Carroll swiftons

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Quantum Gravity, Strings and the Swampland
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FWF

2403.00544 with Ecker, Henneaux, Salgado-Rebolledo

Outline

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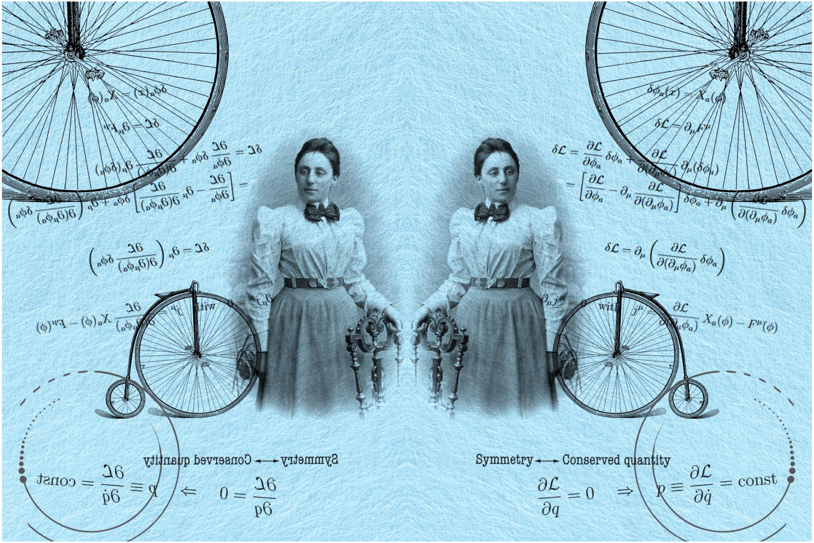
Carrollian symmetries

Symmetries ubiquitious in constraining physics

- Kinematics & Dynamics



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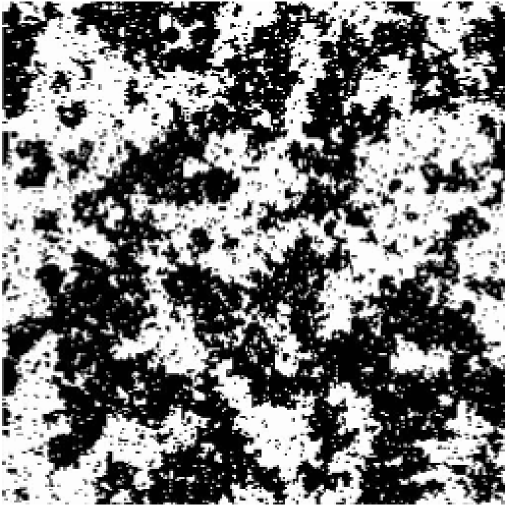
- Correlations functions



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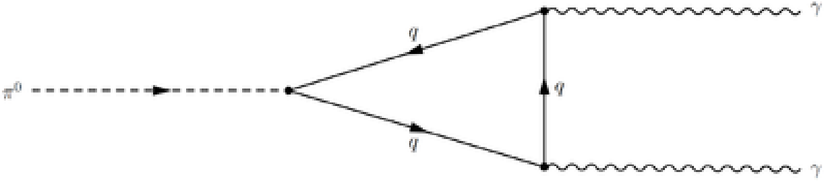
- Correlations functions
- Decay channels



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- Decay channels
- Density of states

$$S_{\text{BH}} = S_{\text{Cardy}}$$

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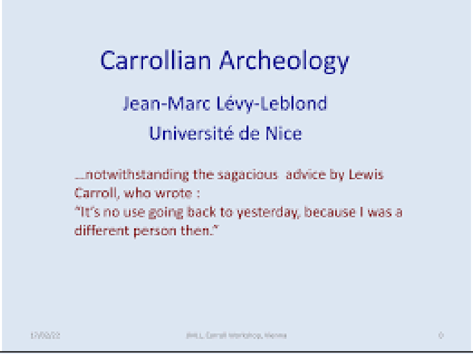
Carrollian symmetries arise in various contexts

- Formally $c \to 0$ limit of Poincaré



The Red Queen offers advice to Alice, who finds herself running intensely, but not actually moving forward: "Now, here, you see," says the Red Queen, "it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

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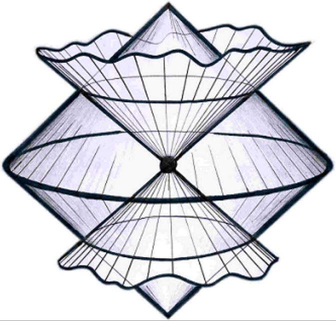
- Symmetries of null hypersurfaces horizons, flat space asymptotics



