a distance proportional to 3. But having fixed the scale $\mu_U$ this way it is still a very nontrivial prediction that e.g. the absolute difference between the $SU(2)$-crossing and the $SU(3)$-crossing is just $3\pi/2 = 4.712385$. This is illustrated on figure 1.

## Character of Our Prediction(s)

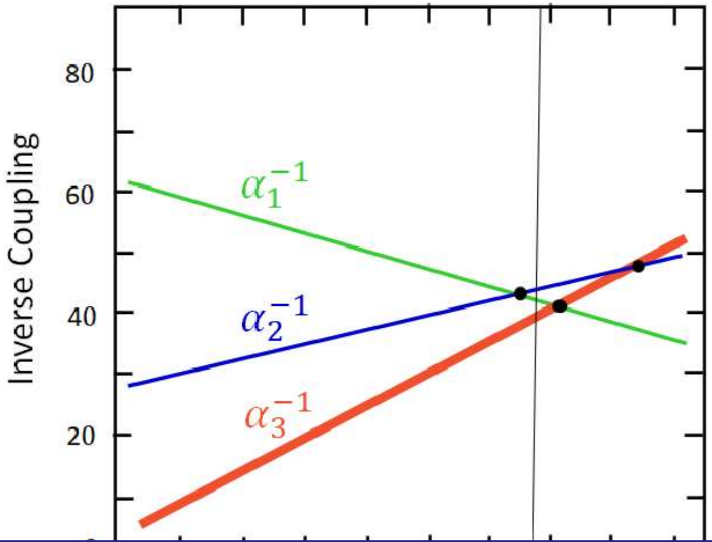


Figure: This is the usual graph representing the three Standard Model inverse fine structure constants with the $\alpha_1^{-1}$ being in the notation suitable for $SU(5)$, meaning it is 3/5 times the natural normalization, $\alpha_{1\ SU(5)}^{-1} = 3/5 * \alpha_{1\ SM}^{-1} = 3/5 * \alpha_{EM}^{-1} cos^2\Theta_W$. The vertical thin line at the energy scale $\mu_U = 5 * 10^{13} GeV$ points out "our unified scale", which is as can be seen not really unifying the couplings, but rather is the scale where the ratio of the two independent differences, $\alpha_2^{-1} - \alpha_{1\ SU(5)}^{-1}$ and $\alpha_{1\ SU(5)}^{-1} - \alpha_3^{-1}$ have just the ratio 2/3 as our model predicts at the " our unification scale". One may note, that this "our unified scale" is actually very close to, where the three inverse couplings are nearest to each other, and in that sense an "approximate" unification scale.

## Character of Our Prediction(s)

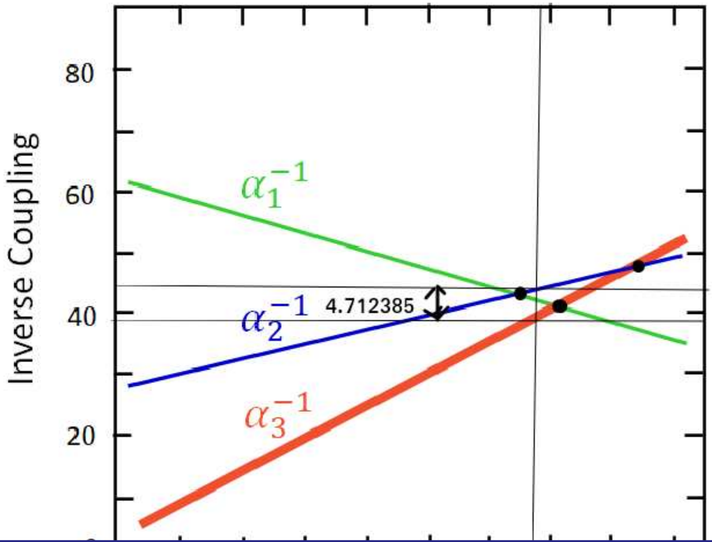


Figure: Same as figure 1, but now with our prediction inserted, marked as the number $4.712385 = 3 * \pi/2$, which is predicted to be at the "our unified scale" the difference $1/\alpha_2 - 1/\alpha_3$. Our prediction is, that just at horizontal thin black line, at $5 * 10^{13} GeV$, corresponding to the scale $\mu_U$, given by our fitted to the green line crossing point dividing the region between the blue and the red in the ratio 2 to 3, we shall have the difference in ordinate between the red and the blue crossing points with the vertical black being $3\pi/2$.

Our formulas for fitting the three inverse finestructure constants in the Standard Model in the for $SU(5)$ adjusted notation, wherein one uses $1/\alpha_{1\ SU(5)} = 3/5 * 1/\alpha_{1\ SM} = 3/5 * cos^2\Theta_W * 1/\alpha_{EM}$ are rather simple, and concerns of course the three Standard Model fine structure by renormalization group transformed to a certain scale $\mu_U$, which is our replacement for the unification scale (because there is of course as we know no unification scale proper unless one involves susy or something else extra). The choice of the scale $\mu_U$ is only indirectly determined in our model, and is essentially just a fitting parameter, although we in section 24 shall speculatively relate $\mu_U$ to the Planck energy scale $E_{Pl}$ by a crude relation $\frac{\ln(\frac{E_{Pl}}{m_t})}{\ln(\frac{\mu_U}{m_t})} \approx 3/2$. Then at this scale - to be fitted -

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 11/5 * q \tag{48}$$
$$\frac{1}{\alpha_2}(\mu_U) = \frac{1}{\alpha_{5\ uncor.}} - 9/5 * q \tag{49}$$
$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 14/5 * q, \tag{50}$$

where the one parameter $\frac{1}{\alpha_{5\ uncor.}}$, which we could also give other names like

$$\frac{1}{\alpha_{5\ bare}} = \frac{1}{\alpha_{5\ classical}} = \frac{1}{\alpha_{5\ uncor.}}, \tag{51}$$

is our replacement for the unified inverse $SU(5)$ fine structure constant. The symbols, which we propose $uncor. = bare = classical$ are to tell that this coefficient in the action functioning as the $SU(5)$ inverse coupling is without the quantum fluctuation couplings, i.e. it is uncorrected (= uncor.) or "bare". We could also define a corrected one

$$\frac{1}{\alpha_{5\ cor.}} = \frac{1}{\alpha_{5\ uncor.}} - 24/5 * q. \quad (52)$$

The other parameter $q$ we believe to have calculated in our model with its 3 families of fermions and in a Wilson lattice in a lowest order approximation:

$$q = \text{"}\#families\text{"} * \pi/2 = 3 * \pi/2 = 4.712385. \quad (53)$$

Using this notation we could equally well use the formulation

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 13/5 * q \quad (54)$$

$$\frac{1}{\alpha_2(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 3 * q \quad (55)$$

$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 2 * q. \quad (56)$$

Here in fact the quatity $\frac{1}{\alpha_{5\ cor.}(\mu_U)}$ is the in the analogous way to our treatment of the Standard Model inverse fine structure constants formally corrected $SU(5)$- inverse coupling to an effective one at the our unified scale $\mu_U$, but of course, since there is no $SU(5)$, this is not so important, and rather formal only.
The requirement of the gauge couplings at the fundamental scale being just on the borderline on one or preferably more phase transitions, that are welcome to be lattice artifacts, was the basic

ingredient in the wroks, of which the present one is a development[9, 10, 11, 12, 8, 13, 18, 20, 15]. In the present work with its approximate $SU(5)$ it may seem natural to require the $SU(5)$ coupling being just on the phase border for the pseudo-unified $SU(5)$ coupling as represented by $\frac{1}{\alpha_{5\ uncor.}}$. In principle the critical coupling depends on the lattice details, and it has to be calculated by lattice computer calculations, but here we have for a beginning just taken an approximate formula for the critical coupling out of our earlier works[19].
The fact, that has always been a bit embarrasing for GUT theories of e.g. $SU(5)$, namely that the unified scale turns out appreciably smaller in energy than the Planck scale, is also embarrasing in our theory, and for rescuing it against this problem, we propose the speculation of a strongly fluctuating lattice. It should fluctuate in the size of the lattice constant, and we should imagine, that in various places and moments the lattice is more or less fine. We

shall below see, that this kind of fluctuations can be used as an excuse for the effective scale for gravity, the Planck energy scale, and that for the Standard Model, the "our" grand unified scale (which is a replacement for the GUT scale) can deviate from each other violently. The parameter giving the our unified scale $\mu_U$, namely the logarithm of it relative to the weak scale $M_Z$, namely $\ln(\frac{\mu_U}{M_Z})$ (or may be use better $m_t$ instead of $M_Z$), is according to our speculation given in terms of the Planck scale, which thus is a needed input to obtain all three parameters to give the three fine structure constants.

The three parameters, with which we fit the three Standard Model fine structure constants come in our present work from rather different speculations, which though all should be sufficiently compatible, that we can have them in the same model. Here we announce, in the below table, the success of our model:

| Parameter | Formula | From $\alpha$'s | Theory | Deviation | Section |
|---|---|---|---|---|---|
| q | $q = 1/\alpha_2(\mu_U) - 1/\alpha_3(\mu_U)$ | 4.618 | 4.712385 | -0.094±0.05 | 16, 1 |
| $1/\alpha_{5\ uncor.}(\mu_U)$ | see above | 51.705 | 45.927 | 5.778± 3.5 | 22 |
| $\ln(\frac{\mu_U}{M_Z})$ | $\ln(\frac{\mu_U}{m_t}) = \frac{2}{3} * \ln(\frac{E_{Pl\ red}}{M_t})$ | 27.04 | 24.76 | 2.28± 1 or 0.02 | 24 |

In the third parameter line we put a somewhat by hand taken uncertainty for the theoretical value, because the scales being diveded,the Planck scale over the scale of the three families ending at low energy taken as the $M_Z$ scale or better top-mass $m_t$, is a ratio of rather illdefined concepts of scales and thus at least give an uncertainty of one unit in the natural logarithm.
Depending on how many of the stories behind the “theory” of these parameters the reader might buy as trustable the reader can decide with how many parameters, we fit the three standard model (inverse) fine structure constants. In fact the “theories” for the three different parameters are rather independent of each other, so that a selections that some are wrong and some are right would

not at all be excluded.
In the following section 15 we describe our assumption of a lattice for the Standard Model **Group**,see [29] $S(U(2) \times U(3))$.
In section 16 we perform calculations of the quantum correctons.
In the section 17 we at least mention the Wilson lattice action and give a bit notation. In section 18 we compared to old similar quantum fluctuation calculations to make sure we got correct normalization. and compare to tadpole improvements [31].
The fitting of the data - the experimentally determined fine structure constants in the Standard Model - comes in section 19.
In section 22 we look at, if the coupling, say the approximate GUT one, is the critical coupling, crucial point in our old works leading up tothis work [9, 10, 11, 12, 8, 13, 18, 20, 15].
We shall in general postpone second order calculation, but crudely mention in section 23.
In section 24 we discuss the most speculative one among the

parmeters in our model, the value of the energy scale $\mu_U$,and thereby the fluctuating lattice scale.
Finally in section 25 we conclude.
Our concrete model is, that we have in Nature a fundamental lattice with an energy scale $\mu_U$ crorresponding to the lattice constant $1/\mu_U$ (with $c = \hbar = 1$), the lattice being the Wilson one, say. This lattice is "tripled up" in the sense, that there is really one Wilson lattice for each family of fermions. Calling the number of families $N_{gen} = 3$ one can think of it as the genuine group being not the Standard Model Group itself $SMG$, but its third power $SMG \times SMG \times SMG$, the true gauge group in our model

$$\begin{aligned} G_{full} &= SMG \times SMG \times SMG \\ \text{where } SMG &= S(U(1) \times U(3)) \\ &= (\mathrm{R} \times SU(2) \times SU(3))/\mathrm{Z}_{app} \\ \text{where } \mathrm{Z}_{app} &= \{(r, U_2, U_3) | \exists n \in \mathrm{Z}[(r, U_2, U_3) = (2\pi, -1, \exp(i2\pi/3)1 \end{aligned}$$

Alternatively one might think of a model like this as there being three usual lattices lying parallel to each other (seperated in an extra dimension, say), It could therefore be tempting to call them "layers" of lattices.
In any case we imagine, that somehow or another the $G_{full}$ is broken down to its diagonal subgroup, which is (isomorphic to) the standard Model group $SMG$. In fact this diagonal subgroup is defined as

$$SMG_{diag} = \text{the subgroup of } G_{full} \text{of elements of form } (g,g,g)$$
$$SMG_{diag} = \{(g,g,g) \in G_{full} = SMG \times SMG \times SMG | g \in G_{SMG}\}.$$

(we tend to use both notations $SMG$ and $G_{SMG}$ for the same,so simply $SMG = G_{SMG} = S(U(2) \times U(3)))$. This breaking down of $G_{full}$ to the diagonal $SMG_{diag}$ can easily be imagined to come about by a little bit of mixing up the different layers locally all over. ( "confusion" [23, 24, 25]). In the section 24 we shall speculate a bit more complicated about the lattice structure, because we shall propose that there is even at the lattice scale diffeomorphism symmetry or at least some symmetry, like the symmetry of a projective space time containing (local or global) scalings. This then means that we imagine the lattice to fluctuate in both size and position, so that even if it is Wilson type very locally, it varies in both orientation and size of the lattice constant very strongly from place to place. If it is so, and it might be unrealistic to imagine that it is not fluctuating, if we shall have a so usual gravity theory with its reparametrisation fluctuating (as one should imagine the gauge of any gauge theory to really fluctuate [37]), e.g. the "fundamental scale" $\mu_U$ we calculate below by fitting, must be at the end considered an average value of the "fundamental scale" while the local fundamental scale fluctuates. But apart from this story of connecting our model to gravity, the fluctuations might be ignored, and a lattice with fixed lattice

constant of order $a \sim 1/\mu_U$ would be o.k. (But remember: we fit "the our unification scale" like the one in usual exact $SU(5)$ to be appeciably lower in energy than the Planckscale.)
The crucial special assumtption for this article is to assume, that the degrees of freedom of the lattice-links representing the element of the standard model group SMG is the matrix elements of a matrix representation of this SMG on a minimal faithfull representation. It is then assumed that these matrix elements are restricted to only (be able to) move quite freely along the image of the SMG into the "small" representation used, while motion in other directions is strongly restricted (perhaps by very high potentials) but at least we shall ignore them, if there is any fluctuations, except along the standard model group, so to speak.
The idea of thinking of such an imbedding is to note, that in such an imbedding we have a way of thinking of an SU(5) representation too, because the "small" reprensentation, we have

in mind, is the one, that is the 5 plet reprentation of $SU(5)$. It is of course also a representation of the $SMG \subset SU(5)$. Now a really crucial point is, that we imagine, that once the $SMG$ has been represented this $SU(5)$ simulating way, it tends to inherit an $SU(5)$ symmetry, even though our model has **no true $SU(5)$ symmetry postulated**. It is only, that it seems a bit similar in its simplest reprentaion. A bit more concretely we may say: we use, that the smoothness assumed also for the Lagrangian density as function of the plaquette- variables - which are also postulated to be formulated in $5 \times 5$ matrices - is a smoothness defined from the $5 \times 5$ matrices. When we then Taylor expand and from that look for the form of the plaquette action, we come to the trace of the $5 \times 5$ matrix just as in the usual $SU(5)$ theory. By this we have thus "sneaked in" an **approximate** $SU(5)$ symmetry. This is really the crux of matter of our model: The $SU(5)$ symmetry is **not a symmetry imposed on Nature** but rather an approximate symmetry of the way, we suggested to be the most natural way to represent the link and plaquette degrees of freedom for a model, that basically is only symmetric under the standard model group SMG. Thus there is of course already in our picture built in a breaking of the $SU(5)$ symmetry. Most importantly the **degrees of freedom from the components in the $SU(5)$ theory fields not also in the Standard Model Group SMG, are lacking.**

For us this then means, that there are no quantum fluctuations in the plaquette or link variables corresponding to these lacking degrees of freedom. The concern of the present article is to evaluate, how these lacking modes lead to lakcing some quantum corrections for the fine structure constants, and these corrections from the lacking modes of oscillations are not quite equally big for the three different Standard model gauge couplings. This is then according to us the reason for breaking in these couplings of the - of course fundamentally non-existing - $SU(5)$ symmetry.

Once we have decided to look for the ratio of the second order and the fourth order terms in the Taylor expansion (76) of the Plaquette action (**??**), we should be able to extract the *relative* correction due to inclusion of the quantum fluctuations - the fourth order term in (76) - in by just putting in some ansatz $h$ for fields alone from one of the three standard model groups, meaning a set up of one of the three standard model sub-group fields at a time, and even the normalization (of $h$) is then not important (becuase both terms are second order in $h$, we only kept them) for this relative size of the two terms, while the size of the fluctuations have to be calculated, though.
Now we want to estimate the three Standard model finestructure constants - or rather their ratios - by putting on a "test field" which for the plaquette action, on which we think, is denoted $h = h_{\square}$, and if we think of a purely spatial plaquette, is really a magnetic field of that plaquette. This magnetic field is thought

upon in the notation with the coupling constant absorbed into the field, so that the action actually has an inverse finestructure constant contained as factor to compensate the absorbed charge-factor $e_0$ say,

$$S = \ldots + \sum_{plaquettes} \frac{1}{2\pi\alpha_0} * ReTr(U(\square)) \quad (60)$$

$$\text{or continuum } S \propto \int \frac{1}{16\pi\alpha_0} F_{\mu\nu} F^{\mu\nu} d^4x. \quad (61)$$

(see section 17 for why just $\frac{1}{2\pi\alpha}$ in front of the $ReTr(U_\square)$.)
Thus the inverse fine structure constant are found from how the action (or say the magnetic energy) varies approximately linearly with the square of the test field imposed $h^2$. If the fluctuation field was $SU(5)$-invariant - as it would of course be in a theory without any breaking of the SU(5)-symmetry, the three fine structure constants in the "SU(5)" invariant notation, which is wellknown to

deviate from the more natural one by the replacement:

$$\left.\frac{1}{\alpha_1}\right|_{natural} = \left.\frac{1}{\alpha_1}\right|_{SU(5)} * \frac{5}{3}, \quad (62)$$

would be equal to each other all three.
The test-fields, we shall use, and which for the non-abelian groups $SU(2)$ and $SU(3)$ corresponds to the coupling definitions

$$S = \int (-\frac{1}{4e_2^2}\frac{1}{2}Tr_{matrix,2\times 2}F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4e_3^2}\frac{1}{2}Tr_{matrix,3\times 3}(F_{\mu\nu}F^{\mu\nu}) + \ldots)$$

could be

$$\text{For } SU(2): h_{SU(2)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (63)$$

$$\text{for } SU(3): h_{SU(3)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (64)$$

$$\text{for } U(1): h_{U(1)} = \frac{1}{\sqrt{30}} \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix} \quad (65)$$

All the three proposed test-matrices $h$ have been normalized, so that their squares

$$h^2_{U(1)} = \frac{1}{30} * diag(9,9,4,4,4) \quad (66)$$

$$h^2_{SU(2)} = \frac{1}{2} diag(1,1,0.0.0) \quad (67)$$

$$h^2_{SU(3)} = \frac{1}{2} diag(0.0.,1,1,0) \quad (68)$$

become of trace equal to unity

$$Tr(h^2_{U(1)}) = 1 \quad (69)$$

$$Tr(h^2_{SU(2)}) = 1 \quad (70)$$

$$Tr(h^2_{SU(3)}) = 1. \quad (71)$$

It is this normalization that ensures that the three couplings all become equal in the exact $SU(5)$ limit. (From the unbroken

symmetry under the Standard model group it will not matter which component under one of the three standard model groups is used as test- field, as long as it is a combination of the components of just that one of the three groups $U(1)$, $SU(2)$ and $SU(3)$.) These fields $h$ are meant to be added to the already fluctuating field, but not to flutuate themselves, and then dividing the thereby achieved (magnetic) energy increase or action decrease we shall obtain (apart from a constant factor) the inverse finestructure constant for the subgroup of the Standard Model in question.
In the very first approximation - the $SU(5)$-invariant one - there is the same amount of fluctuation in all the 24 compponents of the $SU(5)$-Lie algebra, actually each of them have the average of the field squared for one component, so that
$1/2 * < H^2_{\text{one component}} >= \frac{\pi}{2} * \alpha_5$. **But in the philosophy, that only the Standard model components really exist, we must in our model only have fluctuations in these components.**

The difference between our model, in which there truly speaking only is gauge symmetry by the Standard Model, and not even fields corresponding to the full $SU(5)$, and the usual $SU(5)$ theory, comes in by **restricting the fluctuation field $H$ in our model to only fluctuate in Standard Model degrees of freedom.**
Actually the Lie algebra components, which are in the $SU(5)$-Lie-algebra but not in the Standard model one, can be in the notation, we have chosen here (64), be represented by the matrix element being put to zero in the following matrix $5 \times 5$:

$$\begin{bmatrix} \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \end{bmatrix}$$

I.e. the difference between our model and the $SU(5)$ symmetric model is, that the fluctuation in the vacuum fields on the 2 times 6 points in this matrix marked by the 0 's is suppressed in our model, while in the $SU(5)$ symmetric $H$ the fluctuation is the same size in all the matrix element except for the detail that the trace of $H$ is restricted to be zero,

$$tr(H) = 0. \tag{72}$$

In both usual $SU(5)$ and ours the trace is zero, but the 12 element marked with zero are restricted from fluctuating only in our model. The technique to estimate what happens when one puts up in a region a smooth continium field is simply, that we add the field due to the continuum field, F say, translated to the matrix $h$ to the fluctuating field $H$. That is to say we consider the configuration:

$$U(\Box) = \exp(i(H+h)), \tag{73}$$

then to extract magnetic energy or the action of the plaquette, we assume the usual type of real part of the trace action:

$$S_{plaquette} \propto ReTr(U(\Box)), \tag{74}$$

and look for the terms in the action change, which is of second order in the continuum extra field representing the continuum field. The coefficient to this second order $h^2$ to give the action change due to the continuum field is simply proportional to the inverse fine structure constant for the type of field we used.
The Taylor expansion of the exponential is wellknown and we only have to keep the terms of second order in $h$, and we shall not go further than to second order in $H$, so we only need to expand to fourth order in the sum $H + h$.

In fact we generelly have

$$ReTr(\exp(i(H+h)) = ReTr(1) + \frac{1}{2}ReTr((i(H+h))^2) + \frac{1}{24}ReTr((i(H$$

(odd powers give zero).

Dropping but the $h^2$ order terms we get

$$S_{plaquette}|_{h^2-part} = ReTr(U(\Box))|_{h^2-part} \quad (76$$

$$= \frac{1}{2}ReTr(h^2) + \frac{1}{24} * 6ReTr(h^2H^2) \quad (77$$

( provided that $h$ and $H$ commute)

$$\text{Otherwise :} = \frac{1}{2}ReTr(h^2) + \frac{1}{24} * (4ReTr(h^2H^2) + 2ReTr(hHhH)$$

$$= \frac{1}{2}ReTr(h^2) + \frac{1}{6}ReTr(h^2H^2) + \frac{1}{12}ReTr(hHhH). \quad (78$$

For the presentation of the calculation of the quantum fluctuation corrections to the three different fine structure constants in the Standard Model, we divide the fluctuations into four classes. Have in mind that in crudest approximation the vacuum fluctuations in the $SU(5)$ symmetric approximation consists of independent fluctuations after all the 24 basis vectors in a basis for the $SU(5)$ Lie algebra. Imaginig having chosen this basis so that the 12 basis vectors are also basis vectors for the three sub Lie algebras corresponding to the three Standard Model groups, we can divide the fluctuation into four sets, denoted symbolically by $H_1$ for the fluctuation in the single mode of the $U(1)$ subgroup, $H_2$ for the fluctuation in the $SU(2)$ degrees of freedom, and $H_3$ for the $SU(3)$ fluctuations, and then for us the most interesting class $H_{int}$, namely those remaining fluctuations in the $SU(5)$ Lie algebra, which do not fall into any of the three welknown subgroups of $SU(5)$ in the Standard model, and which in our model are declared not to exist in Nature and thus must be removed. I.e. these fluctuations under the name $H_{int}$ are put to zero. With such a classification we can divide the fourth order term into a series in principle of $3 \times 4$ combinations. In fact we can ask for any of the three finestructure constants for which we want to calculate the quantum fluctuation corrections, what the contribution is from one of any of the four fluctuation classses, $H_1$, $H_2$, $H_3$, and $H_{int}$. We want to calculate the shift in the three inverse fine structure constants of the Standrd Model by first calculate the relative changes $\frac{\Delta\alpha_i^{-1}}{\alpha_i^{-1}}$ of these inverse finestructure constants $1/\alpha_i$ for $i = 1, 2, 3$ denoting respctively the subgroups $U(1)$, $SU(2)$, and $SU(3)$. Since we are now computing the "correction" after the very lowest order approximation is considered to be exact $SU(5)$ symmetry, we can in principle be careless with which finstructure constants we use in this calculation, when performed at the unification point of energy scale, because at this scale at zeroth

approximation all three and even the $\alpha_5$ are equal.
We shall first caculate the shifts $\Delta\alpha_i^{-1}(\mu_U)$ from their relative shifts. For this we need the very important
$1/2* < H^2_{\text{one component}} > = \frac{\pi}{2}\alpha_5$ (but it is here we can be careless to our approximation with which $\alpha_1$ you replace this $\alpha_5(\mu_u)$), and the factor $\frac{\pi}{2}$ is explained below in section 17.
Thus the shift of the inverse fine structure constant becomes

$$\Delta\frac{1}{\alpha_i(\mu_U)} = \frac{1}{\alpha_i(\mu_U)} * \frac{ReTr(H^2 h_i^2)}{2ReTr(h_i^2)} \text{(for effective commutativity)}$$

$$= \frac{1}{\alpha_i(\mu_U)} * < H^2_{\text{one component}} > * \frac{ReTr(H^2 h_i^2)}{2ReTr(h_i^2) * < H^2_{\text{one co}}}$$

$$= \frac{\pi}{2} * \frac{ReTr(H^2 h_i^2)}{2ReTr(h_i^2) * < H^2_{\text{one component}} >}.$$

One can think of the fraction $\frac{ReTr(H^2 h_i^2)}{2ReTr(h_i^2) * < H^2_{\text{one component}} >}$ as a kind of counting how many components of the fluctuation contribute to the correction of the $i$th inverse fine structure constant,

$$\text{“Eff. } \# < H^2 > \text{ contributions”} \quad \overset{=}{def} \quad \frac{ReTr(H^2 h_i^2)}{2ReTr(h_i^2) * < H^2_{\text{one component}} >$$

$$= \quad \sum_{j=1,2,3} \text{“Eff. } \# < H^2 > \text{ contributio}$$

Here of course

$$\text{“Eff. } \# < H^2 > \text{ contributions”}|_{H_j} \quad \overset{=}{def} \quad < \frac{ReTr(H_j^2 h_i^2)}{2ReTr(h_i^2) * < H^2_{\text{one compon}}$$

If we include into this sum also the $H_{int}$ fluctuations, we get the corrections under unbroken $SU(5)$ and in this case the sum of these “ Eff. $\# < H^2 >$ contributions” should for all three inverse fine structure constants be 24/5. There are 24 components for full $SU(5)$, but in order to contribute to the trace $Tr$ a factor 1 you need 5 1's (along the diagonal).

By a little thinking of, that we want the average of these fluctuations which are independent, except along the diagonal, and that elements in the matrix related by permuting column number with row number are strongly correlated as must be the case to ensure hermiticity of the fluctuating fields $H = H^{\dagger}$, we find out that one gets the same result whatever the order in the matrix product, so that effectively $h$ and $H$ commute after all.

Let us now list a table these "'Eff. $\# < H^2 >$ contributions" and their calculations:

| From the $H_i$ | $\alpha_1^{-1}$ $h_{U(1)}$ | $\alpha_2^{-1}$ $h_{su(2)}$ | $\alpha_3^{-1}$ $h_{SU(3)}$ |
|---|---|---|---|
| $H_1$ | $\frac{2*81+3*16}{900}$ $=7/30$ | $\frac{2*9}{2*30}$ $=3/10$ | $\frac{4}{30}$ $=2/15$ |
| $H_2$ | $\frac{3*9*2}{3*30}$ $=9/10$ | $\frac{2*3}{2*2}$ $=3/2$ | 0 $=0$ |
| $H_3$ | $\frac{4*3*8}{3*30}$ $=16/15$ | 0 $=0$ | $\frac{8}{3}$ $=8/3$ |
| sum | $11/5$ | $9/5$ | $14/5$ |
| $H_{int}$ | $\frac{54+24}{30}$ $=13/5$ | 3 $=3$ | 2 $=2$ |
| check | $24/5$ | $24/5$ | $24/5$ |
| half s. half $H_{int}$ | $11/10$ $13/10$ | $9/10$ $3/2$ | $7/5$ 1 |

Table: Table of the numbers $\frac{ReTr(H_i^2 h_j^2)}{2*ReTr(h_j^2)<H_i^2>}$ first without the explicit denominator 2, but then at the very two lowest lines the half is taken for sum of the contribution from the Standard Model group fluctuations and for the ones from the $H_{int}$ which is missing in the standard model.