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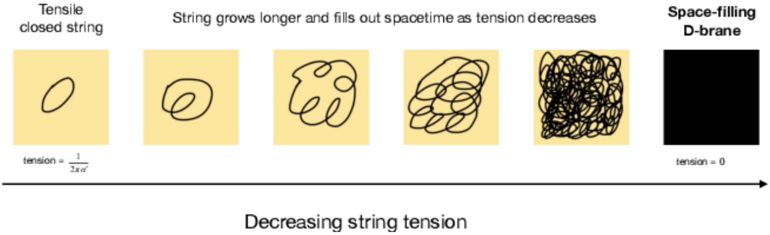

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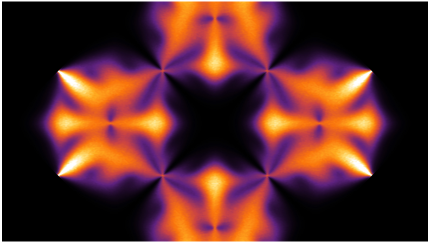

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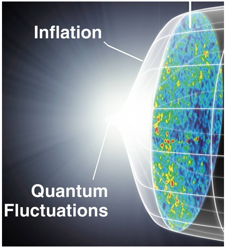

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- Carroll gravity

  Idea: theories with local Carroll boost & diff invariance

  e.g. Carroll Jackiw–Teitelboim model DG, Hartong, Prohazka, Salzer '20; Gomis, Hidalgo, Salgado-Rebolledo '20

  e.g. Carroll black holes Ecker, DG, Hartong, Perez, Prohazka, Troncoso '23

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- Carroll CFTs and flat space holography
  $\text{BMS}_3 \simeq \text{CCA}_2$ Bagchi, Barnich, Detournay, Fareghbal, DG, Simon, . . . '10-'13
  $\text{BMS}_{D+1} \simeq \text{CCA}_D$ Duval, Gibbons, Horvathy '14

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- Carroll QFTs de Boer, Hartong, Obers, Sybesma, Vandoren '23 & refs. therein

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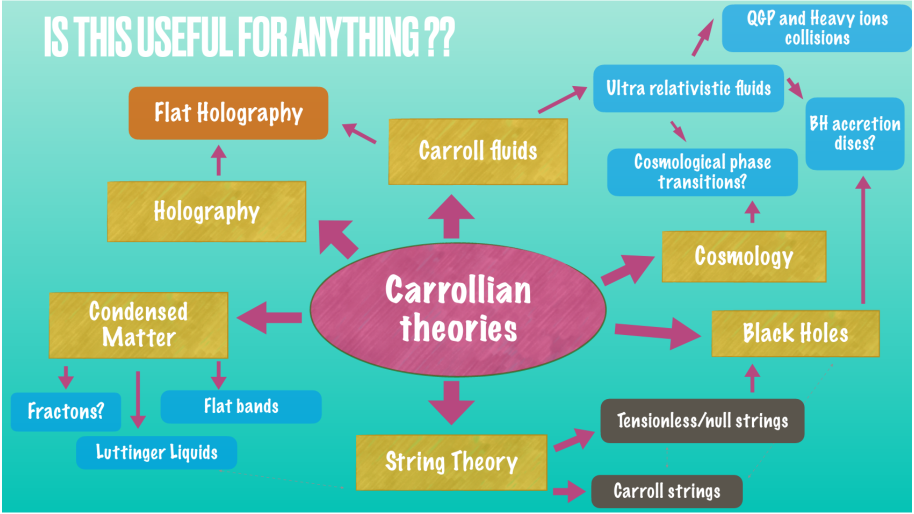
Carrollian symmetries key in numerous recent developments

Landscape of applications of Carrollian physics



slide provided by Arjun Bagchi in Edinburgh 2023

Outline

Formally: take $c \to 0$ limit of Poincaré symmetries

Analogous to Galilean limit but with reversed roles of space and time

- Unchanged: translations $H = \partial_t$, $P_i = \partial_i$, rotations $J_{ij} = x_i \partial_j - x_j \partial_i$

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- Changed: boosts

$$B_i = c^2 t\, \partial_i - x_i\, \partial_t \quad \stackrel{c\to 0}{\to} \quad B_i = -x_i\, \partial_t$$

Carrollian boosts shift time but do not affect space:

$$\text{Carroll boost: } t' = t - \vec{b} \cdot \vec{x} \qquad \vec{x}' = \vec{x}$$

This behavior is opposite to well-known Galilean boosts (limit $c \to \infty$):

$$\text{Galilei boost: } t' = t \qquad \vec{x}' = \vec{x} - \vec{v}\, t$$

Therefore, the Carrollian limit is often dubbed "ultra-relativistic"