The numbers in this table are easily obtained when having in mind when the trace is of the form $Tr(H^2h^2)$ because we can then simply evaluate the traces by using the following diagonal matrices:

$$< H_1^2 > = \frac{1}{30} * diag(9,9,4,4,4) \quad (82)$$

$$< Tr(H_1^2) > = 1 \quad (83)$$

$$< H_2^2 > = \frac{3}{2} * diag(1,1,0,0,0) \quad (84)$$

$$< Tr(H_2^2) > = 3 \quad (85)$$

$$< H_3^2 > = \frac{8}{3} diag(0,0,1,1,1) \quad (86)$$

$$< Tr(H_3^2) > = 8 \quad (87)$$

$$< H_{int}^2 > = diag(3,3,2,2,2) \quad (88)$$

$$< Tr(H_{int}^2) > = 12 \quad (89)$$

combined with the squares of the ansatz matrices

$$h^2_{U(1)} = \frac{1}{30} diag(9,9,4,4,4) \quad (90)$$

$$Tr(h^2_{u(1)}) = 1 \quad (91)$$

$$h^2_{SU(2)} = diag(1/2,1/2,0,0,0) \quad (92)$$

$$tr(h^2_{SU(2)}) = 1 \quad (93)$$

$$h^2_{SU(3)} = diag(0,0,1/2,1/2,0) \quad (94)$$

$$Tr(h^2_{SU(3)}) = 1 \quad (95)$$

The above multiplication to make the table is o.k. if the $h$'s and $H$'s indeed commute. Effectively, however, we can show that by the averaging, we do end up as if they commuted:
The $h$'s, i.e. the ansatz matrices, we can simply choose diagonal, because that is just to select an appropriate basis vector for the group one wants. If the fluctuation field is a diagonal one it is then

indeed commuting, but if we consider an off-diagonal component of an $H_i$ field, then we can argue that it leads to a product of the two diagonal elements in the $h$ and this leads in the special cases we consider to taking trace of an $h$ which is zero. So in pracsis it is as if we had commutation, almost by accident.
We shall use the notation for the single layer (our model has three layers corresponding to three families) Wilson lattice, being related to a continuum theory(we here leave the gauge group open) and with the charge absorbed into the field $F^{\mu\nu}(x)$ (containing magnetic $\vec{B}$ and electric part $\vec{E}$ with their $g$ absorbed):
If we use a notation, in which the $A_\mu(x)$ gauge fields are already Lie-algebra valued fields - or for our $U(N)$ groups of interest here equivalently matrices - and thus can define basis-vector matrices

$\lambda_a$ and $T_a$ so that

$$A_\mu(x) = (\Sigma) A_\mu^a \frac{\lambda_a}{2} \quad (96)$$

$$= (\Sigma) A_\mu^a T_a \quad (97)$$

$$\text{where, say, for off-diagonal } \lambda_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (98)$$

$$\lambda_2 = \begin{bmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (99)$$

$$\text{and with normalization } Tr(\lambda_a \lambda_b) = 2\delta_{ab} \quad (100)$$

$$\text{and } Tr(T_a T_b) = 1/2 * \delta_{ab} \quad (101)$$

you can by interpreting the $A_\mu(x)$ fields as representation in some representation $R$ construct unitary matrices in the crude continuum limit identification

$$U_\mu(x) = \exp(iaA_\mu(x)) \tag{102}$$

in the usual way require the

$$S_{Wilson}[U] = -\frac{\beta}{2N}\Sigma_\Box(W_\Box + W_\Box^*) \tag{103}$$

$$= \frac{a^4\beta}{4N}\int \frac{d^4x}{a^4} trF_{\mu\nu}F_{\nu\mu} + \ldots \tag{104}$$

$$\text{where } W_\Box = tr(U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger(x+\hat{\nu})U^\dagger(x)) \tag{105}$$

$$= tr(\text{oredered product around the plaquette}\Box)$$

obtain using (**??**) $S = \int -\frac{1}{4g^2} F_{\mu\nu} F^{\nu\mu} d^4x$ the relation

$$\frac{\beta}{2N} = \frac{1}{g^2}. \quad (106)$$

$$\text{or } \frac{\beta}{N} = \frac{1}{2\pi\alpha}. \quad (107)$$

And this leads to that the fluctuating part $H = (\Sigma) H^a T_a = (\Sigma) H^a \frac{\lambda_a}{2}$ of the exponent in the plaquette variable

$$U_\square = \exp(i\Sigma H^a \frac{\lambda_a}{2}) \quad (108)$$

goes into the action with

$$\Sigma_{\square} \frac{\beta}{N} Retr \exp(i\Sigma H^a \frac{\lambda_a}{2}) \tag{109}$$

$$= \Sigma_{\square} \frac{1}{2\pi\alpha} Retr \exp(i\Sigma H^a \frac{\lambda_a}{2}) \tag{110}$$

$$\approx_{\text{second o.}} \frac{1}{2\pi\alpha} \Sigma_{\square} Retr(-\frac{1}{2}(\Sigma_a H^a \frac{\lambda_a}{2})^2) \tag{111}$$

$$= \frac{1}{4\pi\alpha} \Sigma_{\square} \Sigma_a (H^a)^2/2 \tag{112}$$

$$= \Sigma_{\square\ a} \frac{1}{8\pi\alpha} (H^a)^2 \tag{113}$$

So if the plaquettes were not coupled - what they though are -

then in the partition function / the Euclidean path integral which is

$$Z = \int DU \exp(-\beta S[U]) \quad (114)$$

$$\approx \Pi_{\Box\ a} \exp(-\frac{1}{8\pi\alpha} * (H^a)^2) \quad (115)$$

where $DU$ is the Haar measure, the fluctuation of a plaquette variable (exponent) $H^a$ would be given as $< (H^a)^2 >$ (no summation) $=8\pi\alpha/2$ (when restriction between the plaquette variables were neglected), since $\frac{\int x^2 \exp(-Kx^2)dx}{\int \exp(-Kx^2)dx} = 1/(2K)$. But of course they are connected so that there are only half the plaquette variables,which are independent. This can actually be seen to lead to that the distribution of the partition function distribtuion become twise as narrow measure in the square $H^a$ average: So in the lattice partition faction or the Euklideanized path integral the fluctuation is

$$< (H^a)^2 > (\text{no summation}) \quad = \quad 8\pi\alpha/2/2 = 2\pi\alpha. \qquad (116)$$

We here used that the plaquette variables, say $H^a(\square)$ for the different plaquettes $\square$ are not independently integrated over. On the contrary for each cube in the lattice there is a constraint which linearized means that the sum of six plaquaette variable for the plaquettes around the cube is restricted to be zero. Since there in 4 dimensions are 6 plaquettes per site and 4 cubes, this restriction would in first go mean that there per site were only 2 independent plaquette variables, but that is, however, not true, because there is a constraint between the four cube-constraints on the plaquettes. So in reality there is per site 3 independent constrains on 6 a priori plaquette variables. This gives that the average of the square $(H^a)^2$ of a (Gaussian distributed) plaquaette variable get reduced by a factor 6 to (6-3) meaning a factor 2. Simplifying to just 2 variables to get restricted to 1 independent we could just think of a Gaussian distribution about the origo in a plane, and that we then restrict the at first two dimensional to a diagonal - a single dimension - being a restriction symmetric between the two orinal variables thought of as the coordinates. Then the restricted distribtuion on the symmetric diagonal would project into one of the coordinate axes with the average of the saquare diminished by a factor 2.

The meaning of our basis choice for defining our lattice variables $H^a$ could be illustrated by asking, what is now the calculated average of the square of an off diagonal element in the $5 \times 5$ matrix. E.g. for matrix element row 1 column 2 we get

$$< |H_{\text{row 1 column 2}}|^2 > \; = \; < (H^1/2)^2 + (H^2/2)^2 > \quad (117)$$

$$= \; 1/2 * 2\pi\alpha = \pi\alpha. \quad (118)$$

It is such an - most easy off diagonal element we denote by

$H_{\text{one component}}$ and its numerical average square is thus for **one layer**

$$< |H_{\text{one component}}|^2 > |_{\text{one layer}} = \pi\alpha. \quad (119)$$

$$\text{Want } \frac{1}{2} < |H_{\text{one component}}|^2 > |_{\text{one layer}} = \pi/2 * \alpha. \quad (120)$$

The reason we want this half of the average square of the matrix element in the $5 \times 5$ matrix, is that in the Taylor expansion (76) has a factor 2 deviation between the two terms,which we shall compare.

In the calculation of the relative correction to the inverse exact $SU(5)$ fine structure constants we need the ratio of the two terms (76) and the correction term comes from the Taylor expansion as

$$\text{" correction term"} = \frac{1}{4} * tr(h^2 H^2)(\text{if commuting}$$

$$\text{while the corresponding "uncorrected"} = \frac{1}{2} tr(h^2).$$

Use the numbers from thee table being just traces of the products of the diagonal matrices, which are noramlized so that their traces are 1 for the $h^2$ and the dimension of the Lie Group for the $H_i^2$ - normalizes the difference $\frac{1}{\alpha_2} - \frac{1}{\alpha_3}$ to one "unit" ignoring yet the factor 3 of number of families, and the hereby absorbed denominator 2, being

$$\text{The "unit"} \quad = \quad \frac{\pi}{2} \tag{122}$$

now in the notation with "Re Tr" (in which it would at first have been $\pi$. So the prediction will be that the difference at the unifying scale of the two nonabelian inverse fine strucutre constants - which had number 1 (when the explicit $1/2$ not included - will be $\frac{\pi}{2}$ for only one family, but $3 * \pi/2$ for three families.
Since it is so crucial for our prediction that we calculate the absolute size of the quantum correction,our $q = 3 * \pi/2$ correctly and that it is indeed such a quantum correction effect,we shall here

compare it to an old work with Don Bennett, though only calculating this correction for simple groups SU(3) and SU(2), but it checks the absolute size. That the physics of this type of quantum correction works even with a background of an extensive computer calculation is seen in the next subsection 1 In my old work with Don Bennett [13] arXiv:hep-ph/9311321v1 "Predictions for Nonabelian Fine Structure Constants from Multicriticality" we in fact presented the same correction, which we use here and even had the normalization included and used that the correction to the inverse fine structure constants are

$$\frac{1}{\alpha} \rightarrow \frac{1}{\alpha}(1 - C_f \pi \alpha) \quad (123)$$

$$= \frac{1}{\alpha} - C_f \pi \quad (124)$$

where $C_f$ means the quadratic Cassimir in the fundamental representation of the group in question. In fact we find in this article:

$$C_f^{SU(2)} = \frac{3}{4} \quad (125)$$

$$C_f^{SU(3)} = \frac{4}{3} \quad (126)$$

In fact the quatity $< H_i^2 >$, which is so crucial to us to get estimated, is a quantity needed to make the so called tad-pole improvements for lattice calculations[30]. In the calculation by Niyazi eta. [31] we find some computer study, that also reach the quantity $u_0$ defined by

$$u_0^4 = \left\langle \frac{1}{N} Tr(U_p(\Box)) \right\rangle , \quad (127)$$

or as being the average value in the fluctuating lattice (in vaccum) for a link variable. They present as a result of their numerical studies in a region of $\beta$'s around $\beta = 7.5$ in their nottation meaning $1/\alpha_3 = 7.5/5 * 2\pi = 9.42477$:

$$u_0(\beta) = 0.87010 + 0.03721\Delta\beta - 0.01223(\Delta\beta)^2. \quad (128)$$

where $\Delta\beta = \beta - 7.3$.

On the basis of the crudest approximations as speculated in our section 17 we expect the $u_0(\beta)$ to be of the form

$$
\begin{aligned}
u_0^4(\beta) &= 1 - \frac{C}{\beta} && (129)\\
\text{needing then } C &= 7.3 * (1 - 0.87010^4) && (130)\\
&= 7.3 * (1 - 0.057316) && (131)\\
&= 3.1159. && (132)\\
\text{If so, shift } \Delta\frac{1}{\alpha_3} &= C/3 * 2\pi * (1 - \frac{2*4}{20}) && (133)\\
&= C * 2\pi/5 && (134)\\
&= 3.9155 && (135)\\
&\approx 4.1888 && (136)\\
&= 8/3 * \pi/2 && (137)
\end{aligned}
$$

(here the correction factor comes from our (105) , correction for a $N_c = 3$ in the notation of Niyazi et a.,and correction because the continuum coupling - the $\alpha_3$ -gets a contribution from a lattice action term with double plaquettes having a coefficient $\beta/20$ in first approximation and contribtuiing 8 times as much as the "main Wilson term") If the inverse $\beta$ type fitting here is correct, then the derivative being the coefficiient on the second term $0.03721\Delta\beta$

should be

$$\frac{d}{d\beta}u_0(\beta) = \frac{d}{d\beta}\sqrt[4]{1-\frac{C}{\beta}} \tag{138}$$

$$= \frac{1}{4}(1-\frac{C}{\beta})^{-3/4} * (\frac{C}{\beta^2}) \tag{139}$$

$$= \frac{1}{4}u_0^{-3} * C/\beta^2 \tag{140}$$

$$= \frac{1}{4}C/7.3^2/0.87010^3 \tag{141}$$

$$= C * 0.00712 \tag{142}$$

$$= 0.022190. \tag{143}$$

This is a little bit lower than the 0.03721.
From formula (2) in reference [31] we see that Niayzi et al. uses the $N$ included in the action explicitely so that for $SU(3)$ their $\beta = 3\beta_{without\ theN}$,so e.g. the $\beta = 7.3$ where they worked would

mean in the notation without the $N$ included in the definition $7.3/3 = 2.4333$. Then since in the usual notation which Niayzy et al.seems to use one has e.g. according to [32] $\beta = \frac{2N_c}{g_s^2}$ implying

$$\frac{1}{\alpha_3} = \frac{4\pi}{g_s^2} = 2\pi\beta_{\text{with no } N_c \text{ notation}} \tag{144}$$

But there is a further point in extracting the fine structure constant used in the work by Nyaizi et al: They use L$'$uscher-Weisz action which even in the large $\beta = \beta_{pl}$ limit has an extra term consisting of double plachette actions with a coefficient which according to [33] is given by the

$$\beta_{rt} = -\frac{\beta_{pl}}{20u_0^2} * (1 + 0.4905\alpha_3) \quad (145)$$

$$S[U] = \beta_{pl}\Sigma_{rt}\frac{1}{3}ReTr(1 - U_{pl}) \quad (146)$$

$$+ \beta_{rt}\Sigma_{rt}\frac{1}{3}Re(1 - U_{rt}) \quad (147)$$

$$+ \beta_{pg}\Sigma_{pg}\frac{1}{3}ReTr(1 - U_{pg}) \quad (148)$$

$$\text{so } \beta_{eff}|_{lowest\ order} = \beta_{pl} * (1 - \frac{1}{20} * 4 * 2) \quad (149)$$

$$= \beta_{pl} * \frac{3}{5} \quad (150)$$

So this would mean we shall use (106), but with $\beta/N$ put to $3/5 * \beta_{pl}/3$. The case $\beta = 7.3$ in the notation of [31] corresponds then to

$$2\pi * 2.433333 = \frac{1}{\alpha_3} \quad (151)$$
$$\text{giving } 1/\alpha_3 = 15.2890 \text{ forgetting the } 3/5$$
$$\text{so the } u_0^4 = 0.87010^4 = 0.573161057 \quad (152)$$
$$\text{will correct by} 15.2890 * (1 - 0.87010^4) = 6.25597 \quad (153)$$
$$\text{which should be } \pi/2 * 8/3 = 4.18878 \quad (154)$$

However,when we now remember the inclusion of the effect of the double plaquette term at least in the weak coupling limit giving the factor 3/5, then instead to Niyazi et al. 's $\beta = 7.3$:

$$\beta_{true} = 7.3/3 * 3/5 \quad (155)$$
$$= 7.3/5 \quad (156)$$
$$= 1.46 \quad (157)$$
$$\text{giving } \frac{1}{\alpha_3} = 2\pi\beta_{true} \quad (158)$$
$$= 9.1734 \quad (159)$$
$$\text{and shift by } 9.1734 * (1 - 0.87010^4) = 3.91556 \quad (160)$$
$$\text{again compare to } 8/3 * \pi/2 = 4.188787 \quad (161)$$

Now there is very little difference,so that we can consider it, that this extraction from the calculation of the $u_0$ became a test of our calculation of the correction from loop corrections being so crucial for the present work.

Let us take yet an example namely $\beta_{Nyaizi} = 7.7$,it gives

$\beta = \beta_{Nyaizi}/3 * 3/5 = 1.54$ and $1/\alpha_3 = 2\pi * 1.54 = 9.6761$. Now we had for 7.7, $u_0 = 0.8803$ and thus $1 - u_0^4 = 1 - 0.8803^4 = 0.399486$ giving the change of the 9.6761 by 3.8655. Still close to 4.1887 (But I do not like it got further away from this 4.1887, when the coupling got weaker, because we expect our values exact in the weak coupling limit).

The first step in our fitting of our model is to calculate the "unifying" scale $\mu_u$, at which the ratios between the differences between the inverse fine structure constants for the three subgroups of the Standard Model group is the one predicted from our calculation of the quantum fluctuation corrections. In fact the three inverse fine structure constants shall lie on the number axis as the numbers $(2, 13/5, 3)$ corresponding to the subgroups $(SU(3), U(1), SU(2)$, where we have chosen the $SU(5)$-normalisation for the $U(1)$-finstructure constant. The relation is expressed in terms of the two independent differences, that can be formed. Let us , e.g., say

$$\frac{\frac{1}{\alpha_2} - \frac{1}{\alpha_{1\ SU(5)}}}{3 - 13/5} = \frac{\frac{1}{\alpha_{1\ SU(5)}} - \frac{1}{\alpha_3}}{13/5 - 2} \quad (162)$$

$$\Rightarrow \frac{1}{\alpha_2} - \frac{1}{\alpha_{1\ SU(5)}} = \frac{2}{3} * (\frac{1}{\alpha_{1\ SU(5)}} - \frac{1}{\alpha_3}) \quad (163)$$

$$\Rightarrow \frac{1}{\alpha_2} - \frac{5}{3} * \frac{1}{\alpha_{1\ SU(5)}} + 2/3 * \frac{1}{\alpha_3} = 0 \quad (164)$$

Expressing the $\frac{1}{\alpha_i}$'s as

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln\left(\frac{\mu}{M_Z}\right) + \ldots \quad (165)$$

$$\text{with } b_i^{SM} = (41/10, -19/6, -7), \quad (166)$$

this relation for the $\alpha_i(\mu_u)$ 's is written for the $M_Z$-scale finestructure constants as

$$\frac{1}{\alpha_2(M_Z)} - \frac{5}{3} * \frac{1}{\alpha_{1\ SU(5)}(M_Z)} + 2/3 * \frac{1}{\alpha_3(M_Z)} = (b_2 - \frac{5}{3} b_1 + 2/3 * b_3$$

Inserting the values obtained for the $M_Z$ inverse fine structure constants this becomes:

$$29.57 \pm 0.06\% - \frac{5}{3} * 59.00 \pm 0.02\% + \frac{2}{3} * 8.446 \pm 0.6\% = \frac{-19/6 - 5}{}$$

$$* \quad \ln \frac{\mu_u}{M_Z}$$

$$-63.10 = -44/3/6.2$$

$$\Rightarrow \ln \frac{\mu_u}{M_Z} = 27.03$$

$$\Rightarrow \frac{\mu_u}{M_Z} = 5.482 * 10^1$$

$$\text{Using } M_Z = 91.1876 Ge$$

$$\text{thus } \mu_u = 5.00 * 10^{13}$$

In the table 1 we go through the calculation of first determine the our unification scale by requiring the ratios of the two relative deviations from true $SU(5)$ symmetry to be in the ratio required by our model. This we have shown to be done by requiring the linear combination of the three inverse finestructure constants at this unifying scale to make zero the linear combination of the inverse fine structure constants having the coefficients $(-5/3, 1, 2/3)$ for respectively $(1/\alpha_{1\ SU(5)}, 1/\alpha_2, 1/\alpha_3)$. As a check of our model we work out, by correcting for the quantum fluctuations in the inverse fine structure constant, to reproduce the two $1/\alpha_5$'s, namely the one without quantum corrections - the bare $SU(5)$ inverse fine structure constant - and the 'èffective " $SU(5)$ inverse fine structure constant which has been corrected for these quantum corrections. The test is that these two formal$SU(5)$ (inverse) couplings shall be the same whichever of the three standard model fine structure constants are uused for the calculation of them, provided our model agrees with the data used.

## Table for inverse fine structure constants and our fitting

| | | | | | |
|---|---|---|---|---|---|
| 1. | | $1/\alpha_{1\ SM}$ | $1/\alpha_{1\ SU(5)}$ | $1/\alpha_2$ | $1/\alpha_3$ |
| 2. | Formula | $1/\alpha_{EM} cos^2\Theta_W$ | $3/5 * 1/\alpha_{EM} cos^2\theta_W$ | $1/\alpha_{EM} * sin^2\Theta_W$ | $\alpha_3^{-1}$ |
| 3. | Start $\#'s$ | $127.916 * 0.76884$ | $\frac{3}{5} * 127.916 * 0.76884$ | $127.916 * 0.23116$ | $0.1184^{-1}$ |
| 4. | Value | 98.347 | 59.008 | 29.569 | 8.446 |
| 5. | Uncertainty | $\pm$ 0.02 | $\pm$ 0.013 | $\pm$ 0.017 | $\pm$ 0.05 |
| 6. | Coefficient | | -5/3 | 1 | 2/3 |
| 7. | Contribution | | -98.347 | 29.569 | 5.631 |
| 8. | Uncertainty | | $\pm$ 0.02 | $\pm$ 0.017 | $\pm$ 0.034 |
| 9. | | **SUM:** | | | |
| 10. | Sum | -63.147 | | | |
| 11. | Uncertainty | $\pm$ 0.04 | | | |
| 12. | b's | 41/6 | 41/10 | -19/6 | -7 |
| 13. | b-contribution | | -5/3*41/10 | 1*(-19/6) | 2/3*(-7) |
| 14. | | | = -41/6 | = -19/6 | =-14/3 |
| 15. | Sum | (-41-19-28)/6 | | | |
| 16. | | =-44/3 | | | |
| 17. | b-contr./$2\pi$ | -2.33420017 | -1.087559696 | -0.503991079 | -0.742723695 |
| 18. | $\ln(\frac{\mu_U}{M_Z})$ | **Ratio:** $\frac{-63.147}{-2.33420017}$ | | | |
| 19. | | =**27.053** | | | |
| 20. | Uncertainty | $\pm$ 0.02 | | | |
| 21. | Scale $\mu_U$ | $5.116 * 10^{13} GeV$ | | | |
| 22. | Uncertainty | $\pm 0.1 * 10^{13} GeV$ | | | |
| 23. | b's/$2\pi$ | | 0.652535818 | -0.503991079 | -1.114085543 |
| 24. | -13.634 | -30.139 | | | |
| 25. | Uncertainty | | $\pm$ 0.01 | $\pm$ 0.01 | $\pm$ 0.02 |

What we are really interested in is the magnitude of the deviation from $SU(5)$ being accurate at the our "unified scale" $\mu_u$, and we should like to develop the expression for this deviation in terms of the original variables at $M_Z$ even. But to get an overview it is better first obtain the deviations by simply calculating the three inverse finstructure constants at the our “unification scale” $\mu_u$:

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_u)} = 59.00 \pm 0.02 - 0.65254 * 27.0566 \quad (172)$$
$$= 59.00 - 17.66 \quad (173)$$
$$= 41.34 \quad (174)$$
$$\frac{1}{\alpha_2(\mu_u)} = 29.57 + 0.50399 * 27.0566 \quad (175)$$
$$= 29.57 + 13.64 \quad (176)$$
$$43.21 \quad (177)$$
$$\frac{1}{\alpha_3(\mu_u)} = 8.446 + 1.11409 * 27.0566 \quad (178)$$
$$= 8.446 + 30.143 \quad (179)$$
$$= 38.59 \quad (180)$$

We may note down the differences and check that they are in the right ratio:

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_{1\ SU(5)}(\mu_u)} = 43.21 - 41.34 \quad (181)$$
$$= 1.87. \quad (182)$$
$$\frac{1}{\alpha_{1\ SU(5)}(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = 41.34 - 38.59 \quad (183)$$
$$= 2.75 \quad (184)$$
$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = 43.21 - 38.59 \quad (185)$$
$$= 4.62 \quad (186)$$

The test now is if

$$2/5 * 4.62 \stackrel{?}{=} 1.87 \quad (187)$$

$$\text{In fact } 2/5 * 4.62 = 1.85 \quad (188)$$

$$\text{and } 3/5 * 4.62 \stackrel{?}{=} 2.75 \quad (189)$$

$$\text{In fact } 3/5 * 4.62 = 2.77 \quad (190)$$

Now our question is how big is this 4.62 in units of $\pi/2 = 1.5708$. We find

$$\frac{\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)}}{\pi/2} \quad (191)$$

$$= \frac{4.62}{\pi/2} \quad (192)$$

$$= 2.94 \approx 3 = \#families! \quad (193)$$

This is remarkably close to 3, the number of families! (with an order of magnitude uncertainty $\pm 0.1$ it the inverse finestructure constants, a deviation of only 0.06 is very good!) This is in itself a remarkable coincidence, in spirit with our old work stories about critical inverse finestructure constants getting multiplied by the number of families, because of the antiGUt theory behind.
Corresponding to this spacing, we can now with the above calculations being used find in fact two $SU(5)$ inverse couplings, namely one before the effect of the quantum fluctuations $< H^2 >$ of $H$ are taken into account and one after they are taken into account for the - in our theory non-existent - whole $SU(5)$.
Using the table 1 we find, that using as unit $1/\alpha_2(\mu_u) - 1/\alpha_3(\mu_u) = 4.62 \approx 3\pi/2$, the two a bit different inverse unified couplings $1/\alpha_{5\ bare}$ and $1/\alpha_{5\ cont}$ (for $SU(5)$

formally) at our unification scale $\mu_u$ are given as

The "bare":

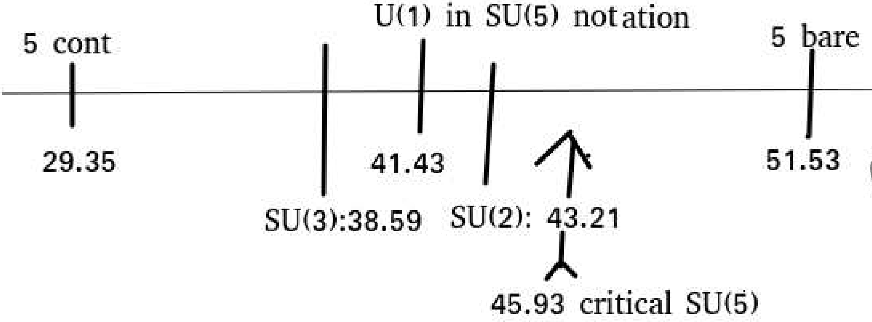
$$1/\alpha_{5\ bare} = 1/\alpha_{1\ SU(5)}(\mu_U) + 11/5 * 4.62 = 41.34 + 10.164 = 5$$
$$\text{or} \quad 1/\alpha_2(\mu_u) + 9/5 * 4.62 = 43.21 + 8.316 = 51.526$$
$$\text{or} \quad 1/\alpha_3(\mu_U) + 14/5 * 4.62 = 38.59 + 12.936 = 51.526$$

The corrected:

$$1/\alpha_{5\ cont}(\mu_U) = 1/\alpha_{1\ SU(5)}(\mu_U) - 13/5 * 4.62 = 41.34 - 12.012 = 2$$
$$\text{or} \quad 1/\alpha_2(\mu_U) - 3 * 4.62 = 43.21 - 13.86 = 29.35$$
$$\text{or} \quad 1/\alpha_3(\mu_U) - 2 * 4.62 = 38.59 - 9.24 = 29.35$$

(We used in this table the "experimental" value $q = 4.62$ but it would have made only very little difference to use the theoretical value $q = 3 * \pi/2$, because our agreement is so good)

Inverse finestructure constants at ``àpproximate unified '" sca



*approximate* $SU(5)$ the difference between say $\frac{1}{\alpha_2(\mu_u)}$ and $\frac{1}{\alpha_3(\mu_u)}$ is just the number of families $N_{gen}$ *times the 'unit"* $\frac{\pi}{2}$. It is so to speak the deviation from proper $SU(5)$ symmetry, which seems remarkably to be an integer - the number of families - times the "unit" $\frac{\pi}{2}$, which denotes the amount of shift in an inverse $\alpha$ per unit of quantum fluctuations in the lattice theory of the theory in question.