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Carrollian algebra like Poincaré, except for boosts:

- Hamiltonian commutes with Carrollian boosts Hamiltonian in center of Carroll algebra

$$[B_i,\, H] = 0$$

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- Spatial translations do not commute with Carrollian boosts Heisenberg

$$[B_i, \, P_j] = \delta_{ij} \, H$$

boosts and translations generate subalgebra of Carroll algebra

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$$[B_i,\, P_j] = \delta_{ij}\, H$$

- Angular rotations do not commute with Carrollian boosts vector trafo

$$[B_k,\, J_{ij}] = \delta_{k[i}\, B_{j]}$$

Carrollian limit of Minkowski metric

- Metric degenerates to spatial metric:

$$\mathrm{d}s^2 = \eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -c^2\,\mathrm{d}t^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \quad \stackrel{c\to 0}{\to} \quad \mathrm{d}s^2 = \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

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- Inverse metric degenerates to temporal bi-vector:

$$-c^2\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2\delta^{ij} \end{pmatrix} \quad \stackrel{c\to 0}{\to} \quad v^\mu v^\nu \qquad \text{with } v^\mu = \delta^\mu_t$$

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- Carroll spacetimes require specification of Carrol metric $h_{\mu\nu}$ with signature $(0,+,+,\dots,+)$ and time-like Carroll vector v^μ with

$$h_{\mu\nu}\,v^\nu = 0$$

could envisage generalization to metrics with signature $(0,\dots,0,-,\dots,-,+\dots,+)$

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- Carroll symmetries preserve this Carroll structure

$$\mathcal{L}_\xi h_{\mu\nu} = 0 = \mathcal{L}_\xi v^\mu$$

Carroll symmetries generated by vector ξ^μ through Lie derivative

Outline

Two Carroll limits of Klein–Gordon action

- Direct $c \to 0$ limit of Lagrangean KG action

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Can we do better?

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2. "inverse metric" $h^{\mu\nu}$ not Carroll boost invariant, so cannot raise tensor indices while maintaining Carroll boost invariance

definition:

$$h^{\mu\nu} = \delta^{ij}\delta^{\mu}_{i}\delta^{\nu}_{j}$$

not invariant under Carroll boosts

$$\delta_{\lambda}h^{\mu\nu} = \lambda^{\mu}v^{\nu} + \lambda^{\nu}v^{\mu}$$

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works also for tensors $\theta_{\mu_1 \dots \mu_n}$ transverse in all indices: θ^2 is invariant!

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Example: $\partial_\mu \phi$ is transverse if $v^\mu \partial_\mu \phi = \partial_t \phi = \dot{\phi} = 0 \Rightarrow$ explains on-shell constraint of magnetic theory!

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Swifton action

Baig, Distler, Karch, Raz, Sun '23; Ecker, DG, Henneaux, Salgado-Rebolledo '24

- Technical key observation:

$$B_\nu := v^\mu \left(\partial_\mu\phi_1\partial_\nu\phi_2 - \partial_\mu\phi_2\partial_\nu\phi_1\right)$$

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- Adding this to electric Carroll action for two scalars yields

$$I[\phi_1, \phi_2] = \int \left[\frac{1}{2}\,\dot\phi_1^2 + \frac{1}{2}\,\dot\phi_2^2 + g\,B^2\right]$$

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- energy density is positive definite

$$H = \frac{1}{2\sqrt{h}}\,H^{AB}\,\pi_A\pi_B$$

  $\Rightarrow$ energy bounded from below! (differs from tachyons)

$$H^{AB} = \frac{1}{1 + g(\partial\phi_1)^2 + g(\partial\phi_2)^2 + g^2((\partial\phi_1)^2(\partial\phi_2)^2 - (h^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_2)^2)}\left(\delta^{AB} + g\,h^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B\right)$$

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- Poisson bracket $\{H(x), H(x')\} = 0$, agrees with general arguments
  Bunster, Henneaux '12

Taylor expanded Swiftons

- Taylor expand scalar fields around non-trivial background

$$\phi_1 = \epsilon\,\varphi + \mathcal{O}(\epsilon^2) \qquad\qquad \phi_2 = t + \mathcal{O}(\epsilon^2)$$

Note: $\mathcal{O}(1)$ is exact solution of EOM

$$\ddot{\phi}_1 = g\partial_\mu\big(h^{\mu\nu}B_\nu\dot{\phi}_2\big) - g\partial_t\big(h^{\mu\nu}B_\nu\partial_\mu\phi_2\big)$$
$$\ddot{\phi}_2 = g\partial_t\big(h^{\mu\nu}B_\nu\partial_\mu\phi_1\big) - g\partial_\mu\big(h^{\mu\nu}B_\nu\dot{\phi}_1\big)$$