For testing and for illustrating, that there is truly a content in our prediction, we want now to rewrite this result in terms of the $M_Z$-scale quantities:

Let us begin to write down the difference, that should have the remarkable value $N_{gen} * \frac{\pi}{2}$ (where $N_{gen}$ is the number of families):

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = \frac{1}{\alpha_2(M_Z)} - \frac{1}{\alpha_3(M_Z)} - \frac{b_2 - b_3}{2\pi} \ln \frac{\mu_u}{M_Z},$$

where now

$$\ln \frac{\mu_u}{M_Z} = \frac{1/\alpha_2(M_Z) - 5/3 * 1/\alpha_{1\ SU(5)}(M_Z) + 2/3 * 1/\alpha_3(}{\frac{b_2 - 5/3 * b_1 + 2/3 * b_3}{2\pi}}$$

so that

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = \frac{1}{\alpha_2(M_z)} - \frac{1}{\alpha_3(M_Z)} - $$
$$-\frac{b_2 - b_3}{b_2 - 5/3 * b_1 + 2/3 b_3} *$$
$$*(1/\alpha_2(M_Z) - 5/3 * 1/\alpha_{1\ SU(5)}(M_Z) + 2/3 * 1/\alpha$$

Here the ratio of the $b_i$'s becomes:

$$\frac{b_2 - b_3}{b_2 - 5/3 * b_1 + 2/3 * b_3} = \frac{-19/6 - (-7)}{-19/6 - 5/3 * (41/10) + 2/3 *}$$

$$= \frac{23}{88}$$

*Numerically*

$$\frac{-3.166 + 7.000}{-3.166 - 5/3 * 4.100 - 2/3 * 7.000}$$

$$= -0.26143(\text{agree with}\frac{23}{88})$$

Our difference is

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = \left(\frac{111}{88\alpha_2} - \frac{115}{264\alpha_{1\ SU(5)}} - \frac{218}{264\alpha_3}\right)|_{M_Z}$$
$$= \left(\frac{1}{\alpha_{EM}} * \left(\frac{201}{132} sin^2\Theta - \frac{69}{264}\right) - \frac{218}{264} * \frac{1}{\alpha_3}\right)|_{M_Z}$$

Let us use

$$\frac{1}{\alpha_{EM}(M_Z)} = 127.916 \pm 0.015 \quad (202)$$
$$sin^2\Theta = 0.23116 \pm 0.00013 \quad (203)$$
$$\alpha_3(M_Z) = 0.1184 \pm 0.0007 \quad (204)$$

Then our difference becomes:

$$
\begin{aligned}
\text{``}\textit{difference}\text{''} &= \frac{1}{\alpha_2(\mu_U)} - \frac{1}{\alpha_3(\mu_U)} \\
&= \left(\frac{1}{\alpha_{EM}} * \left(\frac{201}{132} sin^2\Theta - \frac{69}{264}\right) - \frac{109}{132} * \frac{1}{\alpha_3}\right)\Big|_{M_Z} \\
&= ((127.916 \pm 0.015) * (\frac{201}{132} * (0.23116 \pm 0.00013) \\
&- \frac{109}{132} * \frac{1}{0.1184 \pm 0.0007}) \\
&= 4.6187 \pm 0.0014(\text{from } \alpha_{EM}) \pm 0.025(\text{from } sin^2\theta) \\
&\stackrel{?}{=} 3 * \pi/2 = 4.7124 \\
deviation &= 0.0937 \pm 0.046 \\
deviation & \textit{ is about } 2s.d.
\end{aligned}
$$

If you would like to blame all our deviation on the strong $\alpha_3$, we would get, that in stead of the used 0.1184 a number 2.3 standard deviation higher, meaning the replacement,

$$\alpha_3(M_Z) = 0.1184 \pm 0.0007 \quad \rightarrow \quad 0.1200 \tag{210}$$

$$\text{A strengthning by } 0.0016 \quad \textit{meaning} \quad 2.3s.d. \tag{211}$$

One can of course also check our prediction by calculating what scale of "our unification " is needed for each of the three differences between the inverse fine structure constants to fit exactly with our prediction, the predictions are found in table 2 in line 3.and 4.. The first to compute instead of the unification scale directly is the natural logarithm of it after dividing with the $Z^0$ mass, because we conventionally find in the litterature the running couplings at thisscale $Z^0$-mass.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | | $1/\alpha_2 - 1/\alpha_{1\ SU5}$ | $\pm$ | $1/\alpha_2 - 1/\alpha_3$ | $\pm$ | $1/\alpha_{1\ SU5} - 1/\alpha_3$ | $\pm$ |
| 2. | $dif\|_{M_Z}$ | $-29.4390$ | 0.03 | 21.1232 | 0.05 | 50.5623 | 0.05 |
| 3. | $dif\|_{\mu_U\ pred.}$ | $2/5 * 3 * \pi/2$ | | $1 * 3 * \pi/2$ | | $3/5 * 3 * \pi/2$ | |
| 4. | | =1.88495 | | =4.71239 | | =2.82743 | |
| 5. | dist to run | 31.32405 | | $-16.3993$ | | $-47.7349$ | |
| 6. | Run rate | $\frac{19/6+41/10}{2\pi}$ | | $\frac{19/6-7}{2\pi}$ | | $\frac{-41/10-7}{2\pi}$ | |
| 7. | | =1.156526 | | =-0.6101 | | =-1.76662 | |
| 8. | $\ln(\frac{\mu_U}{M_Z})$ | 27.0846 | 0.03 | 26.8797 | 0.1 | 27.02046 | 0.03 |
| | as av.+dev. | 27.04+0.0446 | | 27.04-0.1203 | | 27.04-0.01954 | |
| 9. | $\|\frac{dif\|_{\mu_U\ pred.}}{\text{Run rate}}\|$ | $\|\frac{1.88}{1.15}\|$ | | $\|\frac{4.71}{-0.610}\|$ | | $\|\frac{2.827}{-1.7666}\|$ | |
| 10. | | =1.629 | | =7.724 | | = 1.6005 | |
| 11. | rel.dev. | 0.052 | | -0.015 | | 0.011 | |
| 12. | $\ln(\frac{\mu_U}{M_Z})$ | 1.5 | | 1.6 | | 0.6 | |
| | s.d.f. av. | | | | | | |
| 13. | d. fr. 27.03 | 0.05 | | -0.15 | | -0.01 | |

Table: Table of results for three - not indenpent - ways of using the by us predicted differences between the running inverse fine structure constants at "unfied scale in our model" (which is the scale at which the three running differences should be equal to the numbers in line 3 or 4. These predictions are to be fullfilled at this "unified scale" which using each of the three differences is written in line 8, and the success of our model is really that these three numbers agree. They deviate from their average 27.04 by the numbers of standard deviations (s.d.) given in line 12. The "small" deviations agree within accuracy. But more important is to compare these deviations from the common average to the ratios given in line 9. which should be the contribution from our prediction numbers translated into the numbers in $\ln(\frac{\mu_U}{M_Z}$, which we gave in line 8. Here it turns out that the deviations from the average of the three numbers as written in line 12 in terms of standard deviations, when compared to these predictions divided by the running rate are relatively small as seen in line 11. In fact these numbers in line 11 are at most of the order of 1/20, while the two smaller ones of them are only of the order of 1/70. This means that our prediction values turned out correctly to better than 5%. A similar conclusion would be reached by instead of the average of the three $\ln(\frac{\mu_U}{M_Z})$ fits using the value of the $\ln(\frac{\mu_U}{M_Z})$ fitted by directly insisting on the ratio of the differences of th the inverse fine structure constants being the one we require. This insisting on the ratio of the differences directly lead to 27.03, which is only deviating by 0.01 from the average here in the table which was 27.04,when wieighting with uncertaitties wereused in evaluating the average (the naive average is rather 27.00) . The difference o.o3 is only 1.5 s.d. and quite small compared the to the predictions corresponding shifts in the $\ln(\frac{\mu_U}{M_Z})$ as seen in line 9.or 10. Again this fact ensures that our agreement although not perfect (yet) is remarkably good.

By accident the naive average(i.e. counting the three as having same uncertainty) of the three values for $\ln(\frac{\mu_U}{M_Z})$ turns out to be exactly 27.00 within our uncertainty. The 11th line in the table gives the deviation from this average relative to the part of the $\ln(\frac{\mu_U}{M_Z})$, which is due to our prediction value, so it give the order of magnitude of the failure of our prediction relatively. Remark, that even the biggest of these three deviation measures relative to our predictions is **only 0.052** meaning that even this deviation is only so well fitting by accident in one out of 24 cases.
Now we have without using but the lattice theory philosophy - see

the old works and [41], also connection to our several phase speculations [42] - reached to an understanding in our picture of the deviations from the $SU(5)$ symmetry. It would of course be natural first to look for, if the unifying coupling should be the critical one for $SU(5)$ corrected of course for the factor that is the number of families. This is though not at all obviously the correct thing to do in our philosophy, because we have in the philosophy of the present article no true $SU(5)$ theory. It is only approximate, but lacks half of the degrees of freedom. nevertheless let us for first orientation look for comparing the expression for the $SU(5)$ critical coupling given by Laperashvili, Ryzhikh, and Das [16, 18]

$$\alpha_{N\,crit}^{-1} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha_{U(1)crit}^{-1} \qquad (212)$$

where we for the critical $U(1)$ coupling take the lattice value for

Wilson and Villain actions:

$$\alpha_{crit}^{lat} \approx 0.2 \pm 0.015. \quad (213)$$

This gives

$$\alpha_{5\ crit}^{-1} = 0.2^{-1} * 5/2 * \sqrt{3/2} = 5 * 5/2 * 1.2247 \quad (214)$$

$$= 15.309 \quad (215)$$

With the family factor $N_{gen} = 3$ this would let us expect $15.309 * 3 = 45.927$ to be compared with the estimates from data above. (Presumably) the value to compare with is the 51.5 for the unified coupling not corrected by the quantum fluctuations, which we considered so much in this paper. Now we must remember, that the $U(1)$-critical coupling was $0.2 \pm 0,015$ meaning 7.5% uncertainty. These 7.5% means $\pm 3.45$ for the 46 we predict. So the "experimental" 51.5 from our fit is only off by $\frac{5.5}{3.45} = 1.6$ s.d..

If there is an uncertainty in the critical coupling formula, we used, in addition to the one from the uncertainty in the critical coupling for $U(1)$, then the deviation in standard deviations will be even smaller than the 1.6.
So formally we must count the hypotesis, that indeed the critical inverse unified finestructure constant should be just 3 times the critical one, is very successfull! One should have in mind, that in reality the "the critial finstructure constant" is not quite well defined, because it depends on the details of the lattice theory.
If we accept this agreement, we can say, that we fitted all three Standar Model fine structure constants with **only the unification scale**, i.e. **one** paramter. The unification value of the fine structure constant for the $SU(5)$ was determined by the "critcallity".
Actually we shall even below in section 24 claim that we can relate the approximate unification scale - the lacking parameter to predict at this stage in the article - to the top-mass and the Planck scale,so

that at the end we shall have predicted all three parameters. Thinking a bit deeper: We should really not take a formula for the $SU(5)$ critical coupling without correction, because we have been claiming all through the article that in our model the $SU(5)$ symmetry and all its degrees and freedom do not exist. Rather we should look for correcting the number for the critical $\alpha_5$ to the critial coupling for the lattice standard Model group coupling: Very crudely we think of the critial coupling for groups like the ones we look at to be the transition between two phases described as

- 1. An essentially classical phase, wherein the coupling is so weak - i.e. $1/\alpha$ so large, that at the scale we consider (the lattice links cale) all the plaquette variables are so close to unity, that the quantum effects can be considered just perturbations, but that basically we have the classical theory working.
- 2. A "confined" phase, in which we rather have that to first

approximation the plaquette variables are distributed uniformly all over the group volume,as the Haar measure, we could say. Of course it will be still be more likely to find the plaquette variables closer to the unit element in the group until the inverse coupling $1/\alpha$ reaches zero. But now it is the variation of the probability density over the group that is the "small" perturbation.

If the standard model group lie as a **dence network** inside the $SU(5)$ in the 5-plet vector representation space, then the a bit smeared volume of the standard model group would be similar to that of $SU(5)$ proper, and the value of the (inverse) fine structure constant, at which one or the other one of the two approximations above will shift their dominance (i.e. the critical value), will be (roughly) the same as for full $SU(5)$. But of course the denceness of the net formed by the standard model group is not perfect, and thus it will require that one goes to a somewhat stronger coupling (i.e. smaller inverse $1/\alpha$) to give the " confinement phase" enough weight in the partition function to (barely) compete with the "classical phase". So thus we expect

$$\frac{1}{\alpha_{SMG\ crit}} \leq \frac{1}{\alpha_{5\ crit}} \textbf{but only a bit}. \tag{216}$$

But now we have - to be fair - to remember that the standard model group, never had the quantum fluctuating degrees of freedom, which the full $SU(5)$ lattice gauge theory has. It lacks at least the 12 degrees of freedom, we refered to by $H_{int}$ in our calculation. So going from the standard model "total" coupling, if such a thing existed, to the various subgroups $SU(2),\ SU(3),\ and\ U(1)$ would not corresond to taking away so many fluctuations as, if one went from the full $SU(5)$. So the critical $\frac{1}{\alpha_{crit\ SMG}}$ should not be identified with the above fitted $\frac{1}{\alpha_{5\ bare}}$, but rather with an inverse fine structure constant of a type, that shall not have had it fluctuations in the set $H_{int}$ type ones removed,as we did in our formalism when constructing this "bare" inverse $SU(5)$ fine structure constant. So what we should rather identify as the implimentation of the critical coupling assumption, is to say, that a "fitted" $\frac{1}{\alpha_{SMG}}$ is the one you get by not counting that the refered to by $H_{int}$ modes be included, but only the other ones, is to be identified by the $3 * \frac{1}{\alpha_{smg\ crit}}$ which by (215) is - actually only a bit - smaller than $\frac{1}{\alpha_{5\ bare\ crit}}$. The "fitted" quantity $\frac{1}{\alpha_{SMG}}$ comes actually very close to be an average of the three inverse fine structure constants from the standard model,which is rather expected, since it is the standard model genuine gauge group. Then if the dence net-work with which the standard model group $G_{SMG}$ covers the $SU(5)$, there will only be little difference between the two sides in (215) and we now expect, that the average of the three standard model group inverse fine structure constants at "our unification scale" say essentially being $\frac{1}{\alpha_{SMG}}$ shall be a bit smaller than the critical SMG inverse fine structure constant times the 3, which again is just a bit smaller than the 3 times the critical inverse fine structure constant for $SU(5)$:

$$41.34 = \frac{1}{\alpha_{1\ SU(5)}(\mu_U)} (\text{taken as average } 1/\alpha_i) \quad (217)$$

$$\approx \frac{1}{\alpha_{SMG}(\mu_U)} \quad (218)$$

$$= 3 * \frac{1}{\alpha_{SMG\ ccrit}} \quad (219)$$

$$\underset{\text{a bit}}{<} 3 * \frac{1}{\alpha_{5\ bare\ crit}} \quad (220)$$

$$= 3 * 15.3 = 45.9 \quad (221)$$

We did in principle the above calculations only up to first order approximation in a preturbative scheme, in which the 0th order approximation is the exact $SU(5)$, wherein all three standard fine structure constants are equal to each other, and the first order approximation is the one, in which our corrections are considered small of first order, so that the squares of the corrections can be considered negligible. The numerical order of the first order quantities are

$$\text{“first order size”} \approx \frac{\alpha}{1 \text{ or } 3 * \pi/2} \quad (222)$$

$$\approx 1/10. \quad (223)$$

One unneccesary ignorance of second order terms, which are expected to be of the order $(1/10)^2$ times the main term is, that we above let the $\alpha$ appearing as a factor in the $< H^2 >$'s cancel with the $1/\alpha$ whichever among the $1/\alpha_i$'s we meet. Actually it was tempting to think, that by using this lucky trick of getting rid of the parameters in the estimate of our corrections, we were likely actually to get a better result with respect to agreeing for our calculations. Once we look for accuracies of the second order, there may be more corrections, such as the $H$ distribtuion being not even just Gaussian, and the whole program of doing the second order deserves a further article. Here we shall only make a very crude attempt to estimate the effect of seeing what $\alpha$ (among the three one say) comes into which of the fluctuations $< H^2 >$. We shall make the assumption that the $\alpha$ to be used for an $H_i$ where it is the fuctuation in one of the basis vectors for the subgroup $i$ of the $SU(5)$, is $\alpha_i$. Then we see from table 5 that we have had relativley good luck by letting the two $\alpha$'s cancel each other, because the mostly contrbuting $< H_i^2 >$ to the correction for the inverse fine structure constant $1/\alpha_j$, for the standard model subgroup denoted $j$ is actually mostly $i = j$ itself. In fact e.g.

according to the table 5 the correction to

$$\text{Fraction of SU(2)-inverse coupling not } H_2 \; \frac{3/10}{3/2} = \frac{1}{10} \quad (224)$$

$$\text{Fraction of SU(3)-inverse coupling not in } H_3 \; \frac{2/15}{8/3} = \frac{1}{20} \quad (225)$$

For the $U(1)$ inverse fine structure constant the dominant cotrbutuion to the corrections comes from the two nonabelian groups, i.e. from $H_2$ and $H_3$, but it has a bigger number from the $H_1$ than any of the other two groups, namely $7/30$. But since the $U(1)$ coupling correction is so mixed, to take all the same $\alpha$ is not so bad.

In any case it looks that it is only about $1/10$ of the correction for the SU(2) coupling and $1/20$ for the SU(3) coupling, that would be changed by being a bit more carefull with which $\alpha$ to use. The change to the more correct $\alpha$ to use would thus increase difference

$1/\alpha_2 - 1/\alpha_3$ percentwise by

$$\text{Decrease of } 1/\alpha_2 - 1/\alpha_3 = \frac{1/10 + 1/20}{2} * 4.7/2/40 \quad (226)$$
$$= 3/40 * 0.06 \quad (227)$$
$$= 0.045 \text{ relatively} \quad (228)$$

This is now to be compared with the deviation of of the $3 * \pi/2 = 4.712385$ from the number in (185) which is 4.62 and thus smaller than or prediction $3 * \pi/2 = 4.712385$ by 0.09 which relatively is 0.0190. This agrees only modulo a factor 2.
The observed by renorm group developping the fine structure constants to the "our unification scale" defined from the ratios of the two independent differences of inverse couplings to be 2:3 was 4.62, i.e. smaller than the theoretical 4.71, but now the effect of pushing the inverse finstructure constants predicted down from their starting point in the SU(5)-symmetric limit $1/\alpha_{5\ \textit{naive}}$ is

getting increased for the $SU(2)$-inverse fine structure constant, because for that the changed $H_1$ contribution is getting increased by our second order correction because the $\alpha_{1\ SU(5)}$ is correctly stonger than what we used at first. For the $1/\alpha_3$ oppsitely the $1/\alpha_{1\ SU(5)}$ is above the $1/\alpha_3$ at the "our unification " so that for the $1/\alpha_3$ the $H_1$ contribution corresponds to a weaker $1/\alpha_{1\ SU(5)}$ thus giving a lower suppression compared to naive inverse SU(5) coupling, $1/\alpha_{5\ naive}$. Thus the theoretical 4.712385 should be deminshed - since the 3- inverse coupling goes up by the correction and 2-inverse coupling down - relatively by the 0.045. But that would bring the theoretical number to 4.50, close to the 4.62. The deviation from the only to first order result of the number gotten by fitting is of the order of magnitude of the second order estimate. So it is important to estimate this second order approach more carefully.

A major problem and surprize comming, if one takes our suggestion of truly existing lattice at the approximate or ours unification scale $\mu_U = 5.18 * 10^{13} GeV$ seriously is, that it suggests a "fundamental" scale **quite different from the Planck scale**. To seek a way out of this problem we propose to think of a **fluctuating lattice even in size of the lattice constant** in the sense that we speculate, that the general theory of relativity is still perturbatively treatable and rather well understood already - so that no completely speculated quatum gravity theory is needed at the $\mu_U$ scale - so that the whole lattice structure must be in a quantum superpostion state invariant under the reparametrization group from the general relativity. That is to say, with the philosophy, that there is very big quantum fluctuations in the gauge and taking the diffeomorphism of reparametrization symmetry as the gauge symmetry of general relativity, we must take it that the world is in a superposition of all the possible deformations of the lattice - needed for our model for the approximate GUT $SU(5)$ - achieved by reparametrizations.

That is to say, that in a typical component in this superpostion somewhere we find a very small lattice constant and somewherewe find a very big one, so that lattice cannot be exactly a Wilson one e.g.. But locally it could still be close to a Wilson lattice. Then of course the lattice constant value suggested by our parameter $\mu_U$ as lattice constant $a \approx 1/\mu_U$ could only be true in an average sense:

$$\mu_U = \text{Average}\frac{1}{a}, \qquad (229)$$

where $a$ is some local, or may be better single link, lattice constant, i.e. length of the link in the metric of the general relativity, which should still be perturbatively treatable in the range around $1/a \approx 5.18 * 10^{13} GeV$ (which is a small energy relative to the Planck scale).

So the physical model, in which we developped our more primitive lattice model, is in the rest of the article further developped into **some presumably more chaotic lattice theory (a kind of**

**glass), in which the degree of fineness varies from region to region and you find links of all possible sizes, and at least approximate diffeomorphism invariant structure of the lattice.** It is of course only approximately diffeomrphisminvariant by being in superposition of having different fineness of the lattice at any place. From the approximate diffeomorphism invariant structure of the lattice model in this section we cannot avoid, that the density of links of the length around $a$ has to vary approximately like

$$\begin{aligned} \text{“density”}(\ln(a))d\ln(a) &= P(\ln(a) < \ln \text{“link length”} < \ln(a) + d\ln(a)) \\ &= a^{-4} d\ln(a), \end{aligned} \tag{230}$$

where $P(\ln(a) < \ln \text{“link length”} < \ln(a) + d\ln(a))$ is the probablity of finding a random link taken out of our "chaotic lattice" within the scale in logartihm from $\ln(a) < \ln(\text{lattice constant }) < \ln(a) + d\ln(a)$. A similar distribtion of the sizes of the plaquettes found in the "chaotic lattice" of this section, would also have a factor in the density going as the fourth power of the inverse plaquette side size.

There is actually a divergence problem with this "chaotic lattice" as we speculate it: If indeed this density distribution should be fully true the probablity of finding links of a specific order of magnitude would need to be zero and all the contribtuion would come from infinitely small links or infinitely long link. So we have to imagine that there finally must be some cut offs for very long - not so important - and for very short links at least.

To have approximate diffeomorphism symmetry and thus also approximate scale-invariance we should have at most a very slowly varying weight factor depending on the logarithm of the say link length - but only very weakly breaing the scale symmetry in the range of scales we consider relevant, meaning scales between the Planck scale and macroscopic scales.

But if we scall be concrete wewould propose aGaussian weighting

as a function of the logarithm of say the link length. Near the peak in the Gaussian such a Guassian weighting is only very weakly breaking the scaling invariance, but for very large or very small scales the Gaussian distribution of the weighting in the logarithm is enoermous. But somehow we hope that for very small or very big link length we have got the cut off effectively and there are anyway so little chanse for the links having that size that it does not matter so much. But I think we need a cut off in this style of being smooth for some "relevant" region and then very drastically cutting off in the scales ofvery small $a$ (i.e. high energies) becuase ifwe did not have the strong cut off somewhere, then attempting to play simultaneous with the extra factor $(1/a)^4$ for the Standard model approximate $SU(5)$ and an other extra factor $(1/a)^6$ for describeing the generalrelativity Einstein-Hilbert action would unavoidably lead to severe divergensies.
We could say that the proposed Gaussian as function of the

logarithm very robust by being able to cut off at the ends large and big scales any polynomial extra factor. Then in addition to in this way be able to cope with any power extra factor, it can be claimed in the appropriate region of scales to be rather flat, so that is not at such scales drastically breaking scale symmetry.
With this very special "cut off" assumption, it might be felt needed tomake at least a little bit of propaganda for it: Once we preferably should have had invariance under scalings in size,it is suggested that we need a slowly varying weight as function of the logarithm of the scale. We also like at the end a robust cut off that can cut off anything polynomial say and then an exponential of a smooth function

$$\text{``}weight\text{''} \sim \exp(f(\ln(1/a))) \tag{231}$$

is suggested. But then the Gaussian -which may not be socrucial exactly - is gotten by Taylor expanding the function $f$ around the

maximum, which is of course the most important region. One could as propaganda also say, that the cut-off proposed represents a weak coupling to the metric tensor of gravity.
Then depending on whether you have a factor $a^{-4}$ as for the inverse fine structure constants or a factor $a^{-6}$ as for the gravitational $\kappa$ the weighted maximum in the over scale logarithm intergal, will have somewhat different central values, i.e. central logarithms of scales.
These centers of the contributing distrbutions will be the effective lattice scales for the different weightings. So we can indeed get the $\mu_U$ weighted with $a^{-4}$ and the gravitational scale being the central one for weight $a^{-6}$ become different by orders of magnitude. If we just at first give a name to scale $\mu_0$ which one gets with weight 1, then in the Taylor expansion lowest order approximation the drag

shifting will be in the ratio 6 : 4, so that

$$\ln\left(\frac{E_{Pl}}{\mu_0}\right) = 6/4 \ln\left(\frac{\mu_U}{\mu_0}\right). \qquad (232)$$

(whether one shall use the formal Planck constant just made by dimensional arguments from the Newton constant $G$ or some reduced one with an extra factor $8\pi$ extracted might be discussed, but may be just considered an uncertainty)

When we have some part of the continuum lagrangian like the $\frac{2\pi}{\alpha}F_{\mu\nu}F^{\mu\nu}d^4x$, then the contribution to it in the lattice theory - our chaotic one or just a usual Wilson lattice - come from individual plaquettes or whatever combination of the lattice ingredients, that contribute, but you get therefore a bigger contribution the more of these contributing objects there are per hypercubic unit volume to the coefficient in the of the continuum lagrangian density.

Actually we can use simple dimensional arguments to see how the average of the continuum Lagrangian coefficient comes about:

For the inverse fine structure constants you simply get a contribution to the action from each plaquette independent of its size (provided you let the $\beta$ weighting the plaquette in the action be the same whichever the size of the plaquette, especially with our philosophy that it should be critical such *beta* independent of the size is suggested.). So in terms of an integral over the logarithm of the inverse size, say $1/a$, of the laticce constant or link-length we have

$$1/\alpha \propto \int (1/a)^4 \text{"cut off weight "} d\ln(1/a) \quad (233)$$

$$\propto \int (1/a)^3 \text{"cut off weight"} \ d(1/a) \quad (234)$$

$$\propto (1/a)^4|_{\text{at peak for } (1/a)^4 \text{ *weight}} \quad (235)$$

But gravity, extra $1/a^2$:

$$\kappa \propto \int (1/a)^4 * (1/a)^2 \text{"cut off weight"} d\ln(1/a) \quad (236)$$

$$\propto \int (1/a)^5 \text{"cut off weight"} d(1/a) \quad (237)$$

$$\propto (1/a)^6|_{\text{at peak for } (1/a)^6 \text{*weight}} \quad (238)$$

So we see that we **predict** from the "chaotic lattice " model with its approximate scale invariance, by an essentially dimensional

argument, that there shall be different effective lattice scales for the Yang Mills theories $\mu_U$, and for gravity. (But it is of course dependent on our Gaussian in log in some sense special cut off, although it is suggestive.)
In the figure 4 we illustrate, how we after having inserted a strong cut off implementing weight get a distribtuion in the logarithm $\ln(1/a)$ of the scale with a broad peak, (which we imagine Gaussian, in this log, in first approximation).
The main point is that the dominant or peak value for the distributions depend on the exact distribution, and that the one for gravity has got an extra factor $(1/a)^2$. For the Standard Model gauge couplings this peak scale is only of relevance via the renomalization group, while for gravity the very size of the (inverse) coupling $\kappa$ (also) depends on the peak value for the (logarithm of) $1/a$.
It should be clarified, that it is only because of some

"phenomenologically" added " cut off weight " factor that we at all mannage to get a peaking distribution instead of some nonsence divergent one, just increasing monotomously. So the picture we propose is really much dependent on there being some cut off of this type, and this cut off has to be considered some sort of "new physics", even though we escape from assuming many details about it, except that it is smooth in the logarithm of the scale and sufficiently strong to cause the convergence (preferably exponential in form, but with a low coefficient on the function, say $f(\ln(1/a)) =$ "small number" $* (\ln(1/a) - const.)^2$, in the exponant.).