Taylor expanded Swiftons

- Taylor expand scalar fields around non-trivial background

$$\phi_1 = \epsilon\,\varphi + \mathcal{O}(\epsilon^2) \qquad\qquad \phi_2 = t + \mathcal{O}(\epsilon^2)$$

- action quadratic in fluctuations

$$I[\varphi] = \epsilon^2 \int \Big[\underbrace{\frac{1}{2}\dot\varphi^2}_{\text{electric}} + g\, \underbrace{h^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi}_{\text{magnetic}} \Big]$$

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> - Swifton theory Carroll boost invariant
> - Quadratic fluctuations around non-trivial background propagate

Outline

Generalizations without gravity

- bi-scalar swiftons $\Rightarrow$ multi-scalar swiftons

$$I[\phi_i] = \int \Big[\sum_{i=1}^{N} \dot\phi_i^2 + g\, B^2 - V(\phi_i) \Big]$$

with

$$B_{\mu_2\dots\mu_N} = v^\mu B_{\mu_1\dots\mu_N} \qquad\qquad B_{\mu_1\dots\mu_N} = \big(\partial_{[\mu_1}\phi_1\big)\dots\big(\partial_{\mu_N]}\phi_N\big)$$

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- bi-scalar swiftons $\Rightarrow$ multi-scalar swiftons
- electromagnetic model; technical key observation ($F = \mathrm{d}A$)

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$$I[A_\mu] = \int \Big[\frac{1}{2}(v^\mu F_{\mu\nu})^2 + g\,C^2\Big]$$

leads again to non-negative energy density

provided $g > -1/B^2$ with $B^2 \sim h^{\mu\nu} h^{\lambda\kappa} F_{\mu\lambda} F_{\nu\kappa}$

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- propagation of fluctuations above constant electric background

$$F_{ti} = \delta_i^x E + \epsilon \mathcal{E}_i + \mathcal{O}(\epsilon^2) \qquad F_{ij} = \epsilon \mathcal{B}_{ij} + \mathcal{O}(\epsilon^2)$$

yields dispersion relation

$$\omega^2 = c_{\text{eff}}^2 \big(k_y^2 + k_z^2\big)$$

for fluctuations, with effective speed of light

$$c_{\text{eff}}^2 = -g\, E^2$$

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- propagation of fluctuations above constant electric background yields again swiftonic propagating modes with finite propagation speed
- can combine complex scalar swifton and couple to electromagnetic swiftons; as expected, $U(1)$ symmetry local by minimal substitution $\partial_\mu \to \partial_\mu - iA_\mu$

Generalization with gravity I — swiftons on Carroll backgrounds

- focus on generic Carroll dilaton gravity in 2d

$$I_{\mathrm{CDG}} = \int \Big[X \, \mathrm{d}\omega + X_{\mathrm{H}} \big(\mathrm{d}\tau + \omega \wedge e \big) + X_{\mathrm{P}} \, \mathrm{d}e + \tau \wedge e \, \mathcal{V}(X, \, X_{\mathrm{H}}) \Big]$$

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- define Carroll boost invariant spatial derivative

$$\hat\partial := e^\mu\partial_\mu + \frac{X_{\text{P}}}{X_{\text{H}}}\,v^\mu\partial_\mu$$

$$\delta_\lambda\hat\partial = 0$$

Note: singular locus $X_{\text{H}} = 0$ is Carroll extremal surface for Carroll black hole solutions

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$$I[\phi] = \int \Big[\frac{1}{2}\,\dot\phi^2 + \frac{g}{2}\,\big(\hat\partial\phi\big)^2 + h\,\dot\phi\hat\partial\phi\Big]$$

yields propagating modes for negative g

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- example: scalar swifton on Carroll–Schwarzschild background

$$\partial_t^2 \Psi + g\, \partial_{r_*}^2 \Psi = \frac{2gm}{r^3} \Big(1 - \frac{2m}{r} \Big) \Psi$$

$\Psi. = r\,\phi$, Regge–Wheeler coordinate $r_* := r + 2m \ln(\frac{r}{2m} - 1)$

Generalization with gravity II

Dynamical torsion from swifton backreaction

- Couple single scalar swifton action to Carroll dilaton gravity

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- torsion equations have scalar field (!) as source

$$\text{intrinsic torsion:} \qquad \mathrm{d}e = -\frac{g}{X_{\mathrm{H}}}\,\dot{\phi}\,\hat{\partial}\phi\,\tau\wedge e$$

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- Qualitative new features as compared to Lorentzian theories
- To be explored!

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$\epsilon\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}$