Let us now suppose that including this "'new physics" weight there is scale, which we call $\mu_0$ for which the density of plaquettes or links counted per **link-size volume** is maximal. Then if we do not put the factor $(1/a)^4$ or $(1/a)^6$ on as we did above, then the peak of the so to speak just "weight" would be at $\mu_0$ or we should say

$\ln(\mu_0)$, when thinking of the plotting with $\ln(1/a)$ along the abscissa as in figure 4.
Now in the approximation of the "weight" distribtuion being Gaussian in the logarithmic scale and noticing that the extra factors $(1/a)^4$ and $(1/a)^6$ from the logarithmic abscissa point of view are linear terms in the exponent $4\ln(1/a)$ and $6\ln(1/a)$ which will shift the peak from $\ln(\mu_0)$ by amonts proportional torespectively 4 and 6, we see that

$$\frac{\ln(\frac{E_{Pl}}{\mu_0})}{\ln(\frac{\mu_U}{\mu_0})} = \frac{6}{4} = \frac{3}{2} \tag{239}$$

In seeking to guess, what to take for the maximum density scale $\mu_0$, when no extra factor like the $(1/a)^4$ or $(1/a)^6$, we should have in mind that the density of plaquettes in a volume (in four space) of size like the plaquette or link is indeed, what we called the number of "layers", which again were identified with the number of families, or at least this density of plaquettes in the range associated with a plaquette is proportional to the number of layers. Since we identify by our hypotesis the number of layers with the number of families, we take the number of layers at different scales to reflect the number of families being present as fermions with negligible mass at the various scales. That is to say, that in the range of scales of the quark and (charged) lepton masses we have region of scales where as one goes down in energy loose more and more families. With such a philosophy of counting only the effectively massless fermions at the scale we may - using a table like table 2 - extrapolate to a scale with maximal number of families and take that as $\mu_0$; we could take it close to the mass of the mostmassive quark or lepton, the top. Actually as seen in table 4 putting $\mu_0 = m_t$ the top quark mass is close to make our prediction (238) be satisfied. Fitting to make our prediction (238) be exactwould require a slightly higher in energy scale $\mu_0$.

We consider this close to agreement as success for explaining theoretically the “unification scale” $\mu_U$ of our approximate $SU(5)$.

| Name | Mass | $\ln(Mass/GeV)$ | Sums etc. |
|---|---|---|---|
| Quarks: | | | |
| up | 2.16 MeV | -6.137 | |
| down | 4.67MeV | -5.367 | |
| strange | 93.4MeV | -2.371 | |
| charme | 1.27GeV | 0.239 | |
| bottom | 4.18GeV | 1.430 | |
| top | 172.5GeV | 5.150 | |
| sum quraks | | -7.055 | -1.176 |
| "average" | 309 MeV | -1.176 | |
| electron | 0.5109989461MeV | -7.055 | |
| muon | 105.6583745MeV | -2.248 | |
| tau | 1776.86MeV | 0.575 | |
| sum leptons | | -9.252 | -3.084 |
| "average" | 45.78 MeV | -3.084 | |
| av. weight 2:1 | 163 MeV | $-1.812$ | |

Table: Here we just listed the charged quarks and leptons exposing their masses and the natural logarithms of the latter with the purpose of very crudely use them to extrapolate to scale $\mu_0$ at which the number of at that scale effectively massless flavours would be maximal. This scale $\mu_0$ is presumably very close to the top-mass, since just above $m_t$ all the quarks and leptons are effectively massless. But how high above we shall expect the maximum for the purpose of our lattice remain speculations.

In reduced Planck units, the Planck energy $1.22 * 10^{19} GeV$ from unreduced Planck units is divided by $\sqrt{8\pi} = 5.01325$ so as to get

$$E_{Pl\ red} = 1.22 * 10^{19} GeV/5.013225 \quad (240)$$

$$= 2.4335 * 10^{18} GeV \quad (241)$$

Now, however, we must ask: what is it that gives us a scale in the sence the studies of the running couplings tells us? The ratio of the reduced Planck energy $2.43 * 10^{18} GeV$ relative to the

logarithmically averaged charged lepton masses
$m_{average} = 163 MeV$ is

$$\frac{2.43 * 10^{18} GeV}{0.163 GeV} = 1.4930 * 10^{19} \quad (242)$$

$$\text{and has } \ln(\frac{E_{Pl\ red}}{m_{av.ch.fermions}}) = 44.15 \quad (243)$$

$$\text{Further: } \frac{m_Z}{m_{av.ch.fermions}} = \frac{91.1876 GeV}{163 MeV} \quad (244)$$

$$= 559.4 \quad (245)$$

$$\text{and has } \ln(\frac{M_Z}{m_{av.ch.fermions}} = 6.327 \quad (246)$$

$$\text{So for "our" scale } \ln(\frac{\mu_U}{m_{av.ch.fermions}}) = 27.05 + 6.327 \quad (247)$$

$$\text{Thus the ratio } \frac{\ln(\frac{E_{Pl\ red}}{m_{av.cg.fermions}})}{\ln(\frac{\mu_U}{m_{av.ch.fermions}})} = 1.323. \quad (248)$$

Had we not used the reduced Planck energy, but the usual one, we would have got the logarithmic distance from the quark and charged lepton mass scale to the Planck one $\ln(\sqrt{8\pi}) = 1.612$ bigger, so that it would go from the 44.15 up to 44.15 + 1.612 =45.76. Then we would get the ratio changed to

$$\frac{\ln(\frac{E_{pl}}{m_{av.ch.fermions}})}{\ln(\frac{\mu_U}{m_{av.ch.fermions}})} = \frac{44.15 + 1.61}{33.38} \quad (249)$$

$$= 1.371 \quad (250)$$

In fact we think, we can argue for, that this latter choice is not the correct one, because the $8\pi$ or $4\pi$ usually comes from the differnce in the coefficient to a Coulomb field and the charge appearing in the field theory action. When we have just used the fermion masses without any $4\pi$-like correction we associate it with the simple relation $m = g_y \phi >$, while if I would like the Yukawa-field aruond the Higgs particle I would get a $1/(4\pi)$ factor in. So the simple masses correspond we could say to the Yukawa coupling $g_y$ being used for unit, and not the alternative $g_y/(4\pi)$. So to speak

$$G \sim \frac{g_y}{4\pi} \quad (251)$$

$$(4\pi \text{or} 8\pi)G \sim g_y \text{ and thus also } m \quad (252)$$

This argues for, that the reduced $E_{Pl\ red}$ was the right one to use not to introduce unjustified extra factors.
We could also have argued that the nice scheme of the Standard Model with its gauge fields and three families is spoiled, when going down in energy already at the Higgs scale, so that we should not come up with this logarithmically averaged fermion masses, but just use the very $Z^0$ mass $M_Z$ instead, then our ratio would be

a bit simpler to compute:

$$\frac{\ln(\frac{E_{Pl\ red}}{M_Z})}{\ln(\frac{\mu_U}{M_Z})} = \frac{-1.612 + \ln(\frac{1.22*10^{19}GeV}{91.1876GeV})}{27.05} \quad (253)$$

$$= 1.398 \quad (254)$$

The most important outcome of the fluctuating-size-of-links lattice, we propose, is that it gives us te possibility of having a Planck scale very different from te "unification scale" and still claim a "fundamental" lattice at the unification scale. But we would of course like to see, if the order of magnitudes are at all thinkable. We therefore in figure 4 illustrate how we imagine a smooth Gaussian distribution in the logarith of the link length say.

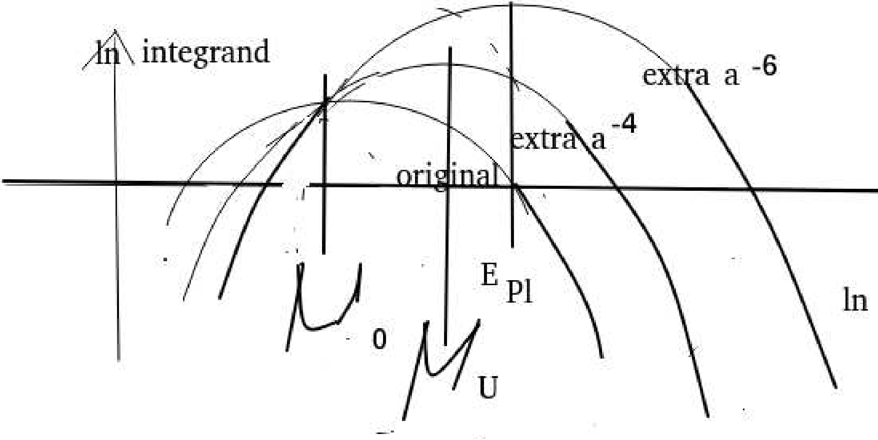



Figure: As function of the logarithm of the scale - say given as energy being the inverse of the link length $1/a$ we give here the 1) density of links perlinks-length to the fourth, 2) this density multiplied by $a^{-4}$,and that is the contribution to the Lagrangian dentity for Yang Mills theories, 3) the first density multiplied by $a^{-6}$, and that is the density of contribution to the Einstein Hilbert Lagrangian density. See also the text. In our approximation we assume these densities to be Gaussian,and with the logarithmic ordinate these Gaussians are parabolas pointing downwards.

**Description of figure4:** Here the number densities of links or of plaquettes, in a small length range of say a percent counted or weighted in different ways. The curve "original" is for counting this number density as the number in 4-cube of size proportional to the link length range which is being counted. In the two other curves the "original" density has been weighted with respectively the inverse fourth power of the link-length $a$ and the sixth power. For all three curves it is the logarithm of the density, which is plotted and a Gaussian behavior as function of the logarithm of the inverse length of the link is assumed as suggestive example. Plotted with logarithmic ordinate of course a Gaussian distribution looks like a downward pointing parabola, and the three curves are meant to be such downward pointing parbolas. It is trivial algebra to see that weighting the density counted the "original" way by further respectively $(1/a)^4$ and $(1/a)^6$, the logarithms of which are linear in $\ln(1/a)$, just leads to displacements of the p parbolas, but leave their shapes the same. For the fine structure constants or say our approximate $SU(5)$ it is the total number of plaquettes equivalent to the weighting with $(1/a)^4$ that counts, and the effective lattice link-size for our approximate $SU(5)$ model should thus be the tip of the distribution with the "extra factor $a^{-4}$". The abscissa of this tip is therefore marked by the symbol $\mu_U$ (with a $\mu$ written by the curve progarm). Because the Einstein Hilbert action has a dimension 2 different behavior from the just counting plaquettes, it is the abscissa of the tip of the parabola, which had an $a^{-6}$

weighting relative to the "original", which means the effective lattice link-size for the extraction of the Planck scale $E_{Pl}$ energy. One shall note from figure or the trivial algebra that denoting the abscissa for the peak of the "original" by $\mu_0$ then the pushing of this tip energy scale by the two different linear extra terms in the logarithm by the $a^{-4}$ and $a^{-6}$ respectively makes displacements in the dominant (energy)scale by terms in the logarithm being in the ration $4 : 6 = 2 : 3$. This means the prediction

$$\frac{\ln(\frac{\mu_U}{\mu_0})}{\ln(\frac{E_{Pl}}{\mu_0})} = \frac{4}{6} = \frac{2}{3}. \qquad (255)$$

But we have to guess e.g $\mu_0 = M_Z$ or $\mu_0 = m_t$ to use this.
In fact the scales $\mu_0$ and also the "Planck scale" do not come in precisely fromour physics and are at best order of magnitudes wise determined. The $\mu_0$ scale should be where the effective number of families is having maximum, but honestly this effective number of

families is 3 from the top-mass and up to infinity? So we only know $\mu_0 \geq m_t$. And for the Planck scale we shallin factclaim that it would be expected that the reduced Planck scale (including the often associated $8\pi$ to $G$ before using dimensional arguments to construct an energy scale) is actually more reasonable to use.

In the table 4 we therefore combine our fit above obtained value of the "our unification scale" $\mu_U = 5.1 * 10^{13}$ GeV with some reasonable suggestions for the two less welldefined scales $\mu_0$ and the "Planck scale":

| Name | $\mu_0$ | $E_{Pl}$ $=1.22*10^{19}GeV$ | $E_{Pl\ red}$ $=2.34*10^{18}G$ |
|---|---|---|---|
| $Z^0$ mass | $M_Z$ $=91.1876GeV$ | 1.4579 | 1.3968 |
| Av. fermion mass | $m_{av.fermions}$ $=163$ MeV | 1.3711 | 1.3216 |
| Top quark | $m_t$ $=172.52$ GeV | 1.4689 | 1.4079 |
| Fitted $\mu_0$ | $\mu_{0\ best}$ $=24.231$ TeV | 1.5769 | 1.5 (exact) |

Table: **Table with $\mu_U = 5.1 * 10^{13}$ GeV of** $\frac{\ln \text{“gravity scale”}}{\ln \text{“unified scale”}} = \frac{\ln(\frac{E_{Pl\ or\ Plred}}{\mu_0})}{\ln(\frac{\mu_U}{\mu_0})}$

Alternative to just guessing on good ideas of what our scale $\mu_0$ at which the density of the size of the scale is maximal counting with its own link length as unit we can simply fit, what we would like this scale to be and then if possible build up a story of, what it should be of that order. Such a fitting of the scale $\mu_0$ would simply mean, that we solve the equation of our prediction, say

$$\ln(\frac{E_{Pl\ red}}{\mu_0}) = \frac{3}{2} * \ln(\frac{\mu_U}{\mu_0}) \quad (256)$$

$$\text{formally giving:} \frac{1}{2}\ln(\mu_0) = \frac{3}{2}\ln(\mu_U) - \ln(E_{Pl\ red}) \quad (257)$$

$$= 31.563 * 3/2 - 42.2967 (\text{using GeV}) \quad (258)$$

$$= 5.048 \quad (259)$$

$$\Rightarrow \mu_0 = 24.23\,TeV. \quad (260)$$

The choice of $\mu_0$ that would make prediction perfect was as seen from the table and our calculation $24\,TeV$, which is higher than the top quark mass only by a factor $\frac{24.23}{172.25\,GeV} = 141$. Very speculatively one could attempt to construct some fitting of the density as a function of the scale fermions still findable as effectively massles at the scales considered. Above the top mass of course the effective number of massless Fermions at the scale correspond to 3 families, but as one asks below the top qaurk mass there is a 1/3 or 1/4 of a family missing, and one could claim that just below the top mass we have some 3-1/4 or 3-1/3 families left, and then crudely estimate in this spirit, if one could fit the (non-integer) number of families to a function reaching a bit above the top-mass a maximum of 3 families. Then this maximum in a curve taken as a function of the logartihm of the scale would have its maximum with value 3 families very close to our wanted $\mu_0 = 24.23 TeV$. (most wellcome to make gravity scale match our model).

We have succeeded in constructing a lattice model picture, in which we fit the three fine structure constants in the Standard

Model by **three parameters, which are with limited accuracy predicted by various assumptions of the model**. What we consider the most important is, that we suggest, that the way with smallest representation used as the link variables for the Standard Model **group** - understood as the global group structure $S(U(2) \times U(3))$ in the O'Raifeartaigh sense [29] and not only the Lie algebra - is in fact links, that also could have been an $SU(5)$ representation, and therefore the model obtains an **approximate** $SU(5)$-symmetry, when we imposed the usual trace-action. We then took it, that this first approximation $SU(5)$-symmetry of the classically treated simple trace action was broken by quantum fluctuations, which are of course only present for those fluctuations, which are true standard model group degrees of freedom, while the degrees of freedom which are only in $SU(5)$, but not in the Standard model group, of course do not contribute quantum corrections to correct the finestructure constants in our model, wherein they do not exist. It is this quantum correction breaking the $SU(5)$ symmetry (The $SU(5)$ relation between the couplings is only valid in the classical approximation) that brings the deviations from $SU(5)$ GUT theories **without help from additions as susy**, and indeed we "predict" in our model not only ratios of the shifts caused by the quantum fluctuation for the three different standard modelinverse fine structure constants, but also **the absolute size of the corrections**. So even, if we used say the ratio of the corrections to fit the pseudo-unified scale, or let us say "our unification scale" $\mu_U$, then it is still a prediction, that we know the **size of the correction** from precise $SU(5)$ unification. This prediction - it must be admitted though - contains a factor 3 being the number of families. Really it is the number of parallel lattices supposed to exist in Nature, that is 3, so the connection to the number of families is, that there would be by assumption one layer (one of the Wilson lattices lying in parallel) for each family of

fermions. (Each family its own "layer".)
The success of this predicting the **deviation** from GUT by quantum corrections fits actually the to experiment fitted fine structure constants at say the $M_Z$ ($Z^0$ -mass scale) **within uncertainties**! And this is quite remarkable, because these uncertainties for the three inverse finstructure constants in the Standard Model are much smaller by a factor of the order of 50 than the corrections due to the quantum fluctuations, we predicted. It is due to the high accuracy, with which the fine structure constants are - now a days - known, that we can find so good agreement compared to our quantum corrections, because these corrections are indeed about 10 times smaller than the typical inverse fine structure constant, which is of order 40, while our correction are of the order of 1 times the important "unit" for our corrections $3 * \pi/2 = 4.7124$. In fact we predict e.g. the difference between the inverse fine structure constants at the "our unification

scale " $(\mu_U)$ such as

$$1/\alpha_2(\mu_u) - 1\alpha_3(\mu_u) \quad \text{“predicted''} \quad 3 * \frac{\pi}{2} = 4.71 \tag{261}$$

$$\text{turned out: } 1/\alpha_2(\mu_u) - 1\alpha_3(\mu_u) \quad \text{“fitted''} \quad 4.62. \tag{262}$$

and the uncertainty in these inverse fine structure such as e.g. the $1/\alpha_3$ is $\pm 0.05$, so the deviation of 0.09 is only 1.8 s. d.(s.d.= standard deviations), and if we count two similar numbers the estimated uncertainty would be $\pm\sqrt{2} * 0.05 = \pm 0.07$ and we would have 1.3 s.d. Our deviation and uncertainty are of the order of a factor 52 smaller than the quantity of deviation 4.62, which we found!

It would in itself be interesting just to leave the two further parameters, namely the unified coupling - for the $SU(5)$ - and the scale of this approximate unification, because we would even then have an interesting relation between the fine structure constants. But we have also formally manged to find assumptions, so that these two further parameters are fitted within the now somewhat smaller accuracies:

- The Unified Coupling as Critical coupling
  We mannaged to be allowed to claim, that the unified coupling is indeed the critical coupling for the non-exitent $SU(5)$ in our model. So in a way there is the little worry with this prediction: that it is for the $SU(5)$ lattice gauge theory, we use the critical coupling, but this $SU(5)$ theory is not truly present in our model. One should possibly replace the $SU(5)$ critical couplng by one for a modified $SU(5)$ with the degrees of freedom cut down to those of the Standard Model -like it is in our model,and then take into account, that we should not correct forthe only in $SU(5)$ present fluctuation corrections if we use indeed a critical coupling being critical witout these non-existent modes. It helpedon the agreement, which was with the critical $SU(5)$ already not so bad.

- Relation of the Unified Scale to the Planck scale
  Our story behind our formally within errors relating in our model our unified scale - at which our corrections are to be applied - to the Planck scale may be a bit too much made up with guesses to be truly convincing. Thus this part of the work should rather than being an attempt to find a third predicted parameter, namely the unification scale - what it though also is -, be taken as a needed story for rescuing our model against a severe problem: Our unification scale $\mu_U$ should as the lattice scale be the fundamental scale in our model. But that is not so good, because this "unification energy scale" is much lower than the presumably fundamental scale of gravity,the Planck energy scale?

The problem with the Planck scale comes about like this:
It is not surprising, that this unified scale turns out, like in all GUT-theories, to be appreciably smaller than the Planck scale, and

in our theory it is even compared to usual unification a bit small:

$$\mu_U = 5.13 * 10^{13} GeV. \quad (263)$$

However, the real problem is that we suggest to have a lattice that is taken **seriously to exist in Nature,** and we would seemingly loose ordinary continuum manifold physics for smaller distances than $1/\mu_U$ and the seemingly approximate well working generel relativity taken classically at such scales, would be already to be considered as a quantum gravity, and in addition we would find it a priori non-attracktive to have several (two) fundamentalscales ($\mu_U$ and the Planck energy scale).
This may bring us some message about gravity: We have to invent a story of the kind, that gravity is for some reason very weak compared to the fundamental scale expectation. Our above described model namely has as its philosophy, that the unified scale - which remains low compared to the Planck scale in energy -

is to be the " fundamental scale"! You might speculatively think about, that the $g^{\mu\nu}$ (with upper indices) has appeared as kind of spontaneous breaking of e.g. diffeomorphism symmetry, and and thus has a chance to be small (often one finds reltively small spontaneously breaking fields, otherwise it would not be so common with low temperature super conductivity, that it was a big sensation to find high temperature super conductivity). If this $g^{\mu\nu}$ is small compared the our fundamental lattice, then compared to this lattice the $g_{\mu\nu}$ with lower indices will be large and thus the length say of a lattice link would be big. This bigness would be bigness compared to the Planck constant and so getting $g^{\mu\nu}$ by some spontaneous breaking story would help bringing about the lack of coincidence of our fundamental scale with the Planck one[22].

Although this idea of having $g^{\mu\nu}$ representing a smontaneous symmetry break down and being "small" for that reason, seems attrcktive to me, we shall in this article rather seek to solve the problem with the Planck scale being different from "the our unified one" $\mu_U$ by the idea of fluctuating lattice link size described in next subsection.

A priori it seems somewhat embarrasing, that our theory taken seriously wants a fundamental scale with lattice already at the approximately unification scale $5 * 10^{13} GeV$, while we a priori would expect the fundamenntal scale at the Planck scale, especially for the gravity itself, when we even seek to uphold a principle of critical coupling constants . If a lattice gravity should have in one sense or another a critical coupling, then the lattice should be of the Planck scale lattice constant roughly. The speculation solution, that almost has to be needed is, that of the in scale fluctuating lattice like this or something similar:

At around the "unifying scale" the gravitational fields must behave classically to a very good approximation, except though that a gauge degree of freedom would tend to fluctuate infinitely (actually Ninomiya Förster and myself[37] even would let such strong quantum fluctuations be the reason for the exact gauge symmetry.) because there is lacking terms in the Lagrangian, that can keep the gauge to a fixed one, except the by hand put in gauge fixing terms, but they are of course not physical.

This then means, that we must think in a gravity containing theory as the lattice fluctuating being dense with small lattice constant somewhere in the Riemann space-time and large somewhere else. In that case we must imagine that the "observed" lattice scale (as for our model the $5.13 * 10^{13} GeV$) will be some appropriate average over a highly fluctuating lattice constant size. We would expect the local lattice scale to fluctuate with a distribution that would be an approximately flat distribution in the logarithme of the lattice constant, because the diffeomorphism group contains scalings and the Haar measure for a pure scaling symmetry subgroup would suggest a smooth in logarithm distribtution. But now, while the averaging of the Yang Mills Lagrangiam over a distribution of scales with a smooth distribution in the logarithm would be weighted in slowly varying way, the gravity action, the Einstein Hilbert one varies with a power law with the scale of the lattice, if you, as we had success with, assumed a critial coupling. This would then lead to that the average size of the lattice link or plaquette structures contributing dominantly to gravity action would be much smaller than the ones contributing to the Yang Mills fields action.

This could suggest a mechanism for the seeming fundamental scale (= lattice constant size scale) for gravity would be much higher in energy than for the Yang Mills theories.

A fluctuating lattice might provide a natural explanation for the much smaller Planck length than length scale at the Yang Mill.

Our theory is in danger of inheriting baryon decay in analogy to the usual $SU(5)$ grand unification theories, but at least the gauge particles in the $SU(5)$ theory which are not in one of the standard modelgroups, also are supposed not to exist in our scheme, so the obvious diagram with an exchange of such an $SU(5)$ gauge particle is missing in our model. Actually it is in our model some four fermion interaction, that could give the baryon violation, but such an interaction would have a dimension similar to that of the Einstein Hilbert action, and thus the interaction of such a type violating baryon number conservation would be suppressed as a term in Lagrangian of high order with Planck energy as the energy unit. At least that is, what happens in our model, just using our cut off scheme as we did with gravity (fluctuating lattice scale). Whether our Gaussian in log weighting can be assumed sufficiently consistently to suppress the baryonnumber violation suffently to cope with bounds on proton decay may deserve study in a later work, but at first it looks like working and giving sufficient suppression. .

One way of looking at the progress of the present work is to think of it as an updated version of the work by Don Bennett and myself[14], which seeks to get all the three fine structure constant from criticallity at Planck scale and the antiGUT type of model with the gauge group being a cross product of 3 isomorphic Standard Model groups. But in the old works we had to help by extra assumptions about the $U(1)$ fine structure constant. In the present article this helping the $U(1)$ has been replaced by the approximate $SU(5)$, so that it seems more natural, and not so specially just making some story for $U(1)$ alone.

Of course behind such fittings of finestructure constants is the holy gral dream of finding the mathematical formula for the (electrodynamics) fine structure constant, because that is so well known - many decimals - that it contains so much infromation[39] that one could hope to justify a theory to be correct, if it fitted the

fine structure constant in a sufficiently simple way (with the many decimals). A work like the present would suggest restrictions on the form of the formula for the fine structure constant, and thereby make an a bit more complicated formula be acceptable as convincingly right provided it were of the right form.
But to make a formula without from phenomenology included expressions possible we would of course need to have the Higgs and the fermion masses connected, and for the time being the usual philosophy is, that the Higgs scale is a pure mystery and, that it needs a solution of the hirarchy problem to be possible at all. Some different philosophy e.g. a coupling of the weak scale or Higgs scale to the development of the renorm group (for e.g. the top quark mass) is needed, one example is our [35, 34] applying the complex action theory[43, 44] also in [36]. It is characteristic of the our unified scale $\mu_U$ for the only approximate that it is a bit to the low side in energy to even unified scales in other models

(especially if it is models with susy), and further it is the spirit of our model that since our unification scale is a lattice scale - or some dominating average in a fluctuating lattice link size -. It is only $5.13 * 10^{13} GeV$. So it puts us in the direction of asking if the see-saw mass scale could be the same as our unification scale? The neatrino mass square differences are for the atmospheric netrino mass square difference and the solar one

$$\Delta m_A^2 \approx 1.4 * 10^{-3} eV^2 \text{ to } 3.3 * 10^{-3} eV^2 \quad (264)$$

$$\Delta m_{sol}^2 \approx 7.3 * 10^{-5} eV^2 \text{ to } 9.1 * 10^{-5} eV^2 \quad (265)$$

indicating masses of the order of magnitudes $(4 \text{ to } 5) * 10^{-2} eV$ and $3 * 10^{-3} eV$. With say a typical charged fermion mass in the Standard Model being of mass $1 GeV$, you would expect by dimensional arguments a see saw neutrino mass of the order

$$\begin{aligned} \text{“see saw scale”} &\approx \frac{(1GeV)^2}{10^{-2}eV} && (266)\\ &= 10^{11}GeV && (267)\\ \text{Not so far from our}\mu_U &= 5.13 * 10^{13}GeV. && (268) \end{aligned}$$

If we take it that the spread in the charged fermion masses from the electron mass $0.5 * 10^{-3}GeV$ and the top quark $174GeV$ implies that our typical charged fermion mass shall be considered to have 2 to 3 orders of magnitude uncertaity, implying by the squaring in going to the see-saw mass a doubling in the numbers of orders of magnitude, then the see-saw scale is

$$\begin{aligned} \text{“see saw scale”} &\approx 10^{11}GeV * 10^{\pm 5} && (269)\\ \text{having inside errors } \mu_U &= 5.13 * 10^{13}GeV. && (270) \end{aligned}$$

So if we believe in a lattice already at the $5.13 * 10^{13} GeV$, we can look for replacement of the see-saw neutrinos by some lattice effects. If our model were right one would look for understanding the charged fermion masses along the lines of our old work with Yasutaka Takanishi and Colin Froggatt [15], while the neutrino oscillations would be related to the lattice of effective lattice scale only $5.13 * 10^{13} GeV$. I am thankfull to the Niels Bohr Institute for allowing me as emeritus, and then I am of course thankful for all the discussions during the earlier related works, although some of it is now many years ago to Niels Brene, Don Bennett, Larisa Laperashvili, Ivica Picek and the important collaborators together with Larisa were very close the same subject of combining Anti-GUT and GUT, D. A. Ryzhikh, C.R.Das...
I remember, that Svend Erik Rugh was the first to tell me, that in fact the SU(5) GUT coupling was crittical (for SUSY-GUT) the unified coupling inverted is down to $1/\alpha_5(10^{16} GeV) \sim 25$ and

closer to just one of the above mentioned critical (214), $15.3 \pm 7.5\%$, but without the factor 3.
After finishing the arXiv version Athanasios Chatzistavrakidis told me about the workby Senjanovic et al.[45].

